Essays on Earnings Management, Investment Efficiency, and Managerial Incentives

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Abstract

In response to accounting scandals, market control systems (e.g. regulations related to internal control systems) have become more stringent in order to restore investors’ confidence in capital markets. Tightening control systems has triggered a fierce debate on its effect on both capital markets and the real economy. My dissertation studies how mitigating earnings management by tightening control systems can affect managerial incentives and a firm’s investment decisions.

Why are CEOs rarely fired? I develop a dynamic agency model to show that costly earnings management can act as an alternative punishment for poor performance and substitute for managerial turnover. The principal can design a contract such that it is incentive compatible for the agent to engage in costly earnings management to avoid being fired when poor performance is realized. Since earnings management can impose a cost on the agent that cannot be replicated by compensation, it can effectively relax the agent’s bankruptcy constraint and, thus, the agent experiences a negative payoff when her performance is poor. Therefore, the principal can not only improve the agent’s ex ante effort incentives but also reduce the use of the threat of turnover (Incentive alignment benefit). The principal, however, may need to pay more to compensate for the cost of earnings management (Wealth transfer cost). Therefore, the trade-off between the incentive alignment benefit and the wealth transfer cost determines whether earnings management is beneficial or harmful to shareholders.

In the second chapter, titled “Earnings Management, Investment, and Managerial Turnover in a Dynamic Agency Model,” I develop a model to investigate how the internal control system influences a
firm’s investment decisions. Contrary to the view that a strong internal control system mitigates CEOs’ incentives to manage earnings and increases investment efficiency, I find that a moderate internal control system, that allows appropriate reporting discretion to CEOs, can improve a firm’s investment decisions when past performance is poor. The essential mechanism is that, in the optimal contract, costly earnings management can act as an alternative punishment for poor performance and, thus, substitute for the threat of turnover. Because the possibility of turnover leads to an underinvestment problem (e.g. because a new CEO needs to learn about the ongoing projects and there are costs associated with searching for a new CEO.), a moderate internal control system can effectively improve investment efficiency. Also, an infinite-horizon dynamic model shows a positive relationship between investment and the level of earnings management for a given internal control system and an inverted U-shape relationship between investment and the internal control system. Finally, calibration results suggest that shareholders’ value under the current level of the internal control systems in the market is 0.4% higher than that under the counterfactual strongest internal control system.

In the third chapter, titled “Accounting Conservatism, Earnings Management, and Investment,” we develop a dynamic model to analyze how accounting conservatism interacts with earnings management to mitigate agency problems and improve investment efficiency. In the presence of CEO turnover, accounting conservatism, which gives more precise but less frequent high signals, can result in more frequent CEO turnover, leading to an underinvestment problem. Costly earnings management helps the firm to maintain a conservative accounting system because it can work as an alternative punishment for poor performance and substitute for the threat of turnover. We find that accounting conservatism, combined with earnings management, can improve contract efficiency by increasing the expected punishment for poor performance in two ways. First, a conservative accounting system increases the probability of CEOs being penalized for poor performance through earnings management. Second, a conservative accounting system enhances the incentive spillback effect and, thus, can penalize CEOs with more negative payoff by further relaxing CEOs’ bankruptcy constraint. Finally, in the extension in which the effect of the accounting system on the
cost of earnings management is introduced, we show that accounting conservatism helps earnings management to be costly enough to provide *ex ante* effort incentives.
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Finally, I want to dedicate the thesis to my parents, brother, and sister for their unconditional love, sacrifice, and support. I also want to express my deepest gratitude to my wife Dainn Wie for consistently encouraging and believing me. This journey would not have been possible without them.
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2 This chapter is based on a joint work with Carlos Corona and Jonathan Glover.
Chapter 1

Earnings Management, Managerial Incentives, and Managerial Turnover

Abstract

Why are CEOs rarely fired? I develop a dynamic agency model to show that costly earnings management can act as an alternative punishment for poor performance and substitute for managerial turnover. The principal can design a contract such that it is incentive compatible for the agent to engage in costly earnings management to avoid being fired when poor performance is realized. Since earnings management can impose a cost on the agent that cannot be replicated by compensation, it can effectively relax the agent’s bankruptcy constraint and, thus, the agent experiences a negative payoff when her performance is poor. Therefore, the principal can not only improve the agent’s \textit{ex ante} effort incentives but also reduce the use of the threat of turnover (Incentive alignment benefit). The principal, however, may need to pay more to compensate for the cost of earnings management (Wealth transfer cost). Therefore, the trade-off between the incentive alignment benefit and the wealth transfer cost determines whether earnings management is beneficial or harmful to shareholders.
... Sports fans are accustomed to baseball managers being fired after one losing season. Few CEOs experience a similar fate after years of underperformance. There are many reasons why we would expect CEOs to be treated differently from baseball managers ... Perhaps corporate directors are providing CEOs with substantial rewards and penalties based on performance... (Jensen & Murphy, 1990)

1.1 Introduction

This paper investigates why CEOs are rarely fired and whether there exists an alternative punishment for poor performance. I develop a dynamic agency model to provide new insights on how earnings management can reduce agency problems. The paper offers policy implications in that standard setters need to understand the effect of earnings management on the flexibility of contracting between shareholders and CEOs and suggests that tightening control systems that aim to reduce CEOs’ discretion in reporting earnings might result in unintended consequences.

It has been documented that the forced turnover rate and pay-performance relation for chief executive officers are too low to be consistent with the agency theory. For instance, at large corporations in the U.S., 2% of CEOs are fired on average each year (Taylor, 2010). Jensen and Murphy (1990) find that CEO wealth changes $3.25 for every $1,000 change in shareholder wealth and suggest that it is difficult to conclude that the threat of dismissal provides meaningful incentives because the economic significance of the turnover-performance relation is fairly small.

The paper questions if the infrequent termination of poorly performing CEOs and the low pay-performance relation imply the absence of incentives. The low force turnover rate and pay-performance relation do not create proper incentives for executives to maximize the value of firms. Perhaps we might not take into account another penalty that is provided to CEOs. This paper offers new rationale for the use of earnings management as an alternative punishment in CEO compensation. Through
optimal contracting, the principal (shareholders) can design a contract that induces the agent (a manager) to engage in costly earnings management to avoid being fired when her performance is poor.\textsuperscript{1} Since costly earnings management can impose a cost on the agent that cannot be replicated by compensation, it can relax the agent’s bankruptcy constraint.\textsuperscript{2} In other words, due to costly earnings management, the agent experiences a negative payoff in the period in which her performance is poor. Therefore, ex ante, the agent has more incentives to work hard to avoid costly earnings management which would be penalty for poor performance. Therefore, considering a disciplining role of earnings management, strengthening control systems, such as regulation and internal control systems that plan to decrease managers’ discretion in reporting earnings, might decrease the flexibility of contracting and, hence, the firm value.

To better understand the consequences of earnings management, the paper introduces a dynamic agency model and derives the optimal contract between the principal and the agent which minimizes the cost of the agency problem. In the model, the risk-neutral principal hires the risk-neutral agent to operate the business. The principal offers a long-term contract specifying compensation and termination decisions as a function of performance history. The agent makes a choice of productive effort and manages earnings. Based on reported earnings, the agent can be dismissed and replaced with a new (identical) agent. The threat of turnover reduces the agent’s rents because of the incentive spillback effect (Glover & Lin, 2015). The incentives provided in the future spillback to the incentives today because the agent can keep working and enjoy the rents in the future only when her performance today is good.

Earnings management has two effects: an incentive alignment benefit and a wealth transfer cost. The incentive alignment benefit means that the agent can be incentivized to work harder in order to avoid the penalty from bearing the cost of earnings management in the case of poor performance. Therefore, the principal can not only motivate the agent to work harder but also decrease the use of the threat of

\textsuperscript{1}The revelation principle is applicable because the compensation contract is implemented after the agent observes the private information. Therefore, earnings management provides a mechanism for the agent to communicate with some cost that she was unlucky and not slacking. This finding contradicts the argument in other papers that earnings management is an outcome of the agent’s lying to the principal.

\textsuperscript{2}This cost is determined by the level of internal control system and the amount of earnings management. The agent’s bankruptcy constraints mean nonnegativity of the agent’s utility.
turnover—a costly incentive device from the principal’s perspectives. This result is consistent with the empirical finding in Weisbach (1988), who finds that intense monitoring by independent boards increases an association between past performance and turnover. The fundamental mechanism in my paper is different, in that costly earnings management works as an alternative punishment and substitutes for turnover. The wealth transfer cost implies that the principal might need to pay more compensation to induce the agent to engage in costly earnings management. In a single period model, the wealth transfer cost always dominates the incentive alignment benefit, and, therefore, earnings management always worsens the principal’s welfare. In a multi-period model, where the principal can motivate the agent through the threat of dismissal, however, the incentive alignment benefit can dominate the wealth transfer cost. More specifically, under moderate control systems that give managers proper discretion in reporting earnings, the incentive alignment benefit can dominates, and, hence, earnings management can improve the principal’s payoff. Thus, which of these two affects dominates determines whether earnings management is beneficial or harmful to the principal.

The paper develops empirical predictions, in that it offers a new explanation for the low forced turnover rate and the low pay-performance relation. Taking into account earnings management, the low forced turnover rate can be better explained as resulting from a substitution of earnings management for turnover following poor actual performance of firms. The low pay-performance relation also can be understood through a disciplining effect of earnings management.

The paper makes related policy contributions. The findings in the paper suggest that standard setters need to carefully take into account the impact of earnings management on the flexibility of contracting as well as on market efficiency. In response to accounting scandals, the Securities and Exchange Commission has been trying to implement tighter control systems to protect participants in the market from receiving misleading information from companies. There is, however, another influence of earnings management through an optimal contract between shareholders and managers. If earnings management can be used to provide the agent with incentives to work harder, it can enhance investors’ welfare in the end, too. I do not
mean to insist that earnings management is all good but instead only that earnings management following performance that would otherwise result in managerial turnover may have a bright side.

The rest of paper is organized as follows. In section 3.2, I review the current literature on earnings management. Section 3.3 develops the two-period agency model and derive the optimal contract between shareholders and the manager including the equilibrium reporting strategy by the agent. Section 1.4 discuss the key trade-off of earnings management and its implications. Section 3.6 concludes.

1.2 Literature Review

Research on executive compensation has been rooted in agency theory (Mirrlees (1974, 1976), Holmstrom (1979) and Grossman and Hart (1983)). Jensen and Murphy (1990) empirically show that pay-performance relation and turnover-performance relation are low. To understand Jensen and Murphy puzzle, Haubrich (1994) studies Jensen and Murphy puzzle using a static agency model and can yield quantitative solutions in line with the empirical observations of Jensen and Murphy. Haubrich, however, predicts zero pay-performance sensitivities for plausible values of the coefficient of constant absolute risk aversion and the CEO takes the least level of effort. Even though small but non-zero pay-performance sensitivities are predicted for lower values of the coefficient of absolute risk aversion, Haubrich needs negative compensation to generate sufficient incentives for the CEO. Using a dynamic agency model, Wang (1997) shows that there exists a compensation rigidity and therefore the CEO’s current compensation can be independent of the firm’s current performance. Wang, however, does not take into account the dismissal of the agent. In contrast, this paper tries to understand the low forced turnover rate and pay-performance sensitivity through the effects of earnings management.

Academic research provides conflicting results regarding the effects of earnings management. Many prior papers show that earnings management can harm managers’ effort incentives and result in a deadweight loss in a contracting setting (Crocker & Slemrod, 2008; Marinovic, Beyer, & Guttmann, 2014). There is, however, another stream of research showing a bright side of earnings management. Arya, Glover, and
Sunder (1998) find that earnings management can make the principal better off in a limited commitment setting by letting the principal to commit to delaying the intervention. In contrast, my paper studies costly earnings management and dynamic moral hazard problems, neither of which are discussed by Arya, Glover, and Sunder.

Prior literature in accounting has not paid much attention to the dynamic setting and, therefore, misses many interesting questions, including how the reversal of accruals influences a firm’s important decisions. This paper studies the effect of earnings management in the presence of managerial turnover using dynamic contract theory.

### 1.3 Model

In this section, I describe the basic framework and present the optimal contract between the principal and the agent, which characterizes the effect of earnings management in a dynamic principal-agent problem. In the model, a risk-neutral principal (shareholders) of the firm hires a risk-neutral agent (a manager) to operate the business. The manager makes a choice of productive effort, which affects earnings of the firm, and manages earnings. I discuss a two-period model to examine the basic intuition about the effect of earnings management on the optimal contract, including a managerial turnover policy. I first introduce the agency problem between shareholders and the manager, and then the optimal contracting problem is specified.

#### 1.3.1 Agency Problem

The agent makes two unobservable actions: productive effort \( a_t \) and earnings management \( b_t \). The agent’s productive effort \( a_t \) affects earnings of the firm, \( e_t \) as follows:

\[
e_t = a_t \bar{e} + \epsilon_t \quad \text{for} \quad t \in \{1, 2\},
\]  

(1.1)
where $E[e_t|a_t] = 0$ and $\bar{e}$ is the productivity parameter. Earnings $e_t$ are continuously distributed according to a probability density function (PDF) $f(e_t|a_t)$ and a cumulative density function (CDF) $F(e_t|a_t)$. I assume that the strict monotone likelihood ratio property (MLRP) holds: for all earnings $e_H$ and $e_L$ with $e_H > e_L$,

$$\frac{f(e_H|a_H)}{f(e_H|a_L)} > \frac{f(e_L|a_H)}{f(e_L|a_L)}$$  \hfill (1.2)

MLRP implies that the distribution of earnings is ordered according to strict first-order stochastic dominance (FOSD): $F(e|a_L) > F(e|a_H)$ for $\forall e$.

The expected earnings are dependent on the agent’s binary action $a_t \in \{a_L, a_H\}$. In other words, the agent can either work, $a_t = a_H$, or shirk, $a_t = a_L$. The cost of effort $a_H$ is $h_t$ and the cost of effort $a_L$ is normalized to 0.

The agent privately observes earnings $e_t$ and reports earnings ($r_t$) with bias $b_t$. The cost of earnings management is $G(m_t + b_t) = \frac{1}{2}c(m_t + b_t)^2$, where $m_t$ is the accumulated discretionary accruals at the beginning of time $t$. This explains well the reversal of discretionary accruals in subsequent periods. Because the cost of earnings management is convex in the accumulated discretionary accruals, the agent has more incentives to reverse earnings management made in the previous periods if she manipulated earnings aggressively.

In the two-period model, I assume that earnings management made in period 1 ($b_1 = b$) is reversed in period 2 ($b_2 = -b$). Thus the cost of earnings management in period 1 and 2 are $\frac{1}{2}cb_1^2$ and 0, respectively, assuming the initial accumulated discretionary accruals $m_1$ are 0. Therefore, earnings reports in period 1 and 2 are

$$r_1 = e_1 + b$$  \hfill (1.3)

$$r_2 = e_2 - b$$  \hfill (1.4)

The agent has no initial wealth and has limited liability, which rules out a negative wage. And the agent
has a higher discount rate than the principal, $\rho_A > \rho_P$, which means that the manager is more impatient than the shareholders. The agent’s reservation utility is normalized to zero, which is related to her outside opportunity if the contract is terminated and the agent is fired. The termination of contract involves replacing the initial agent with a new agent. Figure 3.1 shows the timeline.

### 1.3.2 Optimal Contracting Problem

In a multi-period contract, the principal can motivate the agent with short-term and long-term incentives, including the threat of dismissal.\(^3\) And these incentives determine the return to the agent as follows:

$$
\begin{align*}
    s_1(e_1) + \frac{1}{1 + \rho_A} [s_2(e_1, e_2) - h_2] 1_{[TD(e_1) = 0]} - h_1 \\
    = w(s_1(e_1), s_2(e_1, e_2), r_1(e_1))
\end{align*}
$$

where $w(s_1(e_1), s_2(e_1, e_2), r_1(e_1))$ is the agent’s expected payoff after the cost of effort in period 1, $h_1$, is sunk, $s_1(e_1)$ is the compensation in period 1, $s_2(e_1, e_2)$ is the compensation in period 2, and $TD(e_1) = \{0, 1\}$ is a termination decision such that 1 indicates the termination of contract and 0 indicates a continuation of contract.

I assume that earnings report $r_t$ is observable and contractible. Because the compensation contract is

---

\(^3\)For instance, short-term compensation such as a bonus is provided as a short-term incentive. In addition, long-term compensation, such as deferred compensation, serves as a long-term incentive. The principal uses long-term incentives optimally to smooth the cost of compensation over periods because long-term incentives can make the agent work harder, not only in the current period, but also in the future, and therefore help to reduce short-term incentives.
implemented after the agent observes the private information, the revelation principle is applicable (Myerson, 1979). Therefore, the contract is conditional on the private information \((e_1)\) and must satisfy the incentive constraint

\[
E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1))|e_1] \geq E[w(s_1(\hat{e}_1), s_2(\hat{e}_1, e_2), r_1(\hat{e}_1))|e_1] \quad \text{for } \forall e_1
\] (1.6)

When this incentive compatible contract is offered, the agent who has \(e_1\) would prefer the contract \(\{s_1(e_1), s_2(e_1, e_2), r_1(e_1)\}\) over the alternatives \(\{s_1(\hat{e}_1), s_2(\hat{e}_1, e_2), r_1(\hat{e}_1)\}\) for every \(\hat{e}_1 \neq e_1\).

At the beginning of period 1, the principal offers a contract that specifies the agent’s compensation, \(s_1(e_1)\) and \(s_2(e_1, e_2)\), earnings report, \(r_1(e_1)\), and a termination decision, \(TD(e_1)\). The termination decision \(TD(e_1)\) defines the termination threshold \(\hat{y}\) such that the agent is fired and replaced if \(e_1 \leq \hat{y}\), i.e., \(s_2(e_1, e_2) = 0\) if \(e_1 \leq \hat{y}\). If the agent is replaced, the principal offers a contract to a new (identical) agent that specifies the agent’s compensation \(s_{new}^2(e_{new}^2)\) depending on the earnings in period 2, \(e_{new}^2\).

The principal chooses \(s_1(e_1), s_2(e_1, e_2), r_1(e_1),\) and \(TD(e_1)\) to maximize the following problem:

\[
\max_{s_1, s_2, r_1, TD} \Pi = E[e_1 - s_1(e_1) + \frac{1}{1 + \rho \mu} [(e_2 - s_2(e_1, e_2))|_{TD(e_1) = 0} + (e_{new}^2 - s_{new}^2(e_{new}^2))|_{TD(e_1) = 1}]
\] (1.7)

---

4In a direct revelation mechanism, the agent sends a message \(e_1\) in the first stage, and the incentive compatible contract, which makes the agent reveal her private information truthfully, is assigned (see Crocker & Slemrod 2007).

5Spear and Wang (2005) show the optimal termination in the two-period model. They show that the CEO is also forced out after good performance because she becomes too expensive to motivate under the assumption that the CEO is risk-averse.
\[
\text{s.t. } E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) | a_H] - E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) | a_L] \geq h_1 \
E[s_2(e_1, e_2) | a_H] - E[s_2(e_1, e_2) | a_L] \geq h_2 \\
E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) | a_H] \geq h_1 \\
E[s_2(e_1, e_2) | a_H] \geq h_2 \\
E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) | a_H] - E[w(s_1(\hat{e}_1), s_2(\hat{e}_1, e_2), r_1(\hat{e}_1)) | e_1] \geq 0 \\
s_1(e_1 + \eta) + \frac{1}{1 + \rho_a} E[s_2(e_1 + \eta, e_2)] - [s_1(e_1) + \frac{1}{1 + \rho_a} E[s_2(e_1, e_2)]] \leq \eta \text{ for } \forall \eta > 0 \\
s_2(e_1, e_2 + \eta) - s_2(e_1, e_2) \leq \eta \text{ for } \forall \eta > 0 \\
s_1(e_1), s_1(e_1, e_2) \geq 0
\]

The principal maximizes the expected earnings net of compensation in the equation (2.14). The equations (2.15) and (2.16) are the incentive compatibility constraints for the agent in period 1 and 2, respectively. The equations (2.17) and (2.18) are the individual rationality constraints for the agent in period 1 and 2, respectively. The equation (2.19) is the incentive compatibility constraint for truth-telling and the equations (2.20) and (2.21) are the monotonicity constraints in period 1 and 2, respectively. The monotonicity constraints mean that the principal’s payoff is non-decreasing in earnings $e_t$ in each period.\(^6\) Lastly, the equation (2.22) is the agent’s limited liability constraints.

In the case of turnover, the principal signs a contract with a new agent. The principal chooses $s_{2\text{New}}(e_{2\text{New}})$ to solve the following problem:

\[
\text{Max } s_{2\text{New}}(e_{2\text{New}}) - s_{2\text{New}}(e_{2\text{New}})
\]

\(^6\)Without the monotonicity constraint, the optimal contract of the “live-or-die” form may achieve a “first-best” effort choice (see Innes, 1990).
\begin{align}
\text{s.t. } & E[s^2_{New}(e^2_{New})|a_H] - E[s^2_{New}(e^2_{New})|a_L] \geq H_2 \quad (1.17) \\
& E[s^2_{New}(e^2_{New})|a_H] \geq H_2 \quad (1.18) \\
& s^2_{New}(e^2_{New} + \eta) - s^2_{New}(e^2_{New}) \leq \eta \text{ for } \forall \eta > 0 \quad (1.19) \\
& s^2_{New}(e^2_{New}) \geq 0 \quad (1.20)
\end{align}

The equations (2.24), (2.25), (2.26), and (2.27) are the incentive compatibility constraint, the individual rationality constraint, the monotonicity constraint, and the limited liability constraint, respectively.

### 1.3.3 Model Solution and Optimal Contracting

#### 1.3.3.1 Benchmark

I first consider the benchmark case where the agent’s action is observable. The agent makes a choice of high effort \( a_H \) in both periods and no earnings management is induced (\( b = 0 \)). The principal’s expected payoff is

\[ E[e_1 - h_1 + \frac{1}{1 + \rho_p}[e_2 - h_2]] \quad (1.21) \]

In the benchmark case in which there is no agency concern, the idiosyncratic productivity shock does not have any influence on firm value \( \text{ex ante} \). The next section will show that agency problem will alter these results significantly.

#### 1.3.3.2 Optimal Contract

Now I derive the optimal contract under the condition in which the principal observes earnings reports only, and, thus, the agent has an incentive to manage earnings. I sketch how to derive the optimal contract in this section, and the proof is provided in the Appendix A.

I first derive the optimal compensation in period 2, \( s_2(e_1, e_2) \), in the case of no replacement of the
agent \((TD(e_1) = 0)\). Given the earnings \(e_1\), the principal determines \(s_2(e_1, e_2)\), satisfying the following constraints:

\[
\begin{align*}
\text{s.t. } & E[s_2(e_1, e_2) | a_H] - E[s_2(e_1, e_2) | a_L] \geq h_2 \quad (1.22) \\
& E[s_2(e_1, e_2) | a_H] \geq h_2 \quad (1.23) \\
& s_2(e_1, e_2 + \eta) - s_2(e_1, e_2) \leq \eta \text{ for } \forall \eta > 0 \quad (1.24) \\
& s_2(e_2) \geq 0 \quad (1.25)
\end{align*}
\]

As stated by Innes (1990) and Chaigneau, Edmans, and Gottlieb (2015), the optimal contract is a form of a call option on \(e_2\). The intuition is that the value of information on \(e_2\) is largest in the tails of the distribution of \(e_2\). In other words, \(e_2\) is the most informative about effort in the tail. Because of the limited liability, however, incentives cannot be provided in the left tail, and, thus, only the right tail of \(e_2\) is used for providing incentives. Also, a call option contract satisfies the monotonicity constraint. Therefore, the optimal contract is a form of a call option on \(e_2\) when \(TD(e_1) = 0\).

\[
s_2(e_1, e_2) = \text{Max}[0, e_2 - y_2] \text{ for } e_1 > \hat{y} \quad (1.26)
\]

where \(y_2\) is the unique solution of

\[
\int_{y_2}^{\infty} (e_2 - y_2) [f(e_2 | a_H) - f(e_2 | a_L)] de_2 = h_2 \quad (1.27)
\]

The above equation (2.50) comes from the incentive constraint (2.45), implying that \(y_2\) is the strike price, such that the increased value of the option by high effort \(a_H\) is equal to the cost of effort, \(h_2\).

The optimal contract for the new agent in period 2, \(s_2^{New}(e_2^{New})\), can be derived in the same manner.
Therefore, the optimal contract for the new agent is the same as the one for the initial agent.

\[ s_2^{New}(e_2^{New}) = s_2(e_1, e_2) \text{ for } e_1 > \hat{y} \]  

(1.28)

Now I turn to the principal’s problem to decide \( s_1(e_1) \), satisfying the following constraints:

\[
\begin{align*}
\text{s.t. } E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) | a_H] & - E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) | a_L] \geq h_1 \\
E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) | a_H] & \geq h_1 \\
E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) | e_1] & - E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) | e_1] \geq 0 \\
s_1(e_1 + \eta) + \frac{1}{1 + \rho_A} E[s_2(e_1 + \eta, e_2)] - [s_1(e_1) + \frac{1}{1 + \rho_A} E[s_2(e_1, e_2)]] \leq \eta \text{ for } \forall \eta > 0 \\
s_1(e_1) & \geq 0
\end{align*}
\]  

(1.29) - (1.33)

Similarly, in the optimal contract, \( w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) \) is a form of a call option on \( e_1 \). Because the principal can motivate the agent with long-term incentives as well in a dynamic contract, the agent’s expected payoff over two-periods–after the cost of effort is sunk–should give the agent enough incentives, and all the incentives are concentrated in the right tail of \( e_2 \).

\[
w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) = s_1(e_1) - g(r_1(e_1) - e_1) + \frac{1}{1 + \rho_A} E[(s_2(e_1, e_2) - h_2) 1_{TD(e_1) = 0}] \\
= \text{Max}[0, e_1 - y_1]
\]  

(1.34) - (1.35)

where \( y_2 \) is the unique solution of

\[
\int_{y_1}^{\infty} (e_1 - y_1)[f(e_1 | a_H) - f(e_1 | a_L)]de_1 = h_1
\]  

(1.36)

For the contract to be incentive compatible with truth-telling (2.54), we need to understand the agent’s
incentive to manage earnings.

\[
\begin{align*}
\max_{r_1(e_1)} E[w(s_1(e_1), s_2(e_1, e_2), r_1(e_1))|e_1] &= s_1(e_1) - g(r_1(e_1) - e_1) + \frac{1}{1 + \rho_A} E[[s_2(e_1, e_2) - h_2]_{TD(e_1)=0}] \\
\end{align*}
\]

(1.37)

Considering that the contract is a form of a call option and \(e_1 = e_1\) in the optimal contract, the optimal earnings report is

\[
\begin{align*}
r_1(e_1) &= \begin{cases} 
  e_1 + \frac{1}{c} & \text{if } e_1 > y_1 \\
  e_1 & \text{if } e_1 \leq y_1 
\end{cases} \quad (1.38)
\end{align*}
\]

Thus the optimal earnings management is

\[
\begin{align*}
b(e_1) &= \begin{cases} 
  \frac{1}{c} & \text{if } e_1 > y_1 \\
  0 & \text{if } e_1 \leq y_1 
\end{cases} \quad (1.39)
\end{align*}
\]

Before searching for the threshold for dismissal, \(\hat{y}\), let \(w^*\) be the agent’s continuation payoff for period 2 given \(e_1 > \hat{y}\).

\[
w^* = \frac{1}{1 + \rho_A} E[s_2(e_1, e_2) - h_2|a_H] \\
= \frac{1}{1 + \rho_A} E[\int_{y_2}^{\infty} (e_2 - y_2)f(e_2|a_H)de_2 - h_2] \quad (1.41)
\]

Then

\[
w(s_1(e_1), s_2(e_1, e_2), r_1(e_1), i_1) = \max[0, e_1 - y_1] = s_1(e_1) - g(r_1(e_1) - e_1) + w^* \quad (1.42)
\]

A call option form of \(w(s_1(e_1), s_2(e_1, e_2), r_1(e_1))\) implies that the agent’s expected payoff over two-periods is determined based on \(e_1\). In other words, incentives provided in period 2 spill back to period 1,
and, thus, the agent is incentivized to exert a high effort in period 1 to enjoy not only the compensation in period 1 but also the compensation in period 2 (Glover and Haijin, 2015). Therefore, if the realized $e_1$ is high enough, the agent consumes utilities in both periods 1 and 2. If the realized $e_1$ is low and the principal cannot promise $w^*$ to incentivize the agent in period 2, however, the agent is fired. In the case of dismissal, the agent may consume the utility in period 1, depending on the value of $e_1$. In sum, there exists the threshold for dismissal, $\hat{y}$, such that the agent is fired if $e_1 \leq \hat{y} = y_1 - \frac{1}{2c} + w^*$. When $e_1$ just beats $\hat{y}$, the agent’s utility in period 1 is $-\frac{1}{2c}$ due to the cost of earnings management, and the agent’s expected utility in period 2 is $w^*$. If $e_1$ is between $y_1$ and $\hat{y}$, the agent is fired but consumes the utility in period 1. If $e_1$ is below $\hat{y}$, the agent is fired without any utility. Then the compensation can be computed considering the utility and the cost of earnings management. This can be interpreted to mean that if the principal has to promise a higher continuation payoff for period 2, the principal’s ability to punish the agent in the state of low $x_1$ is weakened. Therefore, the principal should rely more on the threat of dismissal to motivate the agent (Spear and Wang, 2005).

Proposition 1 summarizes the contract.

**Proposition 1.** There exists an optimal contract with

$$s_1(e_1) = \begin{cases} e_1 - \hat{y} & \text{if } e_1 \geq \hat{y} \\ e_1 - y_1 + \frac{1}{2c} & \text{if } y_1 < e_1 < \hat{y} \\ 0 & \text{if } e_1 \leq y_1 \end{cases}$$ (1.43)

$$s_2(e_2) = \max[0, e_2 - y_2]$$ (1.44)

$$s_{New}^2(e_{New}^2) = \max[0, e_{New}^2 - y_{New}^2]$$ (1.45)
where $y_1$ and $y_2$ are the unique solution of

\[ \int_{y_1}^{\infty} (e_1-y_1)[f(e_1|a_H)-f(e_1|a_L)]de_1 = h_1 \]  \hfill (1.46)
\[ \int_{y_2}^{\infty} (e_2-y_2)[f(e_2|a_H)-f(e_2|a_L)]de_2 = h_2 \]  \hfill (1.47)

and $\hat{y}$ is the value $e_1$ such that

\[ e_1-y_1 = \frac{1}{1+\rho_A} E[s_2(e_1, e_2) - h_2] - \frac{1}{2c} \]  \hfill (1.48)

Therefore, the optimal contract satisfies

\[ w(s_1(e_1), s_2(e_1, e_2), r_1(e_1)) = \text{Max}[0, e_1-y_1] \]  \hfill (1.49)

The agent’s optimal earnings report and earnings management are

\[ r_1(e_1) = \begin{cases} e_1 + \frac{1}{c} & \text{if } e_1 > y_1 \\ e_1 & \text{if } e_1 \leq y_1 \end{cases} \]  \hfill (1.50)
\[ b(e_1) = \begin{cases} \frac{1}{c} & \text{if } e_1 > y_1 \\ 0 & \text{if } e_1 \leq y_1 \end{cases} \]  \hfill (1.51)

The optimal contract can be represented on earnings report, $r_1$, as Corollary 1 states.
Corollary 1. The optimal contract on earnings report $r_t$ is

$$s_1(r_1) = \begin{cases} 
    r_1 - \hat{y} - \frac{1}{c} & \text{if } r_1 \geq \hat{y} + \frac{1}{c} \\
    r_1 - y_1 & \text{if } y_1 + \frac{1}{2c} < r_1 < \hat{y} + \frac{1}{c} \\
    0 & \text{if } r_1 \leq y_1 + \frac{1}{2c}
\end{cases}$$

(1.52)

$$s_2(r_2) = \text{Max}[0, r_2 - y_2 + \frac{1}{c}]$$

(1.53)

$$s_2^{New}(r_2^{New}) = \text{Max}[0, r_2^{New} - y_2^{New} + \frac{1}{c}]$$

(1.54)

Therefore, the optimal contract satisfies

$$w(s_1(r_1), s_2(r_1, r_2)) = \text{Max}[0, r_1 - y_1 - \frac{1}{2c}]$$

(1.55)

Figure 2.2 shows earnings report $r_1$, earnings management $b$, compensation $s_1$, and utility $u_1$ in period 1 on $e_1$. Let the utility $u_1$ be the compensation $s_1$ net of the cost of earnings management, $\frac{1}{2}cb^2$. There exists a discontinuity of reported earnings $r_1$ at $e_1 = y_1$, because the agent over-reports only when $e_1 > y_1$. The agent is fired but receives the compensation in period 1, $s_1$, if $e_1 \leq \hat{y}$. At $e_1 = \hat{y}$, the compensation $s_1$ drops by the future incentives (the utility in period 2), $w^* = \frac{1}{1+\rho^2}E[s_2(e_1, e_2) - h_2]$, because the principal can compensate the agent with the future incentives $w^*$ if $e_1 > \hat{y}$. Therefore, the dismissal of the agent is an ex-ante incentive device because the agent can enjoy the future incentives only when the performance in period 1 is relatively higher. Thus, the agent is motivated to work hard in period 1.

1.4 Implications

How does the internal control system of the firm affect the principal’s expected payoff and the investment decisions? The internal control system means how much discretion the agent has in reporting the earnings.
Under a stronger internal control system, for instance, the agent has less discretion in reporting earnings. In this paper, the cost parameter of earnings management, $c$, captures the property of the internal control system. Therefore, the internal control system becomes stronger as $c$ increases.

To understand the effect of the internal control system $c$ on the contract, it is necessary to understand how the strike price $y_1$ changes as $c$ varies. $y_1$ does not change because the principal can infer the amount of earnings management in equilibrium, and, thus, incentives for $a_H$ remain the same. In other words, without any changes in $y_1$, $c$ affects the amount of compensation for each $x_1$ due to additional cost of earnings management. Another parameter implies the severity of the agency problem: the cost of effort $h_1$. $y_1$ strictly decreases with the cost of effort $h_1$ because it is hard to motivate $a_H$. Proposition 2 summarizes the effect of the internal control system $c$ and the cost of effort $h_1$ on the strike price $y_1$.

**Proposition 2.** The strike price $y_1$ on $x_1$ is

(a) independent of $c$ ($\frac{dy_1}{dc} = 0$);

(b) decreasing in $h_1$ ($\frac{dy_1}{dh_1} < 0$)

It is intuitive that the strike price $y_1 + \frac{1}{2c}$ on $r_1$ is strictly decreasing in $c$ because the agent would over-report less as $c$ increases.
Corollary 2. The strike price $y + \frac{1}{2c} \ln r_1$ is strictly decreasing in $c$ ($\frac{d(y + \frac{1}{2c} \ln r_1)}{dc} < 0$).

Then what is the impact of earnings management on the dismissal threshold $\hat{y}$? Note that earnings management, influenced by $c$, has two effects: incentive alignment benefit and wealth transfer cost. Figure 2.2 shows that the agent experiences a negative utility in period 1 from $\hat{y}$ to $\hat{y} + \frac{1}{2c}$ because of the cost of earnings management, which relaxes the agent’s bankruptcy constraint. Therefore, the principal can loosen the threat of dismissal, as the principal can penalize the agent with the cost of earnings management ($\frac{d\hat{y}}{dc} > 0$). That is, the cost of earnings management can be a substitute for the threat of dismissal, leading to an increase in long-term incentives (incentive alignment benefit). Figure 1.3 shows how earnings management affects $\hat{y}$ by comparing with the case of no earnings management ($c \rightarrow \infty$).

The incentive alignment benefit has an influence on the turnover threshold $\hat{y}$. Proposition 3 states the effect of $c$ on the turnover threshold $\hat{y}$.

Proposition 3. As $c$ increases, the threshold for dismissal $\hat{y}$ increases ($\frac{d\hat{y}}{dc} > 0$).

How does the internal control system $c$ have an influence on the principal’s expected payoff? The trade-off between the incentive alignment benefit and the wealth transfer cost determines the consequences of earnings management on the principal’s welfare. First, through the incentive alignment benefit, the principal can lower the probability of turnover and increase long-term incentives, which, in turn, decreases the compensation in period 1 (short-term incentives). In other words, an increase in long-term incentives by penalizing the agent for poor performance through earnings management helps to lower short-term incentives. Second, the wealth transfer cost can increase the cost of compensation because the principal may need to compensate for the cost of earnings management. Therefore, earnings management can be a cheaper incentive device and improve the principal’s welfare if the total gain from the incentive alignment
Figure 1.3: Threshold for Dismissal: No earnings management v.s. Earnings management
benefit dominates the total losses from the wealth transfer cost.

\[
\frac{d\pi}{dc} = -E\left[\frac{s_2(x_1, x_2) - h_2}{1 + \rho_a} \left| a_H \right| f(\hat{y}) | a_H \right] \frac{1}{2c^2} + \left[1 - F(y_1 | a_H)\right] \frac{1}{2c^2} \tag{1.56}
\]

\( \text{effect on short-term incentives} \quad \text{effect on compensation} \quad \text{due to Cost of EM} \)

\( \text{through long-term incentives} \)

Proposition 4 formulates the condition under which the principal’s expected payoff increases as \( c \) decreases.

**Proposition 4.** The expected compensation is strictly decreasing in \( c \). The principal’s expected payoff increases as \( c \) decreases (more earnings management is induced) if and only if

\[
E\left[\frac{s_2(x_1, x_2) - h_2}{1 + \rho_a} \left| a_H \right| f(\hat{y}) | a_H \right] > 1 - F(y_1 | a_H) \tag{1.57}
\]

When does the incentive alignment benefit dominate the wealth transfer cost as \( c \) decreases? The incentive alignment benefit can dominate when long-term incentives are low. In contrast, the wealth transfer cost can dominate when long-term incentives are high. The intuition is that the benefit of increasing long-term incentives decreases as long-term incentives increases. Note that long-term incentives are low when \( \hat{y} \) is large and the probability of turnover is high. Therefore, the incentive alignment benefit dominates when when \( \hat{y} \) is large. Corollary 3 summarizes how \( \hat{y} \) affect the trade-off between the incentive alignment benefit and the wealth transfer cost.

**Corollary 3.** The principal’s expected payoff increases as \( c \) decreases if the probability of turnover is relatively high (\( \hat{y} \) is large). Therefore, there exist \( \bar{y} \), such that

\[
\frac{d\pi}{dc} < 0 \quad \text{if} \quad \hat{y} > \bar{y} \tag{1.58}
\]
The internal control system $c$ also influences the trade-off, because $c$ increases $\hat{y}$ by Proposition 3. Thus, the incentive alignment benefit dominates when $c$ is large. Therefore, Corollary 3 implies that a decrease in $c$ increases the principal’s expected payoff when $c$ is large.

1.5 Conclusion

In response to accounting scandals, for instance, including Enron and WorldCom, control systems have become more stringent to mitigate earnings management and to improve financial reporting quality. Critics, however, question whether tightening control systems strike the right balance between its benefits and costs. To elaborate, this paper examines the effect of earnings management in a dynamic contracting setting. The principal can design an optimal contract such that it is incentive compatible for the agent to engage in costly earnings management in order to avoid being dismissed when her performance is poor. Because earnings management can impose a cost on the agent that cannot be replicated by compensation, it effectively relaxes the agent’s bankruptcy constraint. Thus, the agent can be motivated to work harder to avoid costly earnings management which can be regarded as punishment for poor performance. The principal, however, might need to pay the agent more to compensate for the cost of earnings management. Therefore, the trade-off between the incentive alignment benefit and the wealth transfer cost determines the consequences of earnings management.

The model emphasizes the importance of control systems when evaluating the effect of earnings management. When control systems are very tight, relaxing control systems and giving more flexibility can reduce agency problems and, thus, the firm value. From this perspective, the paper offers a new explanation for why CEOs are seldom fired and why their pay-performance sensitivities are too low to be consistent with the agency model. Taking into account earnings management, the low forced turnover rate can be better explained as a substitution of earnings management for turnover. Also, the low pay-performance sensitivities can be understood through a discipling effect of earnings management.
The paper provides empirical implications, in that the effect of earnings management should be considered in measuring the forced turnover rate and pay-performance sensitivities. As the paper suggests, the tightness of control systems alters the net effect of earnings management and, therefore, empirical research can be benefited by controlling for the tightness of control systems.
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1.6 Appendix

Proof of Proposition 1

Proof of Proposition 1 is based on the proof of Lemma 1 in Chaigneau, Edmans, and Gottlieb (2015). Given that the agent managed earnings per unit of capital by $b$ in period 1, the principal’s problem in period 2 is

$$\begin{align*}
\text{Min} \int s_2(e_2) f(e_2 | a_H) de_2 \\
s.t. \int s_2(e_2)[f(e_2 | a_H) - f(e_2 | a_L)] de_2 \geq h_2 \\
 s_2(e_2 + \eta) - s_2(e_2) \leq \eta \text{ for } \forall \eta > 0 \\
 s_2(e_2) \geq 0
\end{align*}$$

(1.59)

Suppose there exists a contract $\hat{s}_2$ that satisfies the monotonicity constraint, the limited liability, and IC but is not an option contract. Without loss of generality, suppose IC holds with equality because it cannot be optimal otherwise.

$$\int \hat{s}_2(e_2)[f(e_2 | a_H) - f(e_2 | a_L)] de_2 = h_2$$

(1.63)

For any such alternative contract, there exists a unique option contract with the same expected payoff. That is,

$$s^*_2(e_2) f(e_2 | a_H) de_2 = \int \hat{s}_2(e_2) f(e_2 | a_H) de_2$$

(1.64)

Let’s say the option contract has an exercise price of $T$.

$$\int s^*_2(e_2) f(e_2 | a_H) de_2 = \int_{T} (e_2 - T) f(e_2 | a_H) de_2$$

(1.65)
Thus,
\[
\int_{T}^{\infty} (e_2 - T) f(e_2 | a_H) de_2 = \int_{-\infty}^{\infty} \delta_2(e_2) f(e_2 | a_H) de_2
\]  
(1.66)

Applying the Intermediate Value Theorem using the fact that (a). As \( T \to \infty \), \( LHS < RHS \). (b). As \( T \to -\infty \), \( LHS > RHS \). (c). As \( T \to -\infty \), \( \frac{\partial}{\partial T} \int_{T}^{\infty} (e_2 - T) f(e_2 | a_H) de_2 = -[1 - F(T | a_H)] < 0 \), there exists a unique solution \( T \) to the equation (2.128).

Let \( D(e_2) = \delta_2(e_2) - s^2_2(e_2) \) and then \( \int_{-\infty}^{\infty} D(e_2) f(e_2 | a_H) dx_2 = 0 \). Using the fact that \( s^2_2 \) is an option contract and \( \delta_2 \) satisfies the limited liability constraint and the monotonicity constraint, there exists \( \bar{e} \) such that \( D(e_2) \leq 0 \) for \( \forall e_2 > \bar{e} \) and \( D(e_2) \geq 0 \) for \( \forall e_2 < \bar{e} \). Then,

\[
\int_{-\infty}^{\infty} D(e_2) f(e_2 | a_L) de_2 = \int_{-\infty}^{\infty} D(e_2) \frac{f(e_2 | a_L)}{f(e_2 | a_H)} f(e_2 | a_H) de_2
\]  
(1.67)

\[
= \int_{-\infty}^{k} D(e_2) \frac{f(e_2 | a_L)}{f(e_2 | a_H)} f(e_2 | a_H) de_2 + \int_{k}^{\infty} D(e_2) \frac{f(e_2 | a_L)}{f(e_2 | a_H)} f(e_2 | a_H) de_2
\]  
(1.68)

\[
> \int_{-\infty}^{k} D(e_2) \frac{f(e_2 | a_L)}{f(e_2 | a_H)} f(e_2 | a_H) de_2 + \int_{k}^{\infty} D(e_2) \frac{f(e_2 | a_L)}{f(e_2 | a_H)} f(e_2 | a_H) de_2
\]  
(1.69)

\[
= \frac{f(e_2 | a_H)}{f(e_2 | a_L)} \int_{-\infty}^{\infty} D(e_2) f(e_2 | a_H) de_2
\]  
(1.70)

\[
= 0
\]  
(1.71)

The inequality in the third line in the above equation comes from MLRP, \( \frac{f(e_H | a_H)}{f(e_H | a_L)} > \frac{f(e_L | a_H)}{f(e_L | a_L)} \) for \( e_H > e_L \). Therefore,

\[
\int_{-\infty}^{\infty} \delta_2(e_2) f(e_2 | a_L) de_2 > \int_{-\infty}^{\infty} s^2_2(e_2) f(e_2 | a_L) de_2
\]  
(1.72)
This leads to the contradiction that IC does not bind:

\[
\int_{-\infty}^{\infty} s_{2}^{2}(e_2)f(e_2|a_H)de_2 = \int_{-\infty}^{\infty} s_{2}^{2}(e_2)f(e_2|a_H)de_2
\]

\[
= \int_{-\infty}^{\infty} s_{2}^{2}(e_2)f(e_2|a_L)de_2 + h_2
\]

\[
> \int_{-\infty}^{\infty} s_{2}^{2}(e_2)f(e_2|a_L)de_2 + h_2
\]

Therefore, there exists a new contract \( s_{2}^{+} \) with a higher exercise price \( T^{+} \), which has a lower expected payoff and remains incentive compatible:

\[
\int_{-\infty}^{\infty} s_{2}^{+}(e_2)f(e_2|e_H)de_2 < \int_{-\infty}^{\infty} s_{2}^{2}(e_2)f(e_2|e_H)de_2 = \int_{-\infty}^{\infty} s_{2}^{2}(e_2)f(e_2|e_H)de_2
\]

This new option contract \( s_{2}^{+} \) satisfies IC, the monotonicity constraint, the limited liability constraint, and has a lower expected payoff than the initial non-option contract \( s_{2} \), which is contradiction. Hence, the optimal contract is an option contract with an exercise price \( T \), which is the unique solution of

\[
\int_T (e_2 - T)[f(e_2|e_H) - f(e_2|e_L)]dx = h_2
\]

Let \( T = y_2 \). Then

\[
s_{2}(e_2) = Max[0, e_2 - y_2]
\]

On \( r_2 \),

\[
s_{2}(r_2) = Max[0, r_2 - y_2 + b]
\]

To search for the optimal contract, I start with the conjecture that there exists an optimal contract.
satisfying

\[ w(s_1(e_1), s_2(e_1, e_2), b(e_1)) = s_1(e_1) - g(b(e_1)) + \frac{1}{1 + \rho_A} E[(s_2(e_1, e_2) - h_2) | T D(e_1) = 0] \]  

\[ = \text{Max}[0, e_1 - y_1] \]  

(1.80)

(1.81)

where \( w(s_1(e_1), s_2(e_1, e_2), b(e_1)) \) is the agent’s payoff excluding the cost of effort, \( h_1 \).

For the contract to be incentive compatible with earnings management (2.54), we need to understand the agent’s incentive to manage earnings.

\[ \text{Max}_{b(\hat{e}')} E[w(s_1(\hat{e}'), s_2(\hat{e}', e_2), b(\hat{e}')) | e_1] = s_1(\hat{e}) - g(b(\hat{e}')) + \frac{1}{1 + \rho_A} E[(s_2(\hat{e}', e_2) - h_2) | T D(\hat{e}') = 0] \]  

(1.82)

Considering that the contract is a form of a call option, \( b(\hat{e}') = \frac{1}{c} \). And in the optimal contract, \( \hat{e}' = e_1 \).

Therefore, the optimal earnings management is

\[ b(e_1) = \begin{cases} \frac{1}{c} & \text{if } e_1 > y_1 \\ 0 & \text{if } e_1 \leq y_1 \end{cases} \]  

(1.83)

Note that IC becomes

\[ \int_{-\infty}^{\infty} [w(s_1(e_1), s_2(e_1, e_2), b(e_1))] [f(e_1 | a_H) - f(e_1 | a_L)] de_1 \geq h_1 \]  

(1.84)

Therefore, in the optimal contract, \( w(s_1(e_1), s_2(e_1, e_2), b(e_1)) \) is an option contract on \( e_1 \) with the exercise price \( y_1 \) that is unique solution of

\[ \int_{y_1}^{\infty} (e_1 - y_1)[f(e_1 | a_H) - f(e_1 | a_L)] de_1 = h_1 \]  

(1.85)

The call option form of \( w(s_1(e_1), s_2(e_1, e_2), b(e_1)) \) implies that the replacement threshold \( \hat{y} \) is the value
of the scaled earnings $e_1$ such that $e_1 - y_1 = \frac{1}{1 + \rho_1} [s_2(e_1, e_2) - h_2] - \frac{1}{2c}$. Hence, the agent is replaced if $e_1 \leq \hat{y} = \frac{1}{1 + \rho_1} [s_2(e_1, e_2) - h_2] - \frac{1}{2c} + y_1$ because the principal cannot guarantee enough continuation payoff to incentivize the agent in period 2. In other words, incentives provided in period 2 spill back to period 1, and, thus, the agent is incentivized to exert a high effort in period 1 to enjoy not only the compensation in period 1 but also the compensation in period 2.

**Proof of Proposition 2**

The strike price $y_1$ satisfies the equation (2.70)

$$\int_{y_1}^{\infty} (e_1 - y_1) [f(e_1 | a_H) - f(e_1 | a_L)] de_1 = h_1$$

(1.86)

Therefore,

$$\frac{dy_1}{dc} = 0$$

(1.87)

Denote the lower and upper bound of the support of $e_1$ by $\underline{e}$ and $\bar{e}$, respectively. Then, the above equation becomes

$$\int_{y_1}^{\bar{e}} sf(e_1 | a_H) de_1 - \int_{y_1}^{\bar{e}} x_1 f(e_1 | a_L) de_1 - [F(y_1 | a_L) - F(y_1 | a_H)] y_1 = h_1$$

(1.88)

Using integration by parts,

$$\int_{y_1}^{\bar{e}} e_1 f(e_1 | a) de_1 = [e_1 F(e_1 | a) - \int F(e_1 | a) de_1]_{y_1}^{\bar{e}} = \bar{e} - y_1 F(y_1 | a) - \int_{y_1}^{\bar{e}} F(e_1 | a) de_1$$

(1.89)

Then, the equation (2.148) constraint becomes

$$\int_{y_1}^{\bar{e}} [F(e_1 | a_L) - F(e_1 | a_H)] de = h_1$$

(1.90)
Applying the implicit function theorem,

\[
\frac{dy_1}{dh_1} = -\frac{\partial g}{\partial h_1} = - \frac{1}{F (y_1 | a_L) - F (y_1 | a_H)} < 0 \quad (1.91)
\]

Q.E.D.

**Proof of Proposition 3**

\[
\frac{d\hat{y}}{dc} = \frac{\partial \hat{y}}{\partial c} = \frac{1}{2c^2} \quad (1.92)
\]

Q.E.D.

**Proof of Proposition 4**

The expected compensation is

\[
\begin{align*}
E [s_1 (e_1) + \frac{1}{1 + \rho_A} s_2 (e_1, e_2) 1_{|e_1 > \hat{y}|} | a_H] &= \int_{y_1}^{\infty} (e_1 - y_1 + \frac{1}{2c}) f (e_1 | a_H) de_1 + \int_{\hat{y}}^{\infty} \frac{1}{1 + \rho_A} h_2 f (e_1 | a_H) de_1 \\
&= \int_{y_1}^{\infty} (e_1 - y_1) f (e_1 | a_H) de_1 + \int_{\hat{y}}^{\infty} \frac{1}{2c} f (e_1 | a_H) de_1 \\
&+ \int_{\hat{y}}^{\infty} \frac{1}{1 + \rho_A} h_2 f (e_1 | a_H) de_1 \quad (1.93)
\end{align*}
\]

where \( \hat{y} = \frac{1}{1 + \rho_A} E [s_2 (e_1, e_2) - h_2] - \frac{1}{2c} + y_1. \)
Let the lower and upper bound of the support of $e_1$ by $\underline{e}$ and $\bar{e}$, respectively.

\[ \int_{y_1}^{\infty} (e_1 - y_1) f(e_1 | a_H) de_1 = \int_{y_1}^{\infty} e_1 f(e_1 | a_H) de_1 - y_1 [1 - F(y_1 | a_H)] \]  
\hspace{1cm} (1.95)

\[ = \int_{\underline{e}}^{\infty} e_1 f(e_1 | a_H) de_1 - y_1 [1 - F(y_1 | a_H)] - \int_{\underline{e}}^{y_1} e_1 f(e_1 | a_H) de_1 \]  
\hspace{1cm} (1.96)

\[ = \int_{\underline{e}}^{\bar{e}} e_1 f(e_1 | a_H) de_1 - y_1 [1 - F(y_1 | a_H)] - [e_1 F(e_1 | a_H) - \int F(e_1 | a_H) de_1]_{y_1} \]  
\hspace{1cm} (1.97)

\[ = E[e_1 | a_H] - y_1 + \int_{\underline{e}}^{\bar{e}} F(e_1 | a_H) de_1 \]  
\hspace{1cm} (1.98)

Therefore,

\[ E[s_1(e_1) + \frac{1}{1 + \rho_P} s_2(e_1, e_2) 1_{[e_1 > \hat{y}]} | a_H] = E[e_1 | a_H] - y_1 + \int_{\underline{e}}^{\bar{e}} F(e_1 | a_H) de_1 + \int_{y_1}^{\infty} \frac{1}{2c} f(e_1 | a_H) de_1 \]  
\hspace{1cm} + \int_{\underline{y}}^{\bar{y}} \frac{1}{1 + \rho_A} h_2 f(e_1 | a_H) de_1 \]  
\hspace{1cm} (1.99)

The effect of the marginal cost of earnings management is

\[ \frac{d}{dc} E[s_1(e_1) + \frac{1}{1 + \rho_A} s_2(e_1, e_2) 1_{[e_1 > \hat{y}]} | a_H] = \frac{\partial}{dc} E[s_1(e_1) + \frac{1}{1 + \rho_A} s_2(e_1, e_2) 1_{[e_1 > \hat{y}]} | a_H] \]  
\hspace{1cm} (1.100)

\[ = - \frac{1}{2c^2} [1 - F(y_1 | a_H)] - \frac{(1 - \delta + \delta_1) h_2}{1 + \rho_A} \frac{1}{2c^2} f(y_1 | a_H) \]  
\hspace{1cm} (1.101)

\[ < 0 \]  
\hspace{1cm} (1.102)

Let the principal’s expected payoff $\Pi$ be

\[ \Pi = E[e_1 - s_1(e_1) + \frac{1}{1 + \rho_P} [(e_2 - s_2(e_1, e_2)) 1_{[e_1 > \hat{y}]} + (e_2^{new} - s_2^{new}(e_2^{new})) 1_{[e_1 \leq \hat{y}]})] \]  
\hspace{1cm} (1.103)
The effect of \( c \) on the principal’s expected payoff is

\[
\frac{d\pi}{dc} = \frac{\partial \pi}{\partial c} = \frac{1}{2c^2} [F(\hat{\gamma}|a_H) - F(y_1|a_H)] - \left[ \frac{1}{1+\rho_A} E[s_2(e_1, e_2) - h_2] f(\hat{\gamma}|a_H) - (1 - F(\hat{\gamma}|a_H)) \right] \frac{1}{2c^2} \tag{1.104}
\]

\[
= \frac{1}{2c^2} [1 - F(y_1|a_H)] - \left[ \frac{1}{1+\rho_A} E[s_2(e_1, e_2) - h_2] f(\hat{\gamma}|a_H) - (1 - F(\hat{\gamma}|a_H)) \right] \frac{1}{2c^2} \tag{1.105}
\]

because \( \frac{\partial}{\partial \gamma} F(\gamma|a_H) \frac{d\gamma}{dc} = \frac{1}{2c^2} f(\hat{\gamma}|a_H) \) and

\[
E[s_1(e_1)] = \int_{y_1}^{\hat{\gamma}} (e_1 - y_1 + \frac{1}{2c}) f(e_1|a_H) de_1 + \int_{\hat{\gamma}}^{\infty} (e_1 - \hat{\gamma}) f(e_1|a_H) de_1 \tag{1.107}
\]

\[
= \int_{y_1}^{\infty} e_1 f(e_1|a_H) de_1 - \int_{y_1}^{\hat{\gamma}} (y_1 - \frac{1}{2c}) f(e_1|a_H) de_1 - \int_{\hat{\gamma}}^{\infty} \hat{\gamma} f(e_1|a_H) de_1 \tag{1.108}
\]

\[
\frac{\partial}{\partial c} E[s_1(e_1)] = \frac{\partial}{\partial c} E[s_1(e_1)] + \frac{\partial}{\partial \gamma} E[s_1(e_1)] \frac{\partial \gamma}{dc} \tag{1.109}
\]

\[
= -\frac{1}{2c^2} [F(\hat{\gamma}|a_H) - F(y_1|a_H)] + [c - y_1 + \frac{1}{2c}] f(\hat{\gamma}|a_H) - (1 - F(\hat{\gamma}|a_H)) \) \frac{\partial \hat{\gamma}}{dc} \tag{1.110}
\]

\[
= -\frac{1}{2c^2} [F(\hat{\gamma}|a_H) - F(y_1|a_H)] + \left[ \frac{1}{1+\rho_A} E[s_2(e_1, e_2) - h_2] f(\hat{\gamma}|a_H) - (1 - F(\hat{\gamma}|a_H)) \right] \frac{1}{2c^2} \tag{1.111}
\]

Therefore, the principal’s expected payoff decreases as \( c \) increases if and only if

\[
E[s_2(e_1, e_2) - h_2] \frac{f(\hat{\gamma}|a_H)}{1+\rho_A} > 1 - F(y_1|a_H) \tag{1.112}
\]

or

\[
\frac{f(\hat{\gamma}|a_H)}{1 - F(y_1|a_H)} > \left[ E[s_2(e_1, e_2) - h_2] \frac{1+\rho_A}{a_H} \right]^{-1} \tag{1.113}
\]

Q.E.D.
Proof of Corollary 3

The principal’s expected payoff decreases as $c$ increases if and only if

$$\frac{f(\hat{y} | a_H)}{1 - F(y_1 | a_H)} > \left[ E\left\{ \frac{\left( s_2(e_1, e_2) - h_2 \right)}{1 + \rho_A} | a_H \right\} \right]^{-1}$$

(1.114)

where $\hat{y} = \frac{1}{1 + \rho_A} E[s_2(e_1, e_2) - h_2] - \frac{1}{2\sigma} + y_1$.

When $\hat{y}$ is large, $y_1$ is large, too, if everything else is equal.

If the earnings $e_t$ follows the uniform distribution,

$$\frac{\partial}{\partial y_1} \frac{f(\hat{y} | a_H)}{1 - F(y_1 | a_H)} > 0$$

(1.115)

Therefore, there exists large $y_1$ or $\hat{y}$, such that the condition (2.189) holds.

If the earnings $e_t$ follows the normal distribution,

$$\lim_{y_1 \to \infty} \frac{f(\hat{y} | a_H)}{1 - F(y_1 | a_H)} = \lim_{y_1 \to \infty} \frac{f'(\hat{y} | a_H)}{f(y_1 | a_H)} = \infty$$

(1.116)

using L’hopital’s rule.

Therefore, if $y_1$ or $\hat{y}$ is sufficiently large, then the condition (2.189) holds.

Q.E.D.
Chapter 2

Earnings Management, Investment, and Managerial Turnover in a Dynamic Agency Model

Abstract

I develop a model to investigate how the internal control system influences a firm’s investment decisions. Contrary to the view that a strong internal control system mitigates CEOs’ incentives to manage earnings and increases investment efficiency, I find that a moderate internal control system, that allows appropriate reporting discretion to CEOs, can improve a firm’s investment decisions when past performance is poor. The essential mechanism is that, in the optimal contract, costly earnings management can act as an alternative punishment for poor performance and, thus, substitute for the threat of turnover. Because the possibility of turnover leads to an underinvestment problem (e.g. because a new CEO needs to learn about the ongoing

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1This chapter is based on my job market paper. I am deeply indebted to Carlos Corona, Jonathan Glover, Steve Spear, and Jing Li for their guidance and encouragement. I am grateful to Jack Stecher, Austin Sudbury, and workshop participants at Carnegie Mellon University. I thank Yuliy Sannikov for sharing his code. I also thank my fellow students Hyun Hwang, Eunhee Kim, Lufei Ruan, and Ronghuo Zheng. All errors are my own.
projects and there are costs associated with searching for a new CEO.), a moderate internal control system can effectively improve investment efficiency. Also, an infinite-horizon dynamic model shows a positive relationship between investment and the level of earnings management for a given internal control system and an inverted U-shape relationship between investment and the internal control system. Finally, calibration results suggest that shareholders’ value under the current level of the internal control systems in the market is 0.4% higher than that under the counterfactual strongest internal control system.

Key Words: earnings management, internal control system, investment, executive compensation, managerial turnover, dynamic agency model, calibration

... It has become clear, especially in retrospect, that by increasing the regulatory burden, Sarbanes-Oxley has decreased U.S. competitive flexibility... Sarbanes-Oxley is proving unnecessarily burdensome... business leaders have been quite circumspect about embarking on major new investment projects... (Former Federal Reserve Chairman Alan Greenspan, 2003)

2.1 Introduction

The conflict of interest between shareholders of a corporation and its chief executive officer is a standard example of a principal-agent problem. Shareholders design compensation, such as bonuses and a turnover policy, to give CEOs incentives to implement desired actions. If internal control systems and external enforcement mechanisms are not sufficiently stringent, CEOs have incentives to engage in earnings management when contracts are contingent on manipulable performance measures such as earnings reports (Healy, 1985; Degeorge et al., 1999; Murphy & Zimmerman, 1993; Pourciau, 1993). Earnings management influences not only CEOs’ effort incentives but also firms’ real decisions. Kedia and Philippon (2007) and McNichols and Stubben (2008) show that earnings management may induce suboptimal investment decisions. Relatively little attention has been paid to dynamic settings with earnings
management, even though earnings management and investment are history-dependent. Earnings management is an outcome of a manager’s myopic behavior to increase short-term performance at the expense of long-term value. Thus, earnings management is correlated with profits and earnings management in the past, just like a firm’s investment decisions depend on profits and past investment.

In a multi-period setting, CEOs can be replaced when their performance is poor. Turnover, however, results in inefficiencies. In the case of turnover, a loss in productivity of ongoing investments may occur because a new agent needs to learn the ongoing projects or because there are costs associated with searching for a new agent. Thus, the possibility of turnover discourages firms from investing, leading to an underinvestment problem. Because of the associated investment inefficiency, dismissal is a costly incentive device from the principal’s perspective. According to the PricewaterhouseCoopers (PwC) report, any CEO turnover is associated with a median total shareholder return of -2.3% in the preceding year and -3.5% in the year after. When a CEO turnover is forced, companies have lost $112 billion a year, which is roughly $1.8 billion for each company.

In this paper, I show that earnings management that is moderately costly to the agent (e.g., because of a moderate internal control system) can improve investment efficiency. In a single-period or a multi-period model without turnover, earnings management weakens incentives and decreases investment efficiency—in this case, there is only a dark side to earnings management. In a multi-period model with turnover, however, earnings management can mitigate moral hazard problems and substitute for the threat of dismissal. The fundamental mechanism is that, under the optimal contract, the agent is required to engage in costly earnings management when her performance is bad in order to retain her job which brings with it the prospect of future economic rents. As a result, due to the cost of earnings management, the agent experiences

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2 Weisbach (1995) shows that there is an increased probability of divesting a recent acquisition at the time of a management change.

3 “While firing the CEO can be the right call, it’s enormously costly. When you quantify the cost of turnovers, particularly forced ones, you get a strong sense of the importance and payoff involved in getting CEO succession right,” says report co-author Per-Ola Karlsson, Senior Partner at Strategy&. See ‘A Forced CEO Turnover Costs a Large Company $1.8B More in Shareholder Value than a Planned Turnover’ by PwC.

4 The revelation principle is applicable because the compensation contract is implemented after the agent observes the private information. Therefore, earnings management provides a mechanism for the agent to communicate with some cost that she was
a negative utility in the period in which her performance is bad. In other words, earnings management relaxes the agent’s bankruptcy constraints by imposing a cost on the agent.\(^5\) This negative utility can be regarded as punishment for poor performance and, thus, the agent wants to work harder *ex-ante* to avoid the punishment resulting from earnings management. Therefore, the principal can not only motivate the agent to work harder but also reduce the use of a costly incentive device—the threat of dismissal—leading to an increase in long-term incentives. This result is consistent with the empirical finding in Weisbach (1988), who shows that intense monitoring by independent boards increases an association between past performance and turnover. The key mechanism in my paper is different, in that costly earnings management acts as an alternative punishment. Therefore, earnings management can improve a firm’s investment decisions as well as incentives. That is, once the possibility of turnover is introduced, there is a bright side to earnings management—it can substitute for turnover and, hence, reduce the underinvestment problem associated with turnover. There is still a dark side to earnings management—when performance is good and there would be no turnover, the agent may still engage in earnings management. Which of these two affects dominates determines whether earnings management is beneficial or harmful to the principal in my model.

This paper develops a dynamic agency model that encompasses earnings management, investment, and managerial turnover to allow for all of the above complexities. In the model, the principal invests capital and the agent chooses a level of productive effort, which affects the productivity of the firm, and manages earnings. The principal commits to a long-term contract that specifies compensation, such as short-term incentives and long-term incentives (for instance, deferred compensation) to the agent as a function of performance history. Long-term incentives enable the principal to keep providing incentives to the agent more efficiently, in that the principal can reward her for good performance and penalize her for bad performance with future incentives. Therefore, long-term incentives increase with past performance, and

\(^5\)This cost is determined by the level of internal control system and the amount of earnings management. A moderate internal control system helps impose a cost on the agent that cannot be replicated with compensation. The agent’s bankruptcy constraints mean nonnegativity of the agent’s utility whereas the agent’s limited liability constraints mean nonnegativity of the agent’s compensation.
the agency problem decreases with long-term incentives because higher long-term incentives mean the agent has a greater stake in the firm. Consequently, it is optimal to fire the agent when long-term incentives become zero, because the agent has no incentives to work and moral hazard problems are extremely severe.

The investment decision and pay-performance sensitivity (PPS) are contingent on long-term incentives. Investment efficiency increases with long-term incentives because the principal invests less optimally when long-term incentives are low (the probability of turnover is high). Also, pay-performance sensitivity, which captures the level of incentives for effort, increases with long-term incentives in general, because moral hazard problems decrease with long-term incentives. In summary, a lack of long-term incentives can bring in inefficiencies such as the possibility of turnover, the underinvestment problem, and less effective effort incentives.

Earnings management helps to create long-term incentives efficiently in a dynamic model with turnover. Earnings management has two effects: a turnover-reducing benefit and an incentive-weakening cost. The turnover-reducing benefit means that earnings management can substitute for the threat of dismissal and help to create long-term incentives. Therefore, the turnover-reducing benefit improves investment efficiency as well as pay-performance sensitivity. On the other hand, the incentive-weakening cost implies that the cost of compensation can increase because earnings management can weaken incentives. Therefore, the incentive-weakening cost reduces investment efficiency as well as pay-performance sensitivity. As a result, earnings management can improve investment efficiency and pay-performance sensitivity as the turnover-reducing benefit dominates. In contrast, earnings management can worsen investment efficiency and pay-performance sensitivity when the incentive-weakening cost dominates.

The model interprets the cost of earnings management as arising from the internal control system of firms to discuss the effect of earnings management on the optimal contract. The internal control system

\[ \text{For instance, in the extreme, there would be no agency problem if the agent possessed the entire company. In this sense, the inverse of long-term incentives can be interpreted as the severity of moral hazard in the firm.} \]
means how much discretion the agent has in reporting earnings.\footnote{For example, a stronger internal control system implies that the agent has less discretion in reporting earnings and has to report more truthfully.} I investigate the effect of earnings management in two ways. First, I study the relationship between the level of earnings management and investment for a given internal control system within a firm. Second, I consider the relationship between the internal control system and investment across firms with different levels of internal control systems. For a given internal control system within a firm, I find a positive relationship between the level of earnings management and investment. As discussed above, investment and pay-performance sensitivity increase with long-term incentives. Because the marginal benefit of earnings management depends on pay-performance sensitivity, earnings management also increases with long-term incentives. Thus, investment increases with earnings management. This result is consistent with the empirical evidence in Kedia and Philippon (2007) and McNichols and Stubben (2008), who show that earnings management increases a firm’s investment. My paper rationalizes the positive relationship between the level of earnings management and investment by showing that the increase in investment by earnings management is the outcome of the optimal decision for shareholders, whereas their papers claim it is a suboptimal decision for shareholders. The paper also sheds light on why and when CEOs take big baths. Big baths are more likely to be observed when the pay-performance is low and CEOs have managed earnings more aggressively in the previous periods.

Next, I show that the relationship between the internal control system and investment depends on long-term incentives, which summarize past performance. The internal control system affects the trade-off between the turnover-reducing benefit and the incentive-weakening cost. When long-term incentives are high (good performance in the past), the incentive-weakening cost always dominates because the benefit of increasing long-term incentives is small. Therefore, as the internal control system becomes weaker, investment decreases because the incentive-weakening cost dominates. In other words, investment increases with the internal control system when past performance is good. In contrast, when long-term incentives are low (bad performance in the past), the benefit of increasing long-term incentives is large.
Hence, as the internal control system becomes weaker, investment increases because the turnover-reducing benefit initially dominates. The cost of compensation, however, increases because the cost of earnings management increases and, as a result, the incentive-weakening cost dominates as the internal control system becomes weaker than a certain threshold. Therefore, the relationship between the internal control system and investment is an inverted U-shape when past performance is bad. This finding means that earnings management under a moderate internal control system can improve the investment efficiency when past performance is poor.

Consequently, the principal’s welfare increases as the turnover-reducing benefit dominates, because the turnover-reducing benefit improves not only investment efficiency but also the agent’s effort incentives. On the other hand, the principal’s welfare decreases as the incentive-weakening cost dominates, because the incentive-weakening cost decreases not only investment efficiency but also the agent’s effort incentives. Similarly, the principal’s welfare increases with the internal control system when past performance is good, whereas the relationship between the principal’s welfare and the internal control system is an inverted U-shape when past performance is bad. This finding implies that earnings management under a moderate internal control system can improve shareholders’ value when past performance is poor.

I also quantify the results through calibration. The calibration enables us to understand how much the agency problem causes the underinvestment and losses of firm value and to measure how much earnings management improves the agency problem. The calibration results are consistent with the agency problem resulting in an underinvestment problem of 32.4% compared to the first-best investment-capital ratio. And this underinvestment, along with the moral hazard problem, leads to a reduction in firm value of 2.56% on average. A counter-factual analysis is consistent with current levels of earnings management improving shareholders’ value by 0.4% compared to that of counterfactual under the strongest internal control system.

The paper develops empirical predictions. First, the model stresses the (fairly obvious) importance of the strength of the internal control system in calibrating the model. Without taking into account the internal

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8The total cost of earnings management increases as the internal control system becomes weaker, because the amount of earnings management increases faster.
control system, the model may result in misspecification of the parameters of importance, such as cost of effort and productivity parameters, that can significantly alter the agency cost and shareholders’ welfare. Second, the paper emphasizes the (less obvious) importance of past performance in measuring the effect of earnings management on investment efficiency, which may help explain existing mixed evidence on earnings management. By controlling for long-term incentives, which summarize past performance, the effect of earnings management can be tested more accurately. Third, the paper offers a new explanation for the low forced turnover rate. At large corporations in the U.S., on average 2% of CEOs are fired every year. This rate seems to be too low to be consistent with the agency model. Taking into account earnings management and internal control systems, the low forced turnover rate can be better explained as resulting from a substitution of earnings management for turnover following poor actual firm performance.

The paper makes related policy contributions. The results in the paper imply that standard setters need to consider the effect of earnings management on the flexibility of contracting as well as on market efficiency. The Securities and Exchange Commission has been trying to implement stronger control systems (for example, as part of the Sarbanes-Oxley Act) to protect participants in the market from receiving misleading information from companies. There is, however, another effect of earnings management through an optimal contract between shareholders and managers. If earnings management can be used to motivate the agent to work harder, it can also improve investors’ welfare in the end. I do not mean to argue that earnings management is all good but instead only that earnings management following performance that would otherwise result in managerial turnover may have a bright side.

The rest of the paper is organized as follows: In section 3.2, I review the current literature on earnings management, the interaction between earnings management and investment, and the current literature on the dynamic contract on which the model is based. Section 3.3 develops the two-period model and derives the optimal contract to discuss the fundamental trade-off of earnings management. Section 2.4 extends the model to an infinite-horizon dynamic model to explore how the effect of earnings management varies depending on past performance. In section 2.5, I calibrate the model to replicate the empirical moments that
2.2 Literature Review

Academic research provides conflicting results regarding the effects of earnings management. Many prior studies find that earnings management can weaken managers’ incentives and increase the cost of compensation, which results in a deadweight loss in a contracting setting (Crocker & Slemrod, 2008; Marinovic, Beyer, & Guttman, 2014). Arya, Glover, and Sunder (1998) show that earnings management can benefit the principal in a limited commitment setting by helping the principal to commit to delaying the intervention (dismissal). My paper studies costly earnings management and investment, neither of which are studied by Arya, Glover, and Sunder.

Earnings management is not just a change of numbers in financial statements but also has an impact on resource allocation. Two streams of research study the effect of earnings management on investment. The first stream studies the relationship between earnings management and investment within a firm. Wang (2011) shows that misstating firms are likely to invest more in R&D and stock-financed mergers and acquisitions. Kedia and Philippon (2007) show that overstating firms overinvest during the earnings manipulation period in order to pool with better performing firms. McNichols and Stubben (2008) find that earnings manipulation affects a firm’s internal decision and thus incurs a direct cost to investors in the form of inefficient investment. Their papers study how the amount of discretionary accruals are correlated with the investment within a firm. The second stream of literature studies the relationship between internal control systems and investments across firms. This research examines the effect of earnings management on investment across firms with different levels of internal control systems. Bar-Gill and Bebchuk (2003) find that overreporting, by relaxing the internal control system, helps obtain cheaper financing and, as a

See also Bushman and Smith (2001), Healy and Palepu (2001), Lambert et al. (2007), Albuquerque and Zhu (2013), and Biddle, Hilary, and Verdi (2009).
result, increases the distortion in capital allocation. Kang, Liu, and Qi (2010), however, structurally estimate the impact of Sarbanes-Oxley Act (SOX), which was to improve the internal control system, on corporate investment and find that corporate investment has decreased after the adoption in 2002 because the discount rate the managers apply for their investment rose significantly. This paper provides a unified theory that explains the effect of earnings management both within a firm and across firms.

Prior literature in accounting has not paid much attention to the dynamic setting and, therefore, misses many interesting questions, including how the reversal of accruals influences a firm’s important decisions. This paper studies a dynamic interaction between earnings management and a turnover and investment policy of firms using dynamic contract theory. Spear and Srivastava (1987) introduce the agent’s continuation payoff that can be used as a state variable in dynamic programming. Using the flexibility of differential equations, Sannikov (2008) describes the dynamics of the optimal contract in continuous-time to overcome difficulties in the tractability of discrete-time models. Using the continuous-time methodology, DeMarzo and Sannikov (2006) study the optimal contract in a cash-flow diversion model. My paper is closely related to the work of DeMarzo et al. (2012). DeMarzo et al. consider models of corporate investment that integrate dynamic agency frictions into the neoclassic q theory of investment. In their model, investors dynamically manage the agent’s continuation payoff based on the firm’s historical performance. DeMarzo et al., however, do not take into account the effect of earnings management, internal control system, and pay-performance sensitivity. Earnings management, internal control system, and pay-performance sensitivity influence the evolution of the agent’s scaled continuation payoff, and these distinctions lead to important implications for investment, shareholders’ welfare, the pattern of earnings management, and the dismissal policy of the firm.

2.3 Model

In this section, I describe the basic framework and present the optimal contract between the principal and the agent, which characterizes the effect of earnings management in a dynamic principal-agent problem.
In the model, a risk-neutral principal (shareholders) of the firm hires a risk-neutral agent (a manager) to operate the business. The principal provides capital and the manager makes a choice of productive effort, which affects the productivity of the firm, and manages earnings. I discuss a two-period model to examine the basic intuition about the effect of earnings management on the optimal contract, including a managerial turnover policy and the firm’s investment decisions. Later, I extend to an infinite-horizon dynamic model to understand how its effect varies depending on the history of the firm. I first introduce the agency problem between shareholders and the manager, and then the optimal contracting problem is specified.

2.3.1 Agency Problem

The agent makes two unobservable actions: productive effort $a_t$ and earnings management $B_t$. The agent’s productive effort $a_t$ affects the productivity of the firm, $x_t$, as follows:

$$x_t = a_t \bar{x} + \varepsilon_t \quad \text{for } t \in \{1, 2\},$$

where $E[\varepsilon_t | a_t] = 0$ and $\bar{x}$ is the productivity parameter. The productivity $x_t$ is continuously distributed according to a probability density function (PDF) $f(x_t | a_t)$ and a cumulative density function (CDF) $F(x_t | a_t)$. I assume that the strict monotone likelihood ratio property (MLRP) holds: for all productivities $x_H$ and $x_L$ with $x_H > x_L$,

$$\frac{f(x_H | a_H)}{f(x_H | a_L)} > \frac{f(x_L | a_H)}{f(x_L | a_L)} \quad (2.2)$$

MLRP implies that the distribution of the productivity is ordered according to strict first-order stochastic dominance (FOSD): $F(x | a_L) > F(x | a_H)$ for $\forall x$.

The expected productivity is dependent on the agent’s binary action $a_t \in \{a_L, a_H\}$. In other words, the agent can either work, $a_t = a_H$, or shirk, $a_t = a_L$. The cost of effort $a_H$ is $h_t$ per capital (i.e. $H_t = h_t K_t$) and the cost of effort $a_L$ is normalized to 0. The cost of effort $a_H$ is proportional to the current capital stock, because administering a larger firm takes more effort.
The agent privately observes earnings $e_t$, which is a function of $x_t$, and reports earnings ($R_t$) with bias $B_t$. The cost of earnings management is $G(M_t + B_t) = \frac{1}{2} \frac{c}{K_t} (M_t + B_t)^2$, where $M_t$ is the accumulated discretionary accruals and $K_t$ is capital stock at the beginning of time $t$. First, this explains well the reversal of discretionary accruals in subsequent periods. Because the cost of earnings management is convex in the accumulated discretionary accruals, the agent has more incentives to reverse earnings management made in the previous periods if she manipulated earnings aggressively. Second, the cost of managing the earnings per capital is the same across firms with different sizes for the same $c$, i.e., $G(M_t + B_t) = g(m_t + b_t)K_t$, where $m_t(= \frac{M_t}{K_t})$ is the accumulated discretionary accruals per unit of capital and $b_t(= \frac{B_t}{K_t})$ is earnings management. Furthermore, this can be interpreted to mean that the agent in a large firm can manage the earnings at a lower cost or that it is more difficult to audit a large firm.

In the two-period model, I assume that earnings management made in period 1 ($B_1 = B$) is reversed in period 2 ($B_2 = -B$). Thus the cost of earnings management in period 1 and 2 are $\frac{1}{2} \frac{c}{K_1} B^2$ and 0, respectively, assuming the initial accumulated discretionary accruals $M_1$ are 0. Therefore, earnings reports in period 1 and 2 are

$$R_1 = e_1 + B$$
$$R_2 = e_2 - B$$

The agent has no initial wealth and has limited liability, which rules out a negative wage. And the agent has a higher discount rate than the principal, $\rho_A > \rho_P$, which means that the manager is more impatient than the shareholders. The agent’s reservation utility is normalized to zero, which is related to her outside opportunity if the contract is terminated and the agent is fired.

The termination of contract involves replacing the initial agent with a new agent, and replacement of the agent results in a loss in productivity of ongoing investment, which means the dismissal is inefficient ex-post and brings in deadweight losses. For instance, when the agent is replaced, the new agent has to learn the ongoing projects of the firm, or there are costs associated with searching for a new agent. I formulate these
costs as a loss in productivity of ongoing investment. Therefore, if turnover takes place, the productivity in
period 2 is
\[ x_2^{\text{New}} = (1 - \ell)x_2 \]  \hspace{1cm} (2.5)
where \( \ell \in [0, 1) \) reflects a cost of a loss in productivity.

### 2.3.2 Investment and Production Technology

The firm uses physical capital to produce output, and the price of capital is normalized to one. The capital stock \( K_t \) changes as follows:
\[ K_{t+1} = (1 - \delta)K_t + I_t \] \hspace{1cm} (2.6)
where \( I_t \) is the gross investment and \( \delta \geq 0 \) is the rate of depreciation.

Based on the Cobb-Douglas production function, the earnings \( e_t \) are
\[ e_t = x_t K_t \] \hspace{1cm} (2.7)
If the agent is replaced, the earnings \( e_2^{\text{New}} \) are
\[ e_2^{\text{New}} = x_2^{\text{New}} K_2 \] \hspace{1cm} (2.8)
\[ = (1 - \ell)x_2 K_2 \] \hspace{1cm} (2.9)

Investment encompasses the adjustment cost, \( J(I_t, K_t) \). Following Hayashi (1982), I assume that the adjustment cost is homogeneous of degree one in \( I \) and \( K \), which enables the adjustment cost to take the form \( J(I_t, K_t) = j(i_t)K_t \), where \( i_t = \frac{I_t}{K_t} \) is the investment-capital ratio, \( g'(i) > 0 \), and \( g''(i) > 0 \). For simplicity, I assume \( g(i) = \frac{\theta}{2} i^2 \), where \( \theta \) is the degree of the adjustment cost.
Thus, the total cost of investment is

\[ I_t + J(I_t, K_t) = (i_t + j(i_t))K_t \]  \hspace{1cm} (2.10)
\[ = \xi(i_t)K_t \]  \hspace{1cm} (2.11)

where \( \xi \) is the total cost per unit of capital required when the firm invests to grow at rate \( i_t \).

In the two-period model, I assume the capital stock in period 1, \( K_1 \), is given and the principal determines the amount of investment, \( I_1 \), before observing the earnings report in period 1, \( R_1 \).\(^{10}\) Figure 3.1 shows the timeline.

### 2.3.3 Optimal Contracting Problem

In a multi-period contract, the principal can motivate the agent with short-term and long-term incentives, including the threat of dismissal.\(^{11}\) And these incentives determine the return to the agent as follows:

\[
S_1(e_1) - G(R_1(e_1) - e_1) + \frac{1}{1 + \rho_A} [S_2(e_1, e_2) - H_2] 1_{[TD(e_1) = 0]} - H_1
\]

\[= W(S_t(e_1), S_t(e_1, e_2), R_t(e_1), I_t) \]  \hspace{1cm} (2.12)

\(^{10}\)An example would be long-term investment.

\(^{11}\)For instance, short-term compensation such as a bonus is provided as a short-term incentive. In addition, long-term compensation, such as deferred compensation, serves as a long-term incentive. The principal uses long-term incentives optimally to smooth the cost of compensation over periods because long-term incentives can make the agent work harder, not only in the current period, but also in the future, and therefore help to reduce short-term incentives.
where $W(S_1(e_1), S_2(e_1, e_2), R_1(e_1), I_1)$ is the agent’s expected payoff after the cost of effort in period 1, $H_1$, is sunk, $S_1(e_1)$ is the compensation in period 1, $S_2(e_1, e_2)$ is the compensation in period 2, and $TD(e_1) = \{0, 1\}$ is a termination decision such that 1 indicates the termination of contract and 0 indicates a continuation of contract.

I assume that earnings report $R_t$ and capital stock $K_t$ are observable and contractible. Therefore, investment $I_t$ is also contractible. Because the compensation contract is implemented after the agent observes the private information, the revelation principle is applicable (Myerson, 1979). Therefore, the contract is conditional on the private information ($e_1$) and must satisfy the incentive constraint

$$E[W(S_1(e_1), S_2(e_1, e_2), R_1(e_1), I_1)|e_1] \geq E[W(S_1(\hat{e}_1), S_2(\hat{e}_1, e_2), R_1(\hat{e}_1), I_1)|e_1] \text{ for } \forall e_1$$ (2.13)

When this incentive compatible contract is offered, the agent who has $e_1$ would prefer the contract \{ $S_1(e_1), S_2(e_1, e_2), R_1(e_1), I_1$ \} over the alternatives \{ $S_1(\hat{e}_1), S_2(\hat{e}_1, e_2), R_1(\hat{e}_1), I_1$ \} for every $\hat{e}_1 \neq e_1$.

At the beginning of period 1, the principal offers a contract that specifies the agent’s compensation, $S_1(e_1)$ and $S_2(e_1, e_2)$, earnings report, $R_1(e_1)$, the firm’s investment policy $I_1$, and a termination decision, $TD(e_1)$. The termination decision $TD(e_1)$ defines the termination threshold $\hat{Y}$ such that the agent is fired and replaced if $e_1 \leq \hat{Y}$, i.e., $S_2(e_1, e_2) = 0$ if $e_1 \leq \hat{Y}$. If the agent is replaced, the principal offers a contract to a new agent that specifies the agent’s compensation $S_2^{\text{New}}(e_2^{\text{New}})$ depending on the earnings in period 2, $e_2^{\text{New}}$.

---

12 In a direct revelation mechanism, the agent sends a message $e_1$ in the first stage, and the incentive compatible contract, which makes the agent reveal her private information truthfully, is assigned (see Crocker & Slemrod 2007).

13 Spear and Wang (2005) show the optimal termination in the two-period model. They show that the CEO is also forced out after good performance because she becomes too expensive to motivate under the assumption that the CEO is risk-averse.
The principal chooses $S_1(e_1), S_2(e_1, e_2), R_1(e_1), I_1,$ and $TD(e_1)$ to maximize the following problem:

$$\max_{S_1, S_2, R_1, I_1, TD} \Pi = E[e_1 - I_1 - J(I_1, K_1) - S_1(e_1) + \frac{1}{1 + \rho_p} \left[ (e_2 - S_2(e_1, e_2)) 1_{TD(e_1)=0} + (e_2^{New} - S_2^{New}(e_2^{New})) 1_{TD(e_1)=1} \right]$$

$$s.t. E[W(S_1(e_1), S_2(e_1, e_2), R_1(e_1), I_1)|a_H] - E[W(S_1(e_1), S_2(e_1, e_2), R_1(e_1), I_1)|a_L] \geq H_1$$  \hspace{1cm} (2.15)

$$E[S_2(e_1, e_2)|a_H] - E[S_2(e_1, e_2)|a_L] \geq H_2$$  \hspace{1cm} (2.16)

$$E[W(S_1(e_1), S_2(e_1, e_2), R_1(e_1), I_1)|a_H] \geq H_1$$  \hspace{1cm} (2.17)

$$E[S_2(e_1, e_2)|a_H] \geq H_2$$  \hspace{1cm} (2.18)

$$E[W(S_1(e_1), S_2(e_1, e_2), R_1(e_1), I_1)|e_1] - E[W(S_1(\hat{e}_1), S_2(\hat{e}_1, e_2), R_1(\hat{e}_1), I_1)|e_1] \geq 0$$  \hspace{1cm} (2.19)

$$S_1(e_1 + \eta) + \frac{1}{1 + \rho_a} E[S_2(e_1 + \eta, e_2)] - |S_1(e_1) + \frac{1}{1 + \rho_a} E[S_2(e_1, e_2)]| \leq \eta \text{ for } \forall \eta > 0$$  \hspace{1cm} (2.20)

$$S_2(e_1, e_2 + \eta) - S_2(e_1, e_2) \leq \eta \text{ for } \forall \eta > 0$$  \hspace{1cm} (2.21)

$$S_1(e_1), S_1(e_1, e_2) \geq 0$$  \hspace{1cm} (2.22)

The principal maximizes the expected earnings net of compensation in the equation (2.14). The equations (2.15) and (2.16) are the incentive compatibility constraints for the agent in period 1 and 2, respectively. The equations (2.17) and (2.18) are the individual rationality constraints for the agent in period 1 and 2, respectively. The equation (2.19) is the incentive compatibility constraint for truth-telling and the equations (2.20) and (2.23) are the monotonicity constraints in period 1 and 2, respectively. The monotonicity constraints mean that the principal’s payoff is non-decreasing in earnings $e_i$ in each period.\footnote{Without the monotonicity constraint, the optimal contract of the “live-or-die” form may achieve a “first-best” effort choice (see Innes, 1990).}

Lastly, the equation (2.22) is the agent’s limited liability constraints.

In the case of turnover, the principal signs a contract with a new agent. The principal chooses $S_2^{New}(e_2^{New})$
to solve the following problem:

$$\text{Max} \quad S_{2}^{\text{New}}(e_{2}^{\text{New}}) \quad e_{2}^{\text{New}} - S_{2}^{\text{New}}(e_{2}^{\text{New}})$$

(2.23)

s.t. $E[S_{2}^{\text{New}}(e_{2}^{\text{New}})|a_{H}] - E[S_{2}^{\text{New}}(e_{2}^{\text{New}})|a_{L}] \geq H_{2}$

(2.24)

$E[S_{2}^{\text{New}}(e_{2}^{\text{New}})|a_{H}] \geq H_{2}$

(2.25)

$\quad S_{2}^{\text{New}}(e_{2}^{\text{New}} + \eta) - S_{2}^{\text{New}}(e_{2}^{\text{New}}) \leq \eta \text{ for } \forall \eta > 0$

(2.26)

$\quad S_{2}^{\text{New}}(e_{2}^{\text{New}}) \geq 0$

(2.27)

The equations (2.24), (2.25), (2.26), and (2.27) are the incentive compatibility constraint, the individual rationality constraint, the monotonicity constraint, and the limited liability constraint, respectively.

Because of the size-homogeneity property of the model, the principal’s problem is homogeneous of degree one in $K_{1}$, and the problem can be scaled by $K_{1}$. Let $s_{j} = \frac{S_{j}}{K_{1}}$, $s_{2}^{\text{New}} = \frac{S_{2}^{\text{New}}}{K_{2}}$, $\hat{y} = \frac{\hat{Y}}{K_{1}}$, $r_{1} = \frac{R_{1}}{K_{1}}$, and $w = \frac{W}{K_{1}} = s_{1}(x_{1}) - g(r_{1}(x_{1}) - x_{1}) + \frac{(1-\delta+i_{1})}{1+p_{r}} [s_{2}(x_{1}, x_{2}) - h_{2}] 1[TD(x_{1})=0]$. All the decisions and variables are scaled values from now on. The principal chooses $s_{1}(x_{1})$, $s_{2}(x_{1}, x_{2})$, $r_{1}(x_{1})$, $i_{1}$, and $TD(x_{1})$ to maximize the following problem:

Contract with the initial agent:

$$\text{Max} \quad \pi = E[x_{1} - \xi(i_{1}) - s_{1}(x_{1}) + \frac{(1-\delta+i_{1})}{1+p_{r}} [(x_{2} - s_{2}(x_{1}, x_{2})) 1[TD(x_{1})=0] + \left(s_{2}^{\text{New}} - S_{2}^{\text{New}}(x_{2}^{\text{New}})\right) 1[TD(x_{1})=1]]$$

(2.28)
\[
s.t. E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|a_H] - E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|a_L] \geq h_1
\]
\[
E[x_2(x_1, x_2)|a_H] - E[x_2(x_1, x_2)|a_L] \geq h_2
\]
\[
E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|a_H] \geq h_1
\]
\[
E[x_2(x_1, x_2)|a_L] \geq h_2
\]
\[
E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|a_H] - E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|a_H] \geq 0
\]
\[
s_1(x_1 + \eta) + \frac{1}{1 + \rho_A}E[s_2(x_1 + \eta, x_2)] - [s_1(x_1) + \frac{1}{1 + \rho_A}E[s_2(x_1, x_2)]] \leq \eta \text{ for } \forall \eta > 0
\]
\[
s_2(x_1, x_2 + \eta) - s_2(x_1, x_2) \leq \eta \text{ for } \forall \eta > 0
\]
\[
s_1(x_1), s_1(x_1, x_2) \geq 0
\]

Contract with the new agent:

\[
\text{Max } s_2^{New}(x_2^{New}) - s_2^{New}(x_2^{New})\]

\[
s.t. E[s_2^{New}(x_2^{New})|a_H] - E[s_2^{New}(x_2^{New})|a_L] \geq h_2
\]
\[
E[S_2^{New}(x_2^{New})|a_H] \geq h_2
\]
\[
s_2^{New}(x_2^{New} + \eta) - s_2^{New}(x_2^{New}) \leq \eta \text{ for } \forall \eta > 0
\]
\[
s_2^{New}(x_2^{New}) \geq 0
\]

2.3.4 Model Solution and Optimal Contracting

2.3.4.1 Benchmark

I first consider the benchmark case where the agent’s action is observable. The agent makes a choice of high effort \(a_H\) in both periods and no earnings management is induced \((b = 0)\). The principal’s investment
decision solves
\[
\text{Max } E[x_1 - \xi (i_1) - h_1 + \left( \frac{1 - \delta + i_1}{1 + \rho_p} \right) [x_2 - h_2]]
\] (2.42)

Therefore, the optimal investment-capital ratio \( i_1 \) satisfies
\[
\xi'(i_1) = \frac{1}{1 + \rho_p} E[x_2 - h_2]
\] (2.43)

In the above equation, the LHS is the marginal cost of investment and the RHS is the marginal benefit of investment. Lemma 1 summarizes the principal’s expected payoff and the investment-capital ratio \( i_1 \) in the benchmark case, and all the terms in the contract are represented by scaled values.

**Lemma 1.** In the benchmark case, the principal’s expected payoff is
\[
E[x_1 - \xi (i_1^{FB}) - h_1 + \left( \frac{1 - \delta + i_1^{FB}}{1 + \rho_p} \right) [x_2 - h_2]]
\] (2.44)

where the investment-capital ratio \( i_1 \) satisfies \( \xi'(i_1^{FB}) = \frac{1}{1 + \rho_p} [x_2 - h_2] \).

In the benchmark case in which there is no agency concern, the first-best investment-capital ratio \( i_1^{FB} \) is independent of the volatility of productivity \( x_t \). Therefore, the idiosyncratic productivity shock does not have any influence on the firm’s investment or firm value. The next section will show that agency problem will alter these results significantly.

### 2.3.4.2 Optimal Contract

Now I derive the optimal contract under the condition in which the principal observes earnings reports only, and, thus, the agent has an incentive to manage earnings. I sketch how to derive the optimal contract in this section, and the proof is provided in the Appendix A.

I first derive the optimal compensation in period 2, \( s_2(x_1, x_2) \), in the case of no replacement of the
agent \((TD(x_1) = 0)\). Given the earnings per unit of capital, or the productivity in period 1, \(x_1\), the principal determines \(s_2(x_1, x_2)\), satisfying the following constraints:

\[
\begin{align*}
& s.t. \ E[s_2(x_1, x_2) | a_H] - E[s_2(x_1, x_2) | a_L] \geq h_2 \tag{2.45} \\
& E[s_2(x_1, x_2) | a_H] \geq h_2 \tag{2.46} \\
& s_2(x_1, x_2 + \eta) - s_2(x_1, x_2) \leq \eta \text{ for } \forall \eta > 0 \tag{2.47} \\
& s_2(x_2) \geq 0 \tag{2.48}
\end{align*}
\]

As stated by Innes (1990) and Chaigneau, Edmans, and Gottlieb (2015), the optimal contract is a form of a call option on \(x_2\). The intuition is that the value of information on \(x_2\) is largest in the tails of the distribution of \(x_2\). In other words, \(x_2\) is the most informative about effort in the tail. Because of the limited liability, however, incentives cannot be provided in the left tail, and, thus, only the right tail of \(x_2\) is used for providing incentives. Also, a call option contract satisfies the monotonicity constraint. Therefore, the optimal contract is a form of a call option on \(x_2\) when \(TD(x_1) = 0\).

\[
s_2(x_1, x_2) = Max[0, x_2 - y_2] \text{ for } x_1 > \hat{\eta} \tag{2.49}
\]

where \(y_2\) is the unique solution of

\[
\int_{y_2}^{\infty} (x_2 - y_2) [f(x_2 | a_H) - f(x_2 | a_L)] dx_2 = h_2 \tag{2.50}
\]

The above equation (2.50) comes from the incentive constraint (2.45), implying that \(y_2\) is the strike price, such that the increased value of the option by high effort \(a_H\) is equal to the cost of effort, \(h_2\).

The optimal contract for the new agent in period 2, \(s_2^{\text{New}}(x_2^{\text{New}})\), can be derived in the same manner. The only difference between the initial agent and the new agent is a loss in productivity of ongoing investment \((x_2^{\text{New}} = (1 - \ell)x_2)\) which does not affect the information structure of the contract. Therefore, the optimal
contract for the new agent is the same as the one for the initial agent.

\[ s_2^{New}(x_2^{New}) = s_2(x_1, x_2) \text{ for } x_1 > \hat{y} \] (2.51)

Now I turn to the principal’s problem to decide \( s_1(x_1) \), satisfying the following constraints:

\[
\begin{align*}
\text{s.t. } & E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|a_H] - E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|a_L] \geq h_1 \\
& E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|a_H] \geq h_1 \\
& E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|x_1] - E[w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)|x_1] \geq 0 \\
& s_1(x_1) = \eta + \frac{1 - \delta + i_1}{1 + \rho_A} E[s_2(x_1 - \eta, x_2)] - [s_1(x_1) + \frac{1 - \delta + i_1}{1 + \rho_A} E[s_2(x_1 - \eta, x_2)]] \leq \eta \text{ for } \forall \eta > 0 \\
& s_1(x_1) \geq 0
\end{align*}
\] (2.52) - (2.56)

Similarly, in the optimal contract, \( w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1) \) is a form of a call option on \( x_1 \). Because the principal can motivate the agent with long-term incentives as well in a dynamic contract, the agent’s expected payoff over two-periods—after the cost of effort is sunk—should give the agent enough incentives, and all the incentives are concentrated in the right tail of \( x_2 \).

\[
\begin{align*}
& w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1) = s_1(x_1) - g(r_1(x_1) - x_1) + \frac{1 - \delta + i_1}{1 + \rho_A} E[(s_2(x_1, x_2) - h_2)|_{TD(x_1)=0}] \\
& \quad = Max[0, x_1 - y_1]
\end{align*}
\] (2.57) - (2.58)

where \( y_2 \) is the unique solution of

\[
\int_{y_1}^{\infty} (x_1 - y_1)[f(x_1|a_H) - f(x_1|a_L)]dx_1 = h_1
\] (2.59)

For the contract to be incentive compatible with truth-telling (2.54), we need to understand the agent’s
incentive to manage earnings.

\[
\begin{align*}
\max_{r_1(x)} E[&w(s_1(x), s_2(x_1, x_2), r_1(x), i_1)|x_1] = s_1(x) - g(r_1(x) - x) + \frac{1 - \delta + i_1}{1 + \rho_A} E[(s_2(x_1, x_2) - h_2)1_{TD(x_1) = 0}] \\
&
\end{align*}
\]

(2.60)

Considering that the contract is a form of a call option and \(\hat{x}_1 = x_1\) in the optimal contract, the optimal earnings report is

\[
\begin{align*}
  r_1(x) = \begin{cases} 
  x_1 + \frac{1}{c} & \text{if } x_1 > y_1 \\
  x_1 & \text{if } x_1 \leq y_1
\end{cases}
\end{align*}
\]

(2.61)

Thus the optimal earnings management is

\[
\begin{align*}
  b(x) = \begin{cases} 
  \frac{1}{c} & \text{if } x_1 > y_1 \\
  0 & \text{if } x_1 \leq y_1
\end{cases}
\end{align*}
\]

(2.62)

Before searching for the threshold for dismissal, \(\hat{y}\), let \(w^*\) be the agent’s continuation payoff for period 2 given \(x_1 > \hat{y}\).

\[
\begin{align*}
  w^* &= \frac{1 - \delta + i_1}{1 + \rho_A} E[s_2(x_1, x_2) - h_2|a_H] \\
  &= \frac{1 - \delta + i_1}{1 + \rho_A} E[\int_{y_2}^{\infty} (x_2 - y_2)f(x_2|a_H)dx_2 - h_2]
\end{align*}
\]

(2.63)

(2.64)

Then

\[
\begin{align*}
  w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1) &= \max[0, x_1 - y_1] = s_1(x_1) - g(r_1(x_1) - x_1) + w^* \\
  &= s_1(x_1) - g(r_1(x_1) - x_1) + \text{Utility in period 1} + \text{Utility in period 2}
\end{align*}
\]

(2.65)

A call option form of \(w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1)\) implies that the agent’s expected payoff over two-periods is determined based on \(x_1\). In other words, incentives provided in period 2 spill back to period 1,
and, thus, the agent is incentivized to exert a high effort in period 1 to enjoy not only the compensation in period 1 but also the compensation in period 2 (Glover and Haijin, 2015). Therefore, if the realized $x_1$ is high enough, the agent consumes utilities in both periods 1 and 2. If the realized $x_1$ is low and the principal cannot promise $w^*$ to incentivize the agent in period 2, however, the agent is fired. In the case of dismissal, the agent may consume the utility in period 1, depending on the value of $x_1$. In sum, there exists the threshold for dismissal, $\hat{y}$, such that the agent is fired if $x_1 \leq \hat{y} = y_1 - \frac{1}{2c} + w^*$. When $x_1$ just beats $\hat{y}$, the agent’s utility in period 1 is $-\frac{1}{2c}$ due to the cost of earnings management, and the agent’s expected utility in period 2 is $w^*$. If $x_1$ is between $y_1$ and $\hat{y}$, the agent is fired but consumes the utility in period 1. If $x_1$ is below $\hat{y}$, the agent is fired without any utility. Then the compensation can be computed considering the utility and the cost of earnings management.

The investment decision by the principal satisfies the following first order condition for $i_1$:

$$
\xi'(i_1) = \frac{1}{1+\rho_P} \left[ E[x_2 - s_2(x_1, x_2)|a_H][1 - F(\hat{y})] + E[x_2^{New} - s_2^{New}(x_2^{New})|a_H]F(\hat{y}) \right] - \frac{(1 - \delta + i_1)}{1+\rho_H} \left[ E[x_2 - s_2(x_1, x_2)|a_H] - E[x_2^{New} - s_2^{New}(x_2^{New})|a_H] \right] f(\hat{y}) \frac{\partial \hat{y}}{\partial i_1}
$$

(2.66)

The LHS is the marginal cost of investment, and the RHS is the marginal benefit of investment. The marginal benefit of investment is equal to the marginal value of increasing the amount of cash flow per unit of capital (Cashflow Effect) plus the marginal value of increasing the probability of replacement of the agent by increasing the firing threshold $\hat{y}$ (Replacement Effect). The replacement effect comes from the fact that the agent’s rent for period 2, $w^* = \frac{1-\delta + i_1}{1+\rho_H} E[s_2(x_1, x_2) - h_2]$, increases with $i_1$. This can be interpreted to mean that if the principal has to promise a higher continuation payoff for period 2, the principal’s ability to punish the agent in the state of low $x_1$ is weakened. Therefore, the principal should rely more on the threat of dismissal to motivate the agent (Spear and Wang, 2005).

Proposition 1 summarizes the contract, and all the terms in the contract are represented by scaled values.
Proposition 1. There exists an optimal contract with

\[
s_1(x_1) = \begin{cases} 
  x_1 - \hat{y} & \text{if } x_1 \geq \hat{y} \\
  x_1 - y_1 + \frac{1}{2c} & \text{if } y_1 < x_1 < \hat{y} \\
  0 & \text{if } x_1 \leq y_1 
\end{cases}
\]

(2.67)

\[
s_2(x_2) = \text{Max}[0, x_2 - y_2]
\]

(2.68)

\[
s_{2\text{New}}(x_{2\text{New}}) = \text{Max}[0, x_{2\text{New}} - y_{2\text{New}}]
\]

(2.69)

where \(y_1\) and \(y_2\) are the unique solution of

\[
\int_{y_1}^{\infty} (x_1 - y_1)[f(x_1|a_H) - f(x_1|a_L)]dx_1 = h_1
\]

(2.70)

\[
\int_{y_2}^{\infty} (x_2 - y_2)[f(x_2|a_H) - f(x_2|a_L)]dx_2 = h_2
\]

(2.71)

and \(\hat{y}\) is the value \(x_1\) such that

\[
x_1 - y_1 = \frac{1 - \delta + i_1}{1 + \rho_A} \mathbb{E}[s_2(x_1, x_2) - h_2] - \frac{1}{2c}
\]

(2.72)

Therefore, the optimal contract satisfies

\[
w(s_1(x_1), s_2(x_1, x_2), r_1(x_1), i_1) = \text{Max}[0, x_1 - y_1]
\]

(2.73)

The agent’s optimal earnings report and earnings management are

\[
r_1(x_1) = \begin{cases} 
  x_1 + \frac{1}{c} & \text{if } x_1 > y_1 \\
  x_1 & \text{if } x_1 \leq y_1 
\end{cases}
\]

(2.74)
The principal’s optimal investment $i_1$ satisfies

$$b(x_1) = \begin{cases} \frac{1}{c} & \text{if } x_1 > y_1 \\ 0 & \text{if } x_1 \leq y_1 \end{cases}$$

(2.75)

The optimal contract can be represented on earnings report, $r_1$, as Corollary 1 states.

**Corollary 1.** Let $r_j = \frac{R_j}{K_j}$ and $r_2^{\text{New}} = \frac{R_2^{\text{New}}}{K_2}$. The optimal contract on earnings report $r_1$ is

$$s_1(r_1) = \begin{cases} r_1 - \frac{1}{c} \hat{y} - \frac{1}{c} & \text{if } r_1 \geq \hat{y} + \frac{1}{c} \\ r_1 - y_1 & \text{if } y_1 + \frac{1}{2c} < r_1 < \hat{y} + \frac{1}{c} \\ 0 & \text{if } r_1 \leq y_1 + \frac{1}{2c} \end{cases}$$

(2.77)

$$s_2(r_2) = \text{Max}[0, r_2 - y_2 + \frac{1}{c}]$$

(2.78)

$$s_2^{\text{New}}(r_2^{\text{New}}) = \text{Max}[0, r_2^{\text{New}} - y_2^{\text{New}} + \frac{1}{c}]$$

(2.79)

Therefore, the optimal contract satisfies

$$w(s_1(r_1), s_2(r_1, r_2), i_1) = \text{Max}[0, r_1 - y_1 - \frac{1}{2c}]$$

(2.80)

Figure 2.2 shows earnings report $r_1$, earnings management $b$, compensation $s_1$, and utility $u_1$ in period 62.
Let the utility $u_1$ be the compensation $s_1$ net of the cost of earnings management, $\frac{1}{2}cb^2$. There exists a discontinuity of reported earnings $r_1$ at $x_1 = y_1$, because the agent over-reports only when $x_1 > y_1$. The agent is fired but receives the compensation in period 1, $s_1$, if $x_1 \leq \hat{y}$. At $x_1 = \hat{y}$, the compensation $s_1$ drops by the future incentives (the utility in period 2), $w^* = \frac{1-\delta+\mu}{1+\rho_1}E[s_2(x_1, x_2) - h_2]$, because the principal can compensate the agent with the future incentives $w^*$ if $x_1 > \hat{y}$. Therefore, the dismissal of the agent is an *ex-ante* incentive device because the agent can enjoy the future incentives only when the performance in period 1 is relatively higher. Thus, the agent is motivated to work hard in period 1.

2.3.4.3 Implications

Due to the agency problem, the investment-capital ratio $i_1$ is lower than the first-best investment-capital ratio $i_{FB}$ in the benchmark. The principal invests less, because of a loss in productivity of ongoing investments in the case of turnover. Also, the investment-capital ratio $i_1$ depends on the volatility of productivity $x_t$, because the principal cannot distinguish the agent’s effort level from noise. Corollary 2 states the effect of the agency problem on the investment-capital ratio $i_1$.

**Corollary 2.** The investment-capital ratio $i_1$ is
(a) lower than the first-best investment-capital ratio $i^{FB}$;
(b) contingent on the volatility of productivity $x_t$

How does the internal control system of the firm affect the principal’s expected payoff and the investment decisions? The internal control system means how much discretion the agent has in reporting the earnings. Under a stronger internal control system, for instance, the agent has less discretion in reporting earnings. In this paper, the cost parameter of earnings management, $c$, captures the property of the internal control system. Therefore, the internal control system becomes stronger as $c$ increases.

To understand the effect of the internal control system $c$ on the contract, it is necessary to understand how the strike price $y_1$ changes as $c$ varies. $y_1$ does not change because the principal can infer the amount of earnings management in equilibrium, and, thus, incentives for $a_H$ remain the same. In other words, without any changes in $y_1$, $c$ affects the amount of compensation for each $x_1$ due to additional cost of earnings management. Another parameter implies the severity of the agency problem: the cost of effort $h_1$. $y_1$ strictly decreases with the cost of effort $h_1$ because it is hard to motivate $a_H$. Proposition 2 summarizes the effect of the internal control system $c$ and the cost of effort $h_1$ on the strike price $y_1$.

**Proposition 2.** The strike price $y_1$ on $x_1$ is
(a) independent of $c$ ($\frac{dy_1}{dc} = 0$);
(b) decreasing in $h_1$ ($\frac{dy_1}{dh_1} < 0$)

It is intuitive that the strike price $y_1 + \frac{1}{2c}$ on $r_1$ is strictly decreasing in $c$ because the agent would over-report less as $c$ increases.

**Corollary 3.** The strike price $y_1 + \frac{1}{2c}$ on $r_1$ is strictly decreasing in $c$ ($\frac{d(y_1 + \frac{1}{2c})}{dc} < 0$).
Then what is the impact of earnings management on the dismissal threshold \( \hat{y} \) and the investment-capital ratio \( i_1 \)? Note that earnings management, influenced by \( c \), has two effects: turnover-reducing benefit and incentive-weakening cost. Figure 2.2 shows that the agent experiences a negative utility in period 1 from \( \hat{y} \) to \( \hat{y} + \frac{1}{2c} \) because of the cost of earnings management, which relaxes the agent’s bankruptcy constraint. Therefore, the principal can loosen the threat of dismissal, as the principal can penalize the agent with the cost of earnings management (\( \frac{d \hat{y}}{dc} > 0 \)). That is, the cost of earnings management can be a substitute for the threat of dismissal, leading to an increase in long-term incentives (turnover-reducing benefit).

The turnover-reducing benefit has an influence on the marginal benefit of investment through cash flow effect and liquidation effect. Because its impact on the cash flow effect dominates, the marginal benefit of investment increases as \( c \) decreases (more earnings management is induced), leading to an increase of \( i_1 \) (\( \frac{di_1}{dc} < 0 \)). In other words, the principal would invest more as the probability of replacement of the agent—which causes a loss in productivity of ongoing investment—decreases as \( c \) decreases. Proposition 3 states the effect of \( c \) on the turnover threshold \( \hat{y} \) and the investment-capital ratio \( i_1 \).

**Proposition 3.** As \( c \) increases, 
(a) the threshold for dismissal \( \hat{y} \) increases (\( \frac{d \hat{y}}{dc} > 0 \));
(b) the investment-capital ratio \( i_1 \) decreases (\( \frac{di_1}{dc} < 0 \))

In the two-period model, the incentive-weakening cost does not have any influence on \( i_1 \), because there is no cost of earnings management in period 2. In an infinite-horizon dynamic model, however, the incentive-weakening cost decreases \( i_1 \).

How does the internal control system \( c \) have an influence on the principal’s expected payoff? The trade-off between the turnover-reducing benefit and the incentive-weakening cost determines the consequences of earnings management on the principal’s welfare. First, through the turnover-reducing benefit, the principal can lower the probability of turnover and increase long-term incentives, which, in turn, decreases the compensation in period 1 (short-term incentives). In other words, an increase in
long-term incentives helps to lower short-term incentives and improve investment efficiency. Second, the incentive-weakening cost can increase the cost of compensation because the principal may need to compensate for the cost of earnings management. Therefore, earnings management can be a cheaper incentive device and improve the principal’s welfare if the total gain from the turnover-reducing benefit dominates the total losses from the incentive-weakening cost.

\[
\frac{d\pi}{dc} = -(1 - \delta + i_1)E\left[\frac{(x_2 - x_2^{New})}{1 + \rho_P} | a_H | f(\hat{y}|a_H) \frac{1}{2c^2}\right] + E\left[\frac{(s_2(x_1, x_2) - h_2)}{1 + \rho_A} | a_H | f(\hat{y}|a_H) \frac{1}{2c^2}\right] + [1 - F(y_1 | a_H)] \frac{1}{2c^2}
\]

Proposition 4 formulates the condition under which the principal’s expected payoff increases as \( c \) decreases.

**Proposition 4.** The expected compensation is strictly decreasing in \( c \). The principal’s expected payoff increases as \( c \) decreases (more earnings management is induced) if and only if

\[
(1 - \delta + i_1)E\left[\frac{(x_2 - x_2^{New})}{1 + \rho_P} | a_H | f(\hat{y}|a_H)\right] + E\left[\frac{(s_2(x_1, x_2) - h_2)}{1 + \rho_A} | a_H | f(\hat{y}|a_H)\right] > 1 - F(y_1 | a_H)
\]

When does the turnover-reducing benefit dominate the incentive-weakening cost as \( c \) decreases? The turnover-reducing benefit can dominate when long-term incentives are low. In contrast, the incentive-weakening cost can dominate when long-term incentives are high. The intuition is that the benefit of increasing long-term incentives decreases as long-term incentives increases. Note that long-term incentives are low when \( \hat{y} \) is large and the probability of turnover is high. Therefore, the turnover-reducing benefit dominates when when \( \hat{y} \) is large. Corollary 4 summarizes how \( \hat{y} \) affect the trade-off between the turnover-reducing benefit and the incentive-weakening cost.
Corollary 4. The principal’s expected payoff increases as \( c \) decreases if the probability of turnover is relatively high (\( \hat{y} \) is large given \( i_1 \)). Therefore, there exist \( \bar{y}(i_1) \), such that

\[
\frac{d\pi}{dc} < 0 \text{ if } \hat{y} > \bar{y}(i_1)
\]

The internal control system \( c \) also influences the trade-off, because \( c \) increases \( \hat{y} \) by Proposition 3. Thus, the turnover-reducing benefit dominates when \( c \) is large. Therefore, Corollary 4 implies that a decrease in \( c \) increases the principal’s expected payoff when \( c \) is large. In the next section, the infinite-horizon dynamic model will show that the relationship between \( c \) and the principal’s welfare is not monotonic.

The two-period model has some limitations. First, the two-period model does not allow for rebalance of earnings management. Firms sometimes underreport to rebalance earnings management if they have managed aggressively in the past. An example would be big baths. In the two-period model, however, only overstatement is observed. Second, the investment is not dependent on the past investment in the two-period model. Capital stock that reflects the past investment has an impact on investment in the current period. Third, the optimal compensation contract does not depend on the past performance in the two-period model. In practice, past performance affects pay-performance sensitivity and the probability of dismissal is especially contingent on past performance. In the two-period model, the probability of dismissal is determined by the set of parameters given exogenously. The infinite-horizon dynamic model in the next section will discuss how the history of the firm (endogenized long-term incentives) determines the possibility of dismissal. Also, the infinite-horizon model sheds light on how the history of the firm affects the trade-off between the turnover-reducing benefit and the incentive-weakening cost, which has an influence on the firm’s investment decisions as well as the principal’s welfare.
2.4 Dynamic Model

The two-period model shows that the cost of earnings management can be a substitute for the threat of replacement of the agent. Earnings management can relax the bankruptcy constraint by penalizing the agent with the cost of earnings management when her performance is poor and, thus, can help to lower the threshold for replacement. I extend this intuition to an infinite-horizon dynamic model to examine how the effect of earnings management varies, depending on the history of firms over time. More specifically, the dynamic model can explain how the effect of earnings management changes cross-sectionally between firms with different histories, such as past performance and earnings management.

In this section, I describe the basic framework in discrete time but solve the model in continuous time. A continuous time model is a model that is achieved when taking the time interval in a discrete time model to the limit. First of all, it makes the model analytically tractable and enables dynamic insight that we cannot achieve in the two-period model. Second, we can interpret the property of a continuous time model such that the principal is the investors who provide capital to the firm, and the investors decide the amount of capital based on news about the firm, which is released continuously to the market.

In the full dynamic model, all the basic elements of the two-period model remain the same, except that the agent’s productive effort is continuous in \([0, \infty)\) and the cost of effort is \(H(a_t) = \frac{1}{2}c_a K_t a_t^2\).\(^{15}\)

2.4.1 Optimal Contracting Problem

The principal offers a contract that specifies cash compensation \(S_t\), earnings report \(R_t\), the firm’s investment policy \(I_t\), and an endogenous stochastic termination time \(\tau\). I denote the contract by \(\Theta = (S, R, I, \tau)\).

The agent chooses \(a_t\) and \(R_t\) to solve the following problem:

\[
W(\Theta) = \max_{a_t, R_t} E^\lambda \left[ \sum_{t=1}^{\tau-1} \frac{1}{(1 + \lambda)^t} \left( S_t - H(a_t) - G(M_t + (R_t - e_t)) \right) \right] \tag{2.84}
\]

\(^{15}\)The accumulated discretionary accruals \(M_t\) are not assumed to be 0 in the dynamic model. It changes every period according to earnings management in the previous periods.
where $E^A[\cdot]$ is the expectation operator under the probability measure that is induced by an action process 
\$
\{a_t, B_t : 0 \leq t < \tau\}.
\$
The agent maximizes the present discounted value of cash compensation minus the cost of effort and the cost of earnings management.

Given an initial payoff $W_1$ for the agent and capital stock $K_1$, the principal’s problem is

$$V(W_1, K_1) = \max_{\Theta} E^{\Theta} \left[ \sum_{t=1}^{\tau-1} \frac{1}{(1+r)^{t-1}} (e_t - \xi(i_t))K_t + \frac{1}{(1+r)^{\tau-1}} LK\tau - \sum_{t=1}^{\tau-1} \frac{1}{(1+r)^{t-1}} S_t \right]$$

(2.85)

s.t. $\Theta$ is IC

$$W(\Theta) \geq W_1$$

(2.86)

(2.87)

The principal’s objective function maximizes the expectation of discounted value of cash flow, the termination value at time $\tau$, and net of the agent’s cash compensation. The equation (2.87) is the Promise-Keeping constraint. The agent’s expected payoff $W_1$ is determined by the relative bargaining power between the principal and the agent. When the contract is terminated and the agent is replaced, there is a loss in productivity of ongoing projects. I formulate a cost of lost productivity such that the principal’s expected payoff per unit of capital decreases by $\ell$ percent in the case of turnover.

$$L = \max_{w} (1-\ell) \frac{V(W, K_\tau)}{K_\tau}$$

(2.88)

$$= \max_{w} (1-\ell)v(w)$$

(2.89)

where $v(w) = \frac{V(W, K)}{K}$. 
2.4.2 Model Solution and Optimal Contracting

2.4.2.1 Benchmark

I first consider a benchmark case where the agent’s action is observable. It is trivial that no earnings management is induced. The principal’s problem is

\[
\text{Max } E \left[ \sum_{t=1}^{\tau-1} \frac{1}{(1+\rho_P)^{t-1}} (e_t - \xi(i_t)K_t) + \frac{1}{(1+\rho_P)^{\tau-1}} LK_t - \sum_{t=1}^{\tau-1} \frac{1}{(1+\rho_P)^{t-1}} S_t \right]
\]

(2.90)

s.t. \( E^A \left[ \sum_{s=1}^{\tau-1} \frac{1}{(1+\rho_A)^{t-1}} (S_t - H(a_t)) \right] \geq W_1 \)

(2.91)

Then, the optimal effort level \( a_{FB}^t \) and investment-capital ratio \( i_{FB}^t \) satisfy

\[
a_{FB}^t = \frac{\bar{x}}{c_a} \]

(2.92)

\[
\xi'(i_{FB}^t) = \frac{a_{FB}^t \bar{x} - \xi(i_{FB}^t)}{\rho_P - (i_{FB}^t - \delta)}
\]

(2.93)

The equation (2.93) shows that the value of one unit of capital is equal to the perpetuity value of its expected earnings net of the cost of investment with the net growth rate of \( i_{FB}^t - \delta \). I assume that \( \frac{a_{FB}^t \bar{x} - \xi(i_{FB}^t)}{\rho_P - (i_{FB}^t - \delta)} > L \), so that the firm is productive and, thus, replacing the agent is not efficient.

If the principal promises the agent a payoff \( W_1 \), then it is optimal to pay the agent \( W_1 \) in cash immediately because \( \rho_A > \rho_P \). Thus, the principal’s expected payoff is

\[
V_{FB}(W_1, K_1) = E \left[ \sum_{t=1}^{\infty} \frac{1}{(1+\rho_P)^{t-1}} (x_t - \xi(i_t))K_t \right] - W_1
\]

(2.94)

\[
= \frac{a_{FB}^t \bar{x} - \xi(i_{FB}^t)}{\rho_P - (i_{FB}^t - \delta)} K_1 - W_1
\]

(2.95)

The above equation is derived from the fact that the average value of capital is the same as the marginal value of capital \( \frac{a_{FB}^t \bar{x} - \xi(i_{FB}^t)}{\rho_P - (i_{FB}^t - \delta)} \) when the production function has constant returns to scale (Hayashi, 1982).
Also, we can write this on a per unit of capital basis

\[ v^{FB}(w) = \frac{V^{FB}(W_1, K_1)}{K_1} = \frac{a^{FB} \bar{x} - \xi(i^{FB}_t)}{\rho_p - (i^{FB} - \delta)} - w_1 \]  

(2.96)

where \( w_1 = \frac{W_1}{K_1} \). Lemma 2 summarizes the principal’s expected payoff, the effort level, and the investment-capital ratio in the benchmark case.

**Lemma 2.** In the benchmark case, the principal’s scaled expected payoff is

\[ v^{FB}(w) = \frac{a^{FB} \bar{x} - \xi(i^{FB}_t)}{\rho_p - (i^{FB} - \delta)} - w_1 \]  

(2.97)

where the effort level \( a^{FB}_t = \frac{1}{c_w} \) and the investment-capital ratio \( i^{FB}_t \) satisfies \( \xi'(i^{FB}_t) = \frac{a^{FB} \bar{x} - \xi(i^{FB}_t)}{\rho_p - (i^{FB} - \delta)} \).

In the benchmark case where there is no agency concern, the first-best investment-capital ratio is constant over time, regardless of the firm’s history or the volatility of productivity. In other words, idiosyncratic productivity shocks do not affect the firm’s investment or firm value. The next section will show that the agency problem will change these outcomes significantly.

### 2.4.2.2 Optimal Contract

Now I want to derive the optimal contract when the agent’s action is not observable. The optimal contract is incentive compatible with the agent’s productive effort and truth-telling and maximizes the principal’s value function, \( V(W, K) \).

The agent’s continuation payoff, or the discounted expected value of the agent’s future payoff, at time \( t \) is

\[ W_t(\Theta) = E_t \left[ \frac{1}{(1 + \rho_A)^{\tau-t}} (S_t - H(a_t) - G(M_t + (R_t - e_t))) \right] \]  

(2.98)

71
Lemma 3. Given the agent’s cash compensation $S_t$, recommended effort $a_t$ and earnings report $R_t$, the agent’s continuation payoff $W_t$ evolves as follows:

$$W_{t+1} - W_t = \rho A W_t - (S_t - H(a_t) - G(M_t + (R_t - e_t))) + \varphi(W_t)(e_t - a_t \bar{k}_t)$$  \hspace{1cm} (2.99)

where $\varphi$ is the sensitivity of the agent’s continuation payoff to earnings, $e_t$ (pay-performance sensitivity (PPS)).

$\rho A W_t - (S_t - H(a_t) - G(M_t + (R_t - e_t)))$ is the expected change of the agent’s continuation payoff $W_t$, according to the Promise-Keeping constraint when the agent takes the recommended effort $a_t$ and earnings report $R_t$. $W_t$ grows at the interest rate $\rho A$ to compensate for the agent’s time preference and decreases because of the repayments $(S_t - H(a_t) - G(M_t + (R_t - e_t)))$. When the agent takes the recommended effort level $a_t$ and earnings report $R_t$, the mean of $(e_t - a_t \bar{k}_t)$ is 0 and, thus, $\rho A W_t - (S_t - H(a_t) - G(M_t + (R_t - e_t)))$ is the expected change of $W_t$. To be incentive compatible, $W_t$ should be sensitive to earnings $e_t$ through the sensitivity $\varphi(W_t)$. The sensitivity $\varphi(W_t)$ influences the agent’s effort incentives and earnings management. The agent chooses the effort and earnings management
that maximize the expected change of $W_t$ minus the cost of effort and the cost of earnings management:

$$\varphi(W_t)(a_t\bar{x}K_t + B_t) - H(a_t) - G(M_t + (R_t - e_t))$$  \hfill (2.100)

Thus, the optimal action choices are

$$a(W_t) = \frac{\varphi(W_t)\bar{x}}{c_a}$$  \hfill (2.101)
$$R_t(W_t) - e_t = B(W_t) = \frac{\varphi(W_t)K_t}{c} - M_t$$  \hfill (2.102)

It is intuitive that the agent’s effort is strictly increasing in sensitivity $\varphi(W_t)$ and decreasing in $c_a$, and earnings management is strictly increasing in sensitivity $\varphi(W_t)$ and decreasing in $c$ and $M_t$.

After the history of the firm’s performance and earnings management up to time $t$, the relevant state variables are the agent’s continuation payoff $W_t$, accumulated discretionary accruals $M_t$, and capital stock $K_t$. Substituting the optimal effort $a(W_t)$ and earnings report $R(W_t)$ into (2.99), the principal’s problem reduces to the problem with two state variables: $W_t$ and $K_t$. Then the continuation payoff $W_t$ evolves as follows:

$$W_{t+1} - W_t = \rho_A W_t - (S_t - H(\varphi(W_t)\bar{x}) - G(\frac{\varphi(W_t)K_t}{c})) + \varphi(W_t)\bar{x} - \xi(i)K - S + \frac{1}{1+\rho_p}V(W', K')$$  \hfill (2.103)

Now we consider the optimal contract which describes the optimal choice of cash compensation $S(W)$, sensitivity $\varphi(W)$, and investment-capital ratio $i(W)$. Let the value function $V(W, K)$ be the highest profit that the principal can derive when the agent’s continuation payoff $W$ and capital stock is $K$. The principal’s problem is

$$V(W, K) = \max_{\varphi, S, i} E[(x - \xi(i))K - S + \frac{1}{1+\rho_p}V(W', K')]$$  \hfill (2.104)
$$= \max_{\varphi, S, i} E[(\varphi(W)\bar{x} - \xi(i))K - S + \frac{1}{1+\rho_p}E[V(W', K')]]$$  \hfill (2.105)

\footnote{$a_t$ and $B_t$, in the expected change of the agent’s continuation payoff $W_t$, $\rho_A W_t - (S_t - h(a_t) - g(M_t + B_t))$, are the principal’s conjecture on $a_t$ and $B_t$.}
\[
\begin{align*}
\text{s.t. } \dot{w} - w &= \rho_A w - (s - \frac{1}{2} c_a K \left( \frac{\varphi(w) \bar{x}}{c_a} \right)^2 - \frac{1}{2} c \left( \frac{\varphi(w) K}{c} \right)^2) + \varphi(w) K \sigma \varepsilon \\
\end{align*}
\]

\[
\begin{align*}
K' - K &= (i - \delta) K \\
V(W_{\tau}, K_{\tau}) &= L K_{\tau}
\end{align*}
\]

Similar to the model in DeMarzo et al. (2012), the tractability of the model comes from the scale invariance of the firm’s technology, which allows us to reduce the problem to one with a single state variable \( w = \frac{W}{K} \). That is,

\[
V(W, K) = v(w) K
\]

Now the principal’s scaled problem can be represented as follows:

\[
v(w) = \max_{\varphi, s, i} \varphi(w) \bar{x} - \xi(i) - s + \frac{1 - \delta + i}{1 + \rho_p} E[v'(w)]
\]

\[
\begin{align*}
\text{s.t. } \dot{w} - w &= (\rho_A - (i - \delta)) w - (s - \frac{1}{2} c_a \left( \frac{\varphi(w) \bar{x}}{c_a} \right)^2 - \frac{1}{2} c \left( \frac{\varphi(w) K}{c} \right)^2) + \varphi(w) \sigma \varepsilon + n \\\nv(w_{\tau}) &= L = \max_w (1 - \ell) v(w)
\end{align*}
\]

where \( s = \frac{S}{\bar{x}} \), \( b = \frac{B}{\bar{x}} \), \( m = \frac{M}{\bar{x}} \), and \( n \) is a remainder from the Taylor’s expansion.

The agent’s scaled continuation payoff \( w \) grows at the interest rate \( \rho_A \) less the net growth rate \( i - \delta \) of the firm and decreases due to the scaled repayment \( s - \frac{1}{2} c_a \left( \frac{\varphi(w) \bar{x}}{c_a} \right)^2 - \frac{1}{2} c \left( \frac{\varphi(w) K}{c} \right)^2 \) on average.

The dynamic insight comes from the technical advantage of continuous-time methods, which enable us to discuss important properties analytically and lead to a simpler computational procedure. From here, I solve the model using continuous-time methods. The proof is provided in Appendix. Note that the principal cannot provide a negative continuation payoff to the agent given the limited liability \( W_{\tau} \geq 0 \), because the agent can obtain non-negative payoff by deviating from the recommended action. Therefore, the only way to give value 0 to the agent is to fire the agent. In other words, the agent is replaced if \( w \) becomes 0.
The evolution of $w$ is as follows:

$$
dw_t = (\rho_A - (i_t - \delta))w_t dt - (s - \frac{1}{2} c_a(\frac{\varphi(w)}{c_a})^2) \frac{\sigma}{c} dZ_t
$$  \hspace{1cm} (2.113)

where $Z = \{Z_t, F_t; 0 \leq t < \infty\}$ is a standard Brownian motion on a complete probability space.

The Hamilton-Jacobi-Bellman (HJB) equation for the principal’s expected payoff is

$$
\rho_P v(w) = \sup_{\varphi, s, i} \frac{\varphi(w)\bar{x}}{c_a} - \xi(i) - s + v(w)(i - \delta) - v'(w)(s - \frac{1}{2} (\varphi(w))\bar{x})\frac{\sigma}{c} + w v'(w)(\rho_A - (i - \delta)) + \frac{1}{2} v''(w) \ast (\varphi(w))^2 \sigma^2
$$  \hspace{1cm} (2.114)

The RHS consists of instantaneous expected cash flow, the expected change in the value due to capital accumulation, and the expected change in the value due to the drift and volatility of the agent’s scaled continuation payoff $w$.

The optimal choice of the scaled cash compensation $s$ maximizes

$$
-s - v'(w)s
$$  \hspace{1cm} (2.115)

Let $\bar{w}$ be $w$ satisfying $v'(\bar{w}) = -1$. Then, the scaled cash compensation $s$ is 0 when $w$ is in $[0, \bar{w}]$. And for $w > \bar{w}$, $s = w - \bar{w}$ and $v(w) = v(\bar{w}) - (w - \bar{w})$. In other words, in the optimal contract, the scaled cash compensation $s$ is 0 when the agent’s scaled continuation payoff $w$ is small, because the principal can decrease the possibility of replacement of the agent by deferring the agent’s scaled cash compensation. When the scaled cash compensation is deferred, however, the agent requires a higher interest rate, due to $\rho_A > \rho_P$. The trade-off between the benefit and the cost of deferring the agent’s scaled cash compensation yields a threshold $\bar{w}$, such that it is optimal to defer the scaled cash compensation if $w \leq \bar{w}$ and pay the scaled cash compensation otherwise.

Now I would like to discuss optimal sensitivity $\varphi(w)$ and effort $a(w)$. From the equation (2.114), optimal
sensitivity $\varphi(w)$ maximizes

$$\frac{\varphi(w)\bar{x}^2}{c_a} - \nu'(-\frac{\varphi(w)\bar{x}^2}{2c_a} - \frac{1}{2}\frac{(\varphi(w))^2}{c}) - \frac{1}{2}\nu''(w)\varphi(w)(\varphi(w))^2\sigma^2 = 0 \quad (2.116)$$

where $\frac{\varphi(w)\bar{x}^2}{c_a}$ is the expected earnings, $\nu'(-\frac{\varphi(w)\bar{x}^2}{2c_a} - \frac{1}{2}\frac{(\varphi(w))^2}{c})$ is the cost of compensating the agent for the productive effort and earnings management, and $-\frac{1}{2}\nu''(w)\varphi(w)(\varphi(w))^2\sigma^2$ is the cost of exposing the agent to the dismissal, which leads to inefficiency. Then, the optimal sensitivity $\varphi(w)$ is

$$\varphi(w) = \frac{\bar{x}^2}{\nu'(-\frac{\varphi(w)\bar{x}^2}{2c_a} + \frac{1}{c}) - \nu''(w)\sigma^2} \quad (2.117)$$

where $-\nu'(-\frac{\varphi(w)\bar{x}^2}{2c_a} + \frac{1}{c}) - \nu''(w)\sigma^2 > 0$.

From the above HJB equation (2.114), the first order condition for the optimal investment-capital ratio $i(w)$ is given by

$$\xi'(i(w)) = \nu(w) - w\nu'(w) \quad (2.118)$$

The first order condition for the optimal investment-capital ratio shows that the marginal cost of investment per capital (RHS, $\xi'(i(w))$), is equal to the marginal benefit of investment per capital (LHS), $\nu(w) - w\nu'(w)$. In RHS, the first term, $\nu(w)$, is the current value of the principal’s expected payoff per unit of capital and the second term, $w\nu'(w)$, is the marginal effect of decreasing the agent’s scaled continuation payoff $w$ by investing. Proposition 5 summarizes the main results regarding the optimal contract.

**Proposition 5.** The principal’s scaled value function $\nu(w)$ solves the equation (2.114) with boundary conditions $\nu(w_\tau) = M_w(1 - \ell)v(w)$ and $\nu'(\bar{w}) = -1$, for $w \in [0, \bar{w}]$. $\nu(w) = \nu(\bar{w}) - (w - \bar{w})$, for $w > \bar{w}$. The agent’s scaled continuation payoff $w$ evolves according to the equation (2.113). The scaled cash compensation $s$ is 0 for $w \leq \bar{w}$ and $w - \bar{w}$ for $w > \bar{w}$. The optimal sensitivity $\varphi(w)$ satisfies the equation
Figure 2.3: Scaled Value Function \( v(w) \) and investment-capital Ratio \( i(w) \)

The optimal effort level \( a(w) \) is \( \frac{\varphi(w)}{c_a} \). The optimal earnings report \( R(w) = r(w)K \) and earnings management \( B(w) = b(w)K \) satisfy \( r(w) - x_t = b(w) = \frac{\varphi(w)}{c} - m \). The optimal investment is \( I = i(w)K \), where \( i(w) \) solves the equation (2.118). The contract is terminated at \( \tau \) such that \( w_{\tau} = 0 \).

The above HJB equation can be rewritten in the following form that makes computation easier:

\[
v''(w) = \min_{\varphi, i, s} \left[ \rho P v(w) - \frac{\varphi(w)^2}{c_a} + \xi(i) + s - v(w)(i - \delta) + v'(w)(s - 1) \left( \frac{\varphi(w)^2}{c} \right) - wv'(w)(\rho - (i - \delta)) \right] \frac{1}{2}(\varphi(w))^2 \sigma^2
\]

Figure 2.3 and 2.4 describe the value function \( v(w) \), sensitivity \( \varphi(w) \), effort \( a(w) \), and investment-capital ratio \( i(w) \) in the optimal contract. The intuition of the optimal contract is discussed in the next section.

### 2.4.2.3 Implications

Figure 4 shows the concavity of the principal’s scaled value function \( v(w) \), which comes from two effects of the agent’s scaled continuation payoff \( w \): the incentive alignment effect and the wealth transfer effect. First, the incentive alignment effect means a higher \( w \) allows the principal to provide incentives to the agent with a lower probability of turnover. This incentive alignment effect increases the total surplus of the principal’s
Figure 2.4: Sensitivity $\phi$, Effort $a$, Total Discretionary Accruals $b + m$

and the agent’s expected payoff. Second, the wealth transfer effect implies that a higher $w$ decreases the principal’s value, holding the total surplus fixed, because the principal needs to pay more compensation to the agent. The concavity of the principal’s scaled value function $v(w)$ implies that the incentive alignment effect dominates the wealth transfer effect when $w$ is low. The wealth transfer effect, however, dominates the incentive alignment effect when $w$ is high. Because earnings management also has two effects, the turnover-reducing benefit and the incentive-weakening cost, earnings management affects the principal’s welfare through its two effects on the evolution of $w$, which captures long-term incentives. Before discussing the effect of earnings management more precisely, we need to understand the basic properties of the optimal long-term contract.

Figure 2.3 shows that the investment-capital ratio $i(w)$ increases with $w$. The intuition is that the principal invests less when $w$ is low because of the higher probability of turnover. Therefore, the principal increases investment as $w$ increases.

How does the optimal sensitivity (PPS) $\phi(w)$ change as $w$ increases? The sensitivity $\phi(w)$ increases with $w$ as Figure 2.4 depicts. The intuition is that the principal would want $w$ to be sufficiently sensitive to

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17The marginal benefit of investing near termination is $\ell$ per unit of capital ($c'(i(0)) = \ell$) and I assume that $c'(0) > \ell$. Therefore, the principal disinvests near turnover.
performance in order to give incentives to the agent. When \( w \) is low, however, the principal cannot set \( \varphi(w) \) high because the probability of turnover is high. Therefore, the principal can increase \( \varphi(w) \) as \( w \) increases. In other words, the level of incentives (\( \varphi(w) \)) increases as long-term incentives (\( w \)) increase, because moral hazard problems become less severe as \( w \) increases.\(^{18}\) The above intuition is carried over to the pattern of optimal effort \( a(w) \). The optimal level of effort \( a(w) \) increases with \( w \).

The optimal earnings management per unit of capital \( b(w) \) is determined by the agent’s trade-off between the benefit of influencing \( w \) and her current biasing costs. The benefit of earnings management depends on PPS \( \varphi(w) \), whereas the cost of earnings management depends on the accumulated discretionary accruals per unit of capital, \( b(w) + m \), and the internal control system \( c \). Figure 2.4 shows that the accumulated discretionary accruals per unit of capital \( b(w) + m \) increases with \( w \) because \( \varphi(w) \) increases with \( w \). Putting it differently, when \( w \) is high, the agent’s incentives for earnings management to receive the bonuses becomes stronger. Therefore, the agent wants to manage more earnings as \( w \) increases \( \Bar{w} \).

Figure 2.5 shows the relationship between the accumulated discretionary accruals and investment for a given internal control system within a firm. As \( w \) increases (moral hazard problems become less severe), the principal can implement the higher PPS and increase investment. Because the marginal benefit of managing earnings depends on PPS, earnings management also increases with \( w \). Therefore, there is a positive relationship between earnings management and investment that is consistent with the empirical evidence in Kedia and Philippon (2007) and McNichols and Stubben (2008) who show that earnings management increases a firm’s investment. The illustrated mechanism in my paper is different from their explanations in that this paper shows that earnings management enhances investment efficiency and that the increase in investment by earnings management is the outcome of the optimal decision for shareholders, not a suboptimal decision for shareholders.

\(^{18}\)In the maximization problem for \( \varphi(w) \) (the equation (2.116)), the two effects related to costs, the cost of compensating the agent for the effort and earnings management, \( (v'(w)(-\frac{1}{2} \frac{(\varphi(w))}{c} - \frac{1}{2} \frac{(\varphi(w))^2}{c})) \), and the cost of exposing the agent to the dismissal, \( (-\frac{1}{2}v''(w) * (\varphi(w))^2 \sigma^2) \), typically work in opposite directions. \( v'(w) \) decreases in \( w \) due to the concavity of \( v(w) \), whereas \( v''(w) \) increases over some ranges of \( w \). However, with the forms of cost functions of effort and earnings management and the values of parameters provided in the model, the cost of exposing the agent to dismissal dominates the other.
Figure 2.5: The Relationship between Accumulated Discretionary Accruals \((b(w) + m)\) and Investment-Capital Ratio \(i(w)\)

How does the internal control system \(c\) affect the investment-capital ratio \(i(w)\)? This helps to understand the relationship between the internal control system and investment across firms with different levels of internal control systems. The internal control system \(c\) affects not only the turnover-reducing benefit but also the incentive-weakening cost of earnings management. In other words, weakening \(c\) induces more earnings management, which involves the turnover-reducing benefit and the incentive-weakening cost. Specifically, by weakening the internal control system, earnings management can increase \(w\) (long-term incentives) by substituting for the threat of turnover. On the other hand, the cost of compensation also increases as \(c\) decreases. If the turnover-reducing benefit dominates, the investment-capital ratio \(i(w)\) increases. In contrast, if the incentive-weakening cost dominates, the investment-capital ratio \(i(w)\) decreases. In the infinite-horizon dynamic model, the incentive-weakening cost decreases the investment-capital ratio \(i(w)\) because the incentive-weakening cost lowers the principal’s stake in the firm and, thus, reduces the marginal value of investment.\(^{19}\) Figure 2.6 shows the

\(^{19}\)In the two-period model, only the incentive alignment effect has an influence on investment, because earnings management made in period 1 is reversed in period 2 and, thus, there is no cost of earnings management in period 2. In the full dynamic model, however, the agent can manage earnings every period and, thus, trade-off between the incentive alignment effect and the wealth transfer effect over whole periods needs to be considered. As the wealth transfer effect increases, the marginal return of investment decreases because the compensation per unit of capital increases.
investment-capital ratio \(i(w)\) under three different values of the internal control system \(c\), where \(c_H > c_M > c_L\).

When \(w\) is high, the incentive-weakening cost always dominates because the benefit of increasing long-term incentives \((w)\) is small. Therefore, \(i(w)\) increases with \(c\) for high \(w\). Yet, when \(w\) is low, the turnover-reducing benefit can dominate initially as \(c\) decreases. The incentive-weakening cost, however, becomes stronger as well as \(c\) decreases, because the cost of compensation increases due to earnings management. As the internal control system becomes even weaker than a certain point of \(c\), the incentive-weakening cost starts to dominate. Therefore, the relationship between \(i(w)\) and \(c\) is an inverted U-shape for low \(w\), whereas \(i(w)\) increases with \(c\) for high \(w\), as seen by Figure 2.7.

How does the internal control system \(c\) affect sensitivity \(\varphi(w)\) as well as the effort \(a(w)\)? As the equation (2.116) shows, \(\varphi(w)\) is determined by the trade-off between the expected marginal earnings and two costs: the cost of compensating for the effort and earnings management and the cost of exposing the agent to dismissal. The internal control system \(c\) has an influence on these two costs. More specifically, lowering \(c\) decreases the probability of turnover, leading to a decrease of the cost of exposing the agent to dismissal (turnover-reducing benefit). On the other hand, lowering \(c\) increases the cost of earnings management and
thus raises the compensation to the agent (incentive-weakening cost). Similarly, when \( w \) is low, the turnover-reducing benefit of lowering \( c \) dominates initially and thus the principal can increase \( \varphi(w) \). As \( c \) becomes weaker than a certain point of \( c \), the incentive-weakening cost dominates. When \( w \) is high, however, the incentive-weakening cost dominates and thus the principal has to decrease \( \varphi(w) \). Because the effort \( a(w) \) is proportional to \( \varphi(w) \), it shows the same pattern as \( \varphi(w) \). In other words, as the turnover-reducing benefit dominates, it is cheaper to incentivize the agent and, therefore, a higher level of effort is implemented. As the incentive-weakening cost dominates, however, it is more expensive to incentivize the agent, and, thus, a lower level of effort is implemented. Figure 2.8 shows the patterns of sensitivity \( \varphi(w) \) and effort \( a(w) \) under three different values of \( c \), where \( c_H > c_M > c_L \).

The effect of internal control system \( c \) on the principal’s value function \( v(w) \) is summarized in Figure 2.9. \( v(w) \) increases as the turnover-reducing benefit dominates, because the turnover-reducing benefit enhances not only investment efficiency but also the agent’s incentives. On the other hand, \( v(w) \) decreases as the incentive-weakening cost dominates, because the incentive-weakening cost not only reduces investment efficiency but also increases the cost of compensation. Based on similar intuition as above, \( v(w) \) increases as \( c \) increases for high \( w \), whereas the relationship between \( v(w) \) and \( c \) is an inverted U-shape for low \( w \).
Figure 2.8: The Effect of $c$ on Sensitivity $\phi$ and Effort $a$

Figure 2.9: The Effect of $c$ on Principal’s Value Function $v(w)$
Lastly, the pattern of earnings management can be understood from the following equation:

\[ b(w) = \frac{\phi(w)}{c} - m \]  

(2.120)

It is intuitive that the agent overreports as \( \phi(w) \) increases and \( m \) decreases and underreports as \( \phi(w) \) decreases and \( m \) increases. Because \( \phi(w) \) increases with \( w \) in general, the agent overreports as \( w \) increases and \( m \) decreases and underreports as \( w \) decreases and \( m \) increases. Figure 2.10 shows the pattern of earnings management with respect to \( \phi(w) \), \( w \), and \( m \).

As seen by Figure 2.10, the model provides us with an opportunity to study why and when new CEOs take big baths. The new CEOs’ initial continuation payoff \( w \) is determined by the relative bargaining power between shareholders and new CEOs. Because shareholders have more bargaining power than CEOs when signing the contract in general, new CEOs’ initial continuation payoff \( w \) is low. Therefore, pay-performance sensitivity \( \phi(w) \) is low, and, therefore, more big baths are observed when new CEOs are hired. In addition, new CEOs take big baths more often when the previous CEOs have managed earnings more aggressively and carried more accumulated accruals.
2.5 Calibration

2.5.1 Data

I collect financial statements data from COMPUSTAT. The initial sample of data consists of firm-year observations from 1974 to 2013. I remove all regulated (SIC 4900-4999) and financial firms (SIC 6000-6999). Investment is capital expenditures (CAPX) net of sales of property (SPPE). The investment-capital ratio is the investment in year $t$ normalized by gross property and equipment (PPEGT) in year $t - 1$. Discretionary accruals are measured using the model by Jones (1991). Observations with missing current assets, cash and short-term investment, current liabilities, current maturities of long-term debt, income taxes payable, depreciation and amortization, total assets, total revenues, gross property and equipment, capital expenditures, sales of property, and SIC code are excluded.

2.5.2 Calibration

I set the annual interest rate to $\rho_P = 3.48\%$ and the agent’s discount rate $\rho_A = 3.98\%$. And I set the rate of depreciation to $\delta = 10.8\%$ and use quadratic adjustment costs with $\theta = 0.3$. Finally, the loss in productivity in case of replacement is $l = 3\%$, following Bolton, Chen, and Wang (2011) and Nikolov and Schmid (2012). The calibration result is summarized in the Table 2.1.

To reproduce the empirical moments of investment and earnings management, I calibrate the remaining parameters. For a given set of four parameters, including the internal control system $c$, the expected productivity $\bar{x}$, the volatility of productivity $\sigma$, and the cost parameter for the effort $c$, I simulate a panel of 1,000 firms for 100 periods. I compute cross-firm averages of four moments: mean and standard deviation of investment-capital ratio, and mean and standard deviation of earnings management. Finally, the simulated moments are compared with the empirical counterparts. I search for a set of parameter values with which the simulated moments are the closest to the empirical counterparts. I present the calibrated parameter values and moments in Table 2.1.
This table compares the simulated moments with empirical moments of firms’ investment and earnings management. All moments are annualized. Averages and standard deviations of investment-capital ratio and earnings management are the average across firm-level estimates. I employ parameter values for basic inputs from the existing literature: the annual interest rate $\rho_p = 3.48\%$, the agent’s discount rate $\rho_A = 3.98\%$, the rate of depreciation $\delta = 10.8\%$, and quadratic adjustment costs $\theta = 0.3$. The loss in productivity in case of replacement is $\ell = 3\%$. I search for other parameters with which the simulated moments are the closest to the empirical counterparts. The chosen parameters are the internal control system $c = 68$, the expected productivity $\bar{x} = 0.581$, the volatility of productivity $\sigma = 0.15$, and the cost parameter for the effort $c_a = 1.09$.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment-Capital Ratio</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.1014</td>
<td>0.1036</td>
</tr>
<tr>
<td></td>
<td>(0.0256)</td>
<td>(3.3296e−04)</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.0494</td>
<td>0.0155</td>
</tr>
<tr>
<td></td>
<td>(0.0128)</td>
<td>(0.0028)</td>
</tr>
<tr>
<td><strong>Earnings Management</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.0023</td>
<td>−1.0501e − 04</td>
</tr>
<tr>
<td></td>
<td>(0.0380)</td>
<td>(2.2705e−04)</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.1073</td>
<td>0.1301</td>
</tr>
<tr>
<td></td>
<td>(0.0435)</td>
<td>(0.0359)</td>
</tr>
</tbody>
</table>

From the first-best investment-capital ratio in Lemma 2, we can compute the first-best investment-capital ratio using the parameters we used to calibrate the model: the first-best investment-capital ratio is $i^{FB} = 15\%$. Therefore, the agency problem results in underinvestment of 32.4\% on average. The underinvestment, as well as the possibility of contract termination due to the agency problem, leads to a reduction in the value of the firm of 2.56\%. Furthermore, the fact that the average of discretionary accruals is near zero shows the reversal of earnings management in the long run. A more volatile earnings pattern can exist in the short run, but past earnings management is eventually reversed in the long run. A counterfactual analysis shows that earnings management under the current level of the internal control systems in the market improves shareholders’ value by 0.4\% compared to that of counterfactual under the strongest internal control system.


2.6 Conclusion

The paper studies the effect of earnings management on investment efficiency in a dynamic contract setting. In a dynamic contract, the principal can motivate the agent with not only short-term incentives but also long-term incentives, including the threat of turnover. Turnover is, however, a costly incentive device from the principal’s perspective because of its inefficiencies. In other words, the principal invests less than the first-best investment level because turnover results in a loss in productivity of ongoing projects. The principal can design a more efficient contract with earnings management that improves the efficiency of turnover and investment decisions. The paper shows that, under a moderate level of internal control systems, earnings management disciplines the agent and creates long-term incentives, substituting for the threat of dismissal. In this sense, earnings management can be a more efficient punishment than the threat of dismissal. The principal may, however, need to pay more compensation to make up for the cost of earnings management. Therefore, the trade-off between the turnover-reducing benefit and the incentive-weakening cost influences investment efficiency, as well as the principal’s welfare.

The model highlights the importance of past performance (the agent’s continuation payoff or long-term incentives) and internal control systems when evaluating the effect of earnings management. The dynamic model shows that when long-term incentives are high (good past performance), the incentive-weakening cost dominates. Therefore, investment efficiency and the principal’s welfare increase with the internal control system. When long-term incentives are low (bad past performance), however, the turnover-reducing benefit dominates initially, but the incentive-weakening cost dominates in the end as the internal control system becomes weaker. In addition, the dynamic model studies how pay-performance sensitivity is related to long-term incentives. In general, sensitivity increases with long-term incentives, because a low level of incentives is implemented when long-term incentives are low and moral hazard is severe. Because the sensitivities affect the agent’s incentives for earnings management, it is more likely that the firm takes big baths when long-term incentives are low. The agent’s initial continuation payoff (long-term incentives) is determined by the relative bargaining power between the principal and the agent. In many cases, the principal has more
bargaining power and, hence, new CEOs’ continuation payoff is relatively low. Therefore, we observe more big baths in the case of turnover.

The paper presents many implications for future research. First, the effect of earnings management should be taken into account in measuring the turnover-performance relation. As discussed earlier, it has been documented that the pay-performance relation is too low to be consistent with the agency theory. The paper offers a resolution to the Jensen and Murphy puzzle by examining the effect of earnings management in a dynamic contract and finding the importance of internal control systems in measuring the turnover-performance relation. Second, when evaluating the real effect of earnings management, it is important to consider past performance (the agent’s continuation payoff) and internal control systems. So far, empirical studies do not provide clear evidence on the effect of earnings management on investment. As the paper suggests, past performance (the agent’s continuation payoff) and internal control systems significantly alter the net effect of earnings management. Lastly, the paper sheds light on why and when CEOs take big baths. The paper predicts that the probability of taking big baths decreases with the agent’s continuation payoff (long-term incentives).
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2.7 Appendix

2.7.1 Appendix A: Proofs

Proof of Proposition 1

Proof of Proposition 1 is based on the proof of Lemma 1 in Chaigneau, Edmans, and Gottlieb (2015). Given that the agent managed earnings per unit of capital by \( b \) in period 1, the principal’s problem in period 2 is

\[
\min_{s} \int_{-\infty}^{\infty} s_2(x_2) f(x_2|a_H) dx_2
\]

(2.121)

\[
s.t. \int_{-\infty}^{\infty} s_2(x_2)[f(x_2|a_H) - f(x_2|a_L)] dx_2 \geq h_2
\]

(2.122)

\[
s_2(x_2 + \eta) - s_2(x_2) \leq \eta \text{ for } \forall \eta > 0
\]

(2.123)

\[
s_2(x_2) \geq 0
\]

(2.124)

Suppose there exists a contract \( \hat{s}_2 \) that satisfies the monotonicity constraint, the limited liability, and IC but is not an option contract. Without loss of generality, suppose IC holds with equality because it cannot be optimal otherwise.

\[
\int_{-\infty}^{\infty} \hat{s}_2(x_2)[f(x_2|a_H) - f(x_2|a_L)] dx_2 = h_2
\]

(2.125)

For any such alternative contract, there exists a unique option contract with the same expected payoff. That is,

\[
\hat{s}_2^*(x_2)f(x_2|a_H) dx_2 = \int_{-\infty}^{\infty} \hat{s}_2(x_2)f(x_2|a_H) dx_2
\]

(2.126)

Let’s say the option contract has an exercise price of \( T \).

\[
\int_{-\infty}^{\infty} s_2^*(x_2)f(x_2|a_H) dx_2 = \int_{T}^{\infty} (x_2 - T)f(x_2|a_H) dx_2
\]

(2.127)
Thus,
\[
\int_T^{\infty} (x_2 - T)f(x_2 | a_H)dx_2 = \int_{-\infty}^{\infty} \hat{s}_2(x_2) f(x_2 | a_H)dx_2
\]  
(2.128)

Applying the Intermediate Value Theorem using the fact that (a). As \( T \to \infty, \) \( LHS < RHS. \) (b). As \( T \to -\infty, \) \( LHS > RHS. \) (c). \( \frac{\partial}{\partial T} \int_T^{\infty} (x_2 - T)f(x_2 | a_H)dx_2 = -[1 - F(T | a_H)] < 0, \) there exists a unique solution \( T \) to the equation (2.128).

Let \( D(x_2) = \hat{s}_2(x_2) - s_2^*(x_2) \) and then \( \int_{-\infty}^{\infty} D(x_2)f(x_2 | a_H)dx_2 = 0. \) Using the fact that \( s_2^* \) is an option contract and \( \hat{s}_2 \) satisfies the limited liability constraint and the monotonicity constraint, there exists \( \bar{x} \) such that \( D(x_2) \leq 0 \) for \( \forall x_2 > \bar{x} \) and \( D(x_2) \geq 0 \) for \( \forall x_2 < \bar{x} \). Then,

\[
\int_{-\infty}^{\infty} D(x_2)f(x_2 | a_L)dx_2 = \int_{-\infty}^{k} D(x_2) \frac{f(x_2 | a_L)}{f(x_2 | a_H)} f(x_2 | a_H)dx_2 + \int_{k}^{\infty} D(x_2) \frac{f(x_2 | a_L)}{f(x_2 | a_H)} f(x_2 | a_H)dx_2
\]  
(2.129)

\[
= \int_{-\infty}^{k} D(x_2) \frac{f(x | a_L)}{f(x | a_H)} f(x | a_H)dx_2 + \int_{k}^{\infty} D(x_2) \frac{f(x | a_L)}{f(x | a_H)} f(x | a_H)dx_2
\]  
(2.130)

\[
= \frac{f(x | a_H)}{f(x | a_L)} \int_{-\infty}^{\infty} D(x_2)f(x_2 | a_H)dx_2
\]  
(2.131)

\[
= 0
\]  
(2.132)

The inequality in the third line in the above equation comes from MLRP, \( \frac{f(x_H | a_H)}{f(x_H | a_L)} > \frac{f(x_L | a_H)}{f(x_L | a_L)} \) for \( x_H > x_L. \)

Therefore,

\[
\int_{-\infty}^{\infty} \hat{s}_2(x_2) f(x_2 | a_L)dx_2 > \int_{-\infty}^{\infty} s_2^*(x_2) f(x_2 | a_L)dx_2
\]  
(2.134)
This leads to the contradiction that IC does not bind:

\[
\int_{-\infty}^{\infty} s_2^+(x_2)f(x_2|a_H)dx_2 > \int_{-\infty}^{\infty} s_2^+(x_2)f(x_2|a_L)dx_2 + h_2
\]  

(2.135)

\[
= \int_{-\infty}^{\infty} \hat{s}_2(x_2)f(x_2|a_H)dx_2 + \int_{-\infty}^{\infty} s_2^+(x_2)f(x_2|a_L)dx_2 + h_2
\]  

(2.136)

\[
> \int_{-\infty}^{\infty} \hat{s}_2(x_2)f(x_2|a_L)dx_2 + h_2
\]  

(2.137)

Therefore, there exists a new contract \( s_2^+ \) with a higher exercise price \( T^+ \), which has a lower expected payoff and remains incentive compatible:

\[
\int_{-\infty}^{\infty} s_2^+(x_2)f(x_2|e_H)dx_2 < \int_{-\infty}^{\infty} s_2^+(x_2)f(x_2|e_H)dx_2 = \int_{-\infty}^{\infty} \hat{s}_2(x_2)f(x_2|e_H)dx_2
\]  

(2.138)

This new option contract \( s_2^+ \) satisfies IC, the monotonicity constraint, the limited liability constraint, and has a lower expected payoff than the initial non-option contract \( \hat{s}_2 \), which is contradiction. Hence, the optimal contract is an option contract with an exercise price \( T \), which is the unique solution of

\[
\int_{T}^{\infty} (x_2 - T)[f(x_2|e_H) - f(x_2|e_L)]dx = h_2
\]  

(2.139)

Let \( T = y_2 \). Then

\[
s_2(x_2) = \max[0, x_2 - y_2]
\]  

(2.140)

On \( r_2 \),

\[
s_2(r_2) = \max[0, r_2 - y_2 + b]
\]  

(2.141)

To search for the optimal contract, I start with the conjecture that there exists an optimal contract
satisfying

\[ w(s_1(x_1), s_2(x_1, x_2), b(x_1), i_1) = s_1(x_1) - g(b(x_1)) + \frac{1 - \delta + i_1}{1 + \rho_A} E[(s_2(x_1, x_2) - h_2)1_{TD(x_1)=0}] \]  

\[ = \text{Max}[0, x_1 - y_1] \]  

(2.142)  

(2.143)  

where \( w(s_1(x_1), s_2(x_1, x_2), b(x_1), i_1) \) is the agent’s payoff excluding the cost of effort, \( h_1 \).

For the contract to be incentive compatible with earnings management (2.54), we need to understand the agent’s incentive to manage earnings.

\[ \text{Max}_{b(\hat{x}_1)} E[w(s_1(\hat{x}_1), s_2(\hat{x}_1, x_2), b(\hat{x}_1), i_1)|x_1] = s_1(\hat{x}_1) - g(b(\hat{x}_1)) + \frac{1 - \delta + i_1}{1 + \rho_A} E[(s_2(\hat{x}_1, x_2) - h_2)1_{TD(\hat{x}_1)=0}] \]  

(2.144)  

Considering that the contract is a form of a call option, \( b(\hat{x}_1) = \frac{1}{\epsilon} \). And in the optimal contract, \( \hat{x}_1 = x_1 \). Therefore, the optimal earnings management is

\[ b(x_1) = \begin{cases} 
\frac{1}{\epsilon} & \text{if } x_1 > y_1 \\
0 & \text{if } x_1 \leq y_1 
\end{cases} \]  

(2.145)  

Note that IC becomes

\[ \int_{-\infty}^{\infty} [w(s_1(x_1), s_2(x_1, x_2), b(x_1), i_1)] [f(x_1|a_H) - f(x_1|a_L)] dx_1 \geq h_1 \]  

(2.146)  

Therefore, in the optimal contract, \( w(s_1(x_1), s_2(x_1, x_2), b(x_1), i_1) \) is an option contract on \( x_1 \) with the exercise price \( y_1 \) that is unique solution of

\[ \int_{y_1}^{\infty} (x_1 - y_1) [f(x_1|a_H) - f(x_1|a_L)] dx_1 = h_1 \]  

(2.147)
The call option form of \( w(s_1(x_1), s_2(x_1, x_2), b(x_1), i_1) \) implies that the replacement threshold \( \hat{y} \) is the value of the scaled earnings \( x_1 \) such that \( x_1 - y_1 = \frac{1 - i_1}{\bar{\pi} + h_1} (s_2(x_1, x_2) - h_2) - \frac{1}{\gamma} \). Hence, the agent is replaced if \( x_1 \leq \hat{y} = \frac{1 - i_1}{\bar{\pi} + h_1} (s_2(x_1, x_2) - h_2) - \frac{1}{\gamma} + y_1 \) because the principal cannot guarantee enough continuation payoff to incentivize the agent in period 2. In other words, incentives provided in period 2 spill back to period 1, and, thus, the agent is incentivized to exert a high effort in period 1 to enjoy not only the compensation in period 1 but also the compensation in period 2.

The first order condition for \( i_1 \) comes from taking derivatives to the principal’s objective with respect to \( i_1 \).

**Proof of Proposition 2**

The strike price \( y_1 \) satisfies the equation (2.70)

\[
\int_{y_1}^{\infty} (x_1 - y_1) [f(x_1|a_H) - f(x_1|a_L)]dx_1 = h_1
\]

(2.148)

Therefore,

\[
\frac{dy_1}{dc} = 0
\]

(2.149)

Denote the lower and upper bound of the support of \( x_1 \) by \( \underline{x} \) and \( \bar{x} \), respectively. Then, the above equation becomes

\[
\int_{y_1}^{\bar{x}} sf(x_1|a_H)dx_1 - \int_{y_1}^{\bar{x}} x_1 f(x_1|a_H)dx_1 - [F(y_1|a_L) - F(y_1|a_H)]y_1 = h_1
\]

(2.150)

Using integration by parts,

\[
\int_{y_1}^{\bar{x}} x_1 f(x_1|a)dx_1 = [x_1 F(x_1|a) - \int F(x_1|a)dx_1]_{y_1}^{\bar{x}} = \bar{x} - y_1 F(y_1|a) - \int_{y_1}^{\bar{x}} F(x_1|a)dx_1
\]

(2.151)
Then, the equation (2.148) constraint becomes

$$
\int_{y_1}^{\hat{y}} [F(x_1|a_L) - F(x_1|a_H)] dx_1 = h_1
$$

(2.152)

Applying the implicit function theorem,

$$
\frac{dy_1}{dh_1} = - \frac{\frac{\partial g}{\partial h_1}}{\frac{\partial g}{\partial y_1}} = - \frac{1}{F(y_1|a_L) - F(y_1|a_H)} < 0
$$

(2.153)

Q.E.D.

**Proof of Proposition 3**

The capital-investment ratio $i_1$ satisfies

$$
\xi'(i_1) = \frac{1}{1 + \rho p} [E[x_2 - s_2(x_1, x_2)|a_H][1 - F(\hat{y})] + E[x_2^{\text{New}} - s_2^{\text{New}}(x_2^{\text{New}})|a_H]F(\hat{y})] \\
- \frac{(1 - \delta + i_1)}{1 + \rho p} [E[x_2 - s_2(x_1, x_2)|a_H] - E[x_2^{\text{New}} - s_2^{\text{New}}(x_2^{\text{New}})|a_H]]f(\hat{y}) \frac{\partial \hat{y}}{\partial i_1} \xi'(i_1)
$$

(2.154)

First, I assume that the marginal benefit of investment in RHS decreases with $i_1$, in the sense that both the cash flow effect and the replacement effect decrease with $i_1$. Otherwise, the capital investment ratio $i_1$ can exceed the first-best $i_1$ for some sets of parameters.

$$
\frac{\partial \text{Marginal Benefit of Investment}}{\partial i_1} = \frac{\partial \text{Cash Flow Effect}}{\partial i_1} - \frac{\partial \text{Replacement Effect}}{\partial i_1} < 0
$$

(2.155)

Let $L(i_1, c) = \frac{1}{1 + \rho p} [E[x_2 - s_2(x_1, x_2)|a_H][1 - F(\hat{y})] + E[x_2^{\text{New}} - s_2^{\text{New}}(x_2^{\text{New}})|a_H]F(\hat{y})] - \frac{(1 - \delta + i_1)}{1 + \rho p} [E[x_2 - s_2(x_1, x_2)|a_H] - E[x_2^{\text{New}} - s_2^{\text{New}}(x_2^{\text{New}})|a_H]]f(\hat{y}) \frac{\partial \hat{y}}{\partial i_1} - \xi'(i_1)$. Applying the implicit function theorem,

$$
\frac{di_1}{dc} = - \frac{\partial L}{\partial c}
$$

(2.156)
Then,

\[ \frac{\partial L}{\partial i_1} < 0 \quad (2.157) \]

\[ \frac{\partial L}{\partial c} = -\frac{1}{1 + \rho_p} \left[ E[x_2 - s_2(x_1, x_2)|a_H] - E[x_2^* - s_2^*(x_2)|a_H]|f(\hat{y}) \frac{\partial \hat{y}}{\partial c} \right] \]

\[ - \frac{(1 - \delta + i_1)}{1 + \rho_p} \left[ E[x_2 - s_2(x_1, x_2)|a_H] - E[x_2^* - s_2^*(x_2)|a_H]|f'(\hat{y}) \frac{\partial \hat{y}}{\partial c} \right] \]

\[ = -\frac{1}{1 + \rho_p} \left[ E[x_2 - s_2(x_1, x_2)|a_H] - E[x_2^* - s_2^*(x_2)|a_H]\frac{\partial \hat{y}}{\partial c} [f(\hat{y}) + (1 - \delta + i_1)f'(\hat{y}) \frac{\partial \hat{y}}{\partial i_1}] \right] \]

\[ = -\frac{1}{1 + \rho_p} \left[ E[x_2 - s_2^*(x_2)|a_H] \right] \frac{1}{2c^2} [f(\hat{y}) + f'(\hat{y})(1 - \delta + i_1)E[s_2(x_1, x_2) - h_2]] \quad (2.158) \]

The last equality comes from the fact that \( \frac{\partial \hat{y}}{\partial c} = \frac{\partial}{\partial c} \left[ \frac{E(x_2 - s_2(x_1, x_2)|a_H)}{1 + \rho_p} \right] \frac{1}{2c^2} \). Because \( f(\hat{y}) + f'(\hat{y})(1 - \delta + i_1)E[s_2(x_1, x_2) - h_2] > 0 \) from the assumption that \( \frac{\partial \text{Replacement Effect}}{\partial i_1} < 0 \),

\[ \frac{d_i}{d_c} = -\frac{\partial \hat{y}}{\partial c} < 0 \quad (2.162) \]

Also, \( \frac{d_i}{d_c} = \frac{\partial \hat{y}}{\partial c} + \frac{\partial \hat{y}}{\partial i_1} \). Using the fact that \( \frac{\partial \hat{y}}{\partial c} = \frac{1}{2c^2} \) and \( \frac{\partial \hat{y}}{\partial i_1} = \frac{1}{1 + \rho_p} E[s_2(x_1, x_2) - h_2] \), I can show

\[ \frac{d_i}{d_c} > -\frac{1}{1 + \rho_p} E[s_2(x_1, x_2) - h_2] \quad (2.163) \]

Therefore,

\[ \frac{d\hat{y}}{d_c} = \frac{\partial \hat{y}}{\partial c} + \frac{\partial \hat{y}}{\partial i_1} \frac{d_i}{d_c} > 0 \quad (2.164) \]

Q.E.D.

Proof of Proposition 4
The expected compensation is

\[
E[s_1(x_1) + \frac{(1 - \delta + i_1)}{1 + \rho_A} s_2(x_1, x_2) 1_{[x_1 > \hat{y}]} | a_H] = \int_{y_1}^{\infty} (x_1 - y_1) f(x_1 | a_H) dx_1 + \int_{y_1}^{\infty} \frac{(1 - \delta + i_1)}{1 + \rho_A} h_2 f(x_1 | a_H) dx_1
\]

\[
= \int_{y_1}^{\infty} (x_1 - y_1) f(x_1 | a_H) dx_1 + \int_{y_1}^{\infty} \frac{1}{2c} f(x_1 | a_H) dx_1
\]

\[
+ \int_{\hat{y}}^{\infty} \frac{(1 - \delta + i_1)}{1 + \rho_A} h_2 f(x_1 | a_H) dx_1
\]

(2.165)

where \( \hat{y} = \frac{1 - \delta + i_1}{1 + \rho_A} E[s_2(x_1, x_2) - h_2] - \frac{1}{2c} + y_1 \).

Let the lower and upper bound of the support of \( x_1 \) by \( \underline{x} \) and \( \overline{x} \), respectively.

\[
\int_{y_1}^{\infty} (x_1 - y_1) f(x_1 | a_H) dx_1 = \int_{y_1}^{\infty} x_1 f(x_1 | a_H) dx_1 - y_1 [1 - F(y_1 | a_H)]
\]

(2.167)

\[
= \int_{\underline{x}}^{\infty} x_1 f(x_1 | a_H) dx_1 - y_1 [1 - F(y_1 | a_H)] - \int_{\underline{x}}^{y_1} x_1 f(x_1 | a_H) dx_1
\]

(2.168)

\[
= \int_{\underline{x}}^{\infty} x_1 f(x_1 | a_H) dx_1 - y_1 [1 - F(y_1 | a_H)] - [x_1 F(x_1 | a_H) - \int F(x_1 | a_H) dx_1]_{\underline{x}}^{y_1}
\]

(2.169)

\[
= E[x_1 | a_H] - y_1 + \int_{\underline{x}}^{y_1} F(x_1 | a_H) dx_1
\]

(2.170)

Therefore,

\[
E[s_1(x_1) + \frac{(1 - \delta + i_1)}{1 + \rho_A} s_2(x_1, x_2) 1_{[x_1 > \hat{y}]} | a_H] = E[x_1 | a_H] - y_1 + \int_{\underline{x}}^{y_1} F(x_1 | a_H) dx_1 + \int_{y_1}^{\infty} \frac{1}{2c} f(x_1 | a_H) dx_1
\]

\[
+ \int_{\hat{y}}^{\infty} \frac{(1 - \delta + i_1)}{1 + \rho_A} h_2 f(x_1 | a_H) dx_1
\]

(2.171)
The effect of the marginal cost of earnings management is

$$\frac{d}{dc} E[s_1(x_1) + \frac{(1 - \delta + i_1)}{1 + \rho_A} s_2(x_1, x_2) 1_{[x_1 \geq s]} | a_H] = \frac{\partial}{\partial c} E[s_1(x_1) + \frac{(1 - \delta + i_1)}{1 + \rho_A} s_2(x_1, x_2) 1_{[x_1 \geq s]} | a_H] (2.172)$$

$$+ \frac{\partial}{\partial i_1} E[s_1(x_1) + \frac{(1 - \delta + i_1)}{1 + \rho_A} s_2(x_1, x_2) 1_{[x_1 \geq s]} | a_H] \frac{di_1}{dc}$$

$$= - \frac{1}{2c^2} [1 - F(y_1 | a_H)] - \frac{(1 - \delta + i_1)i_2}{1 + \rho_A} \frac{1}{2c^2} f(\hat{y} | a_H)$$

$$+ E[s_2(x_1, x_2)] [1 - F(\hat{y} | a_H)] \frac{di_1}{dc} < 0 (2.175)$$

because \( \frac{d}{dc} < 0 \).

Let the principal’s expected payoff \( \Pi \) be

$$\Pi = E[x_1 - \xi (i_1) - s_1(x_1) + \frac{(1 - \delta + i_1)}{1 + \rho_p} (s_2 - s_2(x_1, x_2)) 1_{[x_1 \geq s]} + (x_2^{New} - s_2^{New} (x_2^{New})) 1_{[x_1 \leq s]}] (2.176)$$

The effect of \( c \) on the principal’s expected payoff is

$$\frac{d\Pi}{dc} = \frac{\partial \Pi}{\partial c} + \frac{\partial \Pi}{\partial i_1} \frac{di_1}{dc}$$

$$= \frac{1}{2c^2} [F(\hat{y} | a_H) - F(y_1 | a_H)] - \frac{1 - \delta + i_1}{1 + \rho_A} E[s_2(x_1, x_2) - h_2] f(\hat{y} | a_H) - (1 - F(\hat{y} | a_H))] \frac{1}{2c^2}$$

$$- \frac{(1 - \delta + i_1)}{1 + \rho_p} E[x_2 - x_2^{New} | a_H] f(\hat{y} | a_H) \frac{1}{2c^2}$$

$$= \frac{1}{2c^2} [1 - F(y_1 | a_H)] - \frac{1 - \delta + i_1}{1 + \rho_A} E[s_2(x_1, x_2) - h_2] f(\hat{y} | a_H)] \frac{1}{2c^2} - \frac{(1 - \delta + i_1)}{1 + \rho_p} E[x_2 - x_2^{New} | a_H] f(\hat{y} | a_H) \frac{1}{2c^2}$$

$$= \frac{1}{2c^2} [1 - F(y_1 | a_H)] - (1 - \delta + i_1) E\left[\frac{x_2 - x_2^{New}}{1 + \rho_p} + \frac{(s_2(x_1, x_2) - h_2)}{1 + \rho_A} | a_H\right] f(\hat{y} | a_H) \frac{1}{2c^2}$$

(2.181)
because \( \frac{\partial}{\partial \hat{y}} F(\hat{y} \mid a_H) \frac{d\hat{y}}{dx} = \frac{1}{2c} f(\hat{y} \mid a_H) \cdot \frac{\partial \pi}{\partial c} = 0 \) and

\[
E[s_1(x_1)] = \int_{y_1}^{\hat{y}} (x_1 - y_1 + \frac{1}{2c}) f(x_1 \mid a_H) dx_1 + \int_{\hat{y}}^{\infty} (x_1 - \hat{y}) f(x_1 \mid a_H) dx_1 \tag{2.182}
\]

\[
= \int_{y_1}^{\infty} x_1 f(x_1 \mid a_H) dx_1 - \int_{y_1}^{\hat{y}} (y_1 - \frac{1}{2c}) f(x_1 \mid a_H) dx_1 - \int_{\hat{y}}^{\infty} \hat{y} f(x_1 \mid a_H) dx_1 \tag{2.183}
\]

\[
\frac{\partial}{\partial c} E[s_1(x_1)] = \frac{\partial}{\partial c} E[s_1(x_1)] + \frac{\partial}{\partial \hat{y}} E[s_1(x_1)] \frac{\partial \hat{y}}{\partial c} \tag{2.184}
\]

\[
= -\frac{1}{2c^2} [F(\hat{y} \mid a_H) - F(y_1 \mid a_H)] + \frac{1 - \delta + i_1}{1 + \rho_A} E[(s_2(x_1, x_2) - h_2) f(\hat{y} \mid a_H)] - (1 - F(\hat{y} \mid a_H))] \frac{\partial \hat{y}}{\partial c} \tag{2.185}
\]

\[
= -\frac{1}{2c^2} [F(\hat{y} \mid a_H) - F(y_1 \mid a_H)] + \frac{1 - \delta + i_1}{1 + \rho_A} E[(s_2(x_1, x_2) - h_2) f(\hat{y} \mid a_H)] - (1 - F(\hat{y} \mid a_H))] \frac{1}{2c^2} \tag{2.186}
\]

Therefore, the principal’s expected payoff decreases as \( c \) increases if and only if

\[
(1 - \delta + i_1) E\left[\frac{x_2 - x_2^{\text{New}}}{1 + \rho_P} + \frac{(s_2(x_1, x_2) - h_2)}{1 + \rho_A} \mid a_H\right] f(\hat{y} \mid a_H) > 1 - F(y_1 \mid a_H) \tag{2.187}
\]

or

\[
\frac{f(\hat{y} \mid a_H)}{1 - F(y_1 \mid a_H)} > [(1 - \delta + i_1) E\left[\frac{x_2 - x_2^{\text{New}}}{1 + \rho_P} + \frac{(s_2(x_1, x_2) - h_2)}{1 + \rho_A} \mid a_H\right]]^{-1} \tag{2.188}
\]

Q.E.D.

**Proof of Corollary 4**

The principal’s expected payoff decreases as \( c \) increases if and only if

\[
\frac{f(\hat{y} \mid a_H)}{1 - F(y_1 \mid a_H)} > [(1 - \delta + i_1) E\left[\frac{x_2 - x_2^{\text{New}}}{1 + \rho_P} + \frac{(s_2(x_1, x_2) - h_2)}{1 + \rho_A} \mid a_H\right]]^{-1} \tag{2.189}
\]

where \( \hat{y} = \frac{1 - \delta + i_1}{1 + \rho_A} E[s_2(x_1, x_2) - h_2] - \frac{1}{2c} + y_1 \).
When $\hat{y}$ is large, $y_1$ is large, too, if everything else is equal.

If the earnings $x_t$ follows the uniform distribution,

$$ \frac{\partial}{\partial y_1} \frac{f(\hat{y} \mid a_H)}{1 - F(y_1 \mid a_H)} > 0 $$

(2.190)

Therefore, given $i_1$ there exists large $y_1$ or $\hat{y}$, such that the condition (2.189) holds.

If the earnings $x_t$ follows the normal distribution,

$$ \lim_{y_1 \to \infty} \frac{f(\hat{y} \mid a_H)}{1 - F(y_1 \mid a_H)} = \lim_{y_1 \to \infty} - \frac{f'(\hat{y} \mid a_H)}{f(y_1 \mid a_H)} = \infty $$

(2.191)

using L’hopital’s rule.

Therefore, if $y_1$ or $\hat{y}$ is sufficiently large given $i_1$, then the condition (2.189) holds.

Q.E.D.

**Proof about $v''(w)$**

From the equation (2.114) and the envelope theorem, we have

$$ v'(w)(\rho_A - \rho_P) + v''(w)[(\rho_A - (i - \delta))w + \frac{1}{2} \frac{(\varphi(w)\bar{\chi})^2}{c_e} + \frac{1}{2} \frac{(\varphi(w))^2}{c}] + \frac{1}{2} v'''(w) * (\varphi(w))^2 \sigma^2 = 0 $$

(2.192)

Due to the concavity of $v(w)$, $v''(w) < 0$. Suppose there exists $w^*$, such that $v'(w^*) = 0$. Then $v''(w)$ increases with $w$ for $w > w^*$. For $w < w^*$, $v''(w)$ increases with $w$, likely because $\rho_A - \rho_P$ is sufficiently small.

Q.E.D.
2.7.2 Appendix B: Continuous - Time Approach

In this section, I describe a dynamic principal-agent model in continuous-time methods.

2.7.2.1 The Agency Problem

The agent makes two unobservable actions: productivity effort $a_t$ and earnings management $B_t$. The agent’s productivity effort affects the productivity process

$$dX_t = a_t \bar{x}dt + \sigma dZ_t \tag{2.193}$$

where $\sigma$ is a constant and $Z = \{Z_t, F_t; 0 \leq t < \infty\}$ is a standard Brownian motion on a complete probability space $(\Omega, F, P)$. The expected rate of output, $\bar{x}$, is dependent on the agent’s action $e_t$. The cost of effort is $H(a_t) = \frac{1}{2} c a_t^2$ and we normalize $H(0) = 0$.

The agent observes earnings ($E_t$) privately and reports earnings ($R_t$) with bias $B_t$. The cost of earnings management is $G(M_t + B_t) = \frac{1}{2} c (M_t + B_t)^2$, where $M_t$ is the accumulated discretionary accruals and $K_t$ is capital stock at the beginning of time $t$. The accumulated discretionary accruals evolve as follows:

$$dM_t = B_t dt \tag{2.194}$$

The principal observes reported earnings that follow

$$dR_t = dE_t + B_t dt \tag{2.195}$$

The agent has no initial wealth and has limited liability that rules out a negative wage, and has a higher discount rate than the principal, $\rho_A > \rho_P$, which means that the agent is more impatient than the principal.
2.7.2.2 Investment and Production Technology

The firm uses physical capital to produce output, and the price of capital is normalized to one. The capital stock \( K_t \) changes as follows:

\[
dK_t = (I_t - \delta K_t)dt, \quad t \geq 0
\]  

(2.196)

where \( I_t \) is the gross investment and \( \delta \geq 0 \) is the rate of depreciation.

The firm’s incremental cash flows over time increment \( dt \) is as follows:

\[
dCF_t = K_t dX_t - I_t dt - J(I_t, K_t)dt, \quad t \geq 0
\]  

(2.197)

where \( J(I_t, K_t) \) is the adjustment cost of investment. Following Hayashi(1982), I assume that the adjustment cost is homogeneous of degree one in \( I \) and \( K \), which enables the adjustment cost to take the form \( J(I_t, K_t) = j(i_t) K_t \), where \( i = \frac{I_t}{K_t} \) is the investment-capital ratio, \( j'(i) > 0 \), and \( j''(i) > 0 \). For simplicity, I assume \( j(i) = \frac{\theta}{2} i^2 \), where \( \theta \) is the degree of the adjustment cost.

The total cost of investment is

\[
I_t + J(I_t, K_t) = (i_t + j(i_t))K_t
\]

\[
= \xi(i_t)K_t
\]

(2.198)

(2.199)

where \( \xi \) is the total cost per unit of capital required when the firm invests to grow at rate \( i_t \).

Therefore, the firm’s incremental cash flow net of cost of investment \( dCF_t \) over time increment \( dt \) is written as

\[
dCF_t = K_t (dX_t - \xi(i_t) dt), \quad t \geq 0
\]  

(2.200)

where \( K_t dX_t \) is the incremental gross output at time \( t \) and \( K_t \xi(i_t) dt \) is the total cost of investment. Thus, the
firm’s incremental reported earnings over time increment $dt$ is

$$dR_t = de_t + B_t dt$$

(2.201)

$$= K_t(dX_t + b_t dt)$$

(2.202)

where $b_t = \frac{B_t}{K_t}$.

Finally, when the contract is terminated and the agent is replaced, there is a loss in productivity of ongoing projects. I formulate a cost of lost productivity in a way that the principal’s expected payoff per unit of capital decreases in the case of turnover.

$$L = \max_w \left[ (1 - \ell) \frac{V(W, K_\tau)}{K_\tau} \right]$$

(2.203)

$$= \max_w (1 - \ell)v(w)$$

(2.204)

where $v(w) = \frac{V(W, K)}{K}$ is the principal’s scaled value function.

I also assume that $\lim_{i_t \to -\infty} \xi'(i_t) \leq \ell$ because the firm can liquidate by disinvesting anytime.

2.7.2.3 Optimal Contracting Problem

The agent chooses an action to solve the following problem:

$$W(\Theta) = \max_{a_t, B_t} E^A \left[ \int_0^\tau e^{-\rho s} (S_u - h(a_u) - g(M_u + B_u)) du \right]$$

(2.205)

where $E^A[\cdot]$ is the expectation operator under the probability measure that is induced by an action process $\{a_t, B_t : 0 \leq t < \tau\}$.

Given an initial payoff $W_0$ for the agent, the investors’ problem is

$$V(W_0, K_0) = \max_\Theta E \left[ \int_0^\tau e^{-\rho s} dCF_s - \int_0^\tau e^{-\rho s} S_u du \right]$$

(2.206)
\[ s.t. \quad \Theta \ is \ IC \quad (2.207) \]

\[ W(\Phi) \geq W_0 \quad (2.208) \]

### 2.7.2.4 Model Solution and Optimal Contracting

**Benchmark** I first consider a benchmark case where the agent’s action is observable. The principal’s problem is

\[ \text{Max} \ E \left[ \int_0^\tau e^{-\rho a} dCF_u - \int_0^\tau e^{-\rho a} S_u du \right] \quad (2.209) \]

\[ s.t. \ E^A \left[ \int_0^\tau e^{-\rho s_u} (S_u - h(u) - g(M_u + B_u)) du \right] \geq W_0 \quad (2.210) \]

The first order conditions for \( a_t, S_t \) and \( i_t \) are

\[ e^{-\rho a} \bar{x}_t - \lambda e^{-\rho s_t} cK_t a_t = 0 \quad (2.211) \]

\[ -e^{-\rho s_t} + \lambda e^{-\rho s_t} = 0 \quad (2.212) \]

\[ -e^{-\rho s_t} \xi'(i_t) K_t + e^{-\rho s_t} \frac{a^F B_t - \xi'(i_t^F)}{\rho p - (i^F - \delta) K_t} = 0 \quad (2.213) \]

Then, the optimal effort level \( a_t \) and capital-investment ratio \( i_t \) satisfy

\[ a^F_t = \frac{\bar{x}_t}{c_a} \quad (2.214) \]

\[ \xi'(i^F_t) = \frac{a^F_t \bar{x}_t - \xi'(i^F_t)}{\rho p - (i^F - \delta)} \quad (2.215) \]

If the principal promises the agent a payoff \( W \), then it is optimal to pay the agent \( W \) in cash immediately because \( \rho_A > \rho_I \).

\[ V^F_B(W, K) = \frac{a^F_t \bar{x}_t - \xi'(i^F_t)}{\rho p - (i^F - \delta) K - W} \quad (2.216) \]
We can also write this on a per unit of capital basis
\[
V^F_B(w) = \frac{V^F_B(W, K)}{K} = \frac{a^F_B x - \xi (i^F_B)}{\rho_p - (i^F_B - \delta)} - w
\] (2.217)

where \( w = \frac{W}{K} \).

**Optimal Contract**  The agent’s continuation payoff at time \( t \) is
\[
W_t(\Theta) = E_t \left[ e^{-\rho_t (u-t)} (S_u - h(a_u) - g(M_u + B_u)) du \right]
\] (2.218)
given the history of performance and earnings management up to time \( t \) and an incentive-compatible contract \( \Theta \). In other words, \( W_t \) is the discounted expected value of the agent’s future compensation under \( \Theta \) when the agent decides to work from time \( t \).

**Lemma C-1.** Given the agent’s cash salary \( S_t \), recommended effort \( a_t \) and earnings management \( B_t \), the agent’s continuation payoff \( W_t \) evolves as follows:
\[
dW_t = \rho_A W_t dt - (s_t - h(a_t) - g(M_t + B_t)) dt + \phi(W_t) (dR_t - B_t dt - a_t \bar{x} dt)
\] (2.219)

where \( \phi \) is the sensitivity of the agent’s continuation payoff to the firm’s earnings.

\( \rho_A W_t - (s_t - h(a_t) - g(M_t + B_t)) \) is the drift of continuation payoff \( W_t \) when the agent takes the recommended effort \( a_t \) and earnings management \( B_t \). \( W_t \) grows at the interest rate \( \rho_A \) to compensate for the agent’s time difference and decreases because of the repayments \( s_t - h(a_t) - g(M_t + B_t) \), due to the Promise-Keeping condition. The sensitivity \( \phi(W_t) \) of the agent’s continuation payoff to earnings has an influence on the agent’s incentives to make recommended effort and earnings management.
The sensitivity $\varphi(W_t)$ affects the agent’s incentives for the effort and earnings management. The agent chooses the effort and the bias in earnings reports that maximize the expected change of $W_t$ minus the cost of effort and earnings management

$$\varphi(W_t)(a_t\bar{x} + B_t) - h(a_t) - g(M_t + B_t)$$ (2.220)

Thus,

$$a_t^* = \frac{\varphi(W_t)\bar{x}}{c_a}$$ (2.221)

$$B_t^* = \frac{\varphi(W_t)K_t}{c} - M_t$$ (2.222)

After the history of the firm’s performance and earnings management up to time $t$, the relevant state variables are the agent’s continuation payoff $W_t$, the accumulated accruals $M_t$, and capital stock $K_t$. After substituting the optimal effort and earnings management chosen by the agent into (2.219), the shareholder’s problem reduces to the problem with two state variables, $W_t$ and $K_t$, and the continuation payoff evolves as follows:

$$dW_t = \rho A W_t dt - (S_t - h(\frac{\varphi(W_t)\bar{x}}{c_a}) - g(\frac{\varphi(W_t)}{c}))dt + \varphi(W_t)K_t\sigma dZ_t$$ (2.223)

$$= \rho A W_t dt - (S_t - \frac{1}{2}K_t(\frac{\varphi(W_t)\bar{x}^2}{c_a} - \frac{1}{2}(\frac{\varphi(W_t)}{c})^2)dt + \varphi(W_t)K_t\sigma dZ_t$$ (2.224)

Similar to the model of DeMarzo et al. (2012), the tractability of the model comes from the scale invariance of the firm’s technology. Therefore, I can reduce the problem to one with a single state variable $w = \frac{W}{K}$.

$$V(W, K) = v(w)K$$ (2.225)

I discuss the important properties of the principal’s scaled value function, $v(w)$. First, the value function
lies below the first-best value function \( v(w) \leq v^{FB}(w) \). Second, \( v'(w) + 1 \geq 0 \), because it costs the principal at most $1 to increase \( w \) by $1. This implies that the total surplus, \( v(w) + w \), is weakly increasing in \( w \). Third, the contract is terminated when \( w \) hits 0 to provide the agent with a payoff that is equal to her outside option (normalized to 0). Thus, \( v(0) = l \). Lastly, \( v(w) \) is concave. As \( w \) increases, the probability of liquidation decreases and the principal’s expected payoff increases. This benefit diminishes as \( w \) increases, because the principal needs to pay more to the agent. Lemma C-2 summarizes the properties of the principal’s scaled value function, \( v(w) \).

**Lemma C-2.** The principal’s scaled value function \( v(w) \) has the following properties:

(a) \( v(w) \leq v^{FB}(w) \)

(b) \( v(w) + 1 \geq 0 \)

(c) \( v(0) = l \)

(d) \( v(w) \) is concave in \( w \)

How does cash compensation \( u = \frac{U}{R} \) varies as \( w \) changes? Due to the presence of moral hazard, by deferring the agent’s compensation, the investors can decrease the possibility of ex-post inefficient liquidation of the firm. Therefore, in the optimal contract, cash compensation \( u \) is 0 when the agent’s scaled continuation payoff \( w_t \) is small. When cash compensation is deferred, however, the agent requires a higher interest rate due to \( \rho_A > \rho_P \). This trade-off between the benefit and the cost of deferring the agent’s compensation gives a threshold \( \tilde{w} \) such that it is optimal to defer compensation if the scaled continuation payoff is lower than the threshold \( \tilde{w} \) and pay cash compensation otherwise. Therefore, for \( w_t > \tilde{w} \),

\[
    u_t = w_t - \tilde{w}, \quad v(w_t) = v(\tilde{w}) - (w_t - \tilde{w}), \quad \text{and} \quad v'(\tilde{w}) = -1.
\]

For \( w_t \in [0, \tilde{w}] \), the agent’s cash compensation is deferred and the agent’s scaled continuation payoff
evolves as follows:

\[
dw_t = (\rho_A - (i_t - \delta))w_t dt + \left( \frac{1}{2} \frac{(\varphi(w_t)\bar{x})^2}{c_a} + \frac{1}{2} \frac{(\varphi(w_t))^2}{c} \right) dt + \varphi(w_t)(dr_t - bt dt - \mu dt) \tag{2.226}
\]

\[
= (\rho_A - (i_t - \delta))w_t dt + \left( \frac{1}{2} \frac{(\varphi(w_t)\bar{x})^2}{c_a} + \frac{1}{2} \frac{(\varphi(w_t))^2}{c} \right) dt + \varphi(w_t)\sigma dZ_t \tag{2.227}
\]

The agent’s scaled continuation payoff grows at the interest rate \( r_A \), less the net growth rate \( i_t - \delta \) of the firm on average. The agent’s scaled continuation payoff increases when the firm receives a positive productivity shock and decreases when the firm receives a negative productivity shock.

We know that the principal’s expected payoff is

\[
V(W, K) = \sup_i E \left[ \int_0^\tau e^{-\rho \tau} dCF_t + e^{-\rho \tau} lK_t - \int_0^\tau e^{-\rho \tau} S_t dt \right] \tag{2.228}
\]

Now the Hamilton-Jacobi-Bellman (HJB) equation for \( w \in [0, \bar{w}] \) is

\[
\rho_P v(w) = \sup_i \frac{\varphi(w)\bar{x}^2}{c_a} - \xi(i) + v(w)(i - \delta) + v'(w)\left( \frac{1}{2} \frac{(\varphi(w)\bar{x})^2}{c_a} + \frac{1}{2} \frac{(\varphi(w))^2}{c} \right) \\
+ vv''(w)(\rho_A - (i - \delta)) + \frac{1}{2} v''(w) (\varphi(w))^2 \sigma^2 \tag{2.229}
\]

Put in words, the first term in the right side is instantaneous expected earnings. The second term is the expected change in the value due to capital accumulation. The remaining terms are the expected change in the value because of the drift and volatility of the agent’s continuation payoff \( w \). More specifically, the third term is the expected change in the value because of the drift affected by effort and earnings management by the agent. From the above HJB equation (2.229), the first order condition for the optimal investment-capital ratio is given by

\[
c'(i(w)) = v(w) - wv'(w) \tag{2.230}
\]
Taking derivatives with respect to $w$ gives

$$i'(w) = -\frac{wv''(w)}{\xi''(i(w))} \geq 0 \quad (2.231)$$

The investment-capital ratio $i(w)$ increases with $w$ because the principal increases the investment as $w$ increases, because the possibility of dismissal becomes lower.

Using the fact that $c(i) = i + \frac{1}{2} \theta i^2$, the optimal investment-capital ratio is

$$i(w) = \frac{v(w) - wv'(w) - 1}{\theta} \quad (2.232)$$

As stated by the first order condition for the optimal investment-capital ratio, the marginal cost of investment per capital, $c'(i(w))$, is the same as the marginal value of investment per capital, $v(w) - wv'(w)$. The first term of the marginal value of investment per capital, $v(w)$, is the current value of the principal’s expected payoff per unit of capital. The second term is the marginal effect of decreasing the agent’s scaled continuation payoff $w$ by investing.

Plugging the policy function for the optimal investment-capital ratio into the HJB equation,

$$(\rho_p + \delta)v(w) = \frac{\varphi(w)\bar{x}^2}{c_a} + \frac{(v(w) - wv'(w) - 1)^2}{2\theta} + wv'(w)(\rho_a + \delta)$$

$$+ v'(w)(\frac{1}{2} \frac{(\varphi(w)\bar{x})^2}{c_a} + \frac{1}{2} \frac{(\varphi(w))^2}{c}) + \frac{1}{2} v''(w)(\varphi(w))^2 \sigma^2 \quad (2.233)$$

Boundary conditions\(^\text{20}\) are

$$v(0) = \ell \quad (2.234)$$

$$v'(\bar{w}) = -1 \quad (2.235)$$

$$v''(\bar{w}) = 0 \quad (2.236)$$

\(^\text{20}\)We have three boundary conditions: the liquidation boundary (2.234), the smooth pasting condition (2.235), and the super contact condition (2.236).
The first boundary condition at the lower end comes from the termination of contract. When the agent’s scaled continuation payoff hits zero, the agent is replaced, and there is a loss in productivity by $\ell$ percent per unit of capital. This gives the first boundary condition $v(0) = \ell$. The remaining boundary conditions come from the fact that investors pay the agent $w_t - \bar{w}$ to bring $w_t$ to $\bar{w}$ when $w_t$ is greater than $\bar{w}$. This gives the Smooth-Pasting condition $v'(\bar{w}) = -1$ and the Super-Contact condition $v''(\bar{w}) = 0$ (A Dixit, 1993).

We can also study the value of the firm in the steady-state using the HJB equation (2.229) and boundary conditions.

$$v(\bar{w}) + \bar{w} = \sup_i \frac{\varphi(w)\ell^2}{c_a} - \xi(i) - (\rho_A - \rho_P)\bar{w} - \left(\frac{1}{2} \frac{(\varphi(w)\ell)^2}{c_a} + \frac{1}{2} \frac{(\varphi(w))^2}{c_a}\right) \frac{\rho_P}{\rho + \delta - i}$$

(2.237)

The left side is the value of the firm at $\bar{w}$ and the right side is discounted expected future cash flow.
Chapter 3

Accounting Conservatism, Earnings Management, and Investment

Abstract

We develop a dynamic model to analyze how accounting conservatism interacts with earnings management to mitigate agency problems and improve investment efficiency. In the presence of CEO turnover, accounting conservatism, which gives more precise but less frequent high signals, can result in more frequent CEO turnover, leading to an underinvestment problem. Costly earnings management helps the firm to maintain a conservative accounting system because it can work as an alternative punishment for poor performance and substitute for the threat of turnover. We find that accounting conservatism, combined with earnings management, can improve contract efficiency by increasing the expected punishment for poor performance in two ways. First, a conservative accounting system increases the probability of CEOs being penalized for poor performance through earnings management. Second, a conservative accounting system enhances the incentive spillback effect and, thus, can penalize CEOs with more negative payoff by further relaxing CEOs’

1This chapter is based on a joint work with Carlos Corona and Jonathan Glover.
bankruptcy constraint. Finally, in the extension in which the effect of the accounting system on the cost of earnings management is introduced, we show that accounting conservatism helps earnings management to be costly enough to provide \textit{ex ante} effort incentives.

**Key Words**: accounting conservatism, earnings management, investment, control systems, managerial turnover, executive compensation, spillback effect

### 3.1 Introduction

This paper investigates the interaction between accounting conservatism and earnings management in a multi-period framework in the presence of managerial turnover. We provide new insights on how accounting conservatism and earnings management together can reduce agency problems and improve investment efficiency. The paper also suggests policy implications by highlighting the intertemporal effect of accounting conservatism and its interaction with regulations that aim to strengthen internal control systems.

An extensive body of research has shown that accounting conservatism can help resolve agency problems (Kwon, Newman, & Suh, 2001; Watts, 2003). Since a conservative accounting system gives more precise but less frequent high signals about true earnings, accounting conservatism helps the principal (shareholders) to reward the agent (a manager) more efficiently. In a multi-period framework in which the agent can be fired and replaced after poor performance, however, accounting conservatism might result in inefficiencies, in that it causes more frequent turnover, which can harm a long-term investment of the firm. Earnings management has often been understood as a misdeed by management, but, as discussed in previous chapters, earnings management can act as punishment for poor performance and substitute for the threat of turnover through optimal contracting. Therefore, earnings management can improve investment efficiency as well as decrease agency problems.

In this paper, we study the synergy between accounting conservatism and earnings management in a dynamic model. Because of managerial turnover, which leads to investment inefficiency, the principal might
not be able to implement a conservative accounting system that the principal would choose otherwise. In other words, an optimal accounting system is determined by the trade-off between reducing agency problems through a conservative accounting system and improving investment efficiency through a liberal accounting system. Earnings management, substituting for turnover, can not only improve investment efficiency but also help the principal to maintain a conservative accounting system. Earnings management combined with accounting conservatism can increase the expected punishment for poor performance and, thus, further diminish agency problems. From this perspective, regulation and other enforcements that aim to strengthen control systems might decrease investment efficiency and the firm value.

The paper develops a two-period agency model that features accounting systems, earnings management, investment, and managerial turnover to consider all of the above complexities. In the model, the principal determines the accounting system and offers a long-term contract that specifies compensation, investment, and termination decisions as a function of performance history. The agent chooses a level of productive effort— influencing the productivity of the firm—and engages in earnings management. Based on the reported earnings, the agent can be fired and replaced with a new agent. The threat of dismissal prevents the agent from extracting rents, because the agent can enjoy the rents in period 2 only when her performance is good, and, thus, the incentives in the second period spillback to the incentives in period 1 (Glover & Lin, 2015). Turnover, however, results in inefficiencies. In the case of turnover, there exists a loss in productivity of ongoing investment, because a new manager needs to learn about the ongoing projects or because there are costs associated with searching for a new manager. Thus, the probability of turnover leads to an underinvestment problem. In other words, turnover is an *ex ante* efficient but *ex post* inefficient incentive device.

In the contract with no earnings management, the principal has an incentive to use a more liberal accounting system in order to improve investment efficiency by decreasing the possibility of turnover. A liberal accounting system, however, can lower a disciplining role of accounting conservatism, which
improves the incentive spillback effect (Glover & Lin, 2015).2

Earnings management helps the principal to use a conservative accounting system because earnings management can act as an alternative punishment for poor performance and substitute for the threat of turnover. A conservative accounting system, along with earnings management, can reduce agency problems by increasing the expected punishment for poor performance in two ways. First, the agent would engage in earnings management more often under a conservative accounting system, and, therefore, accounting conservatism increases the probability of the agent being penalized for poor performance. Second, accounting conservatism brings in the intertemporal shift in the information content of performance measure across periods and enhances the incentive spillback effect. As the spillback effect increases, the principal can compensate less for the cost of earnings management because the spillback effect relaxes the incentive compatibility constraint for earnings management. Therefore, it further relaxes the agent’s bankruptcy constraint in period 1, which implies that, when poor performance is realized, the agent experiences more negative payoff that would be penalty for poor performance. In sum, accounting conservatism and earnings management together can raise the expected punishment for poor performance. Consequently, earnings management can not only improve investment efficiency but also reduce the cost of compensation.

In the base model, we assume the cost of earnings management is independent of the choice of accounting system. In the extension, we introduce the effect of accounting systems on the cost of earnings management. A conservative accounting system constrains the manager’s discretion in reporting earnings and makes earnings management more costly. If earnings management is not costly enough, the agent has no incentives to work hard in period 1, because she can manipulate the outcome easily. In other words, to provide ex ante effort incentives, earnings management needs to be costly enough. Therefore, a conservative accounting system helps to introduce earnings management in the optimal contract, which can

---

2In the presence of managerial turnover, the rents in period 2 spillback to period 1 because the agent works hard in period 1 to keep working in period 2 and enjoy the rents in period 2. Because accounting conservatism shifts the information contents of good performance in early periods to later periods, it increases the spillback effect.
reduce agency problems as well as improve investment efficiency. A conservative accounting system, however, can increase the cost of compensation, because inducing earnings management becomes more costly. Thus, a more conservative accounting system is preferred, as its benefits of improving *ex ante* incentives outweigh its costs of increasing the cost of compensation.

Overall, this paper contributes to the literature in three ways. First, it studies the interaction between accounting conservatism and earnings management and shows how accounting conservatism and earnings management work as complements to improve investment efficiency as well as reduce agency conflicts. Prior research has documented mainly whether accounting conservatism mitigates or induces earnings management but has not considered their complementary relationship to improve shareholders’ welfare. Second, the paper highlights the intertemporal effect of accounting conservatism and shows that accounting conservatism, along with earnings management, can improve shareholders’ welfare. Accounting conservatism has been criticized because of its intertemporal reversals, which means an overstatement of earnings in the future. For instance, standard setters have viewed intertemporal reversals as a justification for de-emphasizing accounting conservatism. This paper offers a new rationale for the use of accounting conservatism in managerial compensation by emphasizing the positive aspect of intertemporal reversals. Third, although numerous studies have investigated accounting conservatism and earnings management, they find conflicting evidence on the relationship between accounting conservatism and the level of earnings management. This paper explains their relationship as that of complements or substitutes depending on the state of the firm.

The rest of the paper proceeds as follows. Section 3.2 reviews the related literature. Section 3.3 develops the model and introduces the sequence of events. Section 3.4 studies the benchmark and derives the optimal contract to discuss the basic trade-off of accounting conservatism and earnings management. Section 3.5 studies an extension in which accounting systems influence the cost of earnings management. Section 3.6 concludes.
3.2 Literature Review

Accounting conservatism has been extensively examined from stewardship and valuation perspectives. Prior research shows that accounting conservatism increases the likelihood ratio for high signals, in that it gives more precise but less frequent high signals (Basu, 1997; Kwon, Newman, and Suh, 2001, Gigler et al., 2009). In a single-period agency model, Kwon, Newman, and Suh (2001) show that accounting conservatism facilitates incentivizing the agent by loosening the limited liability constraint. Glover and Haijin (2015) find that the intertemporal reversal caused by accounting conservatism increases the spillback effect in a two-period model. In contrast, our paper studies the interaction between earnings management and accounting conservatism and their effect on a firm’s investment decisions in a dynamic agency model.

The choice of accounting system has an impact on earnings management, but prior studies show conflicting results regarding the effect of accounting conservatism on the level of earnings management. Chen, Hemmer, and Zhang (2007) suggest that accounting conservatism mitigates earnings management by current shareholders in selling their stakes to future shareholders. Gao (2012) finds that conditional conservatism decreases the \textit{ex ante} marginal benefit of earnings management by undoing the impact of earnings management. Caskey and Laux (2016) and Bertomeu, Darrough, and Xue (2015), however, show a positive relationship between accounting conservatism and the level of earnings management. Caskey and Laux suggest that the manager has a stronger incentive to manage earnings when the accounting system is more conservative, because the manager becomes more concerned about board interventions. Bertomeu, Darrough, and Xue find that accounting conservatism, by decreasing the possibility of high earnings, increases the manager’s incentives to engage in earnings management. Because earnings management introduces noise into earnings reports and makes performance measures less informative, it raises the pay-performance sensitivity and, in turn, increases the level of earnings management. Our paper shows, in a single unified model, that the relationship between accounting conservatism can be complementary or substitutive, depending on the state of the firm, and further examines their effect on a firm’s investment decisions.
Many papers study the implications of the impact of accounting conservatism on investment. Watts (2003) finds that accounting conservatism can reduce ex ante overinvestment because debt holders can receive early information to liquidate the project. Lu and Sapra (2009) examine the consequences of auditor conservatism and suggest that, given auditor conservatism, an unfavorable report triggers overinvestment. Garcia Lara, Garcia Osma, and Penalva (2016) find that accounting conservatism improves investment efficiency—mitigating both overinvestment and underinvestment problems. Our paper also investigates the effect of accounting conservatism on investment efficiency but focuses instead on the effect of the combination of accounting conservatism and earnings management on investment efficiency.

3.3 Model

In the model, a risk-neutral principal (shareholders) of the firm hires a risk-neutral agent (a manager) to operate the business. The principal determines the optimal accounting system and provides capital. The manager makes a choice of productive effort, which affects the productivity of the firm, and manages earnings.

3.3.1 Agency Problem

The agent makes two unobservable actions: productive effort \( a_t \) and earnings management \( B_t \). The agent’s productive effort \( a_t \) affects the productivity of the firm \( x_t \) as follows:

\[
P[x_H|a_H] = p \quad \text{(3.1)}
\]

\[
P[x_H|a_L] = q \quad \text{(3.2)}
\]

where \( p > q \) for \( t \in \{1, 2\} \). The cost of high effort \( a_H \) is \( h \) and the cost of low effort \( a_L \) is normalized to 0.

The agent privately observes accounting information, or unmanaged earnings, \( y_t \) about true earnings \( e_t \).
and reports earnings ($R_t$) with earnings management $B_t$. The cost of earnings management is $C_{EM}$.\(^3\) In a two-period model, I assume that earnings management made in period 1 is reversed in period 2.

The agent has no initial wealth and has limited liability, which rules out a negative wage. And the agent has a higher discount rate than the principal, $\rho_A > \rho_P$, which means that the manager is more impatient than the shareholders. The agent’s reservation utility is normalized to zero, which is related to her outside opportunity if the contract is terminated and the agent is fired.

The termination of contract involves replacing the initial agent with a new agent, and replacement of the agent results in a loss in productivity of ongoing investment, which means the dismissal is inefficient ex-post and brings in deadweight losses. For instance, when the agent is replaced, the new agent has to learn the ongoing projects of the firm, or there are costs associated with searching for a new agent. we formulate these costs as a loss in productivity of ongoing investment. Therefore, if turnover takes place, the productivity in period 2 is

$$x_2^{New} = (1 - \ell)x_2$$

(3.3)

where $\ell \in [0, 1)$ reflects a cost of a loss in productivity.

### 3.3.2 Investment and Production Technology

The firm uses physical capital to produce output, and the price of capital is normalized to one. The capital stock $K_t$ changes as follows:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

(3.4)

where $I_t$ is the gross investment and $\delta \geq 0$ is the rate of depreciation.

Based on the Cobb-Douglas production function, the earnings $e_t$ are

$$e_t = x_t K_t$$

(3.5)

\(^3\)I assume that $C_{EM} \geq \frac{h}{p-q}$ so that the principal can implement contracts preventing the agent from engaging in earnings management.
If the agent is replaced, the earnings \( e_2^{\text{New}} \) are

\[
e_2^{\text{New}} = x_2^{\text{New}}K_2
\]

\[
= (1 - \ell)x_2K_2
\]

Investment encompasses the adjustment cost, \( J(I_t, K_t) \). Following Hayashi (1982), we assume that the adjustment cost is homogeneous of degree one in \( I \) and \( K \), which enables the adjustment cost to take the form

\[
J(I_t, K_t) = j(i_t)K_t,
\]

where \( i_t = \frac{I_t}{K_t} \) is the investment-capital ratio, \( g'(i) > 0 \), and \( g''(i) > 0 \). For simplicity, we assume \( g(i) = \frac{\theta}{2}i^2 \), where \( \theta \) is the degree of the adjustment cost.

Thus, the total cost of investment is

\[
I_t + J(I_t, K_t) = (i_t + j(i_t))K_t
\]

\[
= \xi(i_t)K_t
\]

where \( \xi \) is the total cost per unit of capital required when the firm invests to grow at rate \( i_t \).

In the two-period model, we assume the capital stock in period 1, \( K_1 \), is given and the principal determines the amount of investment, \( I_1 \), before observing the earnings report in period 1, \( R_1 \).

### 3.3.3 Accounting System

The accounting system gives information \( y_t \) about true earnings \( e_t \). The accounting system is subject to bias, and the bias is reversed in the long-run as follows (Glover and Lin, 2015):

\[
p_1 = P[y_1 = Y_H|\varepsilon, a_H] = p + \varepsilon
\]

\[
p_2 = P[y_2 = Y_H|\varepsilon, a_H] = p - \varepsilon
\]

\[
q_1 = P[y_1 = Y_H|\varepsilon, a_L] = q + \varepsilon
\]

\[
q_2 = P[y_2 = Y_H|\varepsilon, a_L] = q - \varepsilon
\]
where \( z = \min[p, q, 1 - p, 1 - q] \).

The principal determines the accounting system \( \varepsilon \in [-z, z] \). If \( \varepsilon \in [-z, 0) \), a conservative accounting system is introduced. If \( z = 0 \), a neutral accounting system is introduced. And if \( \varepsilon \in (0, -z] \), a liberal accounting system is introduced. Figure 3.1 shows the timeline.

### 3.3.4 Optimal Contracting Problem

The principal offers a long-term contract which specifies compensation, investment, and a termination (firing) decision as a function of performance history. Let \( S(y_1) \) and \( S(y_1, y_2) \) be the agent’s compensation in period 1 and 2, respectively. Let \( W_2^y \) be the agent’s expected payoff in period 2 if \( y_1 = Y_j \).

We assume that earnings report \( R_t \) and capital stock \( K_t \) are observable and contractible. Therefore, investment \( I_1 \), or the investment-capital ratio \( i_1 \), is also contractible. The revelation principle is applicable (Myerson, 1979) because the contract is implemented after the agent possesses the private information. Accordingly, the contract is conditional on the private information \( (y_t) \).

The principal offers a contract that specifies the agent’s compensation, \( S(y_1) \) and \( S(y_1, y_2) \), earnings reports, \( R(y_1) \) and \( R(y_1, y_2) \), the firm’s investment policy \( I_1 \), and a termination decision, \( TD(y_1) \). \( TD(y_1) = \{0, 1\} \) defines a firing condition on \( y_1 \) such that 0 means a continuation of contract and 1 means the termination of contract. If the agent is fired, the principal signs a contract with a new agent that specifies \( S(y_2^{New}) \) conditional on \( y_2^{New} \).

Let \( W \) be the agent’s expected payoff after the cost of effort is sunk. The principal chooses
\( \varepsilon, S(y_1), S(y_1, y_2), R(y_t), I_t, \) and \( TD(y_1) \) to maximizes the following problem:

\[
\max_{\varepsilon, S(y_1), S(y_1, y_2), R(y_t), I_t, TD(y_1)} 
E[x_1K_1 - S(y_1) - \xi(i_1)K_1 + \frac{1}{1 + \rho_P} \{[x_2K_2 - S(y_1, y_2)]1_{TD(y_1)=0} + [x_2^{New}K_2 - S(y_2^{New})]1_{TD(y_1)=1}\] 
\]

subject to

\[
E[W(S(y_1), S(y_1, y_2), R(y_1))|a_H] - E[W(S(y_1), S(y_1, y_2), R(y_1))|a_L] \geq h 
\] (3.15)

\[
E[S(y_1, y_2)|a_H] - E[S(y_1, y_2)|a_L] \geq h 
\] (3.16)

\[
E[W(S(y_1), S(y_1, y_2), R(y_1))|a_H] \geq h 
\] (3.17)

\[
E[S(y_1, y_2)|a_H] \geq h 
\] (3.18)

\[
E[W(S(y_1), S(y_1, y_2), R(y_1))|y_1] - E[W(S(y_1), S(y_1, y_2), R(y_1))|y_1] \geq h 
\] (3.19)

\[
S(y_1), S(y_1, y_2) \geq 0 
\] (3.20)

The principal maximizes the expected earnings net of compensation in the equation (3.14). The equations (3.15) and (3.16) are the incentive compatibility constraints for the agent in period 1 and 2, respectively. The equations (3.17) and (3.18) are the individual rationality constraints for the agent in period 1 and 2, respectively. The equation (3.19) is the incentive compatibility constraint for truth-telling. Lastly, the equation (3.20) is the agent’s limited liability constraints.

In the case of turnover, the principal signs a contract with a new agent. The principal chooses \( S(y_2^{New}) \) in the following problem:

\[
\max_{S(y_2^{New})} 
E[x_2K_2 - S(y_2^{New})] 
\] (3.21)
subject to

\[ E[ S(y_{2}^{new}) \mid a_H] - E[ S(y_{2}^{new}) \mid a_L] \geq h \] (3.22)

\[ E[ S(y_{2}^{new}) \mid a_H] \geq h \] (3.23)

\[ S(y_{2}^{new}) \geq h \] (3.24)

The equations (3.22), (3.23), and (3.24) are the incentive compatibility constraint, the individual rationality constraint, and the limited liability constraint, respectively.

### 3.4 Model Solution and Optimal Contracting

#### 3.4.1 Benchmark

We consider the benchmark case where there the agent’s actions are observable. The agent chooses high effort \( a_H \) in both periods and no earnings management is induced \( (B_t = 0) \). The principal chooses \( i_1 \) to maximizes

\[ \max_{\varepsilon, S(y_1), S(y_1, y_2), i_1} E[x_1K_1 - h - \xi (i_1)K_1 + \frac{1}{1 + \rho_p}[(x_2K_2) - h]] \] (3.25)

The optimal investment-capital ratio \( i_1^{FB} \) satisfies

\[ \xi'(i_1^{FB}) = \frac{1}{1 + \rho_p} E[x_2] \] (3.26)

Lemma 1 summarizes the principal’s expected payoff and the investment-capital ratio \( i_1 \) when there is no agency concern.
Lemma 1. When there is no agency concern, the principal’s expected payoff is

\[ E[x_1 K_1 - h - \xi(i_1^{FB})K_1 + \frac{1}{1+\rho_p}[x_2 K_1(1 - \delta + i_1^{FB}) - h]] \] (3.27)

where the investment-capital ratio \( i_1^{FB} \) satisfies \( \xi'(i_1^{FB}) = \frac{1}{1+\rho_p} E[x_2] \).

In the benchmark case, the idiosyncratic productivity shock does not affect the firm’s investment or firm value. The next section will discuss how agency problem will change these results significantly.

3.4.2 Optimal Contract without Turnover

Before investigating the interaction between earnings management and accounting conservatism and its effect on the optimal contract in the presence of managerial turnover, we consider two cases without managerial turnover. The first case examines the effect of accounting conservatism and earnings management on contract efficiency in a single-period model. The second case studies the effect of accounting conservatism and earnings management on contract efficiency as well as investment efficiency in a two-period model without the threat of dismissal.

In the next section, we discuss how earnings management can improve investment efficiency and how earnings management interacts with accounting conservatism to improve shareholders’ welfare.

3.4.2.1 Case 1: Single-Period Model

In a single-period model, the principal maximizes

\[ \max_{\varepsilon, S(y_1)} E[x_1 K_1 - S(y_1)] \] (3.28)
subject to

\[ S(R_L; Y_L) \geq S(R_H; Y_L) - C_{EM} \]  

or \[ S(R_H; Y_L) - C_{EM} \geq S(R_L; Y_L) \] (3.29)

\[ p_1 S(Y_H) + (1 - p_1) S(Y_L) - h \geq q_1 S(Y_H) + (1 - q_1) S(Y_L) \]

or \[ p_1 S(R_H; Y_H) + (1 - p_1)(S(R_H; Y_L) - C_{EM}) - h \geq q_1 S(R_H; Y_H) + (1 - q_1)(S(R_H; Y_L) - C_{EM}) \] (3.30)

\[ p_1 S(Y_H) + (1 - p_1) S(Y_L) - h \geq 0 \] or \[ p_1 S(R_H; Y_H) + (1 - p_1)(S(R_H; Y_L) - C_{EM}) \geq 0 \] (3.31)

\[ S(Y_H), S(Y_L) \geq 0 \] or \[ S(R_H; Y_H), S(R_H; Y_L), S(R_L; Y_L) \geq 0 \] (3.32)

The principal maximizes the expected earnings net of compensation in the equation (3.28). The equation (3.29) are the incentive compatibility constraint for (no) earnings management. The equations (3.30) and (3.31) are the incentive compatibility constraint and the individual rationality constraint, respectively. The equation (3.32) is the agent’s limited liability constraints.

In a single-period model, the contract that does not induce earnings management is optimal because earnings management results in a deadweight loss only. The optimal accounting system is conservative \((\epsilon = -z)\) in a single-period model since accounting conservatism increases the likelihood ratio for high signals and, thus, the principal can reward the manager more efficiently. Proposition 1 summarizes the optimal contract and accounting system in a single-period model.

**Proposition 1.** In the single-period model, the optimal contract satisfies

\[ S(Y_H) = \frac{h}{p - q}, \quad S(Y_L) = 0 \] (3.33)

The agent’s optimal reporting strategy is

\[ R(Y_H) = R_H, \quad R(Y_L) = R_L \] (3.34)
The optimal accounting system $\epsilon^*$ is

$$\epsilon^* = -z$$  \hspace{1cm} (3.35)  

In a single-period model in which there is no managerial turnover, the disciplining role of accounting conservatism dominates. This is consistent with the finding in Kwon, Newman, and Suh (2001). The next case will show that the intertemporal effect of accounting systems in a two-period model can alter these results significantly.

3.4.2.2 Case 2: Two-Period Model without turnover

The second case studies a two-period model in which the principal cannot replace the agent and the agent works for two periods. The principal maximizes

$$\text{Max}_{\epsilon, S(y_1), S(y_1, y_2), i_1} E\left[ x_1 K_1 - S(y_1) - \xi(i_1) K_1 + \frac{1}{1 + \rho P} [x_2 K_2 - S(y_1, y_2)] \right]$$  \hspace{1cm} (3.36)  

subject to the incentive compatibility constraint for (no) earnings management, the incentive compatibility constraints in period 1 and 2, the individual rationality constraints in period 1 and 2, and the agent’s limited liability constraints.

Similar to a single-period model, earnings management results in a deadweight loss only and, thus, the optimal contract does not induce earnings management. The optimal accounting system is irrelevant if $\rho P = 0$ because any change in the expected compensation in period 1 due to accounting systems is offset by a reversed change in the expected compensation in period 2. Therefore, the intertemporal effect of accounting systems makes the choice of accounting systems irrelevant. If $\rho P > 0$, however, the optimal accounting system is conservative ($\epsilon = -z$) in period 1 because its effect in period 1 dominates its reversed effect in period 2. Proposition 2 summarizes the optimal contract and accounting system in a two-period model
Proposition 2. In the two-period model without managerial turnover, the optimal contract satisfies

\begin{align}
S(Y_H) &= \frac{h}{p - q}, \quad S(Y_L) = 0, \quad (3.37) \\
S(Y_H, Y_H) &= \frac{h}{p - q}, \quad S(Y_H, Y_L) = 0, \quad (3.38) \\
S(Y_L, Y_H) &= \frac{h}{p - q}, \quad S(Y_L, Y_L) = 0 \quad (3.39)
\end{align}

The agent’s optimal reporting strategy is

\begin{align}
R(Y_H) &= R_H, \quad R(Y_L) = R_L, \quad (3.40) \\
R(Y_H, Y_H) &= R_H, \quad R(Y_H, Y_L) = R_L, \quad (3.41) \\
R(Y_L, Y_H) &= R_H, \quad R(Y_L, Y_L) = R_L \quad (3.42)
\end{align}

The optimal investment-capital ratio \( i^*_1 \) satisfies

\[ \xi'(i^*_1) = \frac{1}{1 + \rho_P} E[x_2] \quad (3.43) \]

The optimal accounting system \( e^* \) is

\begin{align}
\text{irrelevant} & \quad \text{if } \rho_P = 0 \quad (3.44) \\
-z & \quad \text{if } \rho_P > 0 \quad (3.45)
\end{align}

In a two-period model without managerial turnover, the disciplining role of accounting conservatism decreases because of its intertemporal effect in a subsequent period. The next section will study the
intertemporal effect of accounting systems in the presence of managerial turnover.

3.4.3 Optimal Contract in the Presence of Turnover

3.4.3.1 Contract with No Earnings Management

We consider a two-period model in which the agent is fired and replaced with a new agent if low earnings are reported in the first period. The threat of dismissal reduces the agent’s rent because the incentives in period 2 spillback to incentives in period 1. To capture the intuition on the complementary effect of earnings management and accounting conservatism, we study the contracts that induces and prevents earnings management separately and later derive the optimal contract.

We first study the contract preventing earnings management. The principal maximizes

$$\begin{align*}
\max_{\varepsilon, S(y_1)} & \quad E[x_1 K_1 - S(y_1) - \xi(i_1) K_1 + \frac{1}{1 + \rho_p} \left\{ (x_2 K_2 - S(y_1, y_2))_{y_1 = Y_H} + \{ x_{\text{New}}^2 K_2 - S_{\text{New}}(y_{\text{New}}^2) \}_{y_1 = Y_L} \right\}]
\end{align*}$$

subject to the incentive compatibility constraint for no earnings management, the incentive compatibility constraints in period 1 and 2, the individual rationality constraints in period 1 and 2, and the agent’s limited liability constraints.

In the case of turnover, the principal offers a contract to a new agent. The principal maximizes

$$\begin{align*}
\max_{S_{\text{New}}(y_{\text{New}}^2)} & \quad E[x_{\text{New}}^2 K_2 - S_{\text{New}}(y_{\text{New}}^2)]
\end{align*}$$

subject to the incentive compatibility constraint and the individual rationality constraint.

In a two-period model with managerial turnover, the capital-investment ratio $i_1$ can be lower than the benchmark because there exists the possibility of a loss in productivity of ongoing investment due to turnover. The optimal accounting system $\varepsilon^*$ is determined by the trade-off between the spillback effect and investment efficiency. A conservative accounting system magnifies the spillback effect and, thus, reduces
the cost of providing incentives in period 1 because a conservative accounting system defers much of the incentive compensation from period 1 to period 2. A conservative accounting system, however, decreases investment efficiency since it yields low reporting more often and, thus, increases the possibility of turnover. Therefore, a more liberal accounting system is preferred as improving investment efficiency is more valuable than the spillback effect. Lemma 2 states the contract and the accounting system in a two-period model with managerial turnover and no earnings management.

**Lemma 2.** In a two-period model with CEO turnover and no earnings management, the contract satisfies

\[
S(Y_H) = \frac{h}{p-q}[1 - \frac{q - \epsilon^*}{1 + \rho_A}], \quad S(Y_L) = 0, \quad (3.48)
\]

\[
S(Y_H, Y_H) = \frac{h}{p-q}, \quad S(Y_H, Y_L) = 0, \quad (3.49)
\]

\[
S(Y_{New}^H) = \frac{h}{p-q}, \quad S(Y_{New}^L) = 0 \quad (3.50)
\]

The agent’s reporting strategy is

\[
R(Y_H) = R_H, \quad R(Y_L) = R_L, \quad (3.51)
\]

\[
R(Y_H, Y_H) = R_H, \quad R(Y_H, Y_L) = R_L, \quad (3.52)
\]

\[
R(Y_L, Y_H) = R_H, \quad R(Y_L, Y_L) = R_L \quad (3.53)
\]

The investment-capital ratio \(i_1^*\) satisfies

\[
\xi'(i_1^*) = \frac{1}{1 + \rho_p}[p + \epsilon^* + (1 - p - \epsilon^*)(1 - \ell)]E[x_2] \quad (3.54)
\]

The choice of accounting system \(\epsilon^*\) is

\[
\epsilon^* = \bar{\epsilon} \quad (3.55)
\]
where \( z = f(\ell, h; X_H, X_L, p, q, K_1, \rho_P, \delta, \theta) \) and \( z \geq \bar{z} \geq -z \).

The cost of effort \( h \), which also captures the severity of moral hazard, influences the spillback effect because the compensation in period 2 increases with \( h \). Thus, the accounting system becomes more conservative as \( h \) increases. On the other hand, a loss in productivity of ongoing investment \( \ell \) in the case of turnover has an effect on investment efficiency. In other words, investment efficiency decreases with \( \ell \). Therefore, the accounting system becomes more liberal as \( \ell \) increases. Corollary 1 states the effect of \( h \) and \( \ell \) on the choice of accounting system \( \varepsilon \).

**Corollary 1.** The accounting system \( \varepsilon^* \) is
(a) decreasing in \( h \);
(b) increasing in \( \ell \)

In a two-period model with turnover and no earnings management, the threat of dismissal decreases investment efficiency and reduces the spillback effect by giving the principal incentives to implement a more liberal accounting system. The next section shows how earnings management can improve investment efficiency as well as enhance the spillback effect by helping the principal to adopt a conservative accounting system.

### 3.4.3.2 Contract with Earnings Management

Now we study the contract that induces earnings management when low earnings are reported in period 1.

The principal maximizes

\[
\max_{\varepsilon, S(y_1), S(y_1, y_2), i_1} E[x_1K_1 - S(y_1) - \xi(i_1)K_1 + \frac{1}{1 + \rho_P}[x_2K_2 - S(y_1, y_2)]
\]  

(3.56)
subject to the incentive compatibility constraint for earnings management, the incentive compatibility constraints in period 1 and 2, the individual rationality constraints in period 1 and 2, and the agent’s limited liability constraints.

In a two-period model with managerial turnover, earnings management helps not only improve investment efficiency but also reduce agency problems. As discussed in previous chapters, earnings management can act as an alternative punishment and substitute for the threat of turnover. Thus, it can mitigate an underinvestment problem as well as enable the principal to implement a conservative accounting system. A conservative accounting system combined with earnings management can further decrease agency problems because it can increase the expected punishment for poor performance in two ways. First, a conservative accounting system increases the probability of the agent engaging in earnings management—being penalized for poor performance. Second, a conservative accounting system increases the spillback effect and, hence, it relaxes the incentive compatibility constraint for earnings management. In other words, the principal can compensate less for the cost of earnings management, leading to further relaxation of the agent’s bankruptcy constraint. Accordingly, the agent experiences more negative payoff due to the cost of earnings management, which would be penalty for poor performance. In sum, accounting conservatism raises the expected punishment and, in turn, decreases the expected cost of compensation.

Lemma 3 shows the contract and the accounting system in a two-period model with managerial turnover and earnings management.

**Lemma 3.** In a two-period model with CEO turnover and earnings management, the contract satisfies

\[
S(Y_t) = C_{EM} - \frac{1}{1 + \rho_A} \frac{(q - \epsilon^*)h}{p - q} \quad \text{for } t = H, L, \tag{3.57}
\]

\[
S(Y_H, Y_H) = \frac{2h}{p - q}, \quad S(Y_H, Y_L) = \frac{h}{p - q}, \tag{3.58}
\]

\[
S(Y_L, Y_H) = \frac{h}{p - q}, \quad S(Y_L, Y_L) = 0 \tag{3.59}
\]
The agent’s optimal reporting strategy is

\[ R(Y_H) = R_H, \quad R(Y_L) = R_H, \]  
\[ R(Y_H, Y_H) = R_H, \quad R(Y_H, Y_L) = R_L, \]  
\[ R(Y_L, Y_H) = R_L, \quad R(Y_L, Y_L) = R_{LL} \]  
(3.60)

The investment-capital ratio \( i_1^* \) satisfies

\[ \xi'(i_1^*) = \frac{1}{1 + \rho P} E[x_2] \]  
(3.63)

The optimal accounting system \( \varepsilon^* \) is

\[ \varepsilon^* = -z \]  
(3.64)

The contract can be represented based on reported earnings as stated by Corollary 2.

**Corollary 2.** In a two-period model with CEO turnover and earnings management, the contract can be represented based on reported earnings.

\[ S(R_H) = C_{EM} - \frac{1}{1 + \rho_A} \frac{(q - \varepsilon^*) h}{p - q}, \quad S(R_L) = 0, \quad S(R_H, R_H) = \frac{2h}{p - q}, \quad S(R_H, R_L) = \frac{h}{p - q}, \]  
(3.65)

\[ S(R_H, R_L) = \frac{h}{p - q}, \quad S(R_L, R_{LL}) = 0 \]  
(3.66)

Now we want to search for the optimal contract in the presence of managerial turnover and find the
conditions under which earnings management can improve investment efficiency as well as the principal’s expected payoff. The next section will provide a rationale for the use of earnings management in the optimal dynamic contract.

### 3.4.3.3 Optimal Contract

Searching for the optimal contract in a two-period model with managerial turnover can be achieved by comparing the contract inducing earnings management with the contract preventing earnings management. We search for the conditions under which the benefits of earnings management outweigh the cost of earnings management. Because earnings management can motivate the agent as punishment for poor performance, it substitutes for turnover, which results in investment inefficiency. Also, earnings management combined with accounting conservatism can further decrease the expected cost of compensation because the combination of accounting conservatism and earnings management increases the expected punishment for poor performance in two ways: the increased probability of engaging in earnings management and the enhanced spillback effect. First, a conservative accounting system yields low signals more likely and, thus, it increases the probability of the agent engaging in costly earnings management—punishment for poor performance. Second, a conservative accounting system involves the intertemporal shift in the information content of performance measure across periods and enhance the incentive spillback effect as discussed in Glover and Haijin (2015). As the spillback effect relaxes the incentive compatibility constraint for earnings management, accounting conservatism along with earnings management further relaxes the agent’s bankruptcy constraint. In other words, the agent bears more negative payoff which would be penalty for poor performance. Therefore, earnings management and accounting conservatism together can increase the expected punishment for poor performance, leading to a decrease in the expected cost of compensation. The cost of compensation, however, can increase as inducing earnings management is costly and the principal needs to provide the agent with incentives to manage earnings.
A loss in productivity of ongoing investment $\ell$ influences the net effect of inducing earnings management on investment efficiency and accounting conservatism. As $\ell$ increases in the contract with no earnings management, investment efficiency decreases and a more liberal accounting system is adopted, which reduces the spillback effect. Therefore, the contract inducing earnings management is preferred as $\ell$ increases. However, the cost of compensation can increase as $C_{EM}$ increases. Hence, the contract inducing earnings management is preferred as $C_{EM}$ is not sufficiently large. Proposition 3 discusses the optimal contract in a two-period model in the presence of managerial turnover.

**Proposition 3.** The contract inducing earnings management dominates the contract preventing earnings management if

(a) $\ell$ is not sufficiently small ($\exists T$ such that $l > T$);

(b) $C_{EM}$ is not sufficiently large ($\exists C$ such that $C_{EM} < C$)

In the next section, we relax the assumption on the cost of earnings management $C_{EM}$ and study how it gives more fruitful and interesting results.

### 3.5 Extension

In this section, we take into account the effect of accounting systems on the cost of earnings management. Accounting systems can influence not only the information production but also the cost of earnings management. A more conservative accounting system reduces the manager’s discretion to manage earnings and, thus, increases the marginal cost of earnings management ($\frac{\partial}{\partial \varepsilon} C_{EM}(\varepsilon) < 0$). For simplicity, I assume that $C_{EM} = c(1 - \varepsilon)$, where $c$ represents control systems such as external enforcements and internal control systems.\(^4\) Therefore, the cost of earnings management is influenced by control systems as well as the choice of accounting systems.

\(^4\)I relax the assumption $C_{EM} \geq \frac{h}{p-q}$ in this section so that the choice of accounting system has more influence on the contract.
The effect of the accounting system $\varepsilon$ on $C_{EM}(\varepsilon)$ introduces additional influences on each contract with and without earnings management. In the contract with no earnings management, the accounting system influences the incentive compatibility constraint for no earnings management. The principal can prohibit earnings management through optimal contracting only when the cost of earnings management $C_{EM}(\varepsilon)$ is relatively large. Therefore, the principal might not be able to make the accounting system liberal enough to avoid turnover and improve investment efficiency. In other words, there exists an additional demand for accounting conservatism to prevent earnings management. In the contract with earnings management, the accounting system has two additional effects on the contract. First, it affects the incentive compatibility in period 1 because earnings management which is costly enough can motivate the agent to work hard. If earnings management is not costly enough, the agent has no incentives to work hard because she can manipulate the outcome easily. Therefore, a conservative accounting system enables earnings management to reduce agency problems as well as improve investment efficiency. Second, a conservative accounting system can increase the cost of compensation since inducing earnings management becomes more costly. Thus, in the contract with earnings management, the accounting system introduces additional trade-off between an increase in ex ante incentives through a conservative accounting system and a decrease in the cost of compensation through a liberal accounting system. Lemma 4 and 5 summarize the contract and the accounting system in a two-period model with managerial turnover when the accounting system has an effect on the cost of earnings management.

**Lemma 4.** In a two-period model with CEO turnover and no earnings management, the contract satisfies

\[ S(Y_H) = \frac{h}{p-q}[1-\frac{q-\varepsilon^*}{1+\rho_A}], \quad S(Y_L) = 0 \]  
\[ S(Y_H, Y_H) = \frac{h}{p-q}, \quad S(Y_H, Y_L) = 0 \]  
\[ S(Y_H^{New}) = \frac{h}{p-q}, \quad S(Y_L^{New}) = 0 \]
\[ c \geq \frac{h}{(1 - \varepsilon^*)(p - q)} \left[ 1 - \frac{q - \varepsilon^*}{1 + \rho_A} \right] \] (3.71)

The agent’s reporting strategy is

\[ R(Y_H) = R_H, \quad R(Y_L) = R_L \] (3.72)

\[ R(Y_H, Y_H) = R_H, \quad R(Y_H, Y_L) = R_L \] (3.73)

\[ R(Y_L, Y_H) = R_H, \quad R(Y_L, Y_L) = R_L \] (3.74)

The investment-capital ratio \( i^*_1 \) satisfies

\[ \xi'(i^*_1) = \frac{1}{1 + \rho_P} [p + \varepsilon^* + (1 - p - \varepsilon^*)(1 - \ell)]E[x_2] \] (3.75)

The choice of accounting system \( \varepsilon^* \) is

\[ \varepsilon^* = \tilde{z} \quad if \quad c \geq \frac{h}{(1 - \tilde{z})(p - q)} \left[ 1 - \frac{q - \tilde{z}}{1 + \rho_A} \right] \] (3.76)

\[ \varepsilon^* = z \quad if \quad c < \frac{h}{(1 - z)(p - q)} \left[ 1 - \frac{q - z}{1 + \rho_A} \right] \] (3.77)

where \( \tilde{z} = f(\ell, h; X_H, X_L, p, q, K_1, \rho_P, \rho_A, \delta, \theta) \), \( z = \frac{c(p - q)(1 + \rho_A) - h(1 + \rho_A - q)}{c(p - q)(1 + \rho_A) + h} \) and \( \tilde{z} > z \geq -z \).

Lemma 4 shows that the choice of accounting system is contingent on control systems \( c \). As \( c \) is large, the principal can implement a more liberal accounting system to improve investment efficiency. As \( c \) is small, however, the principal needs to implement a more conservative accounting system to make earnings management costly enough to provide the agent with ex ante incentives to work hard.
Lemma 5. In a two-period model with CEO turnover and earnings management, the contract satisfies

\[ S(Y_t) = 0 \text{ for } t = H, L, \quad \text{if } c \leq \frac{1}{1 + \rho_A} \frac{(q - \epsilon^*)h}{(1 - \epsilon^*)(p - q)} \]  

(3.78)

\[ S(Y_t) = C_{EM}(\epsilon^*) - \frac{1}{1 + \rho_A} \frac{(q - \epsilon^*)h}{p - q} \text{ for } t = H, L, \quad \text{if } c > \frac{1}{1 + \rho_A} \frac{(q - \epsilon^*)h}{(1 - \epsilon^*)(p - q)} \]  

(3.79)

\[ S(Y_H, Y_H) = \frac{2h}{p - q}, \quad S(Y_H, Y_L) = \frac{h}{p - q}, \quad S(Y_L, Y_H) = \frac{h}{p - q}, \quad S(Y_L, Y_L) = 0 \]  

(3.80)

\[ c \geq \frac{h}{(1 - \epsilon^*)(p - q)} \frac{\rho_A}{1 + \rho_A} \]  

(3.82)

The agent’s optimal reporting strategy is

\[ R(Y_H) = R_H, \quad R(Y_L) = R_H \]  

(3.83)

\[ R(Y_H, Y_H) = R_H, \quad R(Y_H, Y_L) = R_L \]  

(3.84)

\[ R(Y_L, Y_H) = R_L, \quad R(Y_L, Y_L) = R_{LL} \]  

(3.85)

The investment-capital ratio \( i_1^* \) satisfies

\[ \xi'(i_1^*) = \frac{1}{1 + \rho_p} E[x_2] \]  

(3.86)

The choice of accounting system \( \epsilon^* \) is

\[ \epsilon^* = z \quad \text{if } c > \max\left[ \frac{h}{(p - q)(1 + \rho_A)}, \frac{h}{(1 - z)(p - q)} \right] \frac{\rho_A}{1 + \rho_A} \]  

(3.87)

\[ \epsilon^* = \tilde{z} \quad \text{if } c > \frac{h}{(1 - z)(p - q)} \frac{\rho_A}{1 + \rho_A} \]  

(3.88)

\[ \epsilon^* = -z \quad \text{if } c > \frac{h}{(p - q)(1 + \rho_A)} \]  

(3.89)

\[ \forall \epsilon^* \text{ satisfying } c \geq \frac{h}{(1 - \epsilon^*)(p - q)} \frac{\rho_A}{1 + \rho_A} \quad \text{if } \frac{1}{1 + \rho_A} \frac{(q - \epsilon^*)h}{(1 - \epsilon^*)(p - q)} \geq c \text{ or } c = \frac{h}{(p - q)(1 + \rho_A)} \]  

(3.90)
where \( \hat{z} = 1 - \frac{1}{c} \frac{h}{p-q} \left( \frac{\rho A}{1+p} \right) \) and \( z > \hat{z} > -z \).

Lemma 5 shows that the trade-off of accounting conservatism depends on control systems \( c \). As \( c \) decreases, the benefits of accounting conservatism outweigh its costs because a more conservative accounting system makes earnings management more costly so that earnings management can be used to give \textit{ex ante} effort incentives. As \( c \) increases, however, the costs of accounting conservatism increase because it increases the cost of compensation.

The contract with earnings management can be represented based on reported earnings as stated by Corollary 3.

**Corollary 3.** In a two-period model with CEO turnover and earnings management, the contract can be represented based on reported earnings.

\[
S(R_t) = \begin{cases} 
0 & \text{for } t = H, L, \text{ if } c \leq \frac{1}{1 + \rho A} \frac{(q - \epsilon^*) h}{(1 - \epsilon^*)(p - q)} \\
C_{EM}(\epsilon^*) - \frac{1}{1 + \rho A} \frac{(q - \epsilon^*) h}{p - q}, & S(R_L) = 0, \text{ if } c > \frac{1}{1 + \rho A} \frac{(q - \epsilon^*) h}{(1 - \epsilon^*)(p - q)}
\end{cases} \tag{3.91}
\]

\[
S(R_H) = C_{EM}(\epsilon^*) - \frac{1}{1 + \rho A} \frac{(q - \epsilon^*) h}{p - q}, \text{ if } c \leq \frac{1}{1 + \rho A} \frac{(q - \epsilon^*) h}{(1 - \epsilon^*)(p - q)} \tag{3.92}
\]

\[
S(R_H, R_H) = \frac{2h}{p-q}, \quad S(R_H, R_L) = \frac{h}{p-q}, \quad S(R_L, R_L) = 0 \tag{3.93}
\]

Earnings management can improve investment efficiency and reduce the cost of compensation through the enhanced spillback effect by adopting a conservative accounting system. Earnings management, however, can increase the cost of compensation since the principal needs to provide the agent with incentives to engage in costly earnings management. Introducing the effect of accounting system on the cost of earnings management brings in additional features of accounting conservatism. A more conservative accounting system can improve \textit{ex ante} effort incentives by making earnings management
costly enough to motivate the agent to work hard. On the other hand, a more conservative accounting system can increase the cost of compensation because inducing earnings management become more costly. Thus, the choice of accounting system is influenced by the trade-off of accounting conservatism between \textit{ex ante} effort incentives and the cost of compensation.

The consequences of earnings management are contingent on $\ell$ and $c$. The same intuition on the effect of $\ell$ on the optimal contract in the previous section is still carried over. Intuitively, the positive effects of earnings management on investment efficiency and the spillback effect increase with $\ell$. As $c$ increases, the positive effect of earnings management diminishes because it not only increases the cost of compensation but also decreases the value of accounting conservatism increasing \textit{ex ante} effort incentives. Consequently, the contract inducing earnings management is preferred as $c$ is not sufficiently large. Proposition 4 discusses the optimal contract when the choice of accounting systems has an effect on the cost of earnings management.

**Proposition 4.** The contract inducing earnings management dominates the contract preventing earnings management if

(a) $\ell$ is not sufficiently small ($\exists \ell$ such that $l > \ell$);

(b) $c$ is not sufficiently large ($\exists c$ such that $c < c$)

Under the assumption that $C_{EM}$ is independent of $\varepsilon$, accounting conservatism influences the expected cost of compensation. Under the assumption that $C_{EM}(\varepsilon)$ is contingent on $\varepsilon$, accounting conservatism not only affects the expected cost of compensation further but also helps earnings management to be costly enough to provide \textit{ex ante} effort incentives. The total benefits of accounting conservatism dominates as $c$ is not sufficiently large.
3.6 Conclusion

This paper examines the interaction between accounting conservatism and earnings management and their effects on a firm’s investment decisions and the firm value. Even though accounting conservatism involves the intertemporal shift in the information content of performance measure across periods, its dynamic insights have not been understood well. We show that earnings management and accounting conservatism not only improve investment efficiency but also reduce agency problems. Earnings management as punishment for poor performance substitutes for managerial turnover, which results in an underinvestment problem, and helps the firm to adopt a conservative accounting system. A conservative accounting system combined with earnings management increases the expected punishment for poor performance. It increases the probability of being penalized for poor performance as well as the amount of punishment—negative payoff—the agent receives for poor performance due to the enhanced spillback effect.

In the extension, we consider the case in which a more conservative accounting system increases the cost of earnings management. Accounting conservatism helps make earnings management costly and, thus, the firm to use earnings management to motivate the agent to work hard as well as improve investment efficiency.

Future studies could explore full dynamic insights using an infinite-horizon dynamic model in which the firm can change its accounting systems over time. The firm might have different incentives to implement different accounting systems depending on past performance of the firm. This will help us to further investigate the effect of intertemporal reversals on a firm’s important decisions.
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3.7 Appendix

Notations

\[ S_1 = S(y_1) = S(R(y_1)) : \text{Compensation in period 1} \]
\[ S_2 = S(y_1, y_2) = S(R(y_1), R(y_1, y_2)) : \text{Compensation in period 2} \]
\[ R_1 = R(y_1) : \text{Earnings report in period 1} \]
\[ R_2 = R(y_1, y_2) : \text{Earnings report in period 2} \]
\[ I_1 : \text{Investment in period 1} \]
\[ K_t : \text{Capital stock in period } t \]
\[ i_1 = \frac{I_1}{K_1} : \text{Investment-capital ratio in period 1} \]
\[ T D = T D(y_1) = T D(R(y_1)) : \text{Termination decision} \]
\[ \varepsilon : \text{Accounting system} \]
\[ \rho_P : \text{The principal’s discount rate} \]
\[ \rho_A : \text{The agent’s discount rate} \]
\[ h : \text{Cost of effort } a_H \]
\[ C_{EM} : \text{Cost of earnings management} \]
\[ c : \text{Control systems} \]
\[ \ell : \text{A loss in productivity of ongoing investment in the case of turnover} \]
\[ \xi(i_t)K_t : \text{Total cost of investment} \]
\[ J(I_t, K_t) = j(i_t)K_t : \text{Adjustment cost of investment} \]
\[ W : \text{The agent’s expected payoff after the cost of effort is sunk} \]
\[ W_2^j : \text{the agent’s expected payoff in period 2 if } y_1 = Y_j. \]
Proof of Proposition 1

In the contract with no earnings management, the principal maximizes:

\[ pX_H + (1 - p)X_L - p_1S(Y_H) - (1 - p_1)S(Y_L) \]  

(3.95)

IC for No EM:

\[ S(Y_L) \geq S(Y_H) - C_{EM} \]  

(3.96)

IC is:

\[ p_1S(Y_H) + (1 - p_1)S(Y_L) - h \geq q_1S(Y_H) + (1 - q_1)S(Y_L) \]  

(3.97)

IR is:

\[ p_1S(Y_H) + (1 - p_1)S(Y_L) - h \geq 0 \]  

(3.98)

For any accounting system \( \varepsilon \in [-d, d] \),

\[ S(Y_H) = \frac{h}{p - q} \]  

(3.99)

\[ S(Y_L) = 0 \]  

(3.100)

\[ C_{EM} \geq \frac{h}{p - q} \]  

(3.101)

The principal’s expected payoff is:

\[ pX_H + (1 - p)X_L - (p + \varepsilon)\frac{h}{p - q} \]  

(3.102)

The optimal accounting system is a conservative accounting system \((\varepsilon^* = -z)\).

In the contract with earnings management, the principal maximizes

\[ pX_H + (1 - p)X_L - p_1S(R_H; Y_H) - (1 - p_1)S(R_H; Y_L) \]  

(3.103)
IC for EM:

\[ S(R_H; Y_L) - C_{EM} \geq S(R_L; Y_L) \]  

(3.104)

IC is:

\[ p_1S(R_H; Y_H) + (1 - p_1)(S(R_H; Y_L) - C_{EM}) - h \geq q_1S(R_H; Y_H) + (1 - q_1)(S(R_H; Y_L) - C_{EM}) \]  

(3.105)

IR is:

\[ p_1S(R_H; Y_H) + (1 - p_1)(S(R_H; Y_L) - C_{EM}) - h \geq 0 \]  

(3.106)

For any accounting system \( \varepsilon \in [-d, d] \),

\[ S(R_H; Y_H) = S(R_H; Y_L) = C_{EM} \]  

(3.107)

\[ S(R_L; Y_L) = 0 \]  

(3.108)

\[ C_{EM} \geq \frac{h}{p - q} \]  

(3.109)

The principal’s expected payoff is:

\[ pX_H + (1 - p)X_L - C_{EM} \]  

(3.110)

The choice of accounting system is irrelevant in the contract with earnings management.

The principal is better off with no earnings management contract.

**Proof of Proposition 2**

We derive the contract with no earnings management only. The contract with earnings management can be derived in the same manner.
The principal maximizes

$$\max_{\varepsilon, S(y_1), S(y_1, y_2), i_1} \mathbb{E}[x_1 K_1 - S(y_1) - \xi(i_1)K_1 + \frac{1}{1+\rho_p} \{[(x_2 K_2 - S(y_1, y_2))1_{[y_1=y_H]} + \{x_2^{\text{New}} K_2 - S^{\text{New}}(y_2^{\text{New}})\}1_{[y_1=y_L]}\]$$

IC at $t = 2$,

$$p_2 S(Y_i, Y_H) + (1 - p_2) S(Y_i, Y_L) - h \geq q_2 S(Y_i, Y_H) + (1 - q_2) S(Y_i, Y_L)$$

Re-arranging this gives

$$S(Y_i, Y_H) - S(Y_i, Y_L) \geq \frac{h}{p_2 - q_2} = \frac{h}{p - q}$$

Thus,

$$S(Y_i, Y_H) = \frac{h}{p - q}$$

$$S(Y_i, Y_L) = 0$$

Let continuation payoff $W_2^i$ at the end of period 1 be

$$W_2^i = \frac{1}{1+\rho_A} [p_2 S(Y_i, Y_H) + (1 - p_2) S(Y_i, Y_L) - h]$$

$$= \frac{1}{1+\rho_A} [p_2 \frac{h}{p - q} - h]$$

IC for No EM,

$$S(Y_H) + W_2^{L, EM} - C_{EM} \leq S(Y_L) + W_2^L$$

where $W_2^{L, EM} = \frac{1}{1+\rho_A} [p_2 S(R_H, R_L; Y_L, Y_H) + (1 - p_2) S(R_H, R_L; Y_L, Y_L) - h] = 0$ is the continuation payoff at the end of period 1 when realized earnings are low and the agent engaged in earnings management.
IC at $t = 1$, 

$$p_1(S(Y_H) + W^H_2) + (1 - p_1)(S(Y_L) + W^L_2) - h \geq q_1(S(Y_H) + W^H_2) + (1 - q_1)(S(Y_L) + W^L_2)$$

(3.119)

Re-arranging this gives

$$S(Y_H) + W^H_2 - (S(Y_L) + W^L_2) \geq \frac{h}{p_1 - q_1} = \frac{h}{p - q}$$

(3.120)

Because $W^H_2 = W^L_2$,

$$S(Y_H) - S(Y_L) \geq \frac{h}{p - q}$$

(3.121)

Thus,

$$S(Y_H, Y_H) = \frac{h}{p - q}$$

(3.122)

$$S(Y_H, Y_L) = 0$$

(3.123)

$$S(R_H, R_L; Y_L, Y_H) = 0$$

(3.124)

$$S(R_H, R_L; Y_L, Y_L) = 0$$

(3.125)

$$S(Y_L, Y_H) = \frac{h}{p - q}$$

(3.126)

$$S(Y_L, Y_L) = 0$$

(3.127)

$$S(Y_H) = \frac{h}{p - q}$$

(3.128)

$$S(Y_L) = 0$$

(3.129)

$$C_{EM} \geq \frac{h}{p - q} \left[ 1 - \frac{(q - \epsilon)}{1 + \rho_A} \right]$$

(3.130)
The principal’s expected payoff is:

\[
pX_HK_1 + (1 - p)X_LK_1 - p_1S(Y_H) - (1 - p_1)S(Y_L) + p_1 \frac{1}{1 + \rho_p} (pX_HK_2 + (1 - p)X_LK_2 - p_2S(Y_H, Y_H)) \\
-(1 - p_2)S(Y_H, Y_L) + (1 - p_1) \frac{1}{1 + \rho_p} (pX_HK_2 + (1 - p)X_LK_2 - p_2S(Y_L, Y_H) - (1 - p_2)S(Y_L, Y_L)) - \xi(i_1)K_1
\]

(3.131)

The optimal investment-capital ratio satisfies

\[
\xi'(i_1) = \frac{1}{1 + \rho_p} [pX_H + (1 - p)X_L]
\]

(3.132)

The choice of accounting system is irrelevant if \( \rho_p = 0 \). Otherwise, the optimal accounting system is a conservative accounting system \( (\epsilon^* = -z) \).

Proof: FOC with respect to \( \epsilon \) is

\[
-\frac{h}{p - q} \left(1 - \frac{1}{1 + \rho_p}\right)
\]

(3.133)

Proof of Lemma 2

IC at \( t = 2 \),

\[
p_2S(Y_H, Y_H) + (1 - p_2)S(Y_H, Y_L) - h \geq q_2S(Y_H, Y_H) + (1 - q_2)S(Y_H, Y_L)
\]

(3.134)

Re-arranging this gives

\[
S(Y_H, Y_H) - S(Y_H, Y_L) \geq \frac{h}{p_2 - q_2} = \frac{h}{p - q}
\]

(3.135)
Thus,

\[ S(Y_H, Y_H) = \frac{h}{p - q} \tag{3.136} \]
\[ S(Y_H, Y_L) = 0 \tag{3.137} \]

Let continuation payoff \( W_2^j \) at the end of period 1 be

\[
W_2^H = \frac{1}{1 + \rho_A} [p_2 S(Y_H, Y_H) + (1 - p_2)S(Y_H, Y_L) - h] \tag{3.138}
\]
\[
= \frac{1}{1 + \rho_A} [p_2 \frac{h}{p - q} - h] \tag{3.139}
\]
\[
W_2^L = 0 \tag{3.140}
\]

IC for No EM,

\[ S(Y_H) + W_2^{L, EM} - C_{EM} \leq S(Y_L) + W_2^L \tag{3.141} \]

where \( W_2^{L, EM} = 0 \).

IC at \( t = 1 \),

\[
p_1 (S(Y_H) + W_2^H) + (1 - p_1)(S(Y_L) + W_2^L) - h \geq q_1(S(Y_H) + W_2^H) + (1 - q_1)(S(Y_L) + W_2^L) \tag{3.142}
\]

Re-arranging this gives

\[ S(Y_H) + W_2^H - S(Y_L) \geq \frac{h}{p_1 - q_1} = \frac{h}{p - q} \tag{3.143} \]
Setting $S(Y_L) = 0,$

$$S(Y_H) + \frac{1}{1 + \rho_A} \left[ p_2 \frac{h}{p - q} - h \right] \geq \frac{h}{p - q}$$

(3.144)

$$S(Y_H) \geq \frac{h}{p - q} + \frac{1}{1 + \rho_A} \frac{(-q + \varepsilon)h}{p - q}$$

(3.145)

$$S(Y_H) \geq \frac{h}{p - q} \left[ 1 - \frac{q - \varepsilon}{1 + \rho_A} \right]$$

(3.146)

Thus,

$$S(Y_H, Y_H) = \frac{h}{p - q}$$

(3.147)

$$S(Y_H, Y_L) = 0$$

(3.148)

$$S(R_H, R_L; Y_L, Y_H) = 0$$

(3.149)

$$S(R_H, R_{LL}; Y_L, Y_L) = 0$$

(3.150)

$$S(Y_{\text{New}}^H) = \frac{h}{p - q}$$

(3.151)

$$S(Y_{\text{New}}^L) = 0$$

(3.152)

$$S(Y_H) = \frac{h}{p - q} \left[ 1 - \frac{q - \varepsilon}{1 + \rho_A} \right]$$

(3.153)

$$S(Y_L) = 0$$

(3.154)

$$C_{EM} \geq \frac{h}{p - q} \left[ 1 - \frac{q - \varepsilon}{1 + \rho_A} \right]$$

(3.155)

The principal’s expected payoff is:

$$p X_H K_1 + (1 - p) X_L K_1 - p_1 S(Y_H) - (1 - p_1) S(Y_L) + p_1 \frac{1}{1 + \rho_P} (p X_H K_2 + (1 - p) X_L K_2 - p_2 S(Y_H, Y_H) - (1 - p_2) S(Y_H, Y_L))$$

$$+(1 - p_1) \frac{1}{1 + \rho_P} (p(1 - \ell) X_H K_2 + (1 - p)(1 - \ell) X_L K_2 - p_2 S(Y_{\text{New}}^H) - (1 - p_2) S(Y_{\text{New}}^L)) - \xi (i) K_1$$

(3.156)
The optimal investment-capital ratio $i_1$ satisfies

$$
\xi'(i_1) = \frac{1}{1+\rho_p}[p_1 + (1-p_1)(1-\ell)][pX_H + (1-p)X_L]
$$

(3.157)

The choice of accounting system $\varepsilon^*$ is

$$
\varepsilon^* = \bar{\varepsilon}
$$

(3.158)

where $\bar{\varepsilon} = f(\ell, h; X_H, X_L, p, q, K_1, \rho_p, \rho_A, \delta, \theta)$ and $z \geq \bar{\varepsilon} \geq -z$. Therefore, a relatively liberal accounting system is preferred.

**Proof of Corollary 1**

To be complete

**Proof of Lemma 3**

IC at $t=2$ if the earnings in period 1 is high,

$$
p_2S(R_H, R_H; Y_H, Y_H) + (1-p_2)S(R_H, R_L; Y_H, Y_L) - h \geq q_2S(R_H, R_H; Y_H, Y_H) + (1-q_2)S(R_H, R_L; Y_H, Y_L)
$$

(3.159)

IC at $t=2$ if the earnings in period 1 is low and have been managed upward,

$$
p_2S(R_H, R_L; Y_L, Y_H) + (1-p_2)S(R_H, R_{LL}; Y_L, Y_L) - h \geq q_2S(R_H, R_L; Y_L, Y_Y) + (1-q_2)S(R_H, R_{LL}; Y_L, Y_L)
$$

(3.160)

Re-arranging this gives

$$
S(R_H, R_H; Y_H, Y_H) - S(R_H, R_L; Y_H, Y_L) \geq \frac{h}{p-q}
$$

(3.161)
\[ S(R_H, R_L; Y_L; Y_H) - S(R_H, R_{LL}; Y_L, Y_L) \geq \frac{h}{p-q} \] (3.162)

Thus,

\[ S(R_H, R_H; Y_H, Y_H) = \frac{2h}{p-q} \] (3.163)
\[ S(R_H, R_L; Y_H, Y_L) = S(R_H, R_L; Y_L, Y_H) = \frac{h}{p-q} \] (3.164)
\[ S(R_H, R_{LL}; Y_L, Y_L) = 0 \] (3.165)

The continuation payoff \( W_{2}^{H} \) at the end of period 1 if the earnings in period 1 is high is

\[
W_{2}^{H} = \frac{1}{1 + \rho}\left[p_2S(R_H, R_H; Y_H, Y_H) + (1 - p_2)S(R_H, R_L; Y_H, Y_L) - h\right]
\]

(3.166)

\[
= \frac{1}{1 + \rho}\left[p_2\frac{2h}{p-q} + (1 - p_2)\frac{h}{p-q} - h\right]
\]

(3.167)

The continuation payoff \( W_{2}^{L, EM} \) at the end of period 1 if the earnings in period 1 is low and have been managed upward is

\[
W_{2}^{L, EM} = \frac{1}{1 + \rho}\left[p_2S(R_H, R_L; Y_L, Y_H) + (1 - p_2)S(R_H, R_{LL}; Y_L, Y_L) - h\right]
\]

(3.168)

\[
= \frac{1}{1 + \rho}\left[p_2\frac{h}{p-q} - h\right]
\]

(3.169)

IC for EM at \( t = 1 \),

\[ S(R_H; Y_L) + W_{2}^{L, EM} - C_{EM} \geq S(R_L; Y_L) + W_{2}^{L} \] (3.170)
Setting $S(R_L; Y_L) = W_2^L = 0$,

$$S(R_H; Y_L) \geq C_{EM} - W_2^{LEM}$$

$$\geq C_{EM} - \frac{1}{1 + \rho_A} \frac{(-\varepsilon + q)h}{p - q}$$

IC at $t = 1$,

$$p_1(S(R_H; Y_H) + W_2^H) + (1 - p_1)(S(R_H; Y_L) + W_2^{LEM} - C_{EM}) - h \geq q_1(S(R_H; Y_H) + W_2^H) + (1 - q_1)(S(R_H; Y_L) + W_2^{LEM} - C_{EM})$$

$$S(R_H; Y_H) + W_2^H) - (S(R_H; Y_L) + W_2^{LEM} - C_{EM}) \geq \frac{h}{p_1 - q_1} = \frac{h}{p - q}$$

Therefore, IC becomes

$$C_{EM} \geq \frac{h}{p - q} \left(\frac{\rho_A}{1 + \rho_A}\right)$$

Thus,

$$S(R_H, R_H; Y_H, Y_H) = \frac{2h}{p - q}$$

$$S(R_H, R_L; Y_H, Y_L) = S(R_H, R_L; Y_L, Y_H) = \frac{h}{p - q}$$

$$S(R_H, R_{LL}; Y_L, Y_L) = 0$$

$$S(R_L, R_H; Y_L, Y_H) = 0$$

$$S(R_L, R_L; Y_L, Y_L) = 0$$

$$S(R_H; Y_H) = S(R_H; Y_L) = C_{EM} - \frac{1}{1 + \rho_A} \frac{(q - \varepsilon)h}{p - q}$$

$$S(R_L; Y_L) = 0$$

$$C_{EM} \geq \frac{h}{p - q} \left(\frac{\rho_A}{1 + \rho_A}\right)$$
The principal’s expected payoff is:

\[
pXH K_1 + (1 - p)X_L K_1 - p_1 S(R_H; Y_H) - (1 - p_1)S(R_H; Y_L) + p_1 \frac{1}{1 + \rho_p} (pXH K_2 + (1 - p)X_L K_2 - p_2 S(R_H, R_H; Y_H, Y_H) - (1 - p_2)S(R_H, R_L; Y_L, Y_H) - \xi(i_1)K_1
\]

The optimal investment-capital ratio \( i_1 \) satisfies

\[
\xi'(i_1) = \frac{1}{1 + \rho_p} [pX_H + (1 - p)X_L]
\]

The choice of accounting system \( \varepsilon^* \) is

\[
\varepsilon^* = -z
\]

**Proof of Proposition 3**

To be complete

**Proof of Lemma 4**

From the proof of Lemma 2

\[
C_{EM}(\varepsilon) = c(1 - \varepsilon) \geq \frac{h}{p - q} \left[ 1 - \frac{q - \varepsilon}{1 + \rho_A} \right]
\]

Thus,

\[
c \geq \frac{h}{(1 - \varepsilon)(p - q)} \left[ 1 - \frac{q - \varepsilon}{1 + \rho_A} \right]
\]

This means that earnings management needs to be costly enough to provide the agent with *ex ante* effort incentives. Therefore, if \( c < \frac{h}{(1 - \varepsilon)(p - q)} \left[ 1 - \frac{q - z}{1 + \rho_A} \right] \), the choice of accounting system should be \( \varepsilon^* = z \) such
that $c \geq \frac{h}{(1-\frac{q}{2})(p-q)}[1 - \frac{q-z}{1+\rho}]$.

**Proof of Lemma 5**

IC at $t=2$ if the earnings in period 1 is high,

\[
p_2S(R_H, R_H; Y_H, Y_H) + (1 - p_2)S(R_H, R_L; Y_H, Y_L) - h \geq q_2S(R_H, R_H; Y_H, Y_H) + (1 - q_2)S(R_H, R_L; Y_H, Y_L)
\]

(3.189)

IC at $t=2$ if the earnings in period 1 is low and have been managed upward,

\[
p_2S(R_H, R_L; Y_L, Y_H) + (1 - p_2)S(R_H, R_{LL}; Y_L, Y_L) - h \geq q_2S(R_H, R_L; Y_L, Y_H) + (1 - q_2)S(R_H, R_{LL}; Y_L, Y_L)
\]

(3.190)

Re-arranging this gives

\[
S(R_H, R_H; Y_H, Y_H) - S(R_H, R_L; Y_H, Y_L) \geq \frac{h}{p-q}
\]

(3.191)

\[
S(R_H, R_L; Y_L, Y_H) - S(R_H, R_{LL}; Y_L, Y_L) \geq \frac{h}{p-q}
\]

(3.192)

Thus,

\[
S(R_H, R_H; Y_H, Y_H) = \frac{2h}{p-q}
\]

(3.193)

\[
S(R_H, R_L; Y_H, Y_L) = S(R_H, R_L; Y_L, Y_H) = \frac{h}{p-q}
\]

(3.194)

\[
S(R_H, R_{LL}; Y_L, Y_L) = 0
\]

(3.195)
The continuation payoff $W_2^H$ at the end of period 1 if the earnings in period 1 is high is

\[
W_2^H = \frac{1}{1 + \rho_A} \left[ p_2 S(R_H, R_H; Y_H, Y_H) + (1 - p_2) S(R_H, R_L; Y_H, Y_L) - h \right] \quad (3.196)
\]

\[
= \frac{1}{1 + \rho_A} \left[ p_2 \frac{2h}{p - q} + (1 - p_2) \frac{h}{p - q} - h \right] \quad (3.197)
\]

The continuation payoff $W_2^{L,EM}$ at the end of period 1 if the earnings in period 1 is low and have been managed upward is

\[
W_2^{L,EM} = \frac{1}{1 + \rho_A} \left[ p_2 S(R_H, R_L; Y_L, Y_H) + (1 - p_2) S(R_H, R_{LL}; Y_L, Y_L) - h \right] \quad (3.198)
\]

\[
= \frac{1}{1 + \rho_A} \left[ p_2 \frac{h}{p - q} - h \right] \quad (3.199)
\]

IC for EM at $t = 1,$

\[
S(R_H; Y_L) + W_2^{L,EM} - C_{EM} \geq S(R_L; Y_L) + W_2^L \quad (3.200)
\]

Setting $S(R_L; Y_L) = W_2^L = 0,$

\[
S(R_H; Y_L) \geq C_{EM} - W_2^{L,EM} \quad (3.201)
\]

Thus, if $C_{EM} - W_2^{L,EM} = C_{EM} - \frac{1}{1 + \rho_A} \frac{(-\varepsilon + q)h}{p - q} \leq 0,$

\[
S(R_H; Y_L) = 0 \quad (3.202)
\]

If $C_{EM} - W_2^{L,EM} = C_{EM} - \frac{1}{1 + \rho_A} \frac{(-\varepsilon + q)h}{p - q} > 0,$

\[
S(R_H; Y_L) = C_{EM} - W_2^{L,EM} \quad (3.203)
\]

\[
= C_{EM} - \frac{1}{1 + \rho_A} \frac{(-\varepsilon + q)h}{p - q} \quad (3.204)
\]
IC at $t = 1$,

$$p_1(S(R_H; Y_H) + W^H_2) + (1 - p_1)(S(R_H; Y_L) + W^{L,EM}_2 - C_{EM}) - h \geq q_1(S(R_H; Y_H) + W^H_2) + (1 - q_1)(S(R_H; Y_L) + W^{L,EM}_2 - C_{EM})$$

$$= \frac{h}{p_1 - q_1} = \frac{h}{p - q} \quad (3.205)$$

If $C_{EM} - W^{L,EM}_2 = C_{EM} - \frac{1}{1 + \rho_A} \left( \frac{-\varepsilon + q}{h} \right) \leq 0$, IC becomes

$$\left( S(R_H; Y_H) + W^H_2 \right) - \left( S(R_H; Y_L) + W^{L,EM}_2 - C_{EM} \right) \geq \frac{h}{p_1 - q_1} = \frac{h}{p - q} \quad (3.207)$$

$$C_{EM} \geq \frac{h}{p - q} \left( \frac{\rho_A}{1 + \rho_A} \right) \quad (3.208)$$

Therefore, if $\frac{h}{p - q} \left( \frac{\rho_A}{1 + \rho_A} \right) \leq C_{EM} \leq \frac{1}{1 + \rho_A} \left( \frac{-\varepsilon + q}{h} \right) \leq 0$ and $\rho_A \leq -\varepsilon + q$,

$$S(R_H, R_H; Y_H, Y_H) = \frac{2h}{p - q} \quad (3.209)$$

$$S(R_H, R_L; Y_H, Y_L) = S(R_H, R_L; Y_L, Y_H) = \frac{h}{p - q} \quad (3.210)$$

$$S(R_H, R_{LL}; Y_L, Y_L) = 0 \quad (3.211)$$

$$S(R_L, R_H; Y_H, Y_H) = 0 \quad (3.212)$$

$$S(R_L, R_L; Y_H, Y_L) = 0 \quad (3.213)$$

$$S(R_H; Y_H) = S(R_H; Y_L) = 0 \quad (3.214)$$

$$S(R_L; Y_L) = 0 \quad (3.215)$$

$$C_{EM} \geq \frac{h}{p - q} \left( \frac{\rho_A}{1 + \rho_A} \right) \quad (3.216)$$

If $C_{EM} - W^{H,EM}_2 = C_{EM} - \frac{1}{1 + \rho_A} \left( \frac{-\varepsilon + q}{h} \right) > 0$, IC becomes

$$\left( S(R_H; Y_H) + W^H_2 \right) - \left( S(R_H; Y_L) + W^{L,EM}_2 - C_{EM} \right) \geq \frac{h}{p_1 - q_1} = \frac{h}{p - q} \quad (3.217)$$

$$C_{EM} \geq \frac{h}{p - q} \left( \frac{\rho_A}{1 + \rho_A} \right) \quad (3.218)$$
Therefore, IC becomes

\[ C_{EM} \geq \frac{h}{p-q} \left( \frac{\rho_A}{1+\rho_A} \right) \]  \hspace{1cm} (3.219)

Thus, if \( C_{EM} \geq \text{Max}[\frac{1}{1+\rho_A} \frac{(-\varepsilon+q)h}{p-q}, \frac{h}{p-q} \left( \frac{\rho_A}{1+\rho_A} \right)] \),

\[ S(R_H, R_H; Y_H, Y_H) = \frac{2h}{p-q} \]  \hspace{1cm} (3.220)
\[ S(R_H, R_L; Y_H, Y_L) = S(R_H, R_L; Y_L, Y_H) = \frac{h}{p-q} \]  \hspace{1cm} (3.221)
\[ S(R_H, R_{LL}; Y_L, Y_L) = 0 \]  \hspace{1cm} (3.222)
\[ S(R_L, R_H; Y_L, Y_H) = 0 \]  \hspace{1cm} (3.223)
\[ S(R_L, R_L; Y_L, Y_L) = 0 \]  \hspace{1cm} (3.224)
\[ S(R_H; Y_H) = S(R_H; Y_L) = C_{EM} - \frac{1}{1+\rho_A} \frac{(q-\varepsilon)h}{p-q} \]  \hspace{1cm} (3.225)
\[ S(R_L; Y_L) = 0 \]  \hspace{1cm} (3.226)
\[ C_{EM} \geq \frac{h}{p-q} \left( \frac{\rho_A}{1+\rho_A} \right) \]  \hspace{1cm} (3.227)

The principal’s expected payoff is:

\[ pX_H K_1 + (1-p)X_L K_1 - p_1 S(R_H; Y_H) - (1-p_1) S(R_H; Y_L) + p_1 \frac{1}{1+\rho_p} (p X_H K_2 + (1-p)X_L K_2) - p_2 S(R_H, R_H; Y_H, Y_H) \]

\[ -(1-p_2) S(R_H, R_L; Y_H, Y_L) + (1-p_1) \frac{1}{1+\rho_p} (p X_H K_2 + (1-p)X_L K_2) - p_2 S(R_H, R_L; Y_L, Y_H) - (1-p_2) S(R_H, R_{LL}; Y_L, Y_L) - \xi (i_1) K_1 \]  \hspace{1cm} (3.228)

The optimal investment-capital ratio \( i_1 \) satisfies

\[ \xi'(i_1) = \frac{1}{1+\rho_p} [p X_H + (1-p)X_L] \]  \hspace{1cm} (3.230)
The optimal accounting system is

$$\varepsilon^* = z \text{ if } c > \text{Max}\left[ \frac{h}{(p-q)(1+\rho_A)}, \frac{h}{(1-z)(p-q)(1+\rho_A)} \right] \quad (3.231)$$

$$\varepsilon^* = \hat{z} \text{ if } \frac{h}{(1-z)(p-q)(1+\rho_A)} > c > \frac{h}{(p-q)(1+\rho_A)} \quad (3.232)$$

$$\varepsilon^* = -\hat{z} \text{ if } \frac{h}{(p-q)(1+\rho_A)} > c \geq \frac{1}{1+z} \text{Max}\left[ \frac{1}{1+\rho_A}, \frac{1}{p-q}, \frac{h}{p-q}(\frac{\rho_A}{1+\rho_A}) \right]$$

\[ (q-\varepsilon^*)h \]

$$\forall \varepsilon^* \text{satisfying } c \geq \frac{h}{(1-\varepsilon^*)(p-q)}(\frac{\rho_A}{1+\rho_A}) \text{ if } \frac{1}{1+\rho_A, (1-\varepsilon^*)(p-q)} \geq c \text{ or } c = \frac{h}{(p-q)(1+\rho_A)} \quad (3.234)$$

where $\hat{z} = 1 - \frac{h}{c(p-q)(1+\rho_A)}$ and $z > \hat{z} > -z$.

**Proof of Proposition 4**

To be complete