DISCLOSURE POLICIES, BANK RUNS, AND MANAGERIAL INCENTIVES

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Abstract

My dissertation studies how financial disclosure policies affect the balance of managerial risk-taking incentives and creditors’ bank runs. The ultimate goal of this research is to contribute on the understanding of how financial crises occur and what policies actually help to reduce the social costs of such crises.

In the first chapter, I study how the release of discount window information affects the provision of managerial incentives and whether the disclosure could trigger bank runs. I propose a model in which banks suffer adverse shocks that require cash infusion in order to continue operating and assets are subject to moral hazard. The bank is financed by depositors that can withdraw money at any time, introducing a collective action problem. I provide conditions that characterize whether disclosure or confidentiality of discount window borrowings maximizes the NPV of bank projects. The main result suggests that disclosure is a better policy when moral hazard is serious relative to the size of the liquidity shock, because it allows a contract that induces the banker to behave. Secrecy is a better tool when the moral hazard is small relative to the liquidity shock, because a run is avoided.

In the second chapter, I study the interaction between incentive provision and inefficient rollover freezes for a bank financed with short-term debt. I propose a dynamic model where rollover freezes occur because of an intertemporal coordination problem and management has discretion over the riskiness of assets. The main result states that when bank assets grow slowly, bank management chooses high risk and creditors run more frequently, increasing the resulting run probability. I characterize how changes in the fundamental’s drift, debt
maturity, liquidation costs, interest rate, and bailout policies change the equilibrium outcome of project riskiness and occurrence of runs, which in turn affects the value of the bank. A bailout friendly environment reduces creditors' incentives to run, which in turn gives the manager room to increase asset risk, ultimately increasing the run probability.

In the third chapter, I study the effects of disclosing financial information on the occurrence of bank runs and on management risk-taking activities. The main trade-off is between the risk of bank runs, which increases with a disclosure delay, and managerial incentives for risk taking, which runs discipline. I find that the main policy consideration is the growth rate of bank assets. If bank assets grow sufficiently slowly, then the optimal policy is to disclose with a lag, in order to balance managerial risk taking and creditors’ coordination problems. When bank assets have high-growth rates, a disclosure lag increases the occurrence of runs and decreases bank value.
Chapter 1

Release of Information and Discount Window Lending

1.1 Introduction

The discount window (DW) refers to the facilities that the Federal Reserve has in order to provide liquidity to eligible institutions, mainly commercial banks. The purpose of the facility is to help alleviate liquidity problems in a depository institution or the banking system as a whole. Under the primary credit facility of the DW, banks in sound financial condition pay a rate above the FOMC’s fed funds target for a term typically overnight. Primary credit may be used for any purpose and there is no need for institutions to seek alternative sources of funds before requesting advances on the DW, no questions asked.

The effectiveness of this policy tool depends on the willingness of institutions to use it. Banks have typically been reluctant to use the DW, not only because it is expensive compared to other forms of credit, but also because of the fear that borrowing from the Fed is perceived as a negative signal by regulators, other banks, depositors or investors. That is, if a bank perceives that borrowing from the DW will lead the market to doubt its solvency, it may avoid the facility even if it is the cheapest form of credit available. A bank could
prefer to sell some assets at depressed prices, fire sale, or even to pay more for other forms of credit. This is known as the stigma problem.

To limit this problem, central banks typically disclose limited information about DW borrowing. Confidentiality aims to make the DW a more effective policy tool by encouraging healthy institutions to borrow when they face stress situations. The Dodd-Frank Financial Reform Act of 2010, requires the Fed to disclose names of participants with a two year delay. Previous to this piece of regulation, the Fed kept the information confidential.

Confidential borrowing also has perverse incentive effects. A banker that knows that she is going to be saved in case of a liquidity shock has little incentive to exert effort and try to avoid the problem. By going to the DW she can solve liquidity problems and investors cannot verify good or bad behavior until it is too late. A confidential DW mechanism acts like a bailout, giving the banker the option to misbehave anticipating help from the central bank.

It is crucial to understand how release of information affects moral hazard problems and the effectiveness of the discount window. On one hand, if a visit to the DW is perceived as a negative signal by other banks, depositors or investors, disclosure of information could trigger bank runs, freeze interbank lending for the institution, and cause negative stock returns. On the other hand, the release of information could reduce uncertainty and increase stock returns. The release could also help investors to take corrective action against moral-hazard problems or risk-taking activities, which in turn would enhance the value for depositors.

In this paper I ask how the release of DW data affects moral hazard problems and the occurrence of bank runs. I consider an economy with a banker, investors and a central bank. The banker holds a project in need of funding. If undertaken, the project is subject to a random adverse shock that requires a cash infusion in order to continue, otherwise the project ends with an inefficient recovery value. An interpretation of the adverse shock is as a reinvestment need, an investment cost overrun, or a shortfall in earnings which requires a cash infusion in order to cover operating expenses.
1.1. **INTRODUCTION**

In the model there is moral hazard over the distribution of the adverse shock. The banker can exert effort or shirk. Shirking yields a private benefit to the banker while exerting effort yields no private benefit and a smaller probability of a liquidity shock occurrence. Equivalently, the banker chooses between a project with less probability of an adverse shock and another project that she prefers, but has a higher probability of an adverse shock. I assume that the adverse shock occurrence is the banker’s private information.

The bank is financed by depositors who can withdraw at any time, introducing a collective action problem as in Diamond and Rajan (2000, 2001). If the banker takes any action that impairs the value of deposits, depositors will choose to run in an attempt to save their money, forcing an inefficient liquidation of the project. The collective action problem created by demand deposits prevents the banker from renegotiating down with depositors.

After an adverse shock occurs, any attempt of the banker to raise cash triggers a run on the bank if depositors anticipate dilution from the issuance of new securities. A confidential DW allows the banker to overcome adverse shocks without triggering inefficient bank runs, because investors do not notice dilution. However, confidentiality of DW borrowing in turn aggravates moral hazard by making it more costly or even impossible to place good incentives on management. Solvency shocks, where the cash required is greater than continuation value, are not financed since I assume that the central bank does not face any credit risk.

Disclosure of DW borrowings allows creditors to design contracts contingent on the occurrence of the liquidity shock. The cost to discipline the manager is lower than under confidentiality because going to the DW serves as a signal of bad behavior. However, when investors see that their share is being diluted by the loan taken from the central bank, they join a line to withdraw their deposits, forcing the banker to liquidate the project inefficiently. These features formalize the trade-off studied in this paper: A confidential DW avoids inefficient liquidation caused by runs but aggravates moral-hazard problems. Moreover, they both have negative effects on the net present value (NPV) of the project: liquidation is inefficient and misbehavior increases the probability of a need for cash.
CHAPTER 1. RELEASE OF INFORMATION AND DISCOUNT WINDOW LENDING

I provide conditions that characterize whether disclosure or privacy is better if the policy-maker maximizes the NPV of the project. The main result suggests that secrecy is a better tool when the change in the expected size of the liquidity shock is greater than the private benefit of shirking. This condition is produced by large liquidity shocks, small benefits for shirking or large discretion in the occurrence of the liquidity shock. Under these conditions the banker requires a compensation smaller than the liquidity shock, and hence depositors are diluted after the issuance of new securities. By avoiding inefficient liquidation under a confidential DW the net present value of the project is higher than under a public DW. The benefit comes at a cost, however, because under secrecy the manager requires a larger share of profits in order to behave.

In contrast, when the private benefit of shirking is large, the liquidity shock is small or the probability of the shock is nearly the same regardless of the banker’s effort, it is not possible to discipline the banker under a confidential DW and hence disclosure is a better policy. Intuitively, the banker cannot be disciplined because the cost induced by the increase in the probability of a liquidity shock is less than the benefit of shirking, no matter how much she is paid. Release of information allows a contract contingent on the liquidity shock that induces good behavior, increasing the NPV of the project. In equilibrium, only the banker is diluted by a liquidity shock and therefore there is no run induced.

I explore the lag in disclosure of DW borrowings as a policy tool. This analysis is motivated by the release of DW borrowings after a freedom of information act request on behalf of Bloomberg LP. Also, The Dodd-Frank Financial Reform of 2010 mandates disclosure of DW loans made after July 21, 2010 with a two-year lag\(^1\). As a policy implication, this work models how welfare is affected by the lag in disclosure because of moral hazard problems and bank runs, which could be useful for the Federal Reserve to decide when to release this information. A larger lag in release of DW information decreases the impact of inefficient liquidation but increases the cost needed to control manager’s incentives. An optimal lag

\(^1\)The Federal Reserve has the option of disclosing the data within the two-year limit.
arises from the balance of these two forces.

The paper is organized in five sections, the first one of which is this introduction. In the next section, I present the basic environment and derive the basic results. The main proposition and a magnitude analysis that links with the data released by the Fed is presented in the third section while the fourth section discusses the lag in the release of the DW information as a policy tool. Conclusions follow.

1.2 Basic Model

1.2.1 Agents, Projects, and Endowments

Consider an economy with a banker and investors. The economy lasts for two periods and three dates - date 0, date 1 and date 2. All agents are risk neutral and the discount rate is zero. Each banker holds a project that requires an initial investment \( I > 0 \) at date 0. If undertaken, the project produces a riskless cash flow of \( R > 0 \) at date 2. At date 1 the banker may experience an adverse shock that needs to be withstood to allow the project to continue and produce the date 2 cash flow. If the adverse shock is not financed, the project ends and assets are liquidated with liquidation value \( X \geq 0 \). With probability \( q \) there is no adverse shock (the bank is intact) and with probability \( 1 - q \) an adverse shock of magnitude \( \rho > 0 \) occurs. A simple interpretation of the liquidity shock is as a reinvestment need, an investment cost overrun, but it can be thought of as a shortfall in earnings at the intermediate stage, in which case a new external cash infusion is needed in order to cover operating expenses.

Assume that the liquidation value is lower than the continuation value in case of a liquidity shock, \( X \leq R - \rho \). The project is illiquid in nature because selling the project at date 1 produces less than the present value of future cash flows of the project. Since \( X \geq 0 \) I conclude that \( \rho \leq R \) and therefore only liquidity shocks are considered. This is without loss of generality because as I will explain, solvency shocks \( (\rho > R) \) are never financed.
The adverse shock is subject to moral hazard. The banker can exert effort or shirk, take a private benefit. Exerting effort yields probability \( q = q_H \) of no adverse shock and no private benefit to the banker, and misbehaving results in probability \( q = q_L < q_H \) of no adverse shock and a private benefit of \( B > 0 \) to the banker. Let \( \Delta q = q_H - q_L \). Equivalently, the banker chooses between a project with lower probability of an adverse shock and another project that she prefers but has a higher probability of an adverse shock.

The banker has funds equal to \( A < I \). There are a large number of identical investors each with less than \( I - A \) units of endowment at date 0, and none at other times, who can finance the banker. Investors behave competitively in the sense that the loan, if any, makes zero profit. The net present value of the project is positive if good behavior can be induced, or

\[
R - (1 - q_H) p - I > 0. \tag{1.1}
\]

### 1.2.2 Private Liquidity Shock and Discount Window

I assume that the adverse shock realization is the banker’s private information. That is, investors and the central bank know that an adverse shock might occur but they cannot verify whether one has occurred at date 1. Since the adverse shock is nonverifiable, it is not possible to design contracts contingent on it.

Banks have access to a discount window facility at the central bank, which is risk neutral. When a bank borrows at the DW, the loan is assumed to have higher seniority than any previous claim. This is indeed the case with the DW at the Fed because any loan taken is considered debtor in possession financing and hence it has a higher seniority than any other claim.\(^2\) Moreover, it also approaches to the need for placing collateral valued at discount, or haircut, before asking for funds at the DW of the Fed. At date 1, the central bank either discloses or not whether a bank has asked for a short term loan and the amount lent. I will

\(^2\) The procedures that the Fed can take if there is a default by a lender at the discount window are presented in Circular 10, remedies upon default, available at: https://www.frbservices.org/files/regulations/pdf/operating_circular_10_071613.pdf
1.2. BASIC MODEL

analyze these cases in detail.

1.2.3 Depositors as Investors

The banker can finance the project by issuing deposits to investors. Under a deposit contract, the investor deposits an amount $d$ at date 0 that can be withdrawn at any time before date 2. Depositors who want to withdraw form a line and they are served by the banker in order. If the banker does not pay the full promised nominal repayment $d$, the depositor has the right to get bank assets equal in market value to $d$, until nothing is left. If bank assets are insufficient to repay all depositors, only the first depositors in line get paid, whereas the last ones get nothing. At date 2, when the cash flow $R$ is realized, depositors are paid an amount $\frac{R_l}{n}$, where $n$ is the number of depositors and $R_l \leq R$ stands for the share of depositors as a whole. The sequence of events is presented in Figure 1.1.

1.2.4 Optimal Contract

A contract first stipulates whether the project is financed. If so, it further specifies how the profit is shared between depositors and the banker. The banker has limited liability. Let $R_b$ be the share of the banker and $R_l$ the share of lenders. $R_b$ and $R_l$ could be function of the
CHAPTER 1. RELEASE OF INFORMATION AND DISCOUNT WINDOW LENDING

An allocation \((R_b, R_l)\) is feasible if \(R_b + R_l = R\). The optimal contract is a triple \((R_b, R_l, A)\) such that \((R_b, R_l)\) is a feasible allocation that maximizes the utility of the banker subject to:

1. Investors make zero profit at the investment amount \(I - A\).
2. Good incentives are in place if possible.

An equilibrium is a triple \((R_b, R_l, A)\) such that \((R_b, R_l, A)\) is an optimal contract and \(A\) is minimal in the set of funds such that \((R_b, R_l, A)\) is an optimal contract. I solve for the equilibrium under disclosure and confidentiality of DW borrowings next.

1.2.5 Disclosure of Discount Window Borrowings

**Liquidity shock** \(\rho \in \left( \frac{B}{\Delta q}, R \right)\)

I consider first the case in which the central bank discloses whether a bank asked for funding at the DW. This disclosure occurs at date 1 as described in the timeline, Figure 1.1. Disclosure at date 2 has the same effect as keeping the borrowing private because date 2 is the final date.

At date 1, the project may suffer an adverse shock. If there is no shock, the project is worth \(R\) and all uncertainty is resolved. If there is a shock, the project is worth \(R - \rho\) after a cash injection of \(\rho\). The banker can seek financing or liquidate the project. In the case of liquidation she receives nothing as I explain below. Therefore, the banker prefers trying to raise the required funds, for which she has three options: i) issue new securities that repay \(\rho\) at date 2, ii) ask depositors to finance the shock, or iii) go to the DW and borrow \(\rho\) in exchange for a promise of \(\rho\) at date 2.

I argue that both i) and ii) trigger a run and that under the disclosure of DW borrowings iii) also triggers a run. A bank run occurs when all depositors panic and try to withdraw their deposits at date 1, forcing the bank to sell assets to meet the requirements. Ultimately,
a bank run forces the liquidation of the bank if the market value of the project is not enough to repay all depositors in the queue.

As in Diamond and Rajan (2000, 2001), the sequential nature of demand deposits creates a collective action problem that prevents the banker from negotiating depositors down. As a result, any attempt to raise \( \rho \) triggers a run.\(^3\) To see why this is the case, suppose that the banker announces that she intends to raise \( \rho \) by issuing new securities. New investors are willing to provide funds to the project provided that there are enough resources in order to be paid back, that is, \( \rho \leq R \), so that there is no inefficient continuation. This implies that a shock that is financed is a liquidity shock and not a solvency shock. After the banker announces her intention to raise \( \rho \) from new investors, current depositors anticipate that the new securities dilute their claim. Seeing this, current depositors can (1) accept the new terms, or (2) join a line to seize the bank’s assets to repay their deposit (run). An individual depositor weakly prefers to run because he has the chance of getting paid back and if he does not get paid back, he is in the same position as if he had accepted the new terms. Therefore, an announcement to raise \( \rho \) from new investors triggers a run and forces the bank to liquidate the project at the market value \( X \). Depositors as a whole are better off by waiting (since \( X \leq R - \rho \)) but the collective action problem induced by the nature of their claim prevents them from coordinating not to withdraw their deposits.

Similarly, options ii) asking depositors to finance the shock and iii) going to the DW, trigger a bank run because they reveal dilution of previous creditors. Depositors prefer to withdraw first and then consider whether to finance or not the shock rather than financing the shock directly. However, this liquidates the bank. Similar to trying to issue new securities, going to the DW reveals dilution of previous depositors and causes a run that liquidates the bank. If the banker attempts to take actions that impair the value of the claim to depositors, depositors will run in an attempt to capture the full promised payment on their claim, no matter that collectively they are better off by waiting.

\(^3\)For a detailed proof see Diamond and Rajan (2001)
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Assuming that the banker works at date 0, the bank liquidates after a run with probability $1 - q_H$, the probability of a shock. Let $R_t$ stand for the stake of depositors in case of no liquidation. The borrowing capacity, the amount that can be raised at date 0, is given by the individual rationality (IR) constraint

$$q_H R_t + (1 - q_H) X \geq I - A,$$

which is, with probability $(1 - q_H)$ a run liquidates the bank and leaves depositors with $X$ and with probability $q_H$ the bank is intact and depositors receive $R_t \leq R$ at date 2.

Consider the incentive problem. Assume for the moment that $X < q_H R_t + (1 - q_H) X$, that is, the liquidation value is less than the value of deposits and hence some depositors and the banker are left with nothing after a run. At date 0, the banker faces a trade-off once financing has been secured: by misbehaving, she obtains private benefit $B$, but she increases the probability of a liquidity shock (where she receives nothing) from $1 - q_H$ to $1 - q_L$. Let $R_b$ be the stake of the banker in case of no liquidity shock. The banker will behave if the following incentive compatibility (IC) constraint is satisfied

$$q_H R_b \geq q_L R_b + B,$$

which says that the expected payoff in the case of good behavior, $q_H R_b$ (the banker receives 0 with probability $1 - q_H$ because of liquidation), needs to be greater or equal than the expected payoff of misbehavior, $q_L R_b + B$.

From the incentive compatibility constraint I infer that in case of no liquidity shock, the lowest income that the banker requires in order to behave is $\frac{B}{\Delta q}$. It is optimal to set $R_b = \frac{B}{\Delta q}$ because once incentives are in place, paying more to the banker reduces the borrowing capacity at date 0. Then, $R_t = R - R_b = R - \frac{B}{\Delta q}$ and from (1.2) the minimum funds required are given by

$$\bar{A} = q_H \frac{B}{\Delta q} - (q_H R + (1 - q_H) X - I),$$

(1.4)
which says that the banker needs to provide funds at least as big as the difference between her expected payoff and the net present value of the project.

Since liquidation is inefficient \( X \leq R - \rho \) then

\[
q_H R + (1 - q_H) X - I \leq R - (1 - q_H) \rho - I
\]  

(1.5)

that is, the net present value of the project when there is disclosure of DW borrowings is less than the original net present value where no inefficient liquidation was considered. Note that since the banker is paid last, only shocks \( \rho \) greater than \( \frac{B}{\Delta q} \) effectively dilute depositors, and hence only in this case creditors will run when the banker attempts to raise \( \rho \). Therefore, the analysis presented applies for \( \frac{B}{\Delta q} < \rho \leq R \). Moreover, with \( R_b = \frac{B}{\Delta q} \) and using (1.2), check that the amount of deposits is indeed bigger than the liquidation value

\[
q_H R_l + (1 - q_H) X = q_H \left( R - \frac{B}{\Delta q} \right) + (1 - q_H) X
\]

\[
> q_H (R - \rho) + (1 - q_H) X
\]

\[
\geq X
\]

(1.6)

and hence some depositors and the banker are indeed left with nothing after a run. The following proposition summarizes the equilibrium derived.

**Proposition 1** Under public disclosure of DW borrowings and if \( \frac{B}{\Delta q} < \rho \leq R \), then the equilibrium contract induces good behavior, \( q = q_H \), the banker is paid \( R_b = \frac{B}{\Delta q} \) and a run occurs after a liquidity shock. The minimum assets in place needed for funding are given by equation (1.4). The NPV of the project is \( q_H R + (1 - q_H) X - I \).

Under a public DW, financing through deposits introduces a collective action problem that results in inefficient liquidation with probability \( 1 - q_H \), which affects the borrowing capacity and decreases the net present value of the project. However, disclosure allows a contract contingent on the occurrence of the liquidity shock that induces the banker to
behave.

Stigma arises naturally in this setting because a run liquidates the bank and leaves the banker with no rent. Banks try to avoid asking for funding if they perceive that it can be interpreted as a bad signal by investors and depositors. As it is explained below, a confidential DW reduces stigma because it does not allow investors to know if there is any dilution, there is no run and hence the banker is willing to use the facility.

**Liquidity shock** \( \rho \in \left(0, \frac{B}{\Delta q}\right) \)

In this case the banker is the only one diluted by a liquidity shock, so that creditors do not run and there is no inefficient liquidation. Since the shock is verifiable because either the banker makes it public when asking for funding or because the DW releases the information of borrowings at date 1, then the banker compensation can be designed contingent on the realization of the shock. Let \( R_b^s, R_b^{-s} \) denote the stake of the banker in case of a liquidity shock and no liquidity shock, respectively. The incentive compatible constraint can be written in this case as

\[
q_H R_b^{-s} + (1 - q_H) (R_b^s - \rho) \geq q_L R_b^{-s} + (1 - q_L) (R_b^s - \rho) + B, \tag{1.7}
\]

the banker receives \((R_b^s - \rho)\) in case of a liquidity shock because his stake is diluted to pay for the liquidity shortage. After rearranging terms I get

\[
R_b^{-s} \geq R_b^s + \frac{B}{\Delta q} - \rho, \tag{1.8}
\]

the simplified incentive compatible constraint.

In order to respect limited liability, the minimum compensation needed in case of a shock is \( R_b^s = \rho \), that in turns gives \( R_b^{-s} = \frac{B}{\Delta q} \). The payment to depositors is contingent on the shock and given by \( R_l^s = R - \rho, R_l^{-s} = R - \frac{B}{\Delta q} \), where \( R_l^s \) and \( R_l^{-s} \) denote the stake of
creditors in case of a liquidity shock and no liquidity shock, respectively. The individual rationality constraint for depositors is given by

\[ q_H R_l^{s} + (1 - q_H) R_t^{s} \geq I - A, \quad (1.9) \]

and hence the minimum funds required for the banker are given by

\[ A = q_H B \frac{B}{\Delta q} - (R - (1 - q_H) \rho - I). \quad (1.10) \]

The net present value of the project is given by \( R - (1 - q_H) \rho - I \) because there is no inefficient liquidation and good incentives are in place. From (1.10), the minimum funds needed to secure financing are equal to the difference in the banker’s expected payment and the NPV of the project. The following proposition summarizes the equilibrium derived.

**Proposition 2.** Under public disclosure of DW borrowings and if \( 0 < \rho \leq \frac{B}{\Delta q} < R \), then the equilibrium contract induces good behavior, \( q = q_H \), the banker is paid \( R_b^{\sim s} = \frac{B}{\Delta q} \), \( R_t^{s} = \rho \) and there is no run. The minimum funds required for the banker to secure funding are given by equation (1.10). The NPV of the project is \( R - (1 - q_H) \rho - I \).

There is no run induced in equilibrium because the banker’s payment is greater than or equal to the size of the liquidity shock and hence no dilution occurs. The banker is rewarded in case of a good signal, no liquidity shock, and is punished in case of bad behavior because \( R_b^{\sim s} = \frac{B}{\Delta q} > \rho = R_b^{s} \).

The DW is not useful in this setting because the banker can raise funds by issuing new securities to new or previous depositors. Previous depositors would finance the shock because they are not diluted.

In summary, disclosure of DW borrowings allows a contract contingent on the liquidity shock that induces the manager to behave at date 0. However, large liquidity shocks dilute depositors and therefore disclosure of DW borrowings triggers runs. Runs are inefficient and
decrease the NPV of the project.

1.2.6 Confidential Discount Window Borrowings

**Liquidity shock** $\rho \in \left( \frac{B}{\Delta q}, R \right]$

I now consider the case in which the central bank keeps secret any amount lent through the DW at date 1. At date 1, the banker may suffer a private adverse shock. If there is no shock, the project is worth $R$ and all uncertainty is resolved. If there is an adverse shock, the project is worth $R - \rho$ after a cash injection of $\rho$ or $X$ if the project is liquidated. Since any payment to the banker occurs at date 2 when cash flow $R$ is realized, the banker prefers to continue with the project and therefore he will try to raise funds equal to $\rho$.

As explained before, the banker has three options to raise funds equal to $\rho$: i) issue new securities with face value equal to $\rho$, ii) ask depositors to finance the shock, iii) go to the DW, borrow $\rho$ and promise to pay back $\rho$ at date 2. Even if the banker could succeed in raising $\rho$ either by i) or ii), new investors never finance a shock greater than $R$, that is, only shocks such that $\rho \leq R$ are financed. Solvency shocks ($\rho > R$) are not financed and there is no inefficient continuation. The collective action problem argument presented in the previous section applies here: any attempt from the banker to reduce or impair the claim of previous depositors triggers a run, so that i) or ii) are not successful in overcoming the liquidity shock.

The banker can finance the liquidity shock without causing a run if she goes to the DW. Because the liquidity shock is only observed by the banker and because the DW never discloses the information about borrowings, it is impossible to know at date 1 if the bank suffered a liquidity shock or not. With a risk neutral discount window and zero interest rate, the bank gets funds equal to $\rho$ at date 1 and promises to repay back $\rho$ at date 2. The central bank does not incur any credit risk and therefore it lends amounts lower than $R$. No inefficient continuation occurs ($\rho \leq R$).

Consider the incentive problem. If there is no liquidity shock, depositors get paid first so
that the stake of the banker, $R_b$, needs to satisfy $0 \leq R_b \leq R - R_l$. In the case of a shock, the central bank gets repaid first, then the depositors and if there is anything left, the banker gets paid, this means that $R_b$ needs to satisfy $0 \leq R_b \leq R - \rho - R_l$. As a result, the banker faces a trade-off at date 0 similar to the one in the public information case: by misbehaving, she obtains private benefit $B$, but she increases the probability of a liquidity shock (where there is less cash flow available) from $1 - q_H$ to $1 - q_L$.

There is a key difference with the public information setting because here it is impossible to verify that a liquidity shock has occurred before the bank fails at date 2. Suppose that a liquidity shock has occurred at date 1 and the banker secured funds for $\rho$ at the DW. When date 2 arrives, the central bank gets paid first, so that after the central bank is paid back there is $R - \rho$ for depositors and the banker. If $R_l \leq R - \rho$, depositors are paid back in full as in the case of no shock. If $R_l > R - \rho$, some depositors are not paid back signaling that there was a liquidity shock. However, with the assumptions made, it is not possible to make the banker liable for a shock since the news of an occurrence comes when it is too late: there are no more resources available. This means that $R_b$ cannot be made contingent on the occurrence of the liquidity shock.

In deciding whether to behave or misbehave, the banker compares the expected utility he gets in each case. The incentive compatible (IC) constraint can be written as

$$q_H R_b + (1 - q_H) (R_b - \rho) \geq q_L R_b + (1 - q_L) (R_b - \rho) + B,$$

(1.11)

hence, a necessary condition for the manager to behave is given by $\rho \Delta q \geq B$. Note that this condition only depends on the parameters of the model and not on $R_b$. However, the banker’s limited liability requires a compensation of at least $\rho$. This means, $R_b = \rho$ is optimal and good incentives are in place, $q = q_H$.

Depositors will be paid $R_l = R - R_b = R - \rho$ in all the states of the world. Contrary to the case of public information there is no run because creditors cannot verify whether
CHAPTER 1. RELEASE OF INFORMATION AND DISCOUNT WINDOW LENDING

a shock occurred. Moreover, no depositor is diluted because the banker absorbs liquidity shocks. Consequently, the individual rationality constraint (IR) is given by

\[ R_l = R - \rho \geq I - A, \] (1.12)

which gives minimum funds required of

\[ \bar{A} = q_H \rho - (R - (1 - q_H) \rho - I). \] (1.13)

Good incentives are in place and there is no inefficient liquidation, consequently, the NPV is equal to \( R - (1 - q_H) \rho - I \). The banker needs funds greater than the difference between his expected net payment \( (q_H \rho) \) and the NPV of the project. The following proposition summarizes the equilibrium derived.

**Proposition 3** Under a confidential DW and if \( 0 \leq \frac{B}{\Delta q} < \rho \leq R \), then the equilibrium contract induces good behavior, \( q = q_H \), the banker is paid \( R_b = \rho \) and there is no run. The minimum funds required for the banker to secure funding are given by equation (1.12). The NPV of the project is \( R - (1 - q_H) \rho - I \).

Compared to the public information case, the cost to provide incentives under a confidential DW is higher since \( \rho > \frac{B}{\Delta q} \). Depositors are not diluted in equilibrium because the shock is completely absorbed with the banker’s payment. A confidential DW prevents inefficient liquidation caused by a run and this in turn increases the NPV of the project at the expense of aggravating moral hazard.

**Liquidity shock** \( \rho \in \left(0, \frac{B}{\Delta q}\right] \)

From the incentive compatible constraint (1.11), when \( \rho \Delta q < B \) the manager misbehaves \((q = q_L)\) no matter how big \( R_b \). In order to respect limited liability for the banker it is necessary to set \( R_b = \rho \) and as a result the minimum funds required to the banker are
\[ \bar{A} = \rho - (R - I) \]. The net present value of the project is \( R - (1 - q_L) \rho - I \), a difference of \( \rho \Delta q \) with the original NPV due to the misbehavior of the manager. The following proposition summarizes this equilibrium.

**Proposition 4** Under a confidential DW and if \( 0 < \rho \leq \frac{R}{\Delta q} \leq R \), then the equilibrium contract cannot induce good behavior, \( q = q_L \), the banker is paid \( R_b = \rho \) and earns a private benefit \( B \). The minimum funds required for the banker are given by

\[ \bar{A} = q_L \rho - (R - (1 - q_L) \rho - I) \]  \hspace{1cm} (1.14)

and the NPV of the project is \( R - (1 - q_L) \rho - I \).

### 1.3 Summary and Implications

Proposition 5 presents a summary of equilibriums derived in the previous section.

**Proposition 5** If liquidation is inefficient \( (X \leq R - \rho) \), \( q_L < q_H \), the adverse shock is a liquidity shock \( (\rho \leq R) \), there are enough resources to control moral hazard \( \left( \frac{R}{\Delta q} \leq R \right) \) and

\[ R - (1 - q_H) \rho - I > 0 \]  \hspace{1cm} (1.15)

\[ R - (1 - q_L) \rho - I > 0 \]

\[ q_H R + (1 - q_H) X - I > 0 \],

that is, the NPV is positive under good behavior, misbehavior and runs, then the equilibrium as a function of the disclosure policy and parameter values is given in Table 1.
Table 1. Equilibrium summary

<table>
<thead>
<tr>
<th></th>
<th>Disclosure</th>
<th>Non-disclosure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho \leq \frac{B}{\Delta q} \leq R$</td>
<td>$q = q_H$ (behave) [NPV: R - (1-q_H)\rho - I] [\bar{A} = q_H \frac{B}{\Delta q} - (R - (1-q_H)\rho - I)]</td>
<td>$q = q_L$ (misbehave) [NPV: R - (1-q_L)\rho - I] [\bar{A} = q_L\rho - (R - (1-q_L)\rho - I)]</td>
</tr>
<tr>
<td>$\frac{B}{\Delta q} &lt; \rho \leq R$</td>
<td>$q = q_H$ (behave) [NPV: q_H R + (1-q_H) X - I] [R_b = \frac{B}{\Delta q}] [\bar{A} = q_H \frac{B}{\Delta q} - (q_H R + (1-q_H) X - I)]</td>
<td>$q = q_H$ (behave) [NPV: R - (1-q_H)\rho - I] [R_b = \rho] [\bar{A} = q_H\rho - (R - (1-q_H)\rho - I)]</td>
</tr>
</tbody>
</table>

I analyze Table 1 by rows. In the first row, where $\rho \leq \frac{B}{\Delta q} \leq R$, making the DW confidential induces the manager to misbehave and this causes the NPV of the project to decrease by $R - (1-q_H)\rho - I - (R - (1-q_L)\rho - I) = \rho \Delta q$. As a result of the banker’s misbehavior, the resources needed to secure funding decrease compared to the public information setting because

$$q_H \frac{B}{\Delta q} - (R - (1-q_H)\rho - I) > q_L\rho - (R - (1-q_L)\rho - I)$$

or after rearranging and simplifying,

$$\frac{B}{\Delta q} > \rho.$$

This analysis assumes that the net present value of the project is still positive in the case of misbehavior. If the NPV is negative in the case of misbehavior $(R - (1-q_L)\rho - I < 0)$, then under a confidential DW there is no funding provided to the banker. This implies that a confidential DW causes a decrease in the expected cashflow that the project generates. Lower cashflow reduces the credit capacity. Credit rationing is aggravated in a confidential DW environment. By making DW information public creditors can design a contract that induces the banker to behave, increasing the NPV and the flow of credit to the economy.
In the second row of Table 1, where $\frac{B}{\Delta q} < \rho \leq R$, making the DW confidential increases the NPV of the project by $(1 - q_H)(R - \rho - X)$ because inefficient liquidation caused by a run is avoided. However, under a confidential DW the manager enjoys a bigger share of the cash flow $\left(\rho > \frac{B}{\Delta q}\right)$ as a consequence of aggravated moral hazard. The funds required for the banker to secure financing by depositors might be higher or smaller depending on the relation between the change in the banker’s payoff $\left(\rho - \frac{B}{\Delta q}\right)$ and the difference between the cash flow after a liquidity shock and the liquidation value $(R - \rho - X)$. For instance, funds required under a public DW are smaller than under a confidential DW setting if

$$(1 - q_H)(R - \rho - X) < q_H \left(\rho - \frac{B}{\Delta q}\right),$$

that is, the expected increase in NPV by avoiding inefficient liquidation is less than the increase in expected cost for placing incentives.

In this model, depositors break even due to the competitiveness of the credit market and therefore the banker gets the NPV of the project. The social planner’s problem is hence equivalent to the maximization of the NPV or the utility of the banker. Welfare is maximized when the NPV of the project is maximized.

According to Table 1 in proposition 5, the policymaker decision rule is to disclose DW information when $\rho \leq \frac{B}{\Delta q}$ and not to disclose when $\frac{B}{\Delta q} < \rho$ in order to maximize welfare. Large liquidity shocks make confidentiality a better policy because runs and inefficient liquidation is avoided. The cost to increase the NPV is that under confidentiality moral hazard is aggravated and as a consequence the banker needs a higher compensation in order to behave. Small liquidity shocks suggest that disclosure is a better policy because no run is induced since the banker’s capital absorbs the shock and good behavior is in place. Disclosure allows a contract contingent on the visit to the DW that serves as a signal of misbehavior.

Under a confidential DW, a high benefit for misbehaving or poor monitoring (large $B$) makes it infinitely costly for depositors to place good incentives, making disclosure a better
policy since it allows to compensate the banker using the occurrence of a liquidity shock as a signal. In contrast, good monitoring or small private benefit make runs more costly relative to the cost in placing incentives and therefore confidentiality of DW lending is a better policy.

Consider what happens when $\Delta q$ changes. When the manager has large control over the occurrence of a liquidity shock ($\Delta q$ is large) creditors interpret the liquidity shock as a signal of misbehavior and run in an attempt to avoid dilution. Privacy is a better policy as it helps to prevent inefficient runs. On the other hand, when the manager has a small influence on the occurrence of a liquidity shock ($\Delta q$ is small), then a shock is less likely signalling misbehavior, so that running is a less effective tool for discipline. Disclosure is a better policy since there is no inefficient liquidation.

### 1.3.1 Numerical Example

A numerical example illustrates how changes in $\Delta q$ affect the NPV, the expected banker’s share and the borrowing capacity. Parameters used in this example are presented in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>1.1</td>
</tr>
<tr>
<td>$I$</td>
<td>1</td>
</tr>
<tr>
<td>$X$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.1</td>
</tr>
<tr>
<td>$q_L$</td>
<td>0.8</td>
</tr>
<tr>
<td>$B$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Table 2. Parameter Values**

These parameters imply a return on the project of $0.1 - (1 - q_H)0.1 = 0.1q_H$. Liquidation value relative to cash flow is $\frac{0.8}{1.1} = 72.7\%$ and liquidity shock relative to cash flow is $\frac{0.1}{1.1} = 9.1\%$. Private benefit relative to cash flow is $\frac{0.01}{1.1} = 0.9\%$. The return on the project in the
case of misbehavior is 8.0%. Figure 1.2 plots the net present value of the project and the expected banker share as a function of the probability of being intact (no liquidity shock) in the case of good behavior, \( q_H \).

Figure 1.2 shows that for small values of \( q_H \) (small \( \Delta q \)) the NPV of the project is higher under disclosure. A visit to the DW is less likely a signal of bad behavior and therefore there is no inefficient run. NPV under disclosure is linear in \( q_H \) with slope \( \rho \). The expected banker’s share is decreasing because \( q_H \frac{B}{B + \Delta q} = \frac{B}{1 - \frac{\Delta q}{s_H}} \) is decreasing in \( q_H \). In this region, NPV under privacy is constant because the manager misbehaves no matter the value of \( q_H \).

When \( \frac{B}{\Delta q} = \rho \), there are some changes. For values of \( q_H \) bigger than \( q_L + \frac{B}{\rho} \), a run is induced under disclosure and this causes the NPV to decrease by the amount \( (1 - q_H) (R - \rho - X) \). The NPV under secrecy jumps by the amount \( \rho \Delta q \) because now good incentives are in place. The slope under secrecy is positive and equal to \( \rho \) but less than the slope under disclosure \( (R - X) \). Under disclosure, the expected share for the banker continues to be a decreasing function of \( q_H \) while for privacy it is increasing with slope \( \rho \). Secrecy is a better policy for high values of \( q_H \), projects with low probability of a shock under good behavior of the banker.

When \( q_H = 1 \), there is no liquidity shock if good behavior is in place, there is no dilution and as a consequence there is no run. The NPV is the same under disclosure and confidentiality and there is no need for a DW since no liquidity shock occurs.

### 1.4 Delayed Release of Discount Window Information

The analysis up to now can provide some intuition on what could be the benefit of releasing DW information with a lag. Suppose that there are \( N + 1 \) dates. Date 0 and date 1 stay as in the basic model. The realization of the cash flow occurs at date \( N \). Suppose all the parameters stay constant but \( B \), the private benefit, is higher the more periods DW is confidential. That is, from periods 3 to \( N \), the private benefit of the banker increases. The
intuition is that the banker can extract a larger surplus the longer his actions are unnoticed. Let $\rho < R$ and let $B(N)$ denote the private benefit when considering $N$ dates. Assume $0 < B(2) = B_2 < \rho \Delta q$, $B'(N) > 0$ and

$$\lim_{N \to \infty} \frac{B(N)}{\Delta q} = R. \tag{1.17}$$

These conditions guarantee that I start with $B(2) < \rho \Delta q$ (that is, row 2 in Table 1) and that eventually I get to cases where $B(t) > \rho \Delta q$ (row 1 in Table 1). Therefore, in a model with few dates (where $B(t) < \rho \Delta q$), it is optimal to keep the borrowings from the DW private in order to avoid inefficient runs (row 2, Table 1). However, in a model with more dates in which $B(t) > \rho \Delta q$ it is optimal to release information about DW borrowings because this allows to avoid misbehavior from the banker without inducing runs (row 1, Table 1).

Now suppose that all parameters stay constant but $X$, the liquidation value, increases with the number of periods. That is, suppose $X(2) < R - \rho$, $X(N) < R - \rho$ for all $N$ and

$$\lim_{N \to \infty} X(N) = R - \rho. \tag{1.18}$$
Under these conditions and for $\frac{B}{\Delta q} < \rho$, disclosure is less harmful the higher the number of periods because the liquidation value approaches the continuation value. Although it is never optimal to disclose, if the central bank has uncertainty on whether $\frac{B}{\Delta q} < \rho$ or $\frac{B}{\Delta q} \geq \rho$, then disclosure is eventually a better policy because the cost of inefficient liquidation is less than the cost of misbehavior from management caused by non-disclosure when $\frac{B}{\Delta q} \geq \rho$.

### 1.5 Conclusion

This paper studies how disclosure of discount window lending and moral hazard have effects on welfare. In the model, a confidential discount window serves as a tool that allows the bank to overcome liquidity shocks. After a liquidity shock occurs, depositors as a whole are better off by allowing the dilution of a new issue of deposits, but the collective action problem induced by deposits prevents them from waiting. A run occurs and the banker is forced to inefficiently liquidate. Confidentiality of discount window serves as insurance by preventing depositors from running.

Confidential borrowing also has perverse incentive effects. This paper models a situation in which the manager has influence over the occurrence or not of a liquidity shock. Anticipating a bailout by the central bank and confidentiality to depositors, the banker misbehaves to earn private benefits. Disclosure of discount window borrowings alleviates moral hazard because borrowing from the discount window is perceived as a signal of misbehavior and can be used in contracting with the banker.

The main result of the paper characterizes what policy is better when the policy maker maximizes the NPV of the project. Confidentiality is a better policy for high liquidity shocks, small benefits from misbehaving and high discretion over the probability of a shock. Under these circumstances the capital of the banker is not enough to absorb a liquidity shock, triggering a run after depositors notice dilution. With a confidential discount window, the banker requires a higher compensation in order to behave but a run is avoided, raising total
CHAPTER 1. RELEASE OF INFORMATION AND DISCOUNT WINDOW LENDING

welfare.

Disclosure of discount window lending is a better policy when the liquidity shock is small, the benefits for misbehaving are big and the banker has small control over the occurrence of the liquidity shock. A confidential discount window induces misbehavior because it is impossible to verify the occurrence of a shock. By disclosing discount window data, compensation that induces high effort can be designed because borrowings from the facility serve as a signal of misbehavior. Total welfare increases. The manager compensation increases, absorbs any liquidity shock and as a result there are no runs.

The paper explores the idea of a lag in disclosure as a policy tool that balances the trade-offs described over time. In the model, if the private benefit of misbehavior increases with the number of periods, then it is eventually optimal to disclose information in order to prevent the manager from misbehaving. Also, if the liquidation value approaches the continuation value as the number of periods increases, disclosure is less harmful. If a central bank ignores the values in the model it might be optimal to eventually disclose because liquidation becomes less harmful compared to misbehavior as the number of periods increases. A dynamic model would be useful in this setting in order to enrich the study of the trade-offs presented.
Chapter 2

Bank Assets, Runs, and Risk Taking

2.1 Introduction

In this chapter, I study the joint determination of bank runs and risk taking for a financial firm financed with short-term debt. This chapter introduces the dynamic debt runs framework by He and Xiong (2012) and extends it by allowing the manager, who is the equity holder, to select asset risk. The framework developed in this chapter is important for this dissertation because of two main reasons. First, it contributes to the literature on bank runs and managerial incentives by providing counter-intuitive results about the determination of asset risk and bank runs in equilibrium. Second, it sets the environment to study information frictions that will be the focus of the third chapter and are the common theme that relates chapter one and three.

The main result of this chapter says that the equilibrium risk choice and run frequency are a function of the composition of assets. Specifically, if assets grow sufficiently slowly, creditors run less frequently. However, the bank manager has greater incentives to take risk, increasing the possibility of insolvency for the bank. The risk-taking effect dominates and the resulting run probability is high and the bank value is low. The manager optimal risk choice increases effectively transferring value from creditors.
The model features two type of agents, a manager and a collection of creditors. Each creditor is small and concerned only with getting repaid. An individual creditor sees his chance of triggering a run as negligible. But creditors are aware of the possibility of runs, and they use the current value of bank projects to form beliefs about how likely a run might be. The strategic concern among creditors creates a coordination problem that may result in a debt rollover freeze, a bank run. The bank manager controls the asset’s risk and holds the equity of the firm. As an equity holder, the manager’s payoff is equal to a call option on the bank’s asset. That call option induces the manager to increase asset risk for low asset values.

This paper relates to the literature on the balance of bank runs and managerial incentives. Cheng and Milbradt (2012) studied how debt maturity affects the tradeoff between incentive provision and bank runs. Diamond and Rajan (2001) studied how runs serve as a commitment device for bank management. He and Xiong (2012) analyzed a dynamic coordination problem among creditors of a firm with staggered debt structure. Morris and Shin (2002) showed the trade-off between market discipline and strategic concerns that cause coordination problems. I provide a bank-specific structure that builds on He and Xiong (2012) and explicitly models the manager decision process.

I characterize the equilibrium by running a full set of comparative statics. I analyze the impact of liquidation costs on the equilibrium run probability and the creditors’ and manager’s optimal choices. A higher liquidation cost of the assets, generates a higher run threshold because creditors recover less in bankruptcy. The manager’s optimal risk choice is unaffected by the liquidation costs because she does not get anything after a run occurs. In equilibrium, high liquidation cost generates a higher run threshold and lower asset risk. The asset risk reduction effect dominates and decreases the run probability, or equivalently, increases bank value.

Debt maturity also has an impact on the equilibrium run probability and bank value. A lower debt maturity worsens the coordination problem among creditors and therefore
increases the run threshold. The intuition is that larger number of creditors is renewing the debt at every instant, increasing the strategic concern among non-maturing creditors. For the manager, a lower debt maturity decreases the incentive to take risk. The equilibrium outcome of these two forces is a higher run threshold but lower risk taking. The risk taking reduction effect dominates, decreasing the resulting run probability.

The interaction between bailout policies and the determination of asset risk and bank runs can be analyzed within this framework. The reliability of a bailout affects both the optimal choice of creditors and the manager. A bailout friendly policy reduces creditors’ incentives to run and therefore decreases the run threshold. For the manager, a bailout friendly policy marginally increases her incentives to take asset risk. In equilibrium, bailouts increases the asset risk and decreases the run threshold. The risk increase dominates and the resulting run probability increases. Bailouts undermine the disciplining role of runs.

This chapter is organized as follows. Section 2 introduces the model, the bank’s project and bank runs. Section 3 presents the analysis for an individual creditor and the manager of the bank. Section 4 introduces relevant parameter restrictions, the equilibrium definition, calibration and the equilibrium determination. Section 5 presents a full set of comparative statics that give rise to the main results of the chapter that are explained in Section 6. Conclusions follow.

2.2 Model

The model builds on He and Xiong (2012) and is set in continuous time with an infinite horizon. The bank invests in a long-term asset by rolling over short-term debt financed by a continuum of small creditors.
2.2.1 Asset and Risk Choice

The bank’s asset holding is normalized to one unit. The bank borrows $1 at time $t = 0$ to buy the asset. Once the asset is in place, it generates a constant stream of cash flow $rdt$ over the interval $[t, t + dt]$. At a random time $\tau_\phi$, which arrives according to a Poisson process with intensity $\phi > 0$, the asset matures with a final payoff of $y_{\tau_\phi}$. The advantage of assuming a random asset maturity is that it makes the expected life of the asset constant and equal to $1/\phi$.

The final payoff of the asset evolves according to a geometric Brownian motion with drift $\mu$ and volatility $\sigma$,

$$\frac{dy_t}{y_t} = \mu dt + \sigma dW_t,$$

where $W(t)$ is a standard Brownian motion, $\mu$ is the growth rate of the final payoff, and $\sigma$ is the instantaneous volatility. The evolution of the process is common knowledge and observed by all participants.

The bank’s asset generates a constant cash flow $rdt$ and the random final payoff $y_{\tau_\phi}$. The value of the asset is the expected discounted future cash flows

$$F(y_t) = \mathbb{E}_t \left[ \int_t^{\tau_\phi} e^{-\rho(s-t)} r ds + e^{-\rho(\tau_\phi-t)} y_{\tau_\phi} \right] = \frac{r}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_t,$$

where $\frac{r}{\rho + \phi}$ is the present value of the constant cash flow and $\frac{\phi}{\rho + \phi - \mu} y_t$ is the present value of the final payoff. Because the fundamental value of the bank is a linear function of $y_t$, I refer to $y_t$ as the bank fundamental.

The bank manager controls the risk of the asset; specifically, she chooses the asset’s final payoff volatility, $\sigma$. The bank manager chooses at time $t = 0$ a risk level $\sigma \in [\sigma_L, \sigma_H]$. Equivalently, the banker chooses a combination of an asset with low risk and one with high risk. The value of $\sigma$ is observable by all agents, but assumed not contractible. The manager’s decision is a one time choice and is kept constant afterwards. Endogenous risk taking is a feature not present in He and Xiong (2012). A different version of endogenous risk taking was
studied by Cheng and Milbradt (2012) to investigate optimal debt maturity that balances bank runs and incentive provision.

### 2.2.2 Debt financing and Runs

The bank financing, runs, and liquidation closely follow He and Xiong (2012). I present a brief review for completeness: The bank finances the asset by issuing short-term debt. Each debt contract lasts for an exponentially distributed amount of time with mean $1/\delta$. The parameter $\delta$ is exogenous. Creditors are paid interest payments at rate $r dt$ until the contract expires. Once an individual contract expires, the creditor chooses whether to roll over the debt or withdraw his funds. The times at which debt matures are independent across creditors so that each creditor expects some other creditors’ debt to mature before his.

In aggregate, a fraction $\delta dt$ of the bank’s debt matures over the time interval $[t, t + dt]$. When these creditors decide to withdraw their funds, a bank run, the bank must find financing from other sources or it will be forced into bankruptcy. I assume that the bank has access to a credit line that supplies the required financing. When a run occurs, there is a probability $\theta \delta dt$ that the credit line will fail to provide the required financing. The parameter $\theta > 0$ measures the reliability of the credit line. A low value of $\theta$ means that the credit line will sustain the run with high probability. If the credit line fails before the bank has recovered, the bank is forced to liquidate the asset in an illiquid secondary market. The asset’s liquidation value is a fraction $0 < \alpha < 1$ of the fundamental value of the asset

$$L(y_t) = \alpha F(y_t) \quad (2.3)$$

$$= L + ly_t,$$

with the constants $L = \frac{a r}{\rho + \phi}$ and $l = \frac{a \phi}{\rho + \phi - \mu}$. 
2.3 Agents Problems

2.3.1 The Creditor’s Problem

I follow He and Xiong (2012) and analyze the rollover decision for an individual creditor by taking as given that other creditors use a monotone strategy. A monotone strategy is a strategy in which all creditors whose debt matures decide to rollover if the bank’s lagged fundamental $x_t$ is greater than a threshold $y^*$. If the bank’s fundamental is lower than the threshold, $y_t \leq y^*$, the creditors’ optimal decision is to run.

There are two possible outcomes for the bank. Either the asset’s final payoff is realized or the bank is prematurely liquidated after a run. These events are not controlled directly by an individual creditor. However, once an individual’s debt matures, he can decide whether to rollover the debt or not. Each creditor receives interest payments at a rate $r$ per unit of time until

$$
\tau = \min \left( \tau_\phi, \tau_\theta \delta, \tau_\delta \right),
$$

(2.4)

which is the earliest of the following three events: asset maturity, forced liquidation after a run, and debt expiration without rollover, respectively. When debt matures, creditors receive the face value of the debt back and have the option to rollover their position by buying the new debt.

With a risk neutral creditor, the value of one unit of debt is given by the value function

$$
V(y_t) = E_t \left[ \int_t^\tau e^{-\rho(t-s)} r ds + e^{-\rho(\tau-t)} \left( \min \{1, y_\tau\} 1_{\{\tau = \tau_\phi\}} + \min \{1, \mathcal{L}(y_\tau)\} 1_{\{\tau = \tau_\theta \delta\}} + \max_{\text{rollover or run}} \{V(y_\tau), 1\} 1_{\{\tau = \tau_\delta\}} \right] \right],
$$

(2.5)

where $1_{\{\}}$ takes the value 1 when the statement in brackets is true and 0 otherwise. The value for an individual creditor has four components, represented by the four terms in the right hand side of equation (2.5). First term, coupon payments are at rate $r$. Second term, the creditor will receive at most $\$1$ when the asset matures, $\min \{1, y_\tau\}$. Third term, creditors’
will be paid at most $1 after a run forces asset liquidation, \( \min (1, \mathcal{L}(y_t)) \). Fourth term, the creditor decides whether to rollover or run when the debt matures. He and Xiong (2012) derive the HJB equation for the value function \( V(y_t) \),

\[
\rho V(y_t; y_*, \sigma) = \mu y_t V_y + \frac{\sigma^2}{2} y_t^2 V_{yy} + r + \phi (\min \{1, y_t\} - V(y_t; y_*, \sigma)) \\
+ \theta \delta 1_{\{y_t < y_*\}} (\min \{1, \mathcal{L}(y_t)\} - V(y_t; y_*, \sigma)) \\
+ \delta \max \{0, 1 - V(y_t; y_*, \sigma)\}
\] (2.6)

with boundary conditions

\[
V(0) = \frac{r + \theta \delta L + \delta}{\rho + \phi + \delta + \theta \delta} \tag{2.7}
\]

\[
\lim_{y \to \infty} V(y) = \frac{r + \phi}{\rho + \phi}.
\]

The left hand side of equation (2.6), \( \rho V(y_t; y_*) \), represents the creditor’s required return. The right hand side represents the expected increments on the continuation value. The first two terms, \( \mu y_t V_y + \frac{\sigma^2}{2} y_t^2 V_{yy} \), capture the change in continuation value caused by the fluctuation in the bank’s fundamental. The next four terms represent the components that appeared in the integral form of the value function, equation (2.5) - that is, the coupon payment rate, \( r \), the value change when the asset matures at time \( \tau_{\phi}, \phi (\min \{1, y_t\} - V(y_t; y_*)) \), the value change caused by a forced liquidation, \( \theta \delta 1_{\{y_t < y_*\}} (\min \{1, \mathcal{L}(y_t)\} - V(y_t; y_*)) \), and the option to run when the debt contract expires, \( \delta \max \{0, 1 - V(y_t; y_*), 0\} \), respectively.

A creditor whose debt matures will choose to rollover whenever the value of doing so is higher than the debt’s face value of $1. In other words, if the value function only crosses 1 at the point \( y' \), \( V(y'; y_*) = 1 \), then \( y' \) is the optimal run threshold for the creditor. If debt matures and \( y_t < y' \), the creditor will not rollover the debt. On the other hand, when \( y_t \geq y' \), the creditor finances a new debt contract. I compute symmetric monotone equilibria for which the threshold used by all creditors is the same, \( y_* \). Therefore, a condition to
determine the equilibrium threshold $y_*$ is that $V(y_*; y_*) = 1$.

### 2.3.2 The Manager’s Problem

The bank manager holds the firm’s equity, the residual claim after creditors are paid. This is a feature not present in He and Xiong (2012). Given a run threshold, $y_*$, the manager maximizes the total value of the residual claim by choosing the asset risk level, $\sigma$. The manager’s choice is made at time $t = 0$ and is held constant afterwards. The value of equity at time $t = 0$ is

$$Q(y_t; y_*, \sigma) = E \left[ e^{-\rho(t-t)} \left( (y_r - 1)^+ 1_{\{r = r_0\}} + (\mathcal{L}(y_r) - 1)^+ 1_{\{r = r_0\}} \right) \right]$$

(2.8)

where $(\cdot)^+ = \max\{\cdot, 0\}$. The value function of the manager has two components, represented by the two terms on the right hand side of (2.8). The first represents the equity payoff when the asset matures, $\max\{y_r - 1, 0\}$, and the second represents the payoff after a forced liquidation, $\max\{\mathcal{L}(y_r) - 1, 0\}$.

Using the same method as in He and Xiong (2012), or as in Cheng and Milbradt (2012) one can derive the HJB equation for the equity value $Q(y_t; y_*, \sigma)$,

$$\rho Q(y_t; y_*, \sigma) = \mu y_t Q_y + \frac{\sigma^2}{2} y_t^2 Q_{yy} + \phi \left( \max\{y_t - 1, 0\} - Q(y_t; y_*, \sigma) \right)$$

(2.9)

$$+ \theta \delta 1_{\{y_t < y_*\}} \left( \max\{\mathcal{L}(y_t) - 1, 0\} - Q(y_t; y_*, \sigma) \right),$$

with boundary conditions

$$Q(0) = 0$$

(2.10)

$$Q(y_t) = \frac{-\phi}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_t \text{ for } y_t \gg 0.$$
The right hand side represents the expected increments on the continuation value. The first two terms, $\mu y_t Q_y + \frac{\sigma^2}{2} y_{tt} Q_{yy}$, capture the change in continuation value caused by the fluctuation in the bank fundamental. The next two terms capture the expected continuation value change after the asset matures or a run. The terms $\max\{y_t - 1, 0\}$ and $\max\{\mathcal{L}(y_t) - 1, 0\}$ represent the manager’s payoff in case of project completion or a run, respectively.

Given a run threshold, $y_*$, and the initial value of the process, $y_0$, the manager chooses the asset risk level that maximizes the value of equity at time $t = 0$,

$$\sigma_* = \arg \max_{\sigma_L \leq \sigma \leq \sigma_H} Q(y_0; y_*, \sigma),$$

subject to (2.9) and (2.10).

### 2.4 Asset Risk and Run Equilibrium

The analysis requires some parameter restrictions in order to be meaningful. I work with the same parameter restrictions as in He and Xiong (2012). First,

$$\rho < r < \rho + \phi,$$  \hspace{1cm} (2.12)

which says that interest payments at rate $r$ are higher than the discount rate but not so high that they justify automatic rollover. Second, the growth rate of the asset is bounded,

$$\mu < \rho + \phi,$$  \hspace{1cm} (2.13)

which ensures that the value of the asset is finite. Third, the liquidation value of the asset when the fundamental $y_t = 1$ is lower than the initial capital raised from creditors,

$$\frac{\alpha r}{\rho + \phi} + \frac{\alpha \phi}{\rho + \phi - \mu} = L + l < 1,$$  \hspace{1cm} (2.14)
which ensures that the creditors have incentives to run when the fundamental deteriorates.

**Definition.** A pair of asset risk and run threshold, \((\sigma_*, y_*)\), is a symmetric equilibrium if:

1. Given the asset risk \(\sigma_*\), each creditor chooses and optimal run threshold \(y_*\). The run threshold is symmetric among creditors. Hence, a condition for \(y_*\) to be optimal is \(V(y_*; y_*, \sigma_*) = 1\), where \(V(y; y_*, \sigma)\) is given by equation 2.6.

2. Given the run threshold \(y_*\), the manager maximizes utility by choosing the asset risk \(\sigma\). The manager solves equation (2.11) subject to (2.9) and (2.10).

### 2.4.1 Equilibrium Analysis

I solve the model numerically. Table 1 presents the baseline parameter values which are calibrated to a typical financial firm during the recent financial crisis, see He and Xiong (2012). The volatility of the final payoff is now endogenous and limited to \(\sigma \in [\sigma_L, \sigma_H]\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho)</td>
<td>1.5%</td>
<td>Discount rate</td>
</tr>
<tr>
<td>(r)</td>
<td>7.0%</td>
<td>Cash flow rate from asset</td>
</tr>
<tr>
<td>(\phi)</td>
<td>0.077</td>
<td>Intensity of terminal value realization</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>55%</td>
<td>Liquidation discount</td>
</tr>
<tr>
<td>(\mu)</td>
<td>1.5%</td>
<td>Asset’s final payoff drift</td>
</tr>
<tr>
<td>(\delta)</td>
<td>10</td>
<td>Intensity at which debt matures</td>
</tr>
<tr>
<td>(\theta)</td>
<td>5</td>
<td>Intensity of credit line failure</td>
</tr>
<tr>
<td>(y_0)</td>
<td>1.4</td>
<td>Fundamental value at time 0</td>
</tr>
<tr>
<td>(\sigma_L)</td>
<td>2%</td>
<td>Low-risk volatility of final payoff</td>
</tr>
<tr>
<td>(\sigma_H)</td>
<td>12%</td>
<td>High-risk volatility of final payoff</td>
</tr>
</tbody>
</table>

Table 1. Baseline parameters.
2.4. ASSET RISK AND RUN EQUILIBRIUM

2.4.2 Equilibrium Determination

Figure 2.1 plots the creditors’ run threshold and the manager’s asset-risk choice, respectively. The solid line plots the optimal asset risk chosen by the manager, \( \sigma_* \), as a function of the creditors’ threshold \( y_* \). As the creditors’ run threshold increases, the manager optimally chooses lower asset risk. This relationship can be understood by studying equation (2.9), which gives the equity value given a run threshold. The manager faces a tradeoff when choosing the asset risk; higher levels of asset risk transfer value from creditors to the manager but also increase the chance of reaching the run threshold, where costly liquidation occurs. As a result, for high values of the run threshold, the manager chooses low levels of asset risk to avoid a run. On the other hand, for low values of the run threshold, the manager chooses high asset-risk because the run probability is low.

The dashed line in Figure 2.1 plots creditors’ equilibrium optimal run threshold, \( y_* \), as a function of the asset’s risk, \( \sigma \). The threshold chosen by creditors increases with asset risk. This relation can be intuitively understood by studying creditors’ final payoff, \( \min \{1, y_t\} \), which is equal to the sum of the payoff of a risk free bond and a short position on a put option,

\[
\min \{1, y_t\} = 1 - \max \{1 - y_t, 0\}. \tag{2.15}
\]

When the manager increases asset risk, the value of the short position on the put option decreases. Creditors react by endogenously increasing the run threshold. A similar analysis applies to the final payoff after a run, \( \min \{1, \mathcal{L}(y_r)\} \).

An equilibrium is a pair \((\sigma_*, y_*)\) where \( \sigma_* \) is consistent with \( y_* \). The intersection of the solid line and dashed line determines the equilibrium in Figure 2.1. The equilibrium is given by the pair \((\sigma_*, y_*) = (3.2\%, 0.91)\), which indicates that \( \sigma_* \) takes a moderate value and runs will occur if the fundamental value \( y_t \) gets below 0.91.
CHAPTER 2. BANK ASSETS, RUNS, AND RISK TAKING

2.5 Comparative Statics

The baseline parameters of Table 1 produce an equilibrium \((\sigma_*, y_*) = (3.2\%, 0.91)\). I characterize the equilibrium behavior by running a full set of comparative statics analysis for the key parameters in the model: the drift \(\mu\), the liquidation recovery \(\alpha\), the expected debt maturity \(1/\delta\), and the project’s cashflow rate \(r\). I also show how the equilibrium changes different combinations of the growth rate \(\mu\) and cashflow \(r\) while keeping the initial value of the asset, \(F(y_0)\) in equation (2.2), fixed. Increasing \(\mu\) and decreasing \(r\) transfers value from the coupon payments to the final payoff and effectively increases the average cash-flow maturity of the asset. The increase in maturity is accompanied by more risk, since a greater proportion of the cash flow comes from the final payoff, which is subject to random shocks from the Brownian motion.

2.5.1 The Impact of the Growth Rate \(\mu\)

Figure 2.2 plots the creditors’ run threshold and the manager’s asset-risk choice for different values of the drift of the asset, \(\mu\). Keeping everything else equal, a low value of \(\mu\) decreases

![Equilibrium determination with the baseline parameters. The solid line plots the optimal asset risk as a function of the run threshold \(y^*\). The dashed line plots the optimal run threshold as a function of asset risk. The equilibrium occurs when the asset risk is consistent with the run cutoff, the intersection of the lines.](image-url)
2.5. COMPARATIVE STATICS

Figure 2.2: Equilibrium changes for different values of the drift. Low values of the drift generate a higher run threshold but a lower asset risk, compared to high drift values.

For the value of the project, see equation 2.2, and therefore creditors react by increasing the run threshold. For management, a lower value of $\mu$ increases the risk taking incentives as more risk is needed to maximize utility. However, in equilibrium, a lower growth rate $\mu$ causes an increase in the run threshold by creditors but a reduction of risk by the manager. The coordination problem worsening dominates the decreasing the manager incentive to take risk. Conversely, a high growth rate $\mu$ increases the value of the project an as a result decreases creditors’ incentives to run. The managers takes advantage of a lower run threshold to increase asset risk, which increases the resulting run probability.

2.5.2 The Impact of Liquidation Costs $1 - \alpha$

Figure 2.3 plots the creditors’ run threshold and the manager’s asset-risk choice for different values of the liquidation recovery proportion, $\alpha$. A lower value for $\alpha$, generates a higher run threshold because creditors recover less in the bankruptcy. The manager’s optimal risk choice is unaffected by the liquidation costs because she does not get anything after a run. In equilibrium, a lower recovery value generates a higher run threshold and lower asset risk. The asset risk reduction dominates and increases bank value.
2.5.3 The Impact of Debt Maturity $1/\delta$

Figure 2.4 plots the creditors’ run threshold and the manager’s asset-risk choice for different values of the debt maturity intensity, $\delta$. The expected debt maturity is equal to $1/\delta$. A lower expected debt maturity worsens the coordination problem among creditors and therefore increases the run threshold. For the manager, a lower debt maturity decreases the incentive to take risk. In equilibrium, a lower debt maturity increases the run threshold but decreases risk taking. The risk reduction effect dominates decreasing the net run probability. Cheng and Milbradt (2012) study in detail the tradeoff between coordination problems and incentive provision when debt maturity changes. In their framework, the drift of the project decreases with higher risk, given rise to an interior solution for the debt maturity.

2.5.4 The Impact of the Cashflow Rate $r$

Figure 2.5 plots the creditors’ run threshold and the manager’s asset-risk choice for different values of the cashflow rate, $r$. A higher cashflow rate increases the payoff for creditors. The reaction of creditors is to decrease the run threshold for all levels of asset risk. The manager’s risk choice is not affected by changes in $r$ because she does not receive any of such payoffs.
2.5. COMPARATIVE STATICS

Figure 2.4: The impact of debt maturity. A shorter debt maturity (higher delta) increases the run threshold as creditors worry about other creditors’ actions. The manager in turn decreases asset risk.

Therefore, in equilibrium the run threshold decreases and the asset risk increases.

2.5.5 The Impact of the Growth Rate $\mu$ Keeping Value Fixed

Figure 2.6 plots the creditors’ run threshold and the manager’s asset-risk choice for different values of the drift $\mu$ while keeping constant the value of the asset. In order to keep constant the asset value, the cashflow rate $r$ decreases as $\mu$ increases. A lower drift value and higher cashflow rate generates an interesting shift in the behavior of creditors. For low values of asset risk, creditors decide to increase the run threshold, as they did when the drift changed in figure 2.2. However, for higher values of asset risk, the higher cashflow rate dominates and induces creditors to reduce the run threshold, as they did when the cashflow rate $r$ changed in figure 2.5. The overall result is that the optimal threshold pivots and it flattens for a lower value of the drift $\mu$, keeping constant the asset value.

The manager’s optimal risk choice is also affected by a lower drift. Similarly to figure 2.2, the manager chooses higher asset risk when the drift in the process is low. The overall result is that, in equilibrium, decreasing the drift increases both the asset risk choice and the run threshold. The run probability increases and hence the bank value decreases as a
Figure 2.5: The impact of the cashflow rate $r$. A higher cashflow rate increases the payoff for creditors, who in turn decrease the run threshold. The manager’s behavior is unchanged by changes in $r$.

result of the lower drift in the final payoff.

2.6 Liquidity and Bailouts

2.6.1 Liquidity, Bank Value, and Run Probability

The black line in Figure 2.7 plots equilibrium pairs for different value of $\mu$ while keeping the asset value constant. The equilibrium with baseline parameters is the point with coordinates $(\sigma, y) = (3.2\%, 0.91)$, bottom left side of the plot. When $\mu$ decreases, decreasing the duration of the asset, the equilibrium moves along the black line. The manager optimally chooses higher risk, and creditors choose a higher run threshold. A higher asset risk and higher run threshold decrease the bank value. The bank value decreases when duration decreases. Similarly, if short duration assets are relatively liquid, the bank value decreases with liquid assets.
2.6. LIQUIDITY AND BAILOUTS

Figure 2.6: Impact of the drift keeping constant the project value.

Figure 2.7: Equilibrium trajectory for different values of \( \mu \), keeping the value of the project constant.
2.6.2 Government Bailouts and Bank Runs

The interaction of bailout policies and the determination of risk taking and bank runs can be analyzed within this framework. Figure 2.8 plots the optimal choices from creditors and the manager for different bailout situations. The parameter $\theta$ controls the reliability of the credit line. A low value in $\theta$ indicates a high probability of a bailout. The reliability of a bailout affects both the optimal choice of creditors and the manager. A low value of $\theta$, high probability of a bailout, reduces creditors’ incentives to run and therefore decreases the run threshold. For the manager, a high probability of a bailout marginally increases her incentives to take asset risk. In equilibrium, a higher probability of a bailout increases asset risk and decreases the run threshold. The risk increase dominates and the resulting run probability increases. Bailouts undermine the disciplining role of runs.

2.7 Conclusion

In this chapter, I study the joint determination of bank runs and risk taking for a financial firm financed with short-term debt. This chapter introduces the dynamic debt runs framework by He and Xiong (2012) and extends it by allowing the manager, who is the equity
The main result of this chapter says that the equilibrium risk choice and run frequency are a function of the composition of assets. Specifically, if assets grow sufficiently slowly, creditors run less frequently. However, the bank manager has greater incentives to take risk, increasing the possibility of insolvency for the bank. The risk-taking effect dominates and the resulting run probability is high and the bank value is low. The manager optimal risk choice increases to maximize utility, effectively transferring value from creditors.

This chapter sets the environment for the information disclosure framework discussed in Chapter 3. The analysis presented here prepares the reader to the asymmetric information analyses of Chapter 3, which explores the consequences of delayed information disclosure. This chapter is related to Chapter 1 because both of them model bank runs and manager’s actions either by moral hazard, Chapter 1, or by risk taking as in this Chapter.
Chapter 3

Lagged Disclosure, Bank Runs, and Risk Taking

3.1 Introduction

Disclosure of information for financial institutions highlights two sides of the same coin: bank runs are costly when they occur, but they serve as an ex-ante disciplining device. Is there an optimal way to trade off the potential of disclosure to trigger bank runs against the benefits of providing information on risk-taking activities? It is known that disclosure can provide creditors with valuable information on a manager’s risk taking, which can enable creditors to find ways to restrict manager activity, e.g., Diamond and Rajan (2001). However, banks operate in environments with multiple frictions, and increasing transparency might be sub-optimal, e.g., Goldstein and Sapra (2014). For instance, disclosures might destabilize the banking system due to strategic interaction among creditors. If a disclosure suggests that a bank is solvent but in a precarious position, creditors may become more inclined to run on the bank. Two recent events where costs and benefits of information disclosure manifest are the publication of borrowers from the Federal Reserve’s discount window and the disclosure of banks’ stress tests results.
3.1. **INTRODUCTION**

Given the costs and benefits of information disclosure for financial institutions, the optimal timing for financial-information disclosure remains unclear. As such, I study the effects of lagged financial-information disclosure on the occurrence of bank runs and manager risk-taking activities. I develop a model of a bank that is subject to runs in the form of debt rollover freezes as in He and Xiong (2012), in which a manager chooses asset risk and creditors receive delayed performance information about the asset.\(^1\)

The main contribution of this paper is that requiring disclosure with a lag maximizes bank value when the bank holds a low-growth rate asset. Under these circumstances, introducing a disclosure lag generates two opposing effects. On the one hand, increased opacity causes creditors to run more frequently. On the other hand, the increase in runs by creditors forces the manager to reduce risk, in order to reduce the run probability. The risk reduction effect dominates, leading to a decrease in the run probability. However, for a large enough disclosure lag, bank runs worsen and the increase the resulting run probability. The bank value is a concave function of the disclosure lag. There is an optimal lag that trades-off the cost of bank runs introduced by coordination problems and risk taking.

Another contribution of this paper is that the model reproduces empirical facts of discount window disclosures. Kleymenova (2016) studied the capital market consequences of mandatory disclosures of discount window participants during the financial crisis.\(^2\) Specifically, she found that following discount window disclosures, banks that accessed the facility exhibited positive stock returns, positive bond returns, and lower asset risk. Through the lens of the model, I interpret these mandatory disclosures as shortening of the disclosure lag, from an infinite lag to 2-4 years. The model qualitatively reproduces these empirical facts and suggests that the asset risk reduction documented could be optimal, as opposed to the result of a managerial short-termism inefficiency.

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\(^1\) A bank in this paper is a financial intermediary that has short-term runnable liabilities: MMF, repo, ABCP, and large uninsured deposits.

A final contribution of this paper shows that bank value decreases as the disclosure lag increases for a bank with a high-growth-rate asset. The introduction of a disclosure lag generates two opposing effects. First, a disclosure lag decreases creditors’ incentives to run, because the asset has a high growth rate relative to the risk. As a result, the manager optimally reacts to the run probability reduction by increasing asset risk. The run probability increases because the manager’s extra risk-taking dominates the reduction in the run frequency by creditors.

Costs and benefits of information disclosure manifest in important situations such as the disclosure of banks’ stress tests results, and the publication of borrowers from the Federal Reserve’s discount window. Emergency lending facilities like the discount window became important tools used by the Federal Reserve during the financial crisis of 2007-2009. The Fed encourages banking institutions to borrow from these emergency funding facilities by keeping these banks’ emergency funding confidential. This confidential borrowing by financial institutions has both social benefits and costs. Benefits include the control of bank runs; costs include potential worsening of risk-taking activities by bank managers. In a similar way, costs and benefits of information disclosure are at the core of banks’ stress test results publication. Stress tests evaluate whether banks have sufficient capital to absorb losses resulting from adverse economic conditions. Having sufficient capital concerns bank creditors, and it is a useful piece of information for creditors to enhance manager performance evaluation. With better information, creditors can enhance manager performance by restricting the money deposits before it is too late. If all creditors decide to restrict the flow of funding at the same time, however, a bank run occurs. Bank runs are costly because they force premature liquidation of the asset, possibly at a large discount.

How long should a disclosure lag be? I study this question in a model of lagged disclosure based on the rollover freeze model of He and Xiong (2012). The model features two type of agents, a manager and a collection of creditors. Each creditor is small and concerned only

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3 See "Lenders Stress Over Test Results" http://on.wsj.com/1PWhsOh
with getting repaid. An individual creditor sees his chance of triggering a run as negligible. But creditors are aware of the possibility of runs, and they use the information they have to form beliefs about how likely a run might be. The strategic concern among creditors creates a coordination problem that may result in a debt rollover freeze, a bank run. The bank manager controls the asset’s risk and holds the equity of the firm. As an equity holder, the manager’s payoff is equal to a call option on the bank’s asset. That call option induces the manager to increase asset risk for low asset values. Finally, creditors receive delayed information about the current value of the bank’s asset. A disclosure lag is the amount of time that information is kept confidential. Creditors are aware of the fact that they are receiving outdated information and form rational expectations about the current asset value.

The creditors’ equilibrium run threshold changes when a disclosure lag is implemented. The change in the run threshold is a function of the bank asset’s growth-to-risk ratio. With a low growth-to-risk ratio for the asset, increasing opacity with the implementation of a disclosure lag causes creditors to run more frequently. Creditors run more frequently because they anticipate that a risky asset could have decreased in value since the disclosure date. On the other hand, creditors’ equilibrium run frequency decreases when the asset has a high growth-to-risk ratio, because a high growth rate increases the probability of asset value appreciation since the last disclosure date.

A reinterpretation of the model gives an application to accounting. Immediate disclosure or no information delay is reminiscent of mark to market accounting. A positive disclosure lag corresponds to historical cost accounting, where asset value is updated after some time. My model predicts that when switching from historical cost to mark to market accounting, value for banks with high growth rate assets will increase, management will reduce asset risk, and stocks and bonds will exhibit positive returns. Moreover, when bank assets have low growth rates, bank value is maximized under a historical cost regime as it balances bank runs and risk taking.\(^4\)

\(^4\)Guillaume et al. (2008) analyzed the tradeoff between imprecise accounting information provided by historical cost and price distortions for illiquid assets that are marked to market.
3.1.1 Related Literature

This paper is related to the literature on disclosure for financial firms. Gigler et al. (2013) studied the frequency of disclosure that should be required for public firms. They showed that when the bank manager can endogenously make decisions, frequent disclosure does not necessarily imply economic efficiency. Similarly, my model shows that a shorter disclosure lag does not necessarily improve bank value, as the manager might take excessive risk. Both Gigler et al. (2013) and this paper illustrate that in a model with multiple frictions and when the bank’s management decision is endogenous, price efficiency does not imply economic efficiency. My paper is also related to Williams (2015), who studied how informative bank stress tests should be and concluded that stress tests should give enough failing grades to keep passing grades credible and avoid runs. Faria-e-Castro et al. (2016) studied the tradeoff between a market breakdown caused by adverse selection and bank runs triggered when information is disclosed. Shapiro and Skeie (2015) studied the optimal bail-out policy that balances bank runs and risk-taking activities.

My paper also relates to the literature on the balance of bank runs and managerial incentives. Cheng and Milbradt (2012) studied how debt maturity affects the tradeoff between incentive provision and bank runs. Diamond and Rajan (2001) studied how runs serve as a commitment device for bank management. He and Xiong (2012) analyzed a dynamic coordination problem among creditors of a firm with staggered debt structure. Morris and Shin (2002) showed the tradeoff between market discipline and strategic concerns that cause coordination problems. I provide a bank-specific structure that builds on He and Xiong (2012) and explicitly models the information disclosure lag. The modeling choice makes it possible to identify what conditions justify the use of a lag by looking at bank characteristics, such as the bank’s underlying assets.

Several empirical studies relate to the considerations studied in this paper. Kleymenova (2016) studied the capital market consequences of mandatory disclosures of discount window participants during the financial crisis. Armantier et al. (2015) provided evidence for the
existence and magnitude of stigma in the interbank market associated with banks borrowing from the discount window. Kleymenova (2016) concluded that there is no stigma in capital markets whereas Armantier et al. (2015) documented the existence of stigma in the interbank market. My model does not study stigma, but qualitatively reproduces the market reaction of stocks, bonds, and the reduction in asset risk after disclosure reported in Kleymenova (2016).

3.2 Model

The model builds on He and Xiong (2012) and is set in continuous time with an infinite horizon. The bank invests in a long-term asset by rolling over short-term debt financed by a continuum of small creditors.

3.2.1 Asset and Risk Taking

The bank’s asset holding is normalized to one unit. The bank borrows $1 at time \( t = 0 \) to buy the asset. Once the asset is in place, it generates a constant stream of cash flow \( r dt \) over the interval \( [t, t + dt] \). At a random time \( \tau_\phi \), which arrives according to a Poisson process with intensity \( \phi > 0 \), the asset matures with a final payoff of \( y_{t, \phi} \). The advantage of assuming a random asset maturity is that it makes the expected life of the asset constant and equal to \( 1/\phi \).

The final payoff of the asset evolves according to a geometric Brownian motion with drift \( \mu \) and volatility \( \sigma \),

\[
\frac{dy_t}{y_t} = \mu dt + \sigma dW_t, \tag{3.1}
\]

where \( W(t) \) is a standard Brownian motion, \( \mu \) is the growth rate of the final payoff, and \( \sigma \) is the instantaneous volatility. The initial value of the process, \( y_0 \), is given and observed by all participants.

The bank’s asset generates a constant cash flow \( r dt \) and the random final payoff \( y_{t, \phi} \). The
value of the asset is the expected discounted future cash flows

\[ F(y_t) = E_t \left[ \int_t^{\tau_y} e^{-\rho(s-t)} r ds + e^{-\rho(\tau_y-t)} y_{\tau_y} \right] = \frac{r}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_t, \quad (3.2) \]

where \( \frac{r}{\rho + \phi} \) is the present value of the constant cash flow and \( \frac{\phi}{\rho + \phi - \mu} y_t \) is the present value of the final payoff. Because the fundamental value of the bank is a linear function of \( y_t \), I refer to \( y_t \) as the bank fundamental.

The bank manager controls the risk of the asset; specifically, she chooses the asset’s final payoff volatility, \( \sigma \). The bank manager chooses at time \( t = 0 \) a risk level \( \sigma \in [\sigma_L, \sigma_H] \). Equivalently, the banker chooses a combination of an asset with low risk and one with high risk. The value of \( \sigma \) is observable by all agents, but assumed not contractible. The manager’s decision is a one time choice and is kept constant afterwards. Endogenous risk taking is a feature not present in He and Xiong (2012). A different version of endogenous risk taking was studied by Cheng and Milbradt (2012) to investigate optimal debt maturity that balances bank runs and incentive provision.

### 3.2.2 Debt Financing, Runs, and Liquidation

The bank financing, runs, and liquidation closely follow He and Xiong (2012). I present a brief review for completeness: The bank finances the asset by issuing short-term debt. Each debt contract lasts for an exponentially distributed amount of time with mean \( 1/\delta \). Creditors are paid interest payments at rate \( rdt \) until the contract expires. Once an individual contract expires, the creditor chooses whether to roll over the debt or withdraw his funds. The debt maturity times are independent across creditors so that each creditor expects some other creditors’ debt to mature before his.

In aggregate, a fraction \( \delta dt \) of the bank’s debt matures over the time interval \([t, t + dt]\). When these creditors decide to withdraw their funds, a bank run, the bank must find financing from other sources or it will be forced into bankruptcy. I assume that the bank
Figure 3.1: Information structure for a disclosure lag $I > 0$. The solid line represents the fundamental process, $y(t)$, observed by the manager. The dashed line represents the lagged process, $x(t)$, which is the information available to the creditors at time $t$.

has access to a credit line that supplies the required financing. When a run occurs, there is a probability $\theta \delta dt$ that the credit line will fail to provide the required financing. The parameter $\theta > 0$ measures the reliability of the credit line. A low value of $\theta$ means that the credit line will sustain the run with high probability. If the credit line fails before the bank has recovered, the bank is forced to liquidate the asset in an illiquid secondary market. The asset’s liquidation value is a fraction $0 < \alpha < 1$ of the fundamental value of the asset

$$\mathcal{L}(y_t) = \alpha F(y_t)$$

$$= L + ly_t,$$

with the constants $L = \frac{\alpha r}{\rho + \phi}$ and $l = \frac{\alpha \phi}{\rho + \phi - \mu}$.

3.2.3 Information Sets

The model departs from He and Xiong (2012) in the creditors’ information structure. A regulator has control over the disclosure of the bank’s fundamental, $y_t$. The regulator dis-
closes the value of the bank’s fundamental with a lag $I \geq 0$. At time $t > 0$ creditors observe the bank’s fundamental value at time $t - I$, $y_{t-I}$. Creditors know that they are receiving outdated information and form rational expectations about the bank’s fundamental value. Define $x_t$, the lagged fundamental process as

$$x_t = y_{t-I}$$

with $I \geq 0$. The lagged fundamental process $x_t$ follows a geometric Brownian motion with the same drift and variance as in equation (3.1)

$$\frac{dx_t}{x_t} = \mu dt + \sigma dW_t, \quad x_I = y_0.$$  

I assume that the bank manager is an insider and observes the bank’s fundamental, $y_t$, with no lag. Moreover, she is aware of the fact that creditors observe a lagged fundamental value, and she will use this information in making optimal choices. Management closeness to the bank’s daily operations motivates this assumption.

Figure 3.1 describes the information structure of the model. The lag in disclosure is set at a value $I > 0$, and is common knowledge to all parties. The solid line represents a realization of the fundamental process, $\{y_u\}_{u \leq t}$, which is observed by the manager as it occurs. The dashed line represents the corresponding lagged process $\{x_u\}_{u \leq t}$, which is the information available to the creditors at each point. Denote by $E_t^I [\cdot] = E [\cdot | x_t]$ the conditional expectation used by creditors when the disclosure lag is set to $I$. The manager receives immediate information and, hence, the appropriate conditional expectation is

$$E_t^0 [\cdot] = E [\cdot | y_t].$$
3.3. A CREDITOR’S AND THE MANAGER’S PROBLEMS

Financing stage

At $t = 0$, financing occurs and the manager chooses the asset risk. Creditors choose an optimal run threshold. A run threshold is a value of the fundamental, $y_*$, such that creditors’ strategy is to run whenever the lagged fundamental is below $y_*$, $x_t < y_*$, and to rollover when $x_t \geq y_*$. The equilibrium determination of this threshold is explained in the next section.

The fundamental process $y_t$ starts at $t = 0$, but no coupons are paid and no actions are allowed until $t = I$. At $t = I$, coupons start to be paid and delayed information starts to be disclosed to creditors. Debt contracts also start to mature, and a bank run could occur. Figure 3.2 shows a timeline of the financing stage of the model.

3.3 A Creditor’s and the Manager’s Problems

3.3.1 An Individual Creditor’s Problem

I follow He and Xiong (2012) and analyze the rollover decision for an individual creditor by taking as given that other creditors use a monotone strategy. A monotone strategy is a strategy in which all creditors whose debt matures decide to rollover if the bank’s lagged

Figure 3.2: Financing stage. At $t=0$, financing occurs, management chooses asset risk and creditors choose a run threshold. $y(t)$ starts at $t=0$ but no coupons are paid until $t=I$. Starting at $t=I$, coupons are paid and delayed information starts to be disclosed to creditors. Also, bank runs could occur.
fundamental $x_t$ is greater than a threshold $y_*$. If the bank’s fundamental is lower than the threshold, $x_t \leq y_*$, the creditors’ optimal decision is to run.

There are two possible outcomes for the bank. Either the asset’s final payoff is realized or the bank is prematurely liquidated after a run. These events are not controlled directly by an individual creditor. However, once an individual’s debt matures, he can decide whether to rollover the debt or not. Each creditor receives interest payments at a rate $r$ per unit of time until

$$\tau = \min (\tau_\phi, \tau_\delta, \tau_\delta),$$

(3.7)

which is the earliest of the following three events: asset maturity, forced liquidation after a run, and debt expiration without rollover, respectively. When debt matures, creditors receive the face value of the debt back and have the option to rollover their position by buying the new debt.

With a risk neutral creditor, the value of one unit of debt is given by the value function

$$V (x_t) = E_t^I \left[ \int_t^\tau e^{-\rho(s-t)} \, r \, ds + e^{-\rho(\tau-t)} \left( \min \{1, y_r\} \mathbb{1}_{\{\tau=\tau_\phi\}} + \min \{1, L (y_r)\} \mathbb{1}_{\{\tau=\tau_\delta\}} + \max_{\text{rollover or run}} \{V (x_r; y_*) , 1\} \mathbb{1}_{\{\tau=\tau_\delta\}} \right) \right],$$

(3.8)

where $\mathbb{1}_{\{\}}$ takes the value 1 when the statement in brackets is true and 0 otherwise. The value for an individual creditor has four components, represented by the four terms in the right hand side of equation (3.8). First term, coupon payments are at rate $r$. Second term, the creditor will receive at most $\$1$ when the asset matures, $\min (1, y_r)$. Third term, creditors’ will be paid at most $\$1$ after a run forces asset liquidation, $\min (1, L (y_r))$. Fourth term, the creditor decides whether to rollover or run when the debt matures. The expectation in (3.8) is conditional on the information available to the creditor at time $t$, that is, the lagged fundamental value process $x_t$. 
The Appendix derives the HJB equation for the value function $V(x_t)$,

\[
\rho V(x_t; y_t, \sigma) = \mu x_t V_x + \frac{\sigma^2}{2} x_t^2 V_{xx} + r + \phi \left( E_t^I \left[ \min \{1, y_t\} \right] - V(x_t; y_t) \right) + \theta \delta 1_{\{x_t < y_t\}} \left( E_t^I \left[ \min \{1, \mathcal{L}(y_t)\} \right] - V(x_t; y_t) \right) + \delta \max \{0, 1 - V(x_t; y_t)\}
\]

(3.9)

with boundary conditions

\[
V(0) = \frac{r + \theta \delta L + \delta}{\rho + \phi + \delta + \theta \delta}; \quad \lim_{x \to \infty} V(x) = \frac{r + \phi}{\rho + \phi}, \quad (3.10)
\]

where $E_t^I [\cdot]$ is the expectation taken with the information available to the creditor at time $t$, see equation (3.6). Even though the creditor knows that the current time is $t$, he needs to estimate the current value of $y_t$ from the information provided by the lagged process, $x_t = y_{t-1}$. Hence, a creditor’s estimate of the payoff when the asset matures is $E_t^I \left[ \min \{1, y_t\} \right] = E_t^I \left[ \min \{1, x_{t+1}\} \right]$. Similarly, the creditor’s estimate of the payoff in case of bankruptcy at time $t$ is $E_t^I \left[ \min \{1, \mathcal{L}(y_t)\} \right] = E_t^I \left[ \min \{1, \mathcal{L}(x_{t+1})\} \right]$. The information structure of the model leads to a generalized version of the HJB equation in He and Xiong (2012), such that I recover their equation when the disclosure lag is set to zero, $I = 0$.

The left hand side of equation (3.9), $\rho V(x_t; y_t)$, represents the creditor’s required return. The right hand side represents the expected increments on the continuation value. The first two terms, $\mu x_t V_x + \frac{\sigma^2}{2} x_t^2 V_{xx}$, capture the change in continuation value caused by the fluctuation in the bank’s fundamental. The next four terms represent the components that appeared in the integral form of the value function, equation (3.8) - that is, the coupon payment rate, $r$, the value change when the asset matures at time $\tau_\phi$, $\phi \left( E_t \left[ \min \{1, y_t\} \right] - V(x_t; y_t) \right)$, the value change caused by a forced liquidation, $\theta \delta 1_{\{x_t < y_t\}} \left( E_t \left[ \min \{1, \mathcal{L}(y_t)\} \right] - V(x_t; y_t) \right)$, and the option to run when the debt contract expires, $\delta \max \{0, 1 - V(x_t; y_t), 1\}$, respectively.
A creditor whose debt matures will choose to rollover whenever the value of doing so is higher than the debt’s face value of $1. In other words, if the value function only crosses 1 at the point $x'$, $V(x'; y_*) = 1$, then $x'$ is the optimal run threshold for the creditor. If debt matures and $x_t < x'$, the creditor will not rollover the debt. On the other hand, when $x_t \geq x'$, the creditor finances a new debt contract. I compute symmetric monotone equilibria for which the threshold used by all creditors is the same, $y_*$. Therefore, a condition to determine the equilibrium threshold $y_*$ is that $V(y_*; y_*) = 1$.

### 3.3.2 The Bank Manager’s Problem

The bank manager holds the firm’s equity, the residual claim after creditors are paid. Given a run threshold, $y_*$, the manager maximizes the total value of the residual claim by choosing the asset risk level, $\sigma$. The manager’s choice is made at time $t = 0$ and is held constant afterwards. The value of equity at time $t = 0$ is

$$Q (y_0; y_*, \sigma) = E_{t=0}^0 \left[ e^{-\rho \tau} \left( (y_\tau - 1)^+ 1_{\{\tau = \tau_{\phi}\}} + (\mathcal{L}(y_\tau) - 1)^+ 1_{\{\tau = \tau_{s \delta}\}} \right) \right]$$

(3.11)

where $(\cdot)^+ = \max \{\cdot, 0\}$ and $E_{t=0}^0 [\cdot]$ is the expectation conditional on the information available to the manager at time $t = 0$, the non-delayed fundamental value $y_0$. The value function of the manager has two components, represented by the two terms on the right hand side of (3.11). The first represents the equity payoff when the asset matures, $\max \{y_\tau - 1, 0\}$, and the second represents the payoff after a forced liquidation, $\max \{\mathcal{L}(y_\tau) - 1, 0\}$.

Figure 3.2 presents a timeline describing the information available to the manager at time $t = 0$. The manager observes the fundamental value with no delay, $\{y_s\}_{s \leq t}$. Also, she is aware that creditors are making optimal choices based on lagged information.
3.3. A CREDITOR’S AND THE MANAGER’S PROBLEMS

The Appendix presents the derivation of the HJB equation for the equity value $Q(y_0; y_*, \sigma)$,

$$
\rho Q(y_0; y_*, \sigma) = \mu y_0 Q_y + \frac{\sigma^2}{2} y_0^2 Q_{yy} + \phi \left( E^0_{t=0} \left[ \max \{ y_I - 1, 0 \} \right] - Q \right) + \theta \delta 1_{\{ y_0 < y_* \}} \left( E^0_{t=0} \left[ \max \{ \mathcal{L}(y_I) - 1, 0 \} \right] - Q \right),
$$

with boundary conditions

$$
Q(0) = 0 \quad (3.13)
$$

$$
Q(y_0) = \frac{-\phi}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_0 \text{ for } y_0 \gg 0.
$$

The left hand side of equation (3.12), $\rho Q(y_0; y_*, \sigma)$, represents the banker’s required return. The right hand side represents the expected increments on the continuation value. The first two terms, $\mu y_0 Q_y + \frac{\sigma^2}{2} y_0^2 Q_{yy}$, capture the change in continuation value caused by the fluctuation in the bank fundamental. The next two terms capture the expected continuation value change after the asset matures or a run. The terms $E^0_{t=0} \left[ \max \{ y_I - 1, 0 \} \right]$ and $E^0_{t=0} \left[ \max \{ \mathcal{L}(y_I) - 1, 0 \} \right]$ represent the manager’s best estimate of the final payoff if the asset ends at time $t = I$. Even though she will see the asset fundamental in real time, she needs to make a decision at time $t = 0$ knowing only the initial value of the fundamental. The manager’s estimate of her final payoff is equal to the value of a call option on the bank’s asset, with time to maturity equal to the disclosure lag.

Given a run threshold, $y_*$, and the initial value of the process, $y_0$, the manager chooses the asset risk level that maximizes the value of equity at time $t = 0$,

$$
\sigma_* = \arg \max_{\sigma_L \leq \sigma \leq \sigma_H} Q(y_0; y_*, \sigma), \quad (3.14)
$$

subject to (3.12) and (3.13).
3.4 Asset Risk and Run Equilibrium

The analysis requires some parameter restrictions in order to be meaningful. I work with the same parameter restrictions as in He and Xiong (2012). First,

\[ \rho < r < \rho + \phi, \]  

(3.15)

which says that interest payments at rate \( r \) are higher than the discount rate but not so high that they justify automatic rollover. Second, the growth rate of the asset is bounded,

\[ \mu < \rho + \phi, \]  

(3.16)

which ensures that the value of the asset is finite. Third, the liquidation value of the asset when the fundamental \( y_t = 1 \) is lower than the initial capital raised from creditors,

\[ \frac{\alpha r}{\rho + \phi} + \frac{\alpha \phi}{\rho + \phi - \mu} = L + l < 1, \]  

(3.17)

which ensures that the creditors have incentives to run when the fundamental deteriorates.

**Definition.** A pair of asset risk and run threshold, \((\sigma_*, y_*)\), is an equilibrium if:

1. Given the asset risk \( \sigma_* \), each creditor chooses and optimal run threshold \( y_* \). The run threshold is symmetric among creditors. Hence, a condition for \( y_* \) to be optimal is \( V(y_*; y_*, \sigma_*) = 1 \).

2. Given the run threshold \( y_* \), the manager maximizes utility by choosing the asset risk \( \sigma \). The manager solves equation (3.14) subject to (3.12) and (3.13).
3.5 Equilibrium Analysis

I solve the model numerically. Table 1 presents the baseline parameter values which are calibrated to a typical financial firm during the recent financial crisis, see He and Xiong (2012). I use a lower drift for the final payoff, $\mu$, and a higher cash flow rate $r$, compared to the calibration in He and Xiong (2012). Also, the volatility of the final payoff is now endogenous and limited to $\sigma \in [\sigma_L, \sigma_H]$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>1.5%</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$r$</td>
<td>8.42%</td>
<td>Cash flow rate from asset</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.077</td>
<td>Intensity of terminal value realization</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>55%</td>
<td>Liquidation discount</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.5%</td>
<td>Asset’s final payoff drift</td>
</tr>
<tr>
<td>$\delta$</td>
<td>10</td>
<td>Intensity at which debt matures</td>
</tr>
<tr>
<td>$\theta$</td>
<td>5</td>
<td>Intensity of credit line failure</td>
</tr>
<tr>
<td>$y_0$</td>
<td>1.4</td>
<td>Fundamental value at time 0</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>2%</td>
<td>Low-risk volatility of final payoff</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>12%</td>
<td>High-risk volatility of final payoff</td>
</tr>
</tbody>
</table>

Table 1. Baseline parameters.

3.5.1 Immediate Disclosure Equilibrium

Consider a situation in which there is immediate disclosure, or $I = 0$. Immediate disclosure implies that the lagged and the actual fundamental process coincide, $x_t = y_t$, and, hence, creditors use the real-time value of the asset. Figure 3.3 plots the creditors’ run threshold and the manager’s asset-risk choice, respectively. The solid line plots the optimal asset risk chosen by the manager, $\sigma_*$, as a function of the creditors’ threshold $y_*$. As the creditors’ run threshold increases, the manager optimally chooses lower asset risk. This relationship
can be understood by studying equation (3.12), which gives the equity value given a run threshold. The manager faces a tradeoff when choosing the asset risk; higher levels of asset risk transfer value from creditors to the manager but also increase the chance of reaching the run threshold, where costly liquidation occurs. As a result, for high values of the run threshold, the manager chooses low levels of asset risk to avoid a run. On the other hand, for low values of the run threshold, the manager chooses high asset-risk because the run probability is low.

The dashed line in Figure 3.3 plots creditors’ equilibrium optimal run threshold, $y_*$, as a function of the asset’s risk, $\sigma$. The threshold chosen by creditors increases with asset risk. This relation can be intuitively understood by studying creditors’ final payoff, $\min\{1, y_t\}$, which is equal to the sum of the payoff of a risk free bond and a short position on a put option,

$$\min\{1, y_t\} = 1 - \max\{1 - y_t, 0\}. \tag{3.18}$$

When the manager increases asset risk, the value of the short position on the put option decreases. Creditors react by endogenously increasing the run threshold. A similar analysis applies to the final payoff after a run, $\min\{1, \mathcal{L}(y_t)\}$.

An equilibrium is a pair $(\sigma_*, y_*)$ where $\sigma_*$ is consistent with $y_*$. The intersection of the solid line and dashed line determines the equilibrium in Figure 3.3. The equilibrium is given by the pair $(\sigma_*, y_*) = (12\%, 0.96)$, which indicates that $\sigma_*$ takes the maximum value and runs will occur if the fundamental value $y_t$ gets below 0.96.

### 3.5.2 Equilibrium with Disclosure Lags, $I > 0$

Figure 3.4 plots the manager’s and creditors’ optimal decisions for immediate disclosure and a two-year lag, $I \in \{0, 2\}$. The black-solid line and the black-dashed line reproduce the optimal responses of the manager and creditors’ when there is immediate disclosure, as in Figure 3.3. As before, the equilibrium with immediate disclosure is given by the intersection
of the lines at the point \((\sigma_*, y_*) = (12\%, 0.96)\). The grey-solid line and the grey-dashed line reproduce the optimal choices of the manager and creditors when there is a two-year lag, respectively. The equilibrium occurs at the intersection of these lines, where \(\sigma_*\) is consistent with \(y_*\) and is given by the pair \((\sigma_*, y_*) = (10.2\%, 0.98)\). The equilibrium for a two-year disclosure lag has a higher run threshold and lower asset risk compared to the immediate disclosure equilibrium.

The equilibrium changes after the introduction of a disclosure lag because it changes both the manager’s and creditors’ expected payoff. The creditors’ final payoff, \(E_t^I \left[ \min (1, y_t) \right]\), is the sum of the payoff of a risk free bond and a short position on a put option written on the asset’s final payoff,

\[
E_t^I \left[ \min \{1, y_t\} \right] = 1 - E_t^I \left[ \max \{1 - y_t, 0\} \right].
\]
Figure 3.4: Two equilibriums: Immediate disclosure and disclosure lags of 2 years. The solid lines plot optimal asset risk as a function of the run threshold $y^*$. The dashed lines plot the optimal run threshold as a function of the asset risk. An equilibrium requires asset risk to be consistent with the run threshold.

The derivative of the short put option value with respect to the lag $I$ is

$$
\frac{\partial E_I}{\partial I} [-\max\{1 - y_t, 0\}] = -N'(d_-) \frac{\sigma}{2\sqrt{I}} + x_t \mu e^{\mu I} N(-d_+),
$$

(3.20)

where

$$
d_{\pm} = \frac{1}{\sigma \sqrt{I}} \left( \ln [x_t] + \left( \mu \pm \frac{1}{2} \sigma^2 \right) I \right),
$$

(3.21)

$N(\cdot)$ is the cumulative of the normal standard distribution and $'$ indicates the first derivative.

The first term in the expected payoff’s change, equation (3.20), is always negative and the second is always positive. Hence, when the asset risk $\sigma$ is high relative to the drift $\mu$ the derivative is negative. A negative derivative indicates that the creditors’ payoff decreases as the lag $I$ increases. When the creditors’ expected payoff decreases, they have more incentives to run. In the opposite scenario, when the asset risk is low relative to the drift, the expected payoff increases with the lag and creditors run less frequently.

Figure 3.4 illustrates that for risk values higher than $\sigma = 6\%$, creditors increase the
run threshold when the disclosure lag increases. The partial derivative (3.20) is negative. Intuitively, there is extra uncertainty created by the delay in information, and creditors protect themselves by running sooner. On the other hand, when the asset risk is lower than $\sigma = 6\%$, creditors’ optimal threshold decreases with a disclosure lag. The creditors’ payoff increases as the lag increases, which reduces their incentives to run. The partial derivative (3.20) is positive.

A disclosure lag also changes the manager’s optimal risk choice, as illustrated in Figure 3.4. The manager takes advantage of the fact that creditors are unable to take action for the first $I$ periods. Formally, the manager’s payoff is equal to the value of a call option written on the fundamental, $y_t$, with maturity equal to the disclosure lag, $I$. The derivative of the call option value with respect to the disclosure lag is given by

$$
\frac{\partial E_0 \left[ \max \{ y_I - 1, 0 \} \right]}{\partial I} = N' \left( d_- \right) \frac{\sigma}{2\sqrt{I}} + y_0 e^{\mu I} N \left( d_+ \right),
$$

with $d_\pm$ as in equation (3.21). Since the derivative of the call option is always positive, the manager’s payoff is increasing with the lag, which allows him to increase asset risk.

The economic mechanism when the disclosure lag increases from 0 to 2 years illustrated in Figure 3.4 can be described as follows. A higher disclosure lag creates extra uncertainty for creditors who, in turn, react by running more frequently. The manager has an incentive to increase risk with more opacity, as her actions are not controlled in the first $I$ periods. However, the increase in the run threshold by creditors dominates and actually makes the manager reduce risk in equilibrium.

The bank value changes as the information environment changes from immediate disclosure to a two-year lag. Holding everything else constant, a lower asset risk increases the bank value because it decreases the probability of the fundamental value crossing the run threshold. In addition, a higher run threshold decreases bank value because runs become more likely. The net change in bank value depends on how the change in $\sigma_*$ and $y_*$ affects
The run probability. With a geometric Brownian motion for $y_t$, the probability of a run is given by

$$P_{y_0} \left( \inf_{0 \leq s \leq T_\delta} y_s \leq y_\ast \right) = \left( \frac{y_\ast}{y_0} \right) \sqrt{\left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{\sigma^2}{\sigma^2} + \left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)}.$$

The run probability is increasing in the run threshold, $y_\ast$, and in asset risk, $\sigma$.

### 3.5.3 Optimal Disclosure Lags

Figure 3.5 plots the bank value and run probability for disclosure lags from 0 to 6 years. The left plot shows that the bank value is a concave function of the disclosure lag. The bank value increases when the lag increases from 0 to 2 years, because the reduction in asset risk dominates the increase of the run threshold, the mechanism illustrated in Figure 3.4. The bank value attains a maximum around 2.5 years and decreases afterwards, where the risk reduction benefit does not compensate the increase in the run threshold.

The right plot in Figure 3.5 shows the run probability as a function of the disclosure lag. The run probability decreases initially as the manager’s risk reduction dominates the
increased run threshold. It attains a minimum around 2.5 years and then increases. The probability of a run is a sufficient statistic for the bank value. Increasing the run threshold increases the probability of a run and decreases bank value. Decreasing asset risk decreases the run probability and increases bank value.

3.6 Disclosure Lags and the Data

The model is able to reproduce qualitatively some empirical facts reported in the literature of discount window disclosures. Kleymenova (2016) studied the capital market consequences of mandatory disclosures of discount window participants during the financial crisis. Banks accessed the discount window from 2007 to 2009, thinking that their participation would remain confidential. Kleymenova (2016) provided evidence that even after disclosing the information with a lag, in 2011, the disclosures contained new information. Specifically, the study found that following the discount window disclosures, banks that accessed the facility had positive cumulative abnormal stock returns of 1.10%. The study also found that after the discount window disclosures, banks that used the discount window placed their bonds at lower yields, on average 1.16% lower. Regarding asset risk, the study found that following discount window disclosures, banks decreased risk taking on their assets.

In the context of my model, I can qualitatively reproduce Kleymenova’s findings by interpreting the mandatory disclosures as a disclosure lag shortening. Under this interpretation, a non-disclosure environment corresponds to a very long disclosure lag, possibly infinite. The disclosure that took place in 2011 corresponds to a disclosure with a lag of 2-4 years. Therefore, by changing the disclosure lag, I can study what the model predicts about asset risk, equity prices, and debt prices and compare these predictions with the empirical evidence.

The top plot in Figure 3.6 shows the equilibrium equity and debt prices for disclosure lags that range from 0 to 10 years. The bottom plot presents asset risk as chosen by the manager for the same range of disclosure lags. These plots are generated by setting the growth rate
CHAPTER 3. LAGGED DISCLOSURE, BANK RUNS, AND RISK TAKING

Figure 3.6: Equity and debt values for different lags on the top plot. Asset risk as chosen by the manager for different lags on the bottom plot.

Figure 3.6: Equity and debt values for different lags on the top plot. Asset risk as chosen by the manager for different lags on the bottom plot.

to $\mu = 1\%$ and the cash flow rate $r = 7.72\%$.\(^5\) The top panel shows that when disclosure lags decrease, implying more frequent disclosure, equity and debt prices increase, consistent with the finding in Kleymenova (2016) about abnormal returns in equity and lower yields in debt. The bottom plot shows that for a shorter disclosure lag, the manager chooses less asset risk in equilibrium, which is also consistent with the empirical evidence.

3.7 Comparative Statics

The baseline parameters of Table 1 produce an optimal lag of around 2.5 years. What other parameters produce an optimal lag? In order to answer this question, I conduct a comparative statics analysis and derive regions in which the lag is optimal and regions in which it is not. I evaluate whether a lag is optimal for different combinations of the growth rate $\mu$ and cashflow $r$ while keeping the initial value of the asset, $F(y_0)$ in equation (3.2), fixed. Increasing $\mu$ and decreasing $r$ transfers value from the coupon payments to the final payoff and effectively increases the average cash-flow maturity of the asset. The increase

\(^5\)Keeping the asset value constant and increasing the drift to $\mu = 1\%$ gives $r = 7.72\%$.\)
Figure 3.7: Equilibrium with immediate disclosure and with a lag of 2 years. The solid lines represent the manager’s best response and the dashed lines represent the creditors’ run cutoff.

in maturity is accompanied by more risk, since a greater proportion of the cash flow comes from the final payoff, which is subject to random shocks from the Brownian motion.

3.7.1 Banks with a High Growth Asset

Figure 3.7 presents the equilibrium analysis for a parametrization with $\mu = 1.5\%$ and $r = 7\%$. This is the parametrization used by He and Xiong (2012). It has a higher growth rate, $\mu$, compared to the baseline parameters of Table 1. The manager’s and creditors’ optimal choice follow the same general behavior as in the baseline calibration, Figure 3.4. That is, the creditors threshold increases with asset risk and the manager’s risk allocation decreases with the run threshold.

There are two equilibrium effects when introducing a disclosure lag for a bank with relatively higher growth rate asset. The first is to increase asset risk from 3.24% to 3.84%. The second is to decrease the run threshold from 0.9 to 0.89. The economic mechanism is as follows. Under immediate disclosure and a high growth asset, the manager chooses a low level of asset risk and creditors choose a low run threshold. The introduction of a lag
eases creditors’ incentives to run, lowering the equilibrium run threshold. However, a lower run threshold gives room to the manager to increase asset risk. The resulting equilibrium with a two-year lag has a higher asset risk and a lower run threshold when compared to the immediate disclosure case. The higher asset risk effect dominates and decreases bank value, despite the lower run threshold.

3.7.2 Asset Characteristics and Optimal Disclosure Lags

The previous example illustrates that a disclosure lag is not beneficial for banks with a high-growth asset. However, disclosure lags are beneficial when the bank’s asset has low growth rates. Figure 3.8 plots equilibrium pairs \((\sigma_*, y_*)\) for two disclosure lags, \(I \in \{0, 2\}\), while varying \(\mu\) and \(r\). The equilibrium with baseline parameters is located on the top right side of the plot. The introduction of a lag causes the equilibrium to change from a high asset risk with a high run threshold \((\sigma_*, y_*) = (12\%, 0.95)\), black line, to a lower asset risk and higher run threshold \((\sigma_*, y_*) = (10\%, 0.98)\), grey line. Equilibriums using the He and Xiong (2012) parameters are located at the bottom left side of Figure 3.8. The introduction of a lag decreases the run threshold and increases the asset risk, black to grey line.

More generally, Figure 3.8 shows that a lag is beneficial when the bank’s asset has relatively low growth rates or low average cashflow maturity. An bank asset with short duration is typically highly liquid, and, hence, another interpretation from Figure 3.8 is that a disclosure lag balances risk taking and runs when banks hold liquid assets. The economic mechanism is as follows; liquid assets have low growth opportunities that induce the bank manager to increase asset risk. This produces an asset with a low growth-to-risk ratio. As the disclosure lag increases, creditors become nervous and tend to run more frequently because of the bad characteristics of the asset. The manager optimally reduces asset risk to compensate for the increased run probability. The risk reduction effect dominates, and the bank value increases with a disclosure lag.
3.8. DISCLOSURE LAG AS AN ALTERNATIVE TO BAILOUTS

Figure 3.8: Equilibrium with Constant Asset Values and Disclosure Lag. High-growth assets are in the bottom left of the graph and low-growth assets are in the top right.

3.7.3 Liquidity, Bank Value, and Run Probability

The black line in Figure 3.8 plots equilibrium pairs for different values of $\mu$ while keeping the asset value constant. The equilibrium with baseline parameters is the black circle with coordinates $(\sigma_*, y_*) = (12\%, 0.95)$, top right side of the plot. When $\mu$ increases, increasing the duration of the asset, the equilibrium moves along the black line. The manager optimally chooses lower risk, and creditors choose a lower run threshold. A lower asset risk and lower run threshold increase the bank value. The bank value increases when duration increases. Similarly, if long duration assets are relatively illiquid, the bank value increases with illiquid assets.

3.8 Disclosure Lag as an Alternative to Bailouts

Another way to affect the balance between runs and risk-taking activities is for the regulator to change the bailout policy. The parameter $\theta$ controls the reliability of bailouts. A higher value for $\theta$ means a lower probability of a bailout. In the model, the regulator could decrease
the run probability by decreasing the reliability of bailouts, a situation shown in Figure 3.9. When the regulator decreases the reliability of bailouts, creditors optimally react by running more frequently, a first order effect. Seeing the increased run probability, the manager reduces the equilibrium asset risk. The risk effect dominates, and the resulting run probability decreases.

Figure 3.9 presents an example of this situation where parameter $\theta$, which measures the intensity of failure during a bank run, increases from 5 to 7. A increase in parameter $\theta$ means that the government sustains the bank run for a shorter period of time. The grey-dashed line shows that creditors increase the run threshold in response to the new, stricter bailout policy. To attain an equilibrium, the manager reduces asset risk. The bank value increases as the run probability decreases.

Under the baseline parameters of Table 1, the regulator has two options to decrease the run probability: First, increase opacity by implementing a disclosure lag, the focus of this paper. Second, decrease the reliability of bailouts. The two options attain the same results
in terms of bank value, run probability, and bailout costs. I argue that increasing opacity via a disclosure lag is relatively easier.

Compared to changing the bailout policy, introducing a disclosure lag might face much less scrutiny and opposition from Wall Street firms and Congress, while attaining the same results. The disclosure lag policy also might be easier to commit to. For example, since the Dodd Frank Financial Reform Act of 2010, discount window disclosures are made with a two-year lag, and changing this would require an act of Congress. My paper shows that decreasing the reliability of bailouts and implementing a disclosure lag are two complementary regulatory tools. A world with no bailouts could be equivalent to a world with some bailouts and a disclosure lag, according to the model. This suggests a rationale for the coexistence of bailout and disclosure lags.

3.9 Conclusion

This paper studies the effects of disclosing financial information on the occurrence of bank runs and management risk-taking activities. In particular, I study how a balance between bank runs and asset risk-taking activities can be attained by delaying information release. I develop a model of a bank that is subject to runs in the form of debt rollover freezes as in He and Xiong (2012), in which a manager chooses asset risk and creditors receive delayed performance information about the asset.

The main contribution of this paper is that a positive disclosure lag that maximizes bank value when the bank holds a low-growth-rate asset. Under this condition, implementing a disclosure lag generates two opposing effects. First, increased opacity causes creditors to run more frequently. As a result, the increase in runs by creditors forces the manager to reduce risk, in an attempt to lower the run probability. The risk reduction effect dominates and the bank value increases. When the disclosure lag is large enough, bank runs worsen and the bank value decreases. There is an optimal lag that trades off the cost of bank runs
introduced by coordination problems and risk taking.

Another contribution is that the model also reproduces features of the data. Kleymenova (2016) studied the capital market consequences of mandatory disclosures of discount window participants during the financial crisis. My model suggests that the asset risk reduction documented in Kleymenova (2016) could be optimal as opposed to the result of managerial short-termism inefficiency.

A final contribution shows that bank value decreases as the disclosure lag increases for a bank with a high-growth-rate asset. Under these circumstances, the introduction of a disclosure lag generates two opposing effects. First, a disclosure lag decreases creditors’ incentives to run. Second, the manager optimally reacts to the run probability reduction by increasing asset risk. The risk increase effect dominates, and the bank value decreases.

The model has applications to accounting. A zero disclosure lag corresponds to a situation in which a bank’s assets are marked to market. A positive disclosure lag corresponds to historical cost accounting regime, where the asset value is updated after some time. The model predicts that when switching from historical cost to mark to market accounting, value for banks with high growth rate assets will increase, management will reduce asset risk, and stocks and bonds will exhibit positive returns. Moreover, when bank assets have low growth rates, bank value is maximized under a historical cost regime as it balances bank runs and risk-taking.

I also analyze how bailout policies interact with disclosure lags. My model shows that good incentives can be provided by either a stricter bailout policy or by implementing a disclosure lag. I argue that the implementation of a disclosure lag is much easier to commit to. My model suggests that disclosure lags are a way of tightening the bailout policy. This suggests a rationale for the coexistence of bailout and disclosure lags.

This paper contributes to understanding how bank runs and incentive provision balance in equilibrium. The model serves as a benchmark to evaluate disclosure policies for financial institutions. However, the model does not address the behavior of the interbank lending
market. By abstracting from the interbank lending market, I am able to study the specificities of the disclosure lag at the cost of other considerations such as adverse selections among heterogeneous banks. These could be important considerations that future research will address, in order to better inform policy makers.
Appendix A

Appendix to Chapter 3

A.1 Creditor’s HJB Equation

In this section I review the creditor’s problem and some of its properties. Recall that the creditor’s problem can be written as the solution of the following value function equation:

\[
V(x_t) = E_t\left[ \int_t^\tau e^{-\rho(s-t)} r ds + e^{-\rho(\tau-t)} \min(1,y_\tau) 1_{\{\tau=\tau_{\phi}\}} \right. \\
+ \min(1,\mathcal{L}(y_\tau)) 1_{\{\tau=\tau_{\theta}\}} \\
+ \left. \max_{\text{run or rollover}} (1,V(x_{\tau};y_\tau)) 1_{\{\tau=\tau_{\delta}\}} \right]
\]

Now, I rewrite the problem in a different way. Fix a threshold \(y_*\). At each point in time \(u \geq t\), the creditor receives interest payments \(r\), and when the asset matures he receives \(\min(1,x_{u+t})\) or \(\min(1,\mathcal{L}(x_{u+t}))\) when the bank is forced into bankruptcy. Therefore, the creditor’s expected payoff at time \(u \geq t\) is given by

\[
r + \frac{\phi}{\phi + \theta \delta 1_{\{x_u < y_*\}}} E_u[\min(1,x_{u+t})] \\
+ \frac{\theta \delta 1_{\{x_u < y_*\}}}{\phi + \theta \delta 1_{\{x_u < y_*\}}} E_u[\min(1,\mathcal{L}(x_{u+t}))]
\]
where $E_u[\min (1, x_{u+1})]$ and $E_u[\min (1, \mathcal{L}(x_{u+1}))]$ have Black-Scholes like formulas. Let $p_1 (x, u + I) = E[\min (1, x_{u+1}) | x_u = x]$ and $p_2 (x, u + I) = E[\min (1, \mathcal{L}(x_{u+1})) | x_u = x]$ and integrate the discounted payoff across possible values of $x$ to get

$$p(t, x_t) = E_{t,x_t}[e^{-\rho(u-t)} \left( r + \frac{\phi}{\phi + \theta \delta 1_{\{x_u < y_u\}} p_1 (x_u, u + I) + \frac{\theta \delta 1_{\{x_u < y_u\}}}{\phi + \theta \delta 1_{\{x_u < y_u\}}} p_2 (x_u, u + I) \right)].$$

I show that $e^{-\rho t} p(x_t, t)$ is a martingale under $P$. Let $s \leq t \leq u$

$$E \left[ e^{-\rho t} p(t, x_t) | F(s) \right]$$

$$= E \left[ e^{-\rho t} E \left[ e^{-\rho(u-t)} \left( r + \frac{\phi}{\phi + \theta \delta 1_{\{x_u < y_u\}} p_1 (x_u, u + I) + \frac{\theta \delta 1_{\{x_u < y_u\}}}{\phi + \theta \delta 1_{\{x_u < y_u\}}} p_2 (x_u, u + I) \right) | F(t) \right] | F(s) \right]$$

$$= E \left[ E \left[ e^{-\rho u} \left( r + \frac{\phi}{\phi + \theta \delta 1_{\{x_u < y_u\}} p_1 (x_u, u + I) + \frac{\theta \delta 1_{\{x_u < y_u\}}}{\phi + \theta \delta 1_{\{x_u < y_u\}}} p_2 (x_u, u + I) \right) | F(t) \right] | F(s) \right]$$

$$= e^{-\rho s} e^{-\rho s} p(s, x_s),$$

by the tower property. Hence, $e^{-\rho t} p(x_t, t)$ is a martingale and $p(x_t, t)$ satisfies the Black-Scholes formula.

The creditor’s value can be written as the Laplace transform of $p(t, x_t)$, or

$$V(x_t; y_*) = \int_0^\infty (\phi + \theta \delta 1_{\{x_u < y_u\}}) e^{-\rho + \theta \delta 1_{\{x_u < y_u\}} u} p(t, x_t) \, du$$

and, hence, I can apply the Laplace transform to both sides of the Black-Scholes PDE to get equation (3.9).
A.2 Manager’s HJB Equation

In this section I show that $Q(y_t; y_*, \sigma)$, satisfies equation (3.12). Recall that

$$Q(y_t; y_*, \Gamma) = E_{t=0}^0 \left[ e^{-\rho(\tau-t)} \left( (y_{\tau} - 1)^+ 1_{\{\tau=\tau_\phi\}} + (\mathcal{L}(y_{\tau}) - 1)^+ \right) 1_{\{\tau=\tau_\phi\}} \right].$$

Fix a time $u \geq I$. At time $u$, the asset could expire with payoff $(y_u - 1)^+$, the bank could be forced into bankruptcy with payoff $(\mathcal{L}(y_u) - 1)^+$, or it could continue.

Creditors’ receive information with a lag and, hence, the intensity relevant at time $u$ depends on the value of the fundamental at time $u - I$. The expected payoff at time $u$ can be therefore written as

$$\phi(y_u - 1)^+ + \theta \delta 1_{\{y_{u-I} < y_*\}} (\mathcal{L}(y_u) - 1)^+$$

where $\phi$ represents the intensity of the asset expiring at time $u$ and $\theta \delta 1_{\{y_{u-I} < y_*\}}$ represents the intensity of the bank failing at time $u$.

Consider the discounted expected payoff at time $u - I$

$$payoff(u - I, y_{u-I}) = E_{u-I} \left[ e^{-\rho I} \left( \phi(y_u - 1)^+ + \theta \delta 1_{\{y_{u-I} < y_*\}} (\mathcal{L}(y_u) - 1)^+ \right) \right]$$

which can be written as

$$payoff(u - I, y_{u-I}) = \phi E_{u-I} [e^{-\rho I} (y_u - 1)^+]$$

$$+ \theta \delta 1_{\{y_{u-I} < y_*\}} E_{u-I} [e^{-\rho I} (\mathcal{L}(y_u) - 1)^+]$$

The expected discounted payoff at time $t = 0$ is therefore

$$c(t = 0, y_t) = E_{t=0} [e^{-\rho(u-I)} payoff(0, y_0)].$$
I show that \( e^{-\rho t} c(t, y_t) \) is a martingale under \( P \). Let \( s \leq t \leq u - I \) and consider

\[
E \left[ e^{-\rho t} c(t, y_t) \mid F(s) \right] = E \left[ E \left[ e^{-\rho(u-I)} \text{payoff}(u-I, y_{u-I}) \mid F(t) \right] \mid F(s) \right] = E \left[ e^{-\rho(u-I)} \text{payoff}(u-I, y_{u-I}) \mid F(s) \right] = e^{-\rho s} c(s, y_s)
\]

which indeed means that \( e^{-\rho t} c(t, y_t) \) is a martingale. Therefore \( c(s, x) \) satisfies the Black-Scholes formula

\[
-\rho c(s, x) + c_t(s, x) + \mu x c_x(s, x) + \frac{1}{2} \sigma^2 x^2 c_{xx}(s, x) = 0
\]

with boundary conditions

\[
c(u - I, x) = \text{payoff}(u - I, x) \quad \forall x \in [0, \infty) \\
c(s, 0) = 0 \quad \forall s \in [t, u - I] \\
\lim_{x \to \infty} c(s, x) = \phi(xe^{(u-I)(u-s)} - e^{-\rho(u-s)}) \quad \forall s \in [0, u - I].
\]

It is convenient to change the time variable to the time to expiration \( \tau = u - I - t \) so that \( g(\tau, x) = c(u - I - t, x) \) satisfies the PDE

\[
-\rho g(\tau, x) - g_\tau(\tau, x) + \mu x g_x(\tau, x) + \frac{1}{2} \sigma^2 x^2 g_{xx}(\tau, x) = 0 \quad \text{(A.1)}
\]

with boundary conditions

\[
g(0, x) = \text{payoff}(u - I, x) \quad \forall x \in [0, \infty) \\
g(s, 0) = 0 \quad \forall s \in [0, u - I] \\
\lim_{x \to \infty} g(s, x) = \phi(xe^{(u-I)(s+I)} - e^{-\rho(s+I)}) \quad \forall s \in [0, u - I].
\]
Since the asset maturity and bank failure after a run are exponentially distributed, the manager’s value is the Laplace transform of \( g(\tau, x) \), or

\[
Q(x; y_*) = \int_0^\infty e^{-(\phi + \theta \delta 1_{\{y_u - t < y_*\}})(\tau)} g(\tau, x) \, d(\tau)
\]

and, hence, I can apply the Laplace transform to both sides of the Black-Scholes PDE, equation (A.1), to get

\[
-\rho Q(x) - \left((\phi + \theta \delta 1_{\{y_u - t < y_*\}}) \, Q(x) - g(0, x)\right) + \mu x Q_x(x) + \frac{1}{2} \sigma^2 x^2 Q_{xx}(x) = 0
\]

where \( Q(\cdot) \) is the Laplace transform of \( g(\tau, x) \).

Finally, replace the boundary condition for \( g(0, x) \) to get equation (3.12)

\[
\rho Q(x) = \mu x Q_x(x) + \frac{1}{2} \sigma^2 x^2 Q_{xx}(x) + \phi \left(E_{u-I}[e^{-\rho t} (y_u - 1)^+] - Q(x)\right) \\
+ \theta \delta 1_{\{y_u - t < y_*\}} \left(E_{u-I}[e^{-\rho t} \mathcal{L}(y_u) - 1)^+] - Q(x)\right)
\]

### A.3 Government’s Expected Bailout Costs

Following Cheng and Milbradt (2012) I can compute the Government’s expected bailout costs by solving the following ODE.

\[
\rho G(y_t; y_*) = \mu y_t G_y + \frac{\sigma^2}{2} y_t^2 G_{yy} - \phi G(y_t; y_*) \\
- \theta \delta 1_{\{y_t < y_*\}} G(y_t; y_*) \\
+ \delta 1_{\{y_t < y_*\}} \left[1 - V(y_t; y_*)\right]
\]
In standard form

\[
\frac{\sigma^2}{2} y_t^2 G_{yy} + \mu y_t G_y - G \left( \rho + \phi + 1 \{y_t < y_*\} \theta \delta \right) = -\delta 1_{\{y_t < y_*\}} \left[ 1 - V(y_t; y_*) \right]
\]

with boundary conditions

\[
\lim_{y \to -\infty} G(y) = 0 \\
G(0) = \frac{\delta (1 - V(0))}{\rho + \phi + \theta \delta}
\]

where \(V(0)\) is the value for the creditors when the fundamental is 0.

## A.4 Call and Put Options Value and Time to Maturity

In this section I present a review of the behavior of call and put options prices with respect to time to maturity. This review will be useful in understanding the results of the model.

Recall the formula

\[
E \left[ \left( K - x e^{\sqrt{\sigma^2} Z - \frac{1}{2} v} \right)^+ \right] = KN(-z_-) - x N(-z_+)
\]

(A.2)

where

\[
z_{+/-} = \frac{1}{\sqrt{v}} \left( \log \left[ \frac{x}{K} \right] \pm \frac{1}{2} v \right)
\]

(A.3)

and \(Z \sim N(0,1)\)

Consider the value of a put option on \(y_t\) with strike 1, time to expiration \(I\) and no discounting. Mathematically, the value of this option is \(E_t \left[ (1 - y_{t+I})^+ \right]\). I can compute this expectation by using equation (A.2) with \(K = 1\), \(x = y_t e^{\mu I}\) and \(v = \sigma^2 I\). I get

\[
p_1(I, y_t) = E \left[ (1 - y_{t+I})^+ \right] = N(-z_-) - y_t e^{\mu I} N(-z_+)
\]
where
\[ z_{+/-1} = \frac{1}{\sigma \sqrt{I}} \left( \log |y_t| + \left( \mu \pm \frac{1}{2} \sigma^2 \right) I \right). \] (A.4)

Compute the derivative of \( p_1 (I, y_t) \) with respect to \( I \),
\[ \frac{\partial p_1}{\partial I} = -\mu y_t e^{\mu I} N (-z_{+1}) + \frac{N'(-z_{-1}) \sigma}{2\sqrt{I}}. \]

Now consider the value of a put option on \( y_t \) with strike \( \frac{1-L}{l} \), time to expiration \( I \) and no discounting. Mathematically, the value of this option is \( E_t \left[ \left( \frac{1-L}{l} - y_{t+1} \right)^+ \right] \). I can compute this expectation by using equation (A.2) with \( K = \frac{1-L}{l} \), \( x = y_t e^{\mu I} \) and \( v = \sigma^2 I \). I get
\[ p_2 (I, y_t) = E \left[ \left( \frac{1-L}{l} - y_{t+1} \right)^+ \right] = \frac{1-L}{l} N (-z_{-2}) - y_t e^{\mu I} N (-z_{+2}) \]

where
\[ z_{+/-2} = \frac{1}{\sigma \sqrt{I}} \left( \log \left[ \frac{ly_t}{1-L} \right] + \left( \mu \pm \frac{1}{2} \sigma^2 \right) I \right). \] (A.5)

Compute the derivative of \( p_2 (I, y_t) \) with respect to \( I \),
\[ \frac{\partial p_2}{\partial I} = -\mu y_t e^{\mu I} N (-z_{+2}) + \frac{(1-L) N'(-z_{-2}) \sigma}{2l \sqrt{I}}. \]

Recall the formula
\[ E \left[ \left( x e^{\sqrt{2} \tau - \frac{1}{2} \nu} - K \right)^+ \right] = x N (z_+) - K N (z_-) \] (A.6)

where \( z_+ \) and \( z_- \) are given by equation (3.14). Consider the value of a call option on \( y_t \) with strike 1 and time to expiration \( I \). Mathematically, the value of this option is \( E_t \left[ e^{-\mu I} (y_{t+1} - 1)^+ \right] = e^{-\mu I} E_t \left[ (y_{t+1} - 1)^+ \right] \). I can compute this expectation by using equation (A.6) with \( K = 1 \), \( x = y_t e^{\mu I} \) and \( v = \sigma^2 I \). I get
\[ c_1 (I, y_t) = E \left[ e^{-\mu I} (y_{t+1} - 1)^+ \right] = y_t N (z_{+1}) - e^{-\mu I} N (z_{-1}) \]
A.4. CALL AND PUT OPTIONS VALUE AND TIME TO MATURITY

where \( z_{+1/-1} \) are given by equation (A.4). The partial derivative of \( c_1 \) with respect to \( I \) is given by

\[
\frac{\partial c_1}{\partial I} = \mu e^{-\mu I} N(z_{-1}) + \frac{y_tN'(z_{+1}) \sigma}{2\sqrt{I}}
\]

which is always positive.

Finally, consider the value of a call option on \( y_t \) with strike \( \frac{1-L}{l} \) and time to expiration \( I \). Mathematically, the value of this option is

\[
E_t \left[ e^{-\mu I} \left( y_{t+I} - \frac{1-L}{l} \right)^+ \right] = e^{-\mu I} E_t \left[ \left( y_{t+I} - \frac{1-L}{l} \right)^+ \right].
\]

I can compute this expectation by using equation (A.6) with \( K = \frac{1-L}{l} \), \( x = y_t e^\mu I \) and \( v = \sigma^2 I \).

I get

\[
c_2(I, y_t) = E \left[ e^{-\mu I} \left( y_{t+I} - \frac{1-L}{l} \right)^+ \right] = y_tN(z_{+2}) - \frac{1-L}{l} e^{-\mu I} N(z_{-2})
\]

where \( z_{+2/-2} \) are given by equation (A.4). The partial derivative of \( c_2 \) with respect to \( I \) is given by

\[
\frac{\partial c_2}{\partial I} = \mu \frac{1-L}{l} e^{-\mu I} N(z_{-2}) + \frac{y_tN'(z_{+2}) \sigma}{2\sqrt{I}}
\]

which is always positive.
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