Essays in Low-Income Housing Policies, Mobility, and Sorting

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Abstract

This dissertation offers a study of the mobility of low-income households, particularly households participating in the two largest federal rental housing assistance programs: public housing and the Housing Choice Voucher Program. It features a restricted-use dataset that follows all households living in public housing over a five-year time span in the Pittsburgh, PA, as well as several snapshots of families taking part in the housing voucher program in the Pittsburgh Metropolitan Area. The detailed focus on a single metropolitan area allows the observation of detailed neighborhood amenities and the identification of the heterogeneity across public housing communities.

The goal of the first essay is to estimate the demand for public housing and to quantify the welfare costs associated with failing to maintain a sufficient supply of public housing communities.¹ We develop a new model of discrete choice with rationing that captures excess demand for public housing in equilibrium. We find that for each family that leaves public housing there are on average 3.85 families that would like to move into the vacated unit. Demolitions of existing units increase the degree of rationing and result in large welfare losses. An unintended consequence of demolitions is that they increase racial segregation in low income housing communities.

In the second essay I study how to optimally design rental subsidies. Voucher households have better housing and neighborhood outcomes than those in public housing, but do not do well compared to eligible but nonparticipating households. To explore this puzzling outcome, I propose and estimate a new model of residential choice and housing demand. Simulating the model, I study several possible rental assistance schemes. Compared to the current voucher program, I find that a rental rebate program would increase participants’ utility, lead to improved neighborhood selection, and significantly lower program costs. A requirement that households locate to areas of low poverty concentration results in the most effective policy for moving participants to neighborhoods with better schools and lower crime rates, but would have to be offset with high levels of compensation, perhaps including counseling and relocation assistance.

¹Chapter 2 is co-authored with Dennis Epple and Holger Sieg.
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Chapter 1

Introduction

Low-income families face affordable and livable housing shortages in the United States. The U.S. Department of Housing and Urban Development (HUD) calculated that in 2009 the nation offered only 36 affordable and decent housing units for every 100 households earning less than 30 percent of their area median income (HUD 2011). The current recession and housing crisis has led to the highest absolute (7.1 million) and percentage-wise (22 percent) level on record of very low-income renters paying more than half of their income for housing or living in severely inadequate housing. To assist some of these households, the government offers a variety of programs (HUD 2011). HUD’s oldest assistance program is public housing, which currently serves about 2.3 million households. The next largest program is the Housing Choice Voucher Program that assists around 2.1 million households and is the largest segment of the Section 8 voucher program. Together these programs account for roughly 42% of HUD’s total outlays, or $25.3 billion in 2010.

This dissertation provides new insights into the impact of housing programs on residential choices of low income households. Housing programs are not entitlement programs, thus there are typically more households that wish to participate in the program than the program can afford. However, it is difficult to measure the full demand for public housing programs because wait lists for housing problems in urban areas tend to be very long and are often closed to new applicants for
long periods. In Chapter 2 we estimate the demand for public housing and explore the consequences of demolishing public housing.¹ In Chapter 3 I study optimal policy design for the Housing Choice Voucher Program with respect to participants’ housing and neighborhood choices. Both chapters propose new models of consumer choice that are estimated by micro-level data on revealed preference. These models enable the counterfactual policy analysis.

I focus the analysis on Pittsburgh, PA, and employ a new, restricted-use data set from the Housing Authority of the City of Pittsburgh (HACP). About 20,000 city residents are housed by HACP programs, about 6.3% of the total city population. The main programs offered by the HACP are public housing as well as the Housing Choice Voucher Program. Currently about half of the population served by the HACP lives in public housing. By focusing on low income households within a single large urban area I can study the behavior of these households without the confounding effects of differences in local housing markets and implementation of housing policies across different urban areas. Despite uniform regulation on rent and eligibility, program implementation varies greatly by locality due to differences in the quantity and quality of housing supply. Also, detailed data on 114 distinct neighborhoods in the city reveal heterogeneity in program benefits across the participant population that is typically very difficult to measure in broader surveys of urban housing programs.

The HACP data include a panel of all households that lived in public housing at any point between 2001 and 2006. In Epple, Geyer, & Sieg (2011) we found that the patterns of households entering and exiting public housing in this panel suggested heterogeneity in the quality of different public housing communities across the city. Moreover, we found a significant number of households that voluntarily move from one public housing community to another. Prior to the work for this dissertation, there was no standard econometric method to use all observed patterns of mobility to uncover program participants preferences for neighborhood selection, which are needed to estimate the benefit of the program. While proportional hazard models could inform about preferences revealed through household choices to move out of public housing, no basic model could inform about preferences revealed through household choices to move into or transfer between public hous-

¹Chapter 2 is co-authored with Dennis Epple and Holger Sieg.
ing structures because no basic model can account for supply constraints. If few households move or transfer into a certain community, the low number of move-ins and transfers could be a result of either that community’s very high, or very low, desirability. In order to estimate community preferences informed by all observable types of mobility, I along with my co-authors propose and estimate a static discrete choice model with rationing that can explain and study the welfare consequences of excess demand in equilibrium.

The empirical approach in Chapter 2 allows us to study the impact of changes in the supply of public housing on household mobility. We find that for each family that leaves public housing there are on average 3.85 families that would like to move into the vacated unit. The fraction of households willing to move into a public housing unit largely depends on the community specific fixed effects and thus reflects the attractiveness of the housing community. However, it also depends on the characteristics of eligible households. Older households and extremely poor households are more willing to move from the private sector to public housing communities. These households suffer the highest welfare costs from policies that restrict the supply. Demolitions of existing units increase the degree of rationing and result in large welfare losses. In our simulations, we find that an unintended consequence of demolitions is that they increase racial segregation in low income housing communities.

In 1996, the U.S. Congress passed Section 202 of the Omnibus Consolidated Rescissions and Appropriations Act (Section 202) that required housing authorities to demolish a public housing unit if its rehabilitation and maintenance costs exceeded the cost of providing the household a 20-year rent subsidy in the private housing market, or housing voucher. This Act has led to the gradual expansion of funds for voucher programs and the demolition of public housing units. For this reason, in Chapter 3 I focus on the outcomes of voucher program participants. The HACP provided data on all households that participated in its Housing Choice Voucher program in 2006.

In addition to its potential cost-savings over the public housing program, the Housing Choice Voucher Program is lauded for the residential mobility it affords recipients. The program’s lauded
features are its allowance for residential choice and mobility, the portability of a voucher across all national housing authorities, and its flexible contract options for the tenant and landlord. In Chapter 3 I examine the HACP voucher population and find that the program participants, compared to their peers in HACP’s public housing program, live in neighborhoods with lower crime and poverty rates and better schools. However, the voucher participants’ average measures of neighborhood quality are lower than those of low-income households eligible, but not enrolled, in the voucher program. Motivated by this finding, I use a model to estimate low income households’ relative preferences for housing services and neighborhood amenities, and to understand how the Housing Choice Voucher Program affects consumption of these differing goods. In very-low income households, demand functions and substitution patterns can exhibit discontinuous behavior due to discontinuities induced by public subsidy programs as well as minimum consumption requirements for housing, food, and other services. Standard econometric methods can not handle these discontinuities appropriately. Thus to perform nominal analysis of multiple housing voucher schemes, the model I propose and estimate takes these discontinuous budget restrictions into account.

With the estimated choice model, I conduct policy analysis to examine how voucher recipients’ choices might change as a result of changes to the voucher program. Parameter estimates suggest that enjoyment of neighborhood amenities accounts for 25 percent of overall utility; however, the types of neighborhoods chosen by voucher participants is not greatly affected by changes to the program-induced budget constraint alone. In analyzing the budget constraint, my analysis suggests that changing the structure of the program to be a rebate instead of a voucher would improve participants’ utility, achieve neighborhood selection similar to a program with an unrestricted voucher amount, and would significantly lower costs. The most effective policy change in achieving improved neighborhood selection would be to impose a requirement that households live in neighborhoods with poverty rates below some acceptable maximum, such as 30 percent.

In summary, I submit two essays for my dissertation committee’s consideration as partial fulfillment of the requirements for the degree of Doctor of Philosophy in economics. This dissertation informs housing policy debate on the welfare consequences of public housing demolition and opti-
mal housing voucher design. The dissertation also advances two new models of consumer behavior. First, I along with my coauthors offer the first discrete choice model that captures excess demand in equilibrium that cannot be cleared by a price mechanism. The model informs us of the welfare consequences to demolishing public housing. The model also provides ample scope for future microeconomic research, such as a dynamic model to assess the relationship between supply restrictions and the decision to exit a means-tested subsidy program. Second, I offer the first housing demand and residential choice model that can include the optimization problem of households with discontinuous budget constraints. In addition to its usefulness in exploring optimal housing policy design, this model could be applied in other fruitful directions, such as a study of discontinuous borrowing constraints of residential mortgages.
Chapter 2

Estimating a Model of Excess Demand for Public Housing

2.1 Introduction

The market for affordable or low income housing is a prime example of a market that is subject to many distortions that often arise due to government regulations and interventions. Despite the overall importance of providing adequate housing and shelter for low income households, very little is known about the quantitative magnitude of these market distortions and the associated welfare implications. The objective of this paper is to estimate a new model that accounts for rationing in equilibrium and to provide a framework for quantifying the welfare costs associated with policies that fail to maintain an adequate supply of affordable housing.

The Department of Housing and Urban Development (HUD) subsidizes the construction and maintenance of affordable housing communities in cities and metropolitan areas in the U.S.¹ Similar government institutions and programs exist in most European countries. Low income households

¹This paper focuses the market for public housing communities. The other main rental assistance program funded by HUD provides vouchers for household to rent in the private market.
are eligible for public housing assistance if their income is below a threshold that depends on family status, number of children, and region. Given the current standards for determining eligibility, there is typically a large number of eligible households in each metro area. Supply of public housing units is primarily determined by the current and past political decisions that have allocated funding for local housing authorities. Since rents in public housing are typically a fixed percentage of household income, there is no price mechanism which guarantees that public housing markets clear. Since the demand for public housing often exceeds supply, there is rationing in equilibrium in many local markets.

The federal government has for all practical purposes stopped financing the construction of new housing projects. Existing units are often inadequately maintained because local housing authorities have limited resources. Since the early 1990s, HUD has given financial incentives under HOPE VI and related programs to tear down projects that are considered to be distressed. In some cases demolished units are replaced by mixed income communities that are built with private partners. In other cases, low income households obtain vouchers that they can use to rent apartments in the private markets. Other programs to encourage construction of low income housing emerged as construction of public housing ceased and demolition of public housing began. As detailed in Eriksen & Rosenthal (2010), the Low Income Housing Tax Credit (LIHTC) program was created in 1986 as part of the Tax Reform Act of 1986 as an alternative to public housing. They observe that "LIHTC has quickly overtaken all previous place-based subsidized rental programs to become the largest such program in the nation’s history.” They find, however, that this program has failed to result in new construction that serves the public housing population, for two reasons. One reason is that "... LIHTC actually targets moderate as opposed to low income tenants.” They note that Wallace (1995) finds that only 28 percent of LIHTC residents were in the HUD classification of very low income families whereas 81 percent of residents of traditional public housing developments were in that classification. The other reason Eriksen and Rosenthal (2010) conclude that LIHTC has failed as a substitute for public housing is their finding that LIHTC crowds out 100 percent of unsubsidized
rental housing, implying no net increase in rental housing.\textsuperscript{2}

It is rather puzzling that recent policies have primarily aimed at reducing the supply of public housing, largely ignoring the fact that there is so much excess demand for living in public housing. If there were strong evidence suggesting large negative spill-over effects (such as higher crime rates and lower educational achievement) associated with living in public housing, then supply reductions could be rationalized as part of paternalistic policy towards the poor. However, Jacob (2004) who considers the impact of demolitions in Chicago finds that there are very few positive effects associated with moving out of the projects using a variety of different outcomes.\textsuperscript{3} We know almost nothing about the welfare implications of failing to maintain an adequate supply of affordable housing. Nevertheless, policies that reduce the supply are being adopted in almost all metropolitan areas that have an aging stock of public housing.

It is almost impossible to obtain reliable panel data describing the characteristics of applicants on wait-lists for public housing in local markets in the U.S. It is well-known that there exist long wait-lists for public housing in many metropolitan areas. We can typically obtain some aggregate summary statistics that broadly measure the degree of excess demand in these markets. However, these aggregate statistics are not sufficient to estimate a model that captures heterogeneity across agents and cannot be used to construct welfare measures. Local housing authorities are not willing to disclose detailed micro level data on wait-lists because they often contain politically sensitive material about racial sorting and segregation. To our knowledge, there is no empirical research in this area that has ever used household-level wait-list data to study the welfare implications of rationing in public housing markets. As a consequence, we know very little about the welfare implications of building, maintaining, or demolishing public housing units. This is disturbing since currently policy has taken a strong stand on reducing the supply of public housing communities.

One key challenge encountered in empirical work is to estimate a model of constrained housing

\textsuperscript{2}Eriksen and Rosenthal (2010) is the most recent study of crowding out due to LIHTC. See Section 2 of their paper for a detailed discussion of other studies of crowding out by LIHTC.

\textsuperscript{3}Evidence that growing up in a poor neighborhood can have adverse effects on outcomes is presented by Oreopoulos (2003).
market choices without relying explicitly on household level wait-list data since these data are not broadly available to researchers.

To accomplish this task, we develop an equilibrium model that incorporates supply restrictions that arise from the administrative behavior of the local housing authority. A household can move into public housing if and only if the housing authority offers the household a vacant apartment. The ability of the housing authority to offer apartments to eligible households is largely determined by voluntary exit decisions of households that currently live in housing communities. Exit from public housing is a stochastic event since it is partially determined by idiosyncratic preference and income shocks that are not observed by the administrators. The housing authority’s objective is to fill all vacant units. If the potential demand exceeds the available units at any point of time, the housing authority has to ration access to public housing.

Eligible households that have not been offered an apartment in an affordable housing community are placed in our model on a wait list. As households move up on the wait list, their priority increases. Each period a fraction of households on the wait list will receive an offer to move into one of the apartments that have recently become available. If total supply of public housing is fixed and vacancy rates are constant over time, the housing authority adjusts the offer probabilities in equilibrium so that the inflow into public housing equals the voluntary outflow. We define an equilibrium for our model and characterize its properties. We show that there exists a unique equilibrium if there are no transfers between public housing communities. If transfers are possible we show that equilibrium is also unique as long as the housing authority adopts an equal treatment policy and does not discriminate among current residents.

We then show how to identify and estimate the parameters of the model using data on observed choices, but unobserved wait lists. Since we do not observe the wait list, we do not know which households received offers to move into housing communities. We only observe those offers that were accepted and resulted in a move.\(^4\) The basic insight of our identification approach is that

\(^4\)This type of selection problem is also encountered in labor search and occupational choice models. For a discussion of identification and estimation of labor search model see, among others, Eckstein & Wolpin (1990) and Postel-Vinay &
offer probabilities are endogenous and are constrained to satisfy equilibrium conditions. Hence, offer probabilities can be expressed as functions of the structural parameters of the housing choice models. Moreover, exit is purely voluntary and does not depend on offer probabilities. As a consequence, exit behavior is informative about the structural parameters of the utility function. Imposing the equilibrium conditions then establishes identification of the structural parameters of the model.

We quantify the importance of supply side restrictions and estimate the welfare costs of reducing the supply of public housing using a unique data set from the Housing Authority of the City of Pittsburgh (HACP).\textsuperscript{5} We supplement these data with a sample of eligible low income households in the Survey of Income and Program Participation which allows us to follow eligible households outside of public housing.

We find that households that are exceedingly poor and headed by single mothers have strong preferences for public housing. African American households also have stronger public housing preferences than whites. The income coefficient shows that there are strong incentives for households to leave public housing as their income grows larger. These incentives are offset by the presence of significant moving costs that constrain potential relocations of households. We find that for each family that leaves public housing there are on average 3.85 families that would like to move into the vacated unit. For seniors, the rationing is more pronounced. For each senior that moves out of a housing community there are 13.2 households that would like to move in.

To shed some insights into the welfare effects of reducing the supply of public housing, we consider demolishing some of the existing public housing units. We find that the welfare costs of demolishing even the least desirable units are rather substantial. Moreover, displaced black females are disproportionately disadvantaged, which raises some serious issues related to the distributional impact of these demolition programs. An unintended consequence is that the resulting equilibrium demographic distribution in the remaining public housing communities exhibits some increase in

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\textsuperscript{5}Robin (2002). Heckman & Honore (1990) discuss identification in the Roy model.

\textsuperscript{5}Olsen, Davis, & Carrillo (2005a) use restricted use data from HUD to study the impact of variations in local housing policies on household behavior.
the proportions of female and black residents, and thus an increase in segregation in these already highly segregated communities.

Our empirical estimates are based on Pittsburgh, a market with relatively moderate rental rates compared to most other metropolitan areas in the U.S. Public housing supply in Pittsburgh is also relatively high compared to large cities such as New York, Boston, or Chicago. Nevertheless, our results suggest that increasing the supply restrictions on housing occupied by the very low income population is problematic, even where there is a substantial supply of moderately priced rental housing. The welfare costs of failing to provide an adequate supply of housing for the very poor are likely to be still larger in cities with tighter housing markets and higher housing rental rates. We, therefore, conclude that it might be time to reconsider existing public housing policies.

The remainder of the paper is organized as follows. Section 2.2 introduces our data set. Section 2.3 provides an equilibrium model that treats public housing as a differentiated product that is subject to rationing. Section 2.4 discusses identification and derives the maximum likelihood estimator for this model. The empirical results are presented in Section 2.5. Section 2.6 reports our estimates of the welfare costs of demolitions. We offer some conclusions in Section 2.7.

2.2 Data

The U.S. Housing Act of 1937 formed the U.S. Public Housing Program that funds local governments in their ownership and management of buildings to house low-income residents at subsidized rents. Olsen (2001) provides a detailed description of the history and current practices of the various different U.S. Public Housing Programs. Currently, the U.S. Department of Housing and Urban Development funds the efforts of hundreds of city and county housing authorities in the United States. In Pennsylvania alone, there are 92 distinct housing authorities. In 2006, the estimated HUD budget for public housing was $24.604 billion. HUD (2006) provides details. Note that this figure does not include housing voucher programs, low-income community development programs, or other none-state owned and managed housing programs.

6Olsen (2001) provides a detailed description of the history and current practices of the various different U.S. Public Housing Programs.
7HUD (2006) provides details. Note that this figure does not include housing voucher programs, low-income community development programs, or other none-state owned and managed housing programs.
maintenance, and even law enforcement.

The empirical analysis presented in this paper focuses on communities owned and managed by the Housing Authority of the City of Pittsburgh, where approximately 70,000 households were eligible for public housing during the period of study. In 2005 HUD provided the HACP with $83.7 million in grants for public housing, housing vouchers, and other programs. In the same year, HACP received $8.3 million from tenant payments. The public housing stock in the City of Pittsburgh during our study includes about 4,500 habitable units across 34 heterogeneous sites. Only a small number of public housing communities were demolished during the course of our survey. As a consequence the supply of public housing has been approximately fixed during our study period.

There is a great variety of sites, or communities, ranging in size from four units (single family houses converted into several apartment units) to over 600 units in various neighborhoods across the city. Some large communities are high rises, others are low-rise housing spread homogenously over several blocks. These communities are usually designated as either ‘family’ communities or ‘senior’ communities, where senior communities target households age 62 or older. There are 34 separate sites. 19 of these sites are family units, 11 are designated for seniors and 4 of them are mixed. There are 16 large communities with more than 100 units, 8 are medium sized, and 10 are small with less than 40 units. Heterogeneity in public housing also arises due to differences in local amenities. The 34 public housing communities in the HACP are located across 19 of Pittsburgh’s 32 wards and across 28 census tracts. These public housing communities also vary in terms of neighborhood amenities such as crime, school quality, property values and demographic characteristics.

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8The number of habitable public housing units varies slightly over time, due to repairs, renovations, and demolition.
9Much of the demolition was motivated by the argument that growing up in public housing might be negative for children, although this conjecture is controversial in the literature (Currie & Yelowitz, 2000). For an analysis of the the impact of public housing demolitions in Chicago see Jacob (2004).
10There is much evidence that suggests that households make residential decisions based on neighborhood characteristics and local public goods. This evidence is based on estimated locational equilibrium models such as Epple & Sieg (1999), Epple, Romer, & Sieg (2001), Sieg, Smith, Banzhaf, & Walsh (2004), Calabrese, Epple, Romer, & Sieg (2006), Ferreyra (2007), Walsh (2007), and Epple, Peress, & Sieg (2010). Bergstrom, Rubinfield, & Shapiro (1982), Rubinfield, Shapiro, & Roberts (1987), Neshem (2001), Bajari & Kahn (2004), Bayer, McMillan, & Rueben (2004), Schmidheiny (2006), Bayer, Ferreira, & McMillan (2007), and Ferreira (2009) are examples of related empirical approaches which are based on more traditional discrete choice models or hedonic frameworks.
The HACP data contain records of household entry, exits, and transfers from June 2001 to June 2006 within the 34 public housing communities actively used during this time period. The data set also includes annual updates of each of these households as well as any non-periodic reports that update information about household composition or pre-rent income that is reported to the HACP. These records contain most of the information fields requested of all U.S. housing authorities including age, race, household composition including age and relationship of family members and housemates, earnings, and income adjustment exclusions including disability, medical, and child-care expenses. We also observe the monthly rent being charged to a particular household, the number of bedrooms of the housing unit, whether the community is targeted to seniors, and the address and unit number. There are 7,070 households observed at least once during this time period; there are 2,907 households that move in for the first time, 3,155 households that move out, and 1,244 that transfer from one public housing unit to another.

Table 2.1: Descriptive Statistics of HACP Demographics

<table>
<thead>
<tr>
<th></th>
<th>All Units</th>
<th>Family Units</th>
<th>Mixed Units</th>
<th>Senior Units</th>
<th>2 Bedroom Apartments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (std dev)</td>
<td>48.86 (20.76)</td>
<td>40.42 (16.98)</td>
<td>49.06 (20.53)</td>
<td>71.15 (11.77)</td>
<td>34.45 (13.36)</td>
</tr>
<tr>
<td>Percent Female</td>
<td>80.59</td>
<td>84.87</td>
<td>83.85</td>
<td>64.90</td>
<td>84.78</td>
</tr>
<tr>
<td>Percent Married</td>
<td>2.66</td>
<td>2.20</td>
<td>2.65</td>
<td>3.93</td>
<td>1.43</td>
</tr>
<tr>
<td>Number of Adults (std dev)</td>
<td>1.16 (0.44)</td>
<td>1.17 (0.45)</td>
<td>1.21 (0.50)</td>
<td>1.06 (0.23)</td>
<td>1.06 (0.24)</td>
</tr>
<tr>
<td>Number of Children (std dev)</td>
<td>0.95 (1.36)</td>
<td>1.00 (1.22)</td>
<td>1.59 (1.71)</td>
<td>0.00 (0.00)</td>
<td>0.76 (0.75)</td>
</tr>
<tr>
<td>Percent With Children</td>
<td>43.95</td>
<td>53.46</td>
<td>58.31</td>
<td>0.00</td>
<td>57.40</td>
</tr>
<tr>
<td>Percent Black</td>
<td>88.53</td>
<td>96.67</td>
<td>97.00</td>
<td>55.59</td>
<td>96.11</td>
</tr>
<tr>
<td>Annual Income (std dev)</td>
<td>9082 (7776)</td>
<td>8516 (8957)</td>
<td>9714 (6968)</td>
<td>9784 (4602)</td>
<td>6305 (6771)</td>
</tr>
</tbody>
</table>

Standard deviations are given in parenthesis.

Table 2.1 summarizes key descriptive statistics for the full sample and for four sub-samples that are differentiated by community type. Although some families live in senior housing and some seniors live in non-senior housing, age and family composition distributions are bimodal with respect
to these two types of communities. In mixed communities, demographic variables look similar to a weighted average of senior and family communities, however there are more cohabiting adults and a higher number of children in mixed housing than in family-only or senior-only housing. The mean age in senior housing is 31 years greater than the mean age in non-senior housing. The majority of households in both senior-only and family-only communities are female, but females are a much larger majority in family-only communities. Blacks households are a very high proportion of residents in family and mixed housing, while senior units have nearly equal proportions of black and white households. Marriage rates are low, 2.20% in family housing and 3.93% in senior housing; there are more cohabiting adults in family housing than in senior housing.\textsuperscript{11} There are fewer households in non-senior housing that have children than one might expect (about 53%).\textsuperscript{12}

Table 2.2: Descriptive Statistics of SIPP Compared to Census and HACP

<table>
<thead>
<tr>
<th></th>
<th>Census All</th>
<th>SIPP All</th>
<th>SIPP Private</th>
<th>SIPP Public</th>
<th>HACP Public</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>50.83</td>
<td>52.70</td>
<td>52.72</td>
<td>52.19</td>
<td>48.86</td>
</tr>
<tr>
<td>Percent Female</td>
<td>54.6%</td>
<td>59.94%</td>
<td>59.06%</td>
<td>76.56%</td>
<td>80.59%</td>
</tr>
<tr>
<td>Percent Married</td>
<td>22.6%</td>
<td>30.79%</td>
<td>32.09%</td>
<td>6.25%</td>
<td>2.66%</td>
</tr>
<tr>
<td>Number of Adults</td>
<td>1.450</td>
<td>1.274</td>
<td>1.284</td>
<td>1.094</td>
<td>1.160</td>
</tr>
<tr>
<td>Number of Children</td>
<td>0.495</td>
<td>0.617</td>
<td>0.616</td>
<td>0.641</td>
<td>0.950</td>
</tr>
<tr>
<td>Percent With Children</td>
<td>24.73%</td>
<td>30.32%</td>
<td>30.27%</td>
<td>31.25%</td>
<td>43.95%</td>
</tr>
<tr>
<td>Percent Black</td>
<td>32.64%</td>
<td>28.28%</td>
<td>27.05%</td>
<td>51.56%</td>
<td>88.53%</td>
</tr>
<tr>
<td>Annual Income</td>
<td>14079</td>
<td>18979</td>
<td>19391</td>
<td>11184</td>
<td>9082</td>
</tr>
</tbody>
</table>

We only observe households that have lived in public housing at some point during the sample period. Once households leave the housing communities, the HACP does not conduct any follow-up surveys. To learn about households that are eligible for public housing, but do not live in one of the housing communities, we turn to the 2001 Survey of Income and Program Participation (SIPP).

\textsuperscript{11}There is a strong incentive for families to not report the existence of a cohabiting adult or partner, as it would lead to an increase in rent if the cohabiting adult earns an income. As a result, the number of cohabiting adults as well as household income are surely larger than our estimates from the data.

\textsuperscript{12}Our sample differs from other studies in that Pittsburgh public housing seems to house a higher percent of black households, female-headed households and households with children; but a much lower percent of married households. For example, Hungerford ‘96’s sample from the 1986-1988 SIPP panel was 52% female, 23% black, 32% married and the mean number of children was 0.21 (Hungerford, 1996).
The SIPP is a survey managed by the U.S. Census Bureau that interviews households every four months for 3 years. Each month, households are asked about their previous four months’ family composition, sources of income, and participation in government programs such as public housing and school lunch programs. We create a sample based on the SIPP that contains households that eligible for housing aid.\(^{13}\)

Table 2.2 provides some descriptive statistics for our SIPP sample used in this analysis and compares it to Census and HACP data. We find that low-income households that rent in the private market are on average more likely to be married, are less likely to be black, and have substantially higher income than households in public housing. Comparing the SIPP with the HACP sample we find that the SIPP sample is slightly older and as a consequence average income is slightly higher and children are fewer than in the HACP. Comparing the SIPP with the Census, the SIPP contains slightly older heads of household, more female heads of household, more married householders, households with more children, and fewer black households. However, the differences between the SIPP sample and the Census sample of eligible households in Pittsburgh are relatively small.\(^{14}\)

### Table 2.3: Transition Matrix

<table>
<thead>
<tr>
<th></th>
<th>Private</th>
<th>PH 1</th>
<th>PH 2</th>
<th>PH 3</th>
<th>PH 4</th>
<th>PH 5</th>
<th>PH 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private</td>
<td>0</td>
<td>677</td>
<td>144</td>
<td>24</td>
<td>300</td>
<td>59</td>
<td>191</td>
</tr>
<tr>
<td>PH 1</td>
<td>855</td>
<td>16264</td>
<td>16</td>
<td>2</td>
<td>75</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>PH 2</td>
<td>233</td>
<td>16</td>
<td>5371</td>
<td>3</td>
<td>17</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>PH 3</td>
<td>44</td>
<td>2</td>
<td>29</td>
<td>1438</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>PH 4</td>
<td>572</td>
<td>16</td>
<td>8</td>
<td>1</td>
<td>12156</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>PH 5</td>
<td>105</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2017</td>
<td>29</td>
</tr>
<tr>
<td>PH 6</td>
<td>302</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>47</td>
<td>37</td>
<td>8129</td>
</tr>
</tbody>
</table>

Rows indicate choices in \(t-1\) and columns in \(t\).

The 34 communities are classified into broad community types: family large (PH 1), family

\(^{13}\)The SIPP contains only 14 households that participate in public housing in Pittsburgh at some point during the sample period. There are 156 Pittsburgh households eligible for public housing in the first quarter. We, therefore, constructed a sample that also includes households from metropolitan areas with similar characteristics. Appendix A details how we constructed this sample.

\(^{14}\)An appendix that contains a more detailed description of how our data set was constructed is available from the authors.
medium (PH 2), family small (PH 3), mixed (PH 4), senior large (PH 5), and senior small (PH6).

These six types of housing units are fairly homogenous, but seem to attract different types of households. Large, medium, and small low-rise non-senior communities primarily house families with children. However, they also include a significant percent of households without children ranging from 36% to 42%. Although the demographics of senior and family housing differ, there is some overlap. Most senior-dominated communities include a significant percentage of non-senior adults without kids ranging from 13% to 37%. Most family-only communities include some senior households ranging from 0 - 20%, about a third of which are caring for children.

Table 2.3 shows the transition matrix for the HACP data. We find that locational choices are persistent since most households stay with their past choices. However, the off-diagonal elements of the transition matrix indicate that there is a fair amount of entry into and exit from public housing. Moreover, there are a number of transitions within public housing communities. These transfers are largely voluntary and indicate that households differentiate among the heterogeneous community types.\textsuperscript{15}

2.3 An Equilibrium Model of Housing Markets with Rationing

2.3.1 The Baseline Model without Transfers

We consider a model with a continuum of low-income households. Each household is eligible for housing aid and can thus, in principle, live in one of the available public housing communities or rent an apartment in the private market. Denote the outside private market option with 0. Let \( J \) be the number of different housing communities that are available in the public housing program. Let \( d_{jt} \in \{0, 1\} \) denote an indicator variable which equals one if the household chooses alternative \( j \) at time \( t \) and zero otherwise.\textsuperscript{16} Let the vector \( d_t = (d_0, ..., d_J) \) characterize choices of a household at

\textsuperscript{15}In the SIPP sample, we observe 89 transitions from private to public housing and 98 transitions from public to private housing.

\textsuperscript{16}In our application, we use quarterly data.
Since the alternatives are mutually exclusive, we have

\[ \sum_{j=0}^{J} d_{jt} = 1 \] (2.1)

Households differ along a number of characteristics \( x_t \) such as income, age, number of kids, number of adults, gender of household head, marital status, and race. We treat these characteristics as exogenous, although it is difficult to endogenize income or family status from a conceptual perspective.\(^{17}\)

Household preferences are subject to idiosyncratic shocks denoted by \( \epsilon_t \). Households face relocation costs if they decide to move. Thus lagged choices, denoted by \( d_{t-1} \), are relevant state variables.

Households have preferences defined over all potential elements in the choice set. We model household preferences using a standard random utility specification.

**Assumption 1** Let \( u(d_t, x_t, d_{t-1}, \epsilon_t) \) denote the household utility function. We assume that the utility function is additively separable in observed and unobserved states and thus allows the following representation:

\[ u(d_t, x_t, d_{t-1}, \epsilon_t) = \sum_{j=0}^{J} d_{jt} \left[ u_j(x_t, d_{t-1}) + \epsilon_{jt} \right] \] (2.2)

This specification implicitly treats public housing as a differentiated product.

A key feature of our model is that all potential choices may not be available to a household at any given point of time. A household that is currently renting in the private market may not have access to public housing even if the household meets all eligibility criteria.\(^{18}\) We, therefore, need to formalize the fact that access to public housing is restricted by a local housing authority.

\(^{17}\)We do not observe labor supply or job market participation in the HACP data. See Jacob and Ludwig (2010) for analysis of the impact of Section 8 vouchers on income.

\(^{18}\)In practice, all eligible households are typically assigned to a waiting list. A household will only receive an offer to move into public housing if it is on top of the waiting list.
**Assumption 2** *The public housing authority does not evict any households that have lost eligibility.*

This assumption is motivated by policies that are typically used by many local housing authorities. It implies that exit from public housing is purely voluntary. To characterize the voluntary outflow, let \( P_{jt} \) denote the fraction of eligible households living in community \( j \) at the beginning of period \( t \).

The outflow from public housing community \( j \) to the private sector, \( OF_{jt} \), is defined as:

\[
OF_{jt} = P_{jt} \int Pr\left(u_0(x_t, d_{t-1}) + \epsilon_{0t} \geq u_j(x_t, d_{t-1}) + \epsilon_{jt}\right) f(x_t | d_{jt-1} = 1) \, dx_t
\]  

(2.3)

where \( f(x_t | d_{jt-1} = 1) \) denotes the conditional density function of households with characteristics \( x_t \) that live in \( j \) at the beginning of period \( t \). As a consequence, the housing authority faces a stream of housing units that become available at each point of time. The authority needs to assign these units to new renters. To model this decision process, we need to model the potential demand for public housing.

Let \( P_{0t} \) denote the fraction of eligible households renting in the private market at the beginning of period \( t \). We make the following assumption:

**Assumption 3** *All eligible households that are renting in the private market are placed on a wait list for public housing.*

We offer four observations regarding this assumption. First, signing up for the wait list is, for all practical purposes, costless in practice.\(^{19}\) Second, it is easy to relax the assumption and allow for systematic differences between households on the wait list and eligible households that have not signed up on the wait list. When we discuss the rationing implications, we relax this assumption and consider a case in which a demand signal triggers households to sign up on the wait list. Third, the assumption can be justified by empirical constraints. We do not observe the characteristics of all households on the wait list and neither does the housing authority. We also do not observe the

\(^{19}\)Of course, it does not matter that all eligible households sign up as long as there are no systematic differences between eligible households and households on the wait-list.
priority ranking of households on the wait-list. Assumption 3 implies that the households that have top priority on the wait-list do not systematically differ from the eligible population. Finally, it is also straightforward to assume that the housing authority has multiple wait lists for households with different family sizes.

Next consider the potential demand for public housing. The probability that a household that is currently living in the private sector prefers $j$ at time $t$ is:

$$Pr(d_{jt} = 1 | x_t, d_{it-1} = 1) = Pr(u_j(x_t, d_{t-1}) + \epsilon_{jt} \geq u_0(x_t, d_{t-1}) + \epsilon_0) \quad (2.4)$$

Let $f(x_t | d_{0t-1} = 1)$ denote the conditional density function of households with characteristics $x_t$ that currently rent in the private market, are eligible for public housing, and thus have been assigned to a wait list. The potential demand for community $j$ is then characterized by the fraction of households on the wait list that prefer $j$ at time $t$:

$$F_{0jt} = P_{0t} \int Pr(d_{jt} = 1 | x_t, d_{0t-1} = 1) f(x_t | d_{it-1} = 1) \, dx_t \quad (2.5)$$

The most interesting case arises if demand exceeds supply. We therefore make the following assumption:

**Assumption 4**  

a) The potential demand exceeds the voluntary outflow for each community at each point of time.  
b) The authority offers the free units to households on the wait list that have the highest priority. The housing authority continues offering units until all available vacant units have been filled with eligible households.

Assumption 4a is not necessary to obtain a well defined equilibrium, but it holds empirically in almost all large markets in the U.S. It implies that the housing authority can not meet the full demand. Instead it can only offer public housing to a fraction of households that are eligible. Assumption 4b

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20 As a consequence, we can solve and estimate the model without observing the conditional distribution of households on the wait list.  
21 We discuss these issues when we estimate the model in Section 5.
implies that that housing authority follows a first-in-first-out policy. Assumptions 2 through 4 imply that there is a fraction of households denoted by, $\Pi_{0jt}$, that will receive offers to move into housing community $j$ at time $t$. The total inflow into public housing is then given by:

$$IF_{jt} = \Pi_{0jt} F_{0jt}$$  \hspace{1cm} (2.6)

To close the model, we need to impose an assumption on the supply of public housing and the vacancy rates.

**Assumption 5** *The supply of public housing is constant in each housing community at each point of time.*

We can relax this assumption and allow for exogenous changes in the supply of public housing due to new construction or demolitions. We discuss these issues in detail when we quantify the impact of demolitions in Section 2.6 of the paper.

Assumption 5 then implies that the outflow must equal the inflow for each housing community at each point of time in equilibrium.\(^{22}\)

$$IF_{jt} = OF_{jt}$$  \hspace{1cm} (2.7)

An equilibrium for the baseline model can, therefore, be defined as follows:

**Definition 1** *Given an initial distribution of household types, an equilibrium for this model consists of a rationing mechanism that determines the fraction of households that receive offers to move into public housing such that*

- Households choose the preferred housing option among the set of available options.

\(^{22}\)The assumption of a constant housing stock is common in many theoretical papers that study housing market equilibrium in urban metropolitan areas. See, for example, Nechyba (1997a, 1997b), Nechyba (2003), Bayer & Timmins (2005), and Ferreyra (2007).
For each housing community \( j \), the housing authority offers apartments to eligible households on the wait list. Thus a fraction of households on the wait list will receive offers to move into public housing.

The inflow of households equals the outflow of households for each housing community.

We have \( J \) offer probabilities and \( J \) market clearing conditions. Moreover, the system of equations which defines the equilibrium is linear in the offer probabilities and can be solved equation by equation. A unique equilibrium for the economy exists since the potential inflow is at least as large as the voluntary outflow for each community. Hence we have the following result:

**Proposition 1** There exists a unique housing market equilibrium with rationing in the baseline model without transfers.

As we will see below, uniqueness of equilibrium is essential for identifying the parameters of the model. Next we generalize our model and allow for transfers between public housing units.

### 2.3.2 An Extended Model with Transfers

Transfers imply that the demand for public housing must be modified since households may have additional options. The probability that a household that lives in community \( i \) at the beginning of the period prefers to move to community \( j \) at time \( t \) is:

\[
Pr(d_{jt} = 1|x_t, d_{it-1} = 1) = Pr(u_j(x_t, d_{t-1}) + \epsilon_{jt} \geq \max \{u_i(x_t, d_{t-1}) + \epsilon_{it}, u_0(x_t, d_{t-1}) + \epsilon_{0t}\}) \tag{2.8}
\]

Note that households only compare options that in the effective choice set, i.e. that are available to them. As before, the potential demand is then characterized by the fraction of households living in community \( i \) that prefer \( j \) at time \( t \):

\[
F_{ijt} = P_{it} \int Pr(d_{jt} = 1|x_t, d_{it-1} = 1) f(x_t|d_{it-1} = 1) \, dx_t \tag{2.9}
\]
In contrast to entry into public housing and exit, there is no stated policy for transfers between public housing units. Nevertheless, we observe a fair number of transfers in practice. A useful modeling approach is then to mimic our assumptions imposed on the (external) wait list to generate a well-defined transfer policy. Suppose that the housing authority also has an internal mechanism that determines transfer offers. In that case, a fraction of households that is currently living in \( i \) are offered the opportunity to transfer to community \( j \).

**Assumption 6** The probability of obtaining an offer from housing community \( j \) while living in public housing \( i \) is given by \( \Pi_{ijt} \). Households get at most one offer at each point of time.

The total realized demand (or inflow) from community \( i \) to community \( j \) at time \( t \) is therefore \( \Pi_{ijt} F_{ijt} \). Summing over all current housing choices other than \( j \) gives the total inflow into housing community \( j \):

\[
IF_{jt} = \sum_{i=0, i \neq j}^{J} \Pi_{ijt} F_{ijt} \quad (2.10)
\]

Similarly, we can modify the equation that characterizes the total voluntary outflow from community \( j \):

\[
OF_{jt} = OF_{j0t} + \sum_{i=1, i \neq j}^{J} \Pi_{ijt} F_{ijt} \quad (2.11)
\]

where the outflow to the private sector, \( OF_{j0t} \), is defined as:
\[ OF_{jt} = P_{jt} \Pi_{jt} \int Pr(u_0(x_t, d_{t-1}) + \epsilon_0t \geq u_j(x_t, d_{t-1}) + \epsilon_{jt}) \frac{f(x_t|d_{jt-1} = 1)}{dx_t} \]

\[ + \sum_{k=1, k \neq j}^{K} \Pi_{jkt} \int Pr(u_0(x_t, d_{t-1}) + \epsilon_0t \geq \max [u_j(x_t, d_{t-1}) + \epsilon_{jt}, u_k(x_t, d_{t-1}) + \epsilon_{kt}]) \frac{f(x_t|d_{jt-1} = 1)}{dx_t} \]  

(2.12)

In the extended model we have \( J^2 \) offer probabilities and \( J \) market clearing conditions. Moreover, the system of equations which defines equilibrium is linear in the offer probabilities. An equilibrium for the economy exists if the linear system of market clearing equations has a solution. These solutions (generically) exist, but are not unique, since the number of equations is smaller than the number of unknowns.\(^{23}\)

The potential for multiplicity in equilibrium arises because we have not sufficiently restricted the ability of the housing authority to allow households to transfer between different units. There are many transfer policies that are consistent with equilibrium in the public housing market. The market clearing conditions alone do not uniquely determine the offer probabilities. To obtain a unique solution to this system of equations, we need to impose additional assumptions. It is plausible that the housing authority does not discriminate based on current residence and uses the same odds ratio for insiders and outsiders. We therefore assume that:

**Assumption 7** The fraction of households that receive an offer to transfer between units in different communities does not depend on current residence:

\[ \Pi_{ijt} = \Pi_{jt} \]  

(2.13)

The odds ratios are the same for household inside and outside of public housing:

\[ \Pi_{0jt} = R_{0t} \Pi_{jt} \]  

(2.14)

\(^{23}\)See, for example, the discussion in Strang (1988).
Note that this assumption is plausible since housing authorities are not allowed to discriminate based on income, race, and gender. As a consequence it is hard to believe that they could discriminate based on residency. The parameter $R_0t$ measures the relative degree of preferential treatment that is given to outsiders. In practice $R_0t >> 1$ and as a consequence households on the wait list get preferential treatment over households that are already in public housing. Substituting Assumption 7 into the definition of equilibrium, we obtain:

$$R_0t \Pi_{jt} F_{0jt} + \sum_{i \neq j} \Pi_{ijt} F_{ijt} = OF_{0jt} + \sum_{i \neq j} \Pi_{jit} F_{jit}$$

(2.15)

which is a system of $J$ equations in $J + 1$ unknowns. Thus the equilibrium conditions define the offer probabilities up to the factor $R_0t$. We thus have shown the following result:

**Proposition 2** For each value of $R_0t$, there exists a unique housing market equilibrium with rationing.

In summary, we have developed an equilibrium model of public housing that generates rationing and excess demand in equilibrium. The model explains transfers within public housing since housing communities are heterogeneous.

### 2.4 Identification and Estimation

We estimate the model using two different samples. The first sample is a choice based sample that is provided by a local authority. This sample tracks households as long as they stay in public housing. The second sample is a random sample of households that are eligible for housing aid. In this section we introduce a parametrization of our model. We then derive the conditional choice probabilities and develop our maximum likelihood estimator. We then discuss the role that equilibrium conditions play in establishing identification of the model. Finally, we show that our approach works in a Monte Carlo study when the data generating process is known.

---

24A few transfers in our sample are due to forced relocations or changes in family structure.
2.4.1 A Parametrization

We assume that the utility associated with community $j$ is given by

$$u_{jt} = \gamma_j + \beta \ln(y_{jt}) + \delta x_t + mc 1\{d_t \neq d_{t-1}\} + \epsilon_{jt} \quad j = 1, \ldots, J$$  (2.16)

The utility of the outside option is normalized to be equal to the following expression:

$$u_{0t} = \ln(y_{0t}) + mc 1\{d_t \neq d_{t-1}\} + \epsilon_{0t}$$  (2.17)

In the equations above, $y_{jt}$ denotes household net income, $mc$ is a moving cost parameter, and $\gamma_j$ is a community specific fixed effect.\(^{25}\) Households that live in public housing typically pay 30\% of their income in rent. As a consequence net income is choice specific due to the implicit tax. As income increases, living outside of public housing should become more attractive. We would, therefore, expect that $\beta < 1$. The community specific fixed effects capture observed and unobserved differences among the public housing communities. The specification also accounts for (psychic) moving costs. Idiosyncratic shocks account for factors not observed by the econometrician. Following McFadden (1974), we assume that the $\epsilon$’s are i.i.d. Type I extreme value distributed.

2.4.2 Conditional Choice Probabilities

Our main data set is from a local housing authority and follows households as long as they are in public housing. This is, therefore, a choice based sample since we only observe households that have chosen to live in one of the housing communities at time $t$. A household that lived in community $j$ at the end of the last time period, has potentially three options. First, the household moves back to the private housing market. Second, the household moves to a different housing community. Third, the household stays in its current community $j$. Given the distributional assumptions on the

\(^{25}\)We are implicitly imposing the budget constraint by using net income in the utility function.
idiosyncratic shocks, the probability of moving to the private sector is then:

\[
Pr\{d_{jt} = 1|d_{jt-1} = 1, x_t\} = \sum_{k=1, k \neq j}^{J} \Pi_{jkt} \frac{\exp(u_k(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t)) + \exp(u_k(x_t))} + \Pi_{jjt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))} \tag{2.18}
\]

The probability of moving from community \(j\) to community \(k\) is given by:

\[
Pr\{d_{kt} = 1|d_{jt-1} = 1, x_t\} = \Pi_{jkt} \frac{\exp(u_k(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t)) + \exp(u_k(x_t))} \tag{2.19}
\]

and the probability of staying in community \(j\) is given by:

\[
Pr\{d_{jt} = 1|d_{jt-1} = 1, x_t\} = \sum_{k=1, k \neq j}^{J} \Pi_{jkt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t)) + \exp(u_k(x_t))} + \Pi_{jjt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))} \tag{2.20}
\]

Finally, we also observe new entrants into public housing. The probability of observing a new household in community \(j\) is

\[
Pr\{d_{jt} = 1|d_{jt-1} = 0, x_t\} = \Pi_{0jt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))} \tag{2.21}
\]

The conditional choice probabilities for the choice based sample are thus defined by equations (2.18), (2.19), (2.20) and (2.21).

Our second sample is a random sample of low income households that tracks households both inside and outside of public housing. In contrast to the choice based sample, this sample does not allow us to identify the exact housing community in which a household lives. As a consequence we only observe a coarser version of the choice set in the random sample. For households that are currently not living in public housing, we have two possible outcomes: 1) the household stays in private housing; 2) the household moves to a public housing unit.
The probability of moving to any of the $J$ public housing communities is given by:

$$Pr\{d_{0t} = 0|d_{0t-1} = 1, x_t\} = \sum_{j=1}^{J} \Pi_{0jt} \frac{exp(u_j(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t))} \quad (2.22)$$

Note that (2.22) is obtained by summing the probabilities in (2.21) over all possible choices. Similarly, the probability of staying in private housing is defined:

$$Pr\{d_{0t} = 1|d_{0t-1} = 1, x_t\} = 1 - \sum_{j=1}^{J} \Pi_{0jt} \frac{exp(u_j(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t))} \quad (2.23)$$

Note that we do not observe whether the household obtained an offer and we also do not observe to which housing unit it moved, if it decided to move.

Next consider a household that currently lives in public housing. Again there are two possible outcomes. The household moves back to private housing. Alternatively the household stays in public housing. Consider the first case, in which the household moves back to private housing. Now we do not observe in the random sample in which unit the household lives. However, we can compute relative frequencies based on the choice based sample which assign probabilities to each community type. Let us denote these probabilities by $Pr\{d_{jt-1} = 1|d_{0t-1} = 0, x_t\}$. The choice probability conditional on living in community $j$ is given by equation (2.18). Summing over all $J$ housing units and properly weighting each conditional choice probability, implies that the probability of moving out of public housing is then:

$$Pr\{d_{0t} = 1|d_{0t-1} = 0, x_t\} = \sum_{j=1}^{J} Pr\{d_{0t} = 1|d_{jt-1} = 1, x_t\}Pr\{d_{jt-1} = 1|d_{0t-1} = 0, x_t\} \quad (2.24)$$

Next consider the case in which a household stays in public housing. We cannot distinguish between the case in which a household stays in the same community or moves to a different housing community within public housing. Thus conditional on living in community $j$, the probability of staying in public housing is the sum of the probabilities in equations (2.19) and (2.20), i.e. the
probability of staying conditional on living in \( j \) at the end of the previous period is

\[
Pr(d_{0t} = 0 | d_{jt-1} = 1, x_t) = Pr(d_{jt} = 1 | d_{jt-1} = 1, x_t) + \sum_{k=1, k \neq j}^{J} Pr(d_{kt} = 1 | d_{jt-1} = 1, x_t)
\] (2.25)

Summing over all \( J \) housing units and properly weighting each conditional choice probability, implies that the probability of staying in public housing is then:

\[
Pr(d_{0t} = 0 | d_{0t-1} = 0, x_t) = \sum_{j=1}^{J} Pr(d_{0t} = 0 | d_{jt-1} = 1, x_t)Pr(d_{jt-1} = 1 | d_{0t-1} = 0, x_t)
\] (2.26)

The conditional choice probabilities for the random sample are thus defined by equations (2.22), (2.23), (2.24) and (2.26).

### 2.4.3 The Likelihood Function under Enriched Sampling

To compute the likelihood function we need to take into account the fact that we use a random and a choice based sample in estimation. This sampling scheme is also called enriched sampling as discussed in detail by Cosslett (1978, 1981).\(^{26}\) Let us denote the corresponding sample sizes with \( N_1 \) and \( N_2 \). Similarly, let \( T_1 \) and \( T_2 \) denote the length of the two panels. Observations are assumed to be independent across samples ruling out sampling the same household in both data sets. The joint likelihood function of observing the two samples is thus the product of the two likelihood functions

\[
L = L_1 L_2
\] (2.27)

The likelihood associated with the random sample \( L_1 \) is given by:

\[
L_1 = \prod_{i=1}^{N_1} \prod_{t=1}^{T_1} l_{int}
\] (2.28)

\(^{26}\) Notice that our sampling scheme satisfies assumptions 9 and 10 in Cosslett (1981) which guarantees a sufficient overlap in the relevant choice sets between the two samples.
where $l_{1nt}$ is given by

$$l_{1nt} = [Pr\{d_{0nt} = 0|d_{0nt-1}, x_{nt}\}]^{1-d_{0nt}} [Pr\{d_{0nt} = 1|d_{0nt-1}, x_{nt}\}]^{d_{0nt}} f(x_{nt}, d_{nt-1}) \quad (2.29)$$

The likelihood for the choice based sample $L_2$ is defined:

$$L_2 = \prod_{t=1}^{T} \prod_{t=1}^{T_2} \frac{Pr\{d_{jnt} = 1|d_{nt-1}, x_{nt}\} f(x_{nt}, d_{nt-1})}{\tilde{Q}_t(J)} \quad (2.30)$$

where

$$\tilde{Q}_t(J) = \sum_{j=1}^{J} Q_t(j) \quad (2.31)$$

$Q_t(j)$ is the unconditional probability that choice $j$ is chosen that is defined as:

$$Q_t(j) = \sum_{j=1}^{J} \int Pr\{d_{jnt} = 1|d_{t-1}, x_t\} f(x_t, d_{t-1}) dx_t d_{t-1} \quad (2.32)$$

$$= \sum_{j=1}^{J} \sum_{i=0}^{J} Pr\{d_{jnt} = 1|d_{it-1} = 1, x_i\} f(x_i|d_{it-1} = 1) Pr\{d_{it-1} = 1\} dx_t$$

We assume that $f(x_t, d_{t-1}, \theta)$ is known up to finite vector of parameters $\theta$ and treat the $Q_t(j)$ as unknown. We then define our enriched sampled maximum likelihood estimator (ESMLE) as the argument that maximizes equation (2.27).\[27\]

2.4.4 Imposing the Equilibrium Constraints

One problem associated with the ESML estimator above is that the offer probabilities are not separately identified from the choice specific intercepts. To obtain identification, we use the equilibrium

\[27\]If the $Q_t(j)$’s are known, we can define a constrained enriched sampled maximum likelihood estimator (CESMLE) as the argument which maximizes equation (2.27) subject to the $J$ constraints in equation (2.32). Finally, one could follow Cosslett (1978,1981) and treat $f(x_t, d_{t-1})$ as unknown and then define Pseudo MLE by concentrating out the weights that characterize the empirical likelihood of the data. These estimators extend the standard choice based estimators discussed in Manski & Lerman (1977).
conditions and express the endogenous offer probabilities as functions of the structural parameters of the choice model. To illustrate the basic ideas, consider first the model without transfers. In that model the structural parameters of the utility function are identified from the exit behavior of households. The conditional exit probability does not depend on the probability of getting an offer to move into public housing. Unattractive housing units will have higher exit rates and lower potential demand than attractive housing communities. Given the voluntary exit rates and potential demand for moving into public housing, the offer probabilities are then uniquely determined by the equilibrium conditions. Solving this linear system of equations, we can express the offer probabilities as functions of the voluntary outflow and the potential demand which only depend on the structural parameters of the utility function. Imposing the equilibrium conditions thus resolves the key identification problem encountered in the model without transfers.

In the model with transfers, the sequential identification argument breaks down since exit probabilities depend on unobserved transfer probabilities. Nevertheless, we can still express the offer probabilities as functions of the structural parameters of the utility function. If a community is attractive, voluntary outflows will be low and potential demand will be high. As a consequence offer probabilities are low. Similarly, if the community is unattractive, voluntary outflows and transfers will be high and the potential inflow will be low. As a consequence, offer probabilities need to be sufficiently large to meet the equilibrium condition. Thus a similar logic for identification applies in the extended model that accounts for transfers.

To provide some additional insights into our approach to identification, we have conducted a Monte Carlo study. We find that our estimator works well under random and enriched sampling. The absolute errors are small and approximately centered around zero. Generally, we find that the estimate for the fixed effects are slightly biased upward and the coefficients on income are slightly biased downward in samples with 2000 observations. Larger samples help reduce the estimation bias. Imposing the equilibrium conditions works well and establishes identification. The estimates of the offer probabilities that are implied by the equilibrium conditions are accurate.

28Details are reported in Appendix B.
2.5 Empirical Results

We implemented our estimator for a number of different model specifications. Table 2.4 reports the parameter estimates and estimated standard errors for three models that capture the essence of our modeling approach. In column I, we estimate the model with transfers using the full sample. We are thus implicitly assuming that the housing authority has only one wait list. This estimator controls for differences in income, race, age, family status and number of children. In column II, we estimate the model for the subsample of households that are eligible for two bedroom non-senior apartment units. In column III we consider the subsample of senior housing units. These two models thus explicitly assume that there are separate wait lists for different family and apartment sizes. We only control for differences in income, gender and race in these estimators.

We find that blacks have stronger preferences for public housing than whites. This result is largely driven by the fact that black households are overrepresented in public housing in Pittsburgh. We also find that age has an impact. Male seniors have stronger preferences for public housing than female seniors. Females with children also have stronger preferences for public housing than other households. In contrast, fathers or married couples with children have lower valuations for public housing than those without children. We also find that there are significant moving costs that constrain potential relocations of households.

The income coefficient shows that there are strong incentives for households to leave public housing as income increases. This finding is consistent with the fact that there are only a few higher income household in our sample that live in public housing. There are only 52 households in our sample that, at some time during the study, exceed the income eligibility limit of approximately

---

29In all models, we use the empirical demographic distributions to estimate $f(x_t, d_{t-1})$. Race (black, white) and age (senior, non-senior) are modeled as a multivariate distribution; sex is a binomial conditional on race-age; number of children is a multinomial conditional on sex and race-age; income is a truncated normal based on number of children, sex, and race-age. We fit a logit model to estimate $Pr(d_{jt-1} = 1|d_{jt-1} = 0, x_t)$, which is needed in equations (2.24), (2.25), and (2.26) for the SIPP likelihood. We calibrate $R_0$ based on the observed ratios of mobility for households inside and outside of public housing.

30We have also estimated a version of the model that only used households in the SIPP that live in Pittsburgh. Using the smaller Pittsburgh subsample largely affects the precision of the estimates, but not the magnitude of the point estimates.
## Table 2.4: Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full Sample</td>
<td>2 BR Subsample</td>
<td>Senior Subsample</td>
</tr>
<tr>
<td>Income</td>
<td>0.329 (0.028)</td>
<td>0.280 (0.084)</td>
<td>0.380 (0.086)</td>
</tr>
<tr>
<td>Moving cost</td>
<td>-3.186 (0.017)</td>
<td>-4.282 (0.065)</td>
<td>-2.355 (0.033)</td>
</tr>
<tr>
<td>Black and non-senior</td>
<td>1.222 (0.071)</td>
<td>0.822 (0.178)</td>
<td></td>
</tr>
<tr>
<td>White and senior</td>
<td>0.209 (0.113)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black and senior</td>
<td>1.000 (0.101)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Children</td>
<td>-0.315 (0.123)</td>
<td>0.253 (0.205)</td>
<td>0.535 (0.537)</td>
</tr>
<tr>
<td>Female</td>
<td>0.053 (0.061)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female and senior</td>
<td>-0.174 (0.094)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female with children</td>
<td>0.426 (0.130)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log likelihood</td>
<td>-688,796</td>
<td>-123,144</td>
<td>-130,184</td>
</tr>
</tbody>
</table>

Estimated standard errors are given in parenthesis.
Model III allows the fixed effects to differ by race.

$45,000.\textsuperscript{31}$ Most of these households are headed by a single, black female. We also estimate community specific fixed effects which are not reported in the table above. Our findings suggest that smaller communities are in general more desirable than larger communities.

Estimating a simple discrete choice models that ignore all supply side restrictions, we find that predicted demand exceeds supply by a factor of 7.7 using the full sample and by a factor of 4.3 using the two bedroom sub-sample. Failure to incorporate the supply side restriction in estimation thus leads to a seriously flawed inference and prediction.

Next we analyze the goodness of fit of our model. One measure of goodness of fit is to compare the residency distribution predicted by the model to the actual residency distribution observed in the sample. We find that the predictions that are based on our preferred model are accurate. Our model, thus, matches the unconditional distributions of households among choices well. A more challenging exercise is to predict the composition of the housing communities using our model. We focus on the composition by gender and family status conditional on race. The results are

\textsuperscript{31}Note that this limit depends on year and size of household.
summarized in Table 2.5. The findings are by and large encouraging. Our model explains the demographic compositions of all communities well.
Table 2.5: Actual vs Estimated Composition of Communities

<table>
<thead>
<tr>
<th></th>
<th>Private</th>
<th>PH1</th>
<th>PH2</th>
<th>PH3</th>
<th>PH4</th>
<th>PH5</th>
<th>PH6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>% Black</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>0.24</td>
<td>0.98</td>
<td>0.94</td>
<td>0.90</td>
<td>0.97</td>
<td>0.56</td>
<td>0.55</td>
</tr>
<tr>
<td>Estimated</td>
<td>0.26</td>
<td>0.95</td>
<td>0.92</td>
<td>0.9</td>
<td>0.95</td>
<td>0.51</td>
<td>0.56</td>
</tr>
<tr>
<td><strong>% Female</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>0.67 / 0.53</td>
<td>0.85 / 0.88</td>
<td>0.89 / 0.75</td>
<td>0.93 / 1.00</td>
<td>0.84 / 0.67</td>
<td>0.63 / 0.53</td>
<td>0.66 / 0.68</td>
</tr>
<tr>
<td>Estimated</td>
<td>0.67 / 0.53</td>
<td>0.82 / 0.67</td>
<td>0.87 / 0.71</td>
<td>0.93 / 0.83</td>
<td>0.84 / 0.64</td>
<td>0.57 / 0.48</td>
<td>0.67 / 0.66</td>
</tr>
<tr>
<td><strong>% Have Kids</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>0.46 / 0.24</td>
<td>0.55 / 0.64</td>
<td>0.62 / 0.43</td>
<td>0.62 / 0.38</td>
<td>0.58 / 0.1</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Estimated</td>
<td>0.42 / 0.24</td>
<td>0.49 / 0.28</td>
<td>0.57 / 0.36</td>
<td>0.60 / 0.37</td>
<td>0.59 / 0.19</td>
<td>0.06 / 0.02</td>
<td>0.05 / 0.02</td>
</tr>
<tr>
<td><strong>Income</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Composition Shown by Race black / white.
We compare the observed mobility with the mobility generated under the model. With the model parameters from our preferred model, the predicted number of move-ins during this whole sample is 1796. The actual number is 1581. The predicted move-outs 2273 (actual is 2106). Finally the predicted number of transfers is 374 compared to 349 observed in the data.\footnote{Some periods in the HACP data were eliminated. Only quarters overlapping with the SIPP data were included in the estimation.}

2.6 The Welfare Costs of Demolitions

We are now in a position to estimate the welfare costs associated with demolitions. As we discussed before, the federal government has for all practical purposes stopped building housing projects. To shed some insights into the effects of reducing the supply of public housing, we consider demolishing some of the least attractive public housing units. We analyze how demolitions affect the demand for public housing, the composition of housing communities, and compute standard welfare measures. We consider demolishing communities with a large number of units. These communities have been the target of demolitions in many cities. Our estimates confirm that they have the lowest fixed effect parameter and are thus the least attractive of all communities. The welfare estimates can, therefore, be interpreted as lower bounds for the welfare estimates associated with demolishing more desirable units.

We consider the demolition of public housing community 1 during the third period of a 12-quarter study. We use the estimates based on our preferred model in column II of Table 2.4. To initialize, the demographic characteristics in the first quarter are the same as those observed in the data. It is well-known that these types of discrete choice models do not yield closed form solutions for compensating variations. We, therefore, follow McFadden (1989, 1995) and adopt a simulation based approach. An additional complication in our model is that we not only need to simulate draws from distributions of the error terms, but also from the equilibrium offer probabilities. For families of varying demographic characteristics, we compute the median compensating variation for...
an evicted household earning $12,000 per year. We find that the estimates range from $11,656 for a white male with kids to $116,010 for a black female with kids. White households require lower compensation to leave public housing than black households. Overall, the estimates suggest that there may be significant welfare losses associated with demolishing existing units. The policy experiment shows a decline in overall welfare for low-income blacks. However for some low-income households earning more than $12,000 a year, there is a small welfare gain.

Compared to the baseline equilibrium, offer probabilities immediately decrease after the eviction because many evicted tenants wish to move back into public housing. Offer probabilities decrease 2.6% for medium communities, 12% for small family communities, 6.3% for mixed family and senior communities, and 16% for mostly senior communities. Over time, the composition of the remaining public housing communities changes. The public housing communities experience an increase of 3% in black households and a 12% decrease in non-black households; there is a 1.3% increase in female-headed households and 2.2% increase in households with children. Average income in the public housing communities decreases 2%. The demolitions of public housing, therefore, lead to an increase in racial and socio-economic segregation.

To better understand the mechanism that drives these welfare costs it is useful to provide a more complete characterization of the rationing process that results in equilibrium. Based on the parameter estimates of our preferred model in column I we estimate the fraction of the population that would like to move into public housing if it was possible. This fraction varies by quarter due to quarterly differences in income and demographic heterogeneity. Table 2.6 shows the percent willing to move for the 12th quarter (a quarter in the middle of the study).

Comparing the fraction of households willing to move into a housing community with the number of available units in that community, we find that this ratio is equal 3.77 for community 1 which is the least attractive community. For the other three family communities this ratio ranges between 7.10 and 72.71. For senior communities this ratio is equal to 37.79 for communities with a small

---

33 Of course, a full cost-benefit analysis would require the inclusion of the cost of maintaining these housing units as well as potential impacts of living in public housing on educational achievements and criminal outcomes.
Table 2.6: Percent of Households in Community $i$ Who Would Accept an Offer to Move to $j$

<table>
<thead>
<tr>
<th>Current Residence:</th>
<th>Private</th>
<th>PH1</th>
<th>PH2</th>
<th>PH3</th>
<th>PH4</th>
<th>PH5</th>
<th>PH6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private</td>
<td>0.006</td>
<td>0.012</td>
<td>0.009</td>
<td>0.008</td>
<td>0.009</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>PH1</td>
<td>0.080</td>
<td>0.067</td>
<td>0.054</td>
<td>0.044</td>
<td>0.055</td>
<td>0.071</td>
<td></td>
</tr>
<tr>
<td>PH2</td>
<td>0.063</td>
<td>0.020</td>
<td>0.029</td>
<td>0.023</td>
<td>0.029</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>PH3</td>
<td>0.075</td>
<td>0.023</td>
<td>0.043</td>
<td>0.028</td>
<td>0.035</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>PH4</td>
<td>0.077</td>
<td>0.031</td>
<td>0.056</td>
<td>0.045</td>
<td>0.046</td>
<td>0.059</td>
<td></td>
</tr>
<tr>
<td>PH5</td>
<td>0.102</td>
<td>0.022</td>
<td>0.041</td>
<td>0.032</td>
<td>0.026</td>
<td>0.043</td>
<td></td>
</tr>
<tr>
<td>PH6</td>
<td>0.085</td>
<td>0.019</td>
<td>0.034</td>
<td>0.027</td>
<td>0.022</td>
<td>0.028</td>
<td></td>
</tr>
</tbody>
</table>

number of units and 18.17 for communities with a large number of units. If we restrict our attention to the subsample of households that are eligible for two bedroom apartments, the demand-supply ratios are 2.65, 3.90, 15.88, and 4.64 for the four types of housing communities. The fraction of households willing to move into a public housing unit largely depends on the community specific fixed effects and thus reflects the attractiveness of the housing community. However, it also depends on the characteristics of eligible households. Older households and extremely poor households are more willing to move from the private sector to public housing communities. These households suffer the highest welfare costs from policies that restrict the supply.

2.7 Conclusions

We developed a new method that can be used to estimate the welfare costs of reducing the supply of public housing. Our estimates are based on an equilibrium model that captures the key supply restrictions. Our empirical analysis of the Pittsburgh metropolitan area, shows that there are significant welfare losses associates with policies that fail to maintain an adequate supply of affordable housing. The welfare effects are likely to be even more pronounced in cities with high housing prices and tight housing markets such as New York or Boston.

We do not dispute that some public housing high rises were in horrible disrepair and contributed
to urban blight, and that a lot of people have benefitted from their demolition. Moreover, vouchers
may provide a more attractive alternative for some families. Still, public housing appears to be
attractive for seniors and very poor households headed by single mothers. Our paper clearly shows
that the demand for public housing remains very strong. Our analysis suggests that relieving some
of the current rationing by constructing new public housing units may be a good policy. A full
cost-benefit analysis of new construction needs to be augmented by estimates of land purchases and
construction costs. Nevertheless, it is straight-forward to conduct such a comprehensive analysis
based on the framework presented in this paper.

The framework presented in this paper can be extended in a number of fruitful directions. In
our model, households maximize current period utility. It is possible to model the dynamic decision
problem faced by forward looking households. Households must now forecast if and when they
will have access to public housing. The value function that corresponds to this problem depends on
current and future offer probabilities. We can then proceed and define demand as before and define
a dynamic equilibrium with forward looking households. Characterizing the equilibrium of this
model and estimating its parameters is, however, more challenging since the equilibrium conditions
are non-linear in the offer probabilities.

It is possible to estimate even richer versions of the model discussed here. We have abstracted
from unobserved heterogeneity in tastes for public housing. It is possible that there is stigma asso-
ciated with living in public housing. Moffitt (1983) has shown that stigma plays a role in explaining
participation in other welfare programs. We can extend our framework and allow for unobserved
heterogeneity in tastes for public housing. Such heterogeneity would provide an alternative expla-
nation for the differential flow rates into and out of public housing. Some households may obtain
a sufficiently strong negative utility from public housing that they effectively are never interested
in the public-sector. Other households might not be affected by stigma and are willing to choose
public housing when they receive a sufficiently strong idiosyncratic shock. However, we can still
define the equilibrium for this modified model. As long as we can express the offer probabilities
as functions of the structural demand parameters, our approach for identification and estimation is

45
valid and can be used to estimate richer specifications of the demand side. Estimating these types of model will allow us to obtain additional insights into the welfare cost of failing to provide an adequate supply of affordable housing in U.S. metropolitan areas.
Chapter 3

Housing Demand and Neighborhood Choice with Housing Vouchers

3.1 Introduction

Low-income families face affordable and livable housing shortages in the United States. The U.S. Department of Housing and Urban Development (HUD) calculated that in 2009 for every 100 household earning less than 30 percent of their area median income, the nation offered only 36 affordable and decent housing units (HUD 2011). The current recession and housing crisis has lead to the highest absolute (7.1 million) and percentage-wise (22 percent) level on record of very low-income renters paying more than half of their income for housing or living in severely inadequate housing. To assist some of these households, the government offers a variety of programs (HUD 2011). One of these programs is the Housing Choice Voucher Program, or Section 8 vouchers. The Housing Choice Voucher Program is significant: in 2009, 4.7 million households received rental assistance from HUD, 2.1 million of whom received vouchers.

Housing vouchers were first introduced by HUD in 1970 in the form of an experimental housing allowance program commissioned by Congress. As a result of the success of that program, in 1974
Congress created the Section 8 voucher program and in the first five years the program expanded to 624,604 households (HUD 2000). As congressional studies continued to show that housing vouchers were more cost-effective than maintaining the country’s public housing stock, the program grew to its current size of 2.1 million households. The program’s most lauded features are its allowance for residential choice and mobility, the portability of a voucher across all national housing authorities, and its flexible contract options for the tenant and landlord.

All local housing authorities receive direction from HUD on how to shape their voucher programs. The federal guidelines specify that participants are selected based on income eligibility. Participants receive a voucher equal to the participant’s metropolitan area’s “Fair Market Rent” (FMR), less a fraction of the participant’s income. Generally HUD sets the FMR at the 40th percentile of local rental rates, based on the Census and the American Housing Survey data on contract rents and inflated using the local CPI index. So long as the participant identifies an amenable landlord and the rental property meets a minimum quality level, the voucher is as flexible as a full-paying tenant’s offer. In most regions including Pittsburgh which is studied here, the relevant housing authorities direct the voucher to the landlords so the voucher household do not have a chance to use it as a general income subsidy. Vouchers offer the possibility that recipients can not only consume a reasonable quality of housing services, but also find housing in decent, safe neighborhoods.

The author is aware of no study that examines voucher recipients’ choices of housing services versus neighborhood quality, although several studies examine relocations. Using a comparison group, Carlson et al (2009) finds that the receipt of a voucher substantially increases the chances of a household’s relocation to a different neighborhood. Feins and Patterson (2005) noted that, nationally, only 14.5% of households receiving vouchers moved within the first two years after initial voucher receipt; 45.3% after five years; 60.8% after eight years; African Americans were the most likely to move, with 75% moving after their current lease expired. Also, Feins and Patterson (2005) found that the fewer the adults, the younger the children and adults, and the higher the

---

1Where local CPI indices are not available, HUD uses data from its own regional Random Digital Dialing Survey.
income, the more likely a voucher recipient household would relocate.²

For relatively few housing voucher initiatives, some voucher recipients receive special assistance in their relocation decisions. These neighborhood counseling programs have demonstrated that counseling has a substantial effect on neighborhood choice (Turner, 1998) (Kling et al., 2007). For example, the Gautreaux Program offered minority families vouchers for use only in predominantly white neighborhoods. In the Moving to Opportunities studies, families in the experimental group receiving a voucher were offered relocation assistance and were required to live in a neighborhood with a poverty rate of less than 10 percent. In another example of relocation decision assistance, several metropolitan areas have seen litigation regarding racial discrimination in public housing that has led to victims’ compensation with vouchers coupled with relocation assistance. Still, relocation counseling is not a primary feature of most voucher programs. For this majority of voucher programs, it is important to understand how households exercise the joint decision of housing services choice and neighborhood choice. Although I do not model search costs, the supply of voucher-friendly housing is taken into account.

One goal of the voucher program, to increase participants’ mobility, stems from the understanding that well-being is impacted by neighborhood quality.³ To study this hypothesis, the recent Moving to Opportunities study randomly assigned vouchers to families in public housing that were interested in the voucher program. In the program evaluation, the researchers found that adults in the treatment groups that received vouchers, compared to the control groups who did not receive vouchers, showed increases in exercise, nutrition, sleep and calmness; and decreases in obesity, distress, depression and anxiety (Kling et al., 2007). Recent research indicates that economic self-sufficiency is less impacted. Klieg et al (2007) found no affect on earnings and welfare participation. In another randomized experiment, Jacob and Ludwig (2008) found that voucher recipients worked and earned

---

²The higher relocation rates in the recent Moving To Opportunities studies are due to several factors; families had a deadline of 4-6 months to move in order to receive the voucher, and families had to volunteer to be in the study. (Kling, Liebman, & Katz, 2007).

³Spatial mismatch, suggesting diminished employment prospects for those living in distressed areas, is a topic explored in economics, for example Kain (1968) and Gobillon, Selod, & Zenou (2007). There is a large literature on this topic in sociology; classics include Wilson (1987) and Kozol (1996).
less than those who did not receive a voucher. This finding is consistent with descriptive statistics indicating lower earnings for voucher recipients (Olsen, Tyler, King, & Carillo, 2005b). Using comparison groups, Susin (2008) found vouchers appear to reduce earnings by 15% and Carlson et al (2009) found a positive effect on employment but negative effect on earnings.

To bridge the expectations of the voucher program with the reality of tenants’ housing and residential choice, the goal of this paper is to compare how different voucher policies would achieve mobility and housing goals. To identify optimal policies, I propose, estimate, and simulate a partial equilibrium residential sorting model. I use the model to estimate low income households’ preferences for housing services relative to their preferences for neighborhood amenities, and to understand how the Housing Choice Voucher Program affects consumption of these differing goods. Housing vouchers are not currently addressed in the general residential sorting literature. The proposed housing and neighborhood choice model complements several empirical household sorting studies that have already made important insights on the equilibrium structure of a metropolitan area, namely the joint stratification of income and public good provision across local jurisdictions in a metropolitan area; many of those achievements were made possible only by abstracting away from the small segment of the population receiving housing subsidies.

The model views households as making a joint discrete-continuous choice. The continuity of housing demand is an important feature for studying slight but important variations in housing consumption that result from policy changes. The discretization of the neighborhood selection problem grants flexibility in parameterizing heterogeneous tastes for various neighborhood characteristics including crime, school quality, and racial composition. In addition, the model allows for the possibility of non-separable preferences for the discrete and continuous choices through co-varying

---

4For example Epple and Sieg (1999), Epple Romer Sieg (2001), Sieg et al (2004), Ferreyra (2007), and Walsh (2007) specify and estimate locational equilibrium models that include the endogenous creation of public goods through political processes. Other studies employ a discrete neighborhood choice framework. Bayer McMillan Reuben (2004), Bayer McMillan (2005), and Ioannides Zabel (2008) explore the effects of income inequality, racial sorting, and neighbors’ effect on housing consumption. This paper is closest to Ioannides and Zabel’s, which offers a joint continuous and discrete choice model with housing services as the continuous choice variable. From this previous work, we might hypothesize that housing subsidies disrupt the stratification of income and neighborhood amenity preferences, that minority subsidy recipients face higher implicit housing prices, and that renters wish to consume a level of housing services similar to their neighbors’, respectfully.

50
parameters. The paper builds on Dubin and McFadden’s (1984) version of a discrete-continuous choice model. I extend this framework to make it applicable to settings where a subset of households faces discontinuities in the budget constraint. In particular, a voucher household may obtain a fixed maximum level of housing subsidy in exchange for a fixed portion of its income. Non-voucher households have regular budget constraints.

The proposed framework can allow for the non-separable preference parameterization as specified in previous work, but I allow a more succinct specification. In the Dubin McFadden model, the non-separability stems from a covariation between discrete goods’-specific unobservable feature and demand elasticity of the continuous good. In my model, housing demand is allowed to vary on observable neighborhood characteristics including public park land and public school quality. The motivation of this choice is threefold. First, it obtains reasonable substitution patterns even if there is limited observable information on household heterogeneity. Second, it makes the estimation results and policy simulations more generalizable to other urban areas. Finally, the covariation of housing requirements and observable neighborhood characteristics (rather than covariation between housing requirements and neighborhood-specific unobservables) decreases the computational burden of simultaneous estimation of both the neighborhood choice probabilities and the related housing demand functions.

I use a restricted-use dataset from 4,341 voucher recipients in Pittsburgh, Pennsylvania, in the year 2006, to estimate the parameters of the model using a new General Method of Moments (GMM) estimator. The estimator imposes the restrictions that arise from observed choices of voucher and non-voucher households, combining aggregate and micro-level data. By focusing on a single city, a detailed specification of neighborhood amenities and housing price decomposition is available. The data suggest that across Pittsburgh’s 114 neighborhoods there is great variation in housing prices, school quality, crime per capita, transportation options, open public spaces, and

\[^5\text{Rapaport (1997) and Ioaniddes and Zabel (2008) employ discrete-continuous choice models to study housing. Rapaport finds that the inclusion of community choice in a housing demand model increases the price elasticity of demand. Ioaniddes and Zabel find that the nonseparability significantly impacts the estimate of the elasticity of housing demand with respect to mean neighbors’ demand.}\]
other features. On average, the voucher households live in more desirable neighborhoods than households in Pittsburgh’s public housing. In addition, voucher households tend to live in neighborhoods with lower crime levels and a higher percentage of home-owners than low-income households receiving no form of housing assistance. However, compared to other low-income households, voucher households tend to live in neighborhoods with lower quality schools and a lower percent of college graduates.

The estimation reveals that crime levels, commute times, public school quality, public open space, racial composition, and street grid density significantly impact households’ preferences for specific neighborhoods. In addition, the availability of public open space serves as a substitute for housing services while public school quality and housing services are complements. The estimation suggests that the price elasticity of housing demand ranges from -.44 to -.83, which is slightly lower than other estimates published in the literature, but reasonable given the fact that we focus only on poor households eligible for housing subsidies.6

With the estimated choice model, I conduct nominal policy analysis to examine how voucher recipients’ choices might change as a result of changes to the voucher program. Parameter estimates suggest that enjoyment of neighborhood amenities accounts for 25 percent of overall utility; however, the types of neighborhoods chosen by voucher participants is not greatly affected by changes to the program-induced budget constraint alone. In analyzing the budget constraint, my analysis suggests that changing the structure of the program to be a rebate instead of a voucher would improve participants’ utility, achieve neighborhood selection similar to a program with an unrestricted voucher amount, and would significantly lower costs. The most effective policy change in achieving improved neighborhood selection would be to impose a requirement that households live in neighborhoods with poverty rates below some acceptable maximum, such as 30 percent.

In light of the welfare consequences of neighborhood quality and the need to assess how vouchers affect access to decent neighborhoods, the rest of the paper is organized as follows. The next

6See, for example, Friedman & Weinberg (1982) who offer a detailed overview, finding that estimates of price elasticity generally range from -0.6 to -1.7
section describes the data sources and offers descriptive information on voucher households. Section 3 describes the residential choice model and Section 4 explains the estimation procedure and addresses identification. Section 5 reviews the estimation results and Section 6 summarizes the findings of three policy evaluations. Conclusions are drawn in Section 7.

3.2 Data

3.2.1 Voucher Households

In the United States low-income households may apply to their jurisdiction’s housing authority for a housing subsidy. Most housing authorities offer a housing voucher program, which is funded by the United States Department of Housing and Urban Development (HUD). HUD stipulates income eligibility limits, maximum levels of subsidy amounts, and a minimum acceptable level of housing services \( h_{\text{min}} \). Income eligibility levels and maximum subsidy amounts vary across metropolitan areas, taking local housing markets and income levels into account. Maximum subsidy amounts are often expressed in terms of Fair Market Rent (FMR) and are adjusted based on the number of bedrooms required by each household.\(^7\) Generally, households participating in the housing voucher program must contribute 30% of their income towards housing expenses;\(^8\) the difference between actual housing expenses and a household’s contribution is subsidized by the housing authority in the form of a voucher, with a maximum subsidy of FMR.

In Pittsburgh, housing subsidies are managed through the HACP. For this research the HACP provided data on the residential location of households with vouchers in 2006. The data set contains the households’ census tract, number of bedrooms, household income, and total rent due to the landlord (the sum of the household’s contribution and the authority’s contribution). Aggregate data on

\(^7\)For example, a parent with two daughters would qualify for a two bedroom while a parent with one daughter and one son would qualified for a three bedroom. Several households enjoy more bedrooms than they qualify for, perhaps because of lagged variables- for example, “empty-nesters”.

\(^8\)If households wish, they may choose to spend an additional 10% of their income towards rent to cover the difference between FMR and a rent that exceeds FMR. However, total household housing expenses may not exceed 40%.
the voucher households’ joint distribution of race and presence of children by census tract were provided by the Allegheny Department of Human Services. At the city level, aggregate data on tenure, income sources and age were obtained from the 2004-2007 Picture of Subsidized Households, a dataset published by HUD. Eliminating the observations with missing values, the HACP data includes 4341 households. The majority (79%) of voucher households in the HACP program are black, 2% are minority but not black, and only 1% of all households are Hispanic. 86% of voucher households with children are non-white, while 71% of voucher households without children are non-white. The majority of households are headed by females (85%) and half of the households include only one adult. Wages or net self-employment revenue is the main income source for only 30% of households; 17% of households obtain the majority of their income from welfare (TANF, Government Assistance, or Public Assistance); the remaining 53% of households receive income from other sources, for example Disability, Social Security, charitable handouts, or no source at all. On average, households have been in the program for 49 months. Most heads of household (62%) are between the ages of 25 and 50; 14% of heads of household are under age 25, 24% are over age 50 (and only half that 12%, are over age 61).

Table 3.1 shows the HACP’s voucher program income limits for 2006, the income levels for which the program prioritized entry into the voucher program, and the mean income of the households in the study. Table 3.1 also shows the FMR, by number of bedrooms, for 2006. The voucher amount is directed towards both rent and utilities. For the housing authority in this study (the HACP), the voucher was directly paid to the landlord and thus could not be exercised as a general income subsidy. Also, not every household received the full voucher amount; rather, the voucher amount depended on the rent level chosen by the participating landlord.

After gaining entry to the HACP program, a prospective tenant initiates a dialogue with a potential landlord. If the landlord agrees to follow-through, the tenant approaches the housing authority with a description of the property and the rental contract specifics. The housing authority then meets with the landlord to inspect the property and review the rent. The HACP considers the FMR as a sum of a maximum allowable rent and the expected utility cost for the unit, where HACP holds
Table 3.1: Income Limits, Median Income, and Housing Subsidies in the HACP Voucher Program, 2006

<table>
<thead>
<tr>
<th></th>
<th>Number of Bedrooms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Income Limit</td>
<td>$20,100</td>
</tr>
<tr>
<td>Priority Limit</td>
<td>$12,050</td>
</tr>
<tr>
<td>Observed Mean Income</td>
<td>$12,043</td>
</tr>
<tr>
<td>Fair Market Rent</td>
<td>$625</td>
</tr>
</tbody>
</table>

a schedule of expected utility payments $m_{ph}$ for each number of bedrooms and apartment types (apartment, town house, or single family home). A household is expected to pay 30% of its income towards housing; the first portion will go towards utility bills and any left over will go towards the contract rent.\(^9\) If a household desires an apartment that exceeds $FMR - m_{ph}$, the housing authority will allow it to spend an additional 10 percent of its income to pay for the higher rent. However, the household’s resulting cost burden must not exceed 40% of its income. Not exceeding FMR, the housing authority computes the voucher amount by subtracting the difference between 30% of the household income minus expected utilities from the contract rent, as in equation (3.1).

\[
\text{Voucher} = \text{Contract Rent} - (0.3 \times \text{Income} - \text{Expected Utilities}) \tag{3.1}
\]

Figure 1 illustrates this budget constraint for a household with income level $y$ and a local housing services price of $p_j$. If a household desires housing services lower than $h_{\text{min}}$, or desires to spend more than 40% of its income towards housing, it cannot participate in the voucher program. Between those values, however, the household has a discontinuous budget constraint induced by program participation. In the voucher program, the vast majority of households choose $h^*_j = (FMR - m_{ph})/p_j$ in housing services and contribute 30% of their income. Taking the expected

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\(^9\) There are several interesting economic implications. First, in the Pittsburgh Metropolitan Area the HUD-determined FMR results in lower voucher benefits the larger the family size. This inequality results because utility costs differences between an efficiency unit and a four bedroom unit (for example) exceed the differences in the FMR for these units. Second, households might be directed towards certain types of housing units and leases in order to exchange more square footage for lower utility costs. For example, a household in this study might choose a 3 bedroom apartment with water and garbage included over a 3 bedroom townhouse where the tenant pays for water and garbage removal in order to purchase $144 in housing services rather than spend that amount in utilities.
utilities expense schedule into account, 50% of households included in this paper’s data set secure an apartment for a rental amount within $15 of the maximum allowable rent; 75% secure an apartment within $90 of the maximum allowable rent. A reasonable conclusion is that the vast majority of the program participants are able to maximize their program benefit. In further analyses, I simplify the voucher household’s general utility optimization problem to solely the discrete choice of a neighborhood $j$ in which to enjoy $h^*$.10

Figure 3.1: Voucher Households’ Budget Constraint

The voucher households’ kinked budget constraint differs from the continuous housing budget constraint of low-income households who do not benefit from a housing subsidy. It is simply $y = ph + b$. The next section describes the data on these unsubsidized households.

3.2.2 Households Without Vouchers

Voucher households are compared to low-income households in Pittsburgh, PA, using the 2000 Census Summary Level 3 files which provide a coarse joint income and rent distribution at the

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10This simplification is not required to estimate the model. The model can easily be extended to the more complex case.
census tract level.\textsuperscript{11} The census data is adjusted by subtracting the number of voucher households in each income category, by census tract.\textsuperscript{12} The housing voucher’s income limit for a family of four was $28,700; due to the coarse income distribution, I consider all households described in the 2000 Census as having an income of less than $35,000. 52\% (about 74,000) of all households in Pittsburgh have a household income level of less than $35,000. Of the households, 7.5\% received Public Assistance payments, 53\% are black (79\% of voucher recipients are black), less than 0.8\% are Hispanic, 53\% are female-headed households with no husband present (84\% of voucher households are female-headed), 34\% are age 65 and over, 14\% are households headed by persons under the age of 25.

Table 3.2 compares demographic characteristics of the HACP voucher households to the population without vouchers. Blacks, adults between the age of 25 and 61, and households with children are overrepresented in the voucher program compared to the general low-income population. Seniors are underrepresented in the voucher program.\textsuperscript{13} The 75th percentile of the income distribution is higher in the general low-income population than in the voucher program population. While the 25th percentile of the income distribution appears lower in the general low-income population than in the voucher program population, this estimate (of $0 annual income) is likely underestimated due to reporting/measurement error. The percentile of the income distribution for low-income households without a voucher was calculated using the publicly available Census Micro-Level data.\textsuperscript{14}

Table 3.3 shows the inflation-adjusted mean gross rent of rental households by income. Table 3.3 also shows the mean budget share of housing for each income group, which is computed two ways. First, Table 3.3 reports the budget share of housing as reported by the Census Bureau. In

\textsuperscript{11}Income levels and housing prices from the 2000 Census are adjusted for inflation based on the housing CPI statistics from the Bureau of Labor Statistics.

\textsuperscript{12}No adjustment in the demographic distribution is considered for the fact that the voucher data are from 2006 instead of 2000. Overall, the population of Pittsburgh was slightly declining over that time period, mostly due to population aging.

\textsuperscript{13}In the HACP data, seniors are identified as those over 61 years of age. In the coarse income and age distribution in the Census Summary 3 Files, I identify seniors as those 65 and over.

\textsuperscript{14}In the model estimation, aggregate data from the joint income and rent distribution are used, as the Census Summary Level 3 data provides finer geographic detail. To be best representative and to avoid an income of zero, the income level used to calculate estimates is the average of upper and lower bounds of the income category.
Table 3.2: Comparison of Demographic Characteristics

<table>
<thead>
<tr>
<th></th>
<th>HACP</th>
<th>Census, Un-Subsidized, less than $35K</th>
</tr>
</thead>
<tbody>
<tr>
<td>% black</td>
<td>79%</td>
<td>53%</td>
</tr>
<tr>
<td>% of white with kids</td>
<td>38%</td>
<td>18%</td>
</tr>
<tr>
<td>% of black with kids</td>
<td>60%</td>
<td>34%</td>
</tr>
<tr>
<td>% under age 25</td>
<td>14%</td>
<td>14%</td>
</tr>
<tr>
<td>% over age 61, 65</td>
<td>12%</td>
<td>34%</td>
</tr>
<tr>
<td>25th %-tile inc</td>
<td>$10,400</td>
<td>$0</td>
</tr>
<tr>
<td>50th %-tile inc</td>
<td>$13,120</td>
<td>$12,115</td>
</tr>
<tr>
<td>75th %-tile inc</td>
<td>$17,040</td>
<td>$22,380</td>
</tr>
</tbody>
</table>

addition, Table 3.3 reports the ratio of the mean rental value to the median income level for each income group; these “suggested” budget ratios are larger than the actual budget shares reported by the Census Bureau. The housing expenses referred to in Table 3.3 do not include heating, electricity, and other utility expenses. Table 3.3 suggests that the voucher household, with voucher amounts for 2006 a maximum of $625 for a one bedroom apartment, is not spending much more on housing than the mean housing expense of low-income households.

Table 3.3: Mean Rent and Rent-Equivalent of All Households in the City of Pittsburgh, by Income

<table>
<thead>
<tr>
<th></th>
<th>&lt;$10K</th>
<th>$10-20K</th>
<th>$20-35K</th>
<th>$35-50K</th>
<th>$50-75K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent of Rental Households</td>
<td>490</td>
<td>590</td>
<td>686</td>
<td>756</td>
<td>846</td>
</tr>
<tr>
<td>Budget Share of Housing, Census</td>
<td>0.349</td>
<td>0.324</td>
<td>0.258</td>
<td>0.197</td>
<td>0.178</td>
</tr>
<tr>
<td>Budget Share of Housing, Suggested</td>
<td>(0.735)</td>
<td>(0.472)</td>
<td>(0.299)</td>
<td>(0.213)</td>
<td>(0.162)</td>
</tr>
</tbody>
</table>

3.2.3 Neighborhood Characteristics

In addition to housing consumption, this research also focuses on neighborhood choice. The inquiry into voucher participants’ neighborhood quality requires observed variation in neighborhood quality across the metropolitan area. For the purposes of this study, the Pittsburgh neighborhoods are defined as census tracts. The density of Pittsburgh, its very small neighborhoods resulting from hilly topology, and the wide variety in neighborhood quality across small geographic areas make census tracts a reasonable boundary. Census tracts where less than 15% of households rent their home are
excluded. After excluding additional census tracts with insufficient data, I estimate the model on 114 census tracts, or neighborhoods, within the city. For the most part, these census tract boundaries are the same as the boundaries describing the Bureau of Police’s 90 distinct neighborhoods. The lowest number of voucher households in an included neighborhood is 0 (3 neighborhoods), the highest number of voucher households is 212, the mean number of voucher households in each neighborhood is 33.5. Similarly, the minimum, maximum, and mean number of low-income households in each neighborhood are 8, 517, and 135.7 respectfully.

This study employs several data sources to describe each neighborhood. The most encompassing source is the 2000 U.S. Census, from which I use information on neighborhood demographics, the percent of commuters who use public transit, the average public transit commute time, and measures of human capital, for example, the percent with a college degree and the percent employed. For a more detailed picture, I turn to local data sources described in the following paragraphs.

As a proxy for the availability of voucher-friendly apartments, I collected six months of apartment listings from the Allegheny County Housing Authority (ACHA) and HACP websites from January 2010 to July 2010. Apartments were listed on these websites if landlords were amenable to accepting vouchers as partial payments and if the landlord contacted the ACHA or the HACP to list the apartment. In Pittsburgh, there were 409 apartments across 121 census tracts (about 50 apartments were excluded because of insufficient information to geocode their addresses). In the estimation, I exclude neighborhoods that do not host a voucher household and do not contain a listing of an apartment available to voucher recipients and do not border a census tract that contains a voucher listing (only 2 census tracts were excluded for those reasons).

The dataset includes geographic data, in particular the percent of land that is dedicated as a park or recreation area for public use, and the number of street intersections per square mile. These data were obtained from the U.S. Geological Survey. Park land might substitute for housing services

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15 Ideally I would have listings from 2006, but those were not available
16 A proxy for low-income housing availability could be the a priori measure of low-income households in a neighborhood. For a discussion on income heterogeneity in U.S. urban places, see Hardman & Ioannides (2004).
as people may substitute private lawns and gardens for public ones. Street intersection density is one measure of street-connectedness; some studies find that street connectedness is correlated with substitution away from motor vehicles (Frank, Sallis, Conway, Chapman, Saelens, & Bachman, 2006).

Current and detailed information on school quality and crime statistics were obtained for neighborhoods in Pittsburgh. School quality is measured by 2006 state-standardized test scores, measured at the individual level but aggregated into census tract means.\(^{17}\) The Pennsylvania System of School Assessment (PSSA) exams are a series of tests administered at all Pennsylvanian schools at the third through eighth grade, as well as eleventh grade. I use the sum of the mathematics and reading scores for eighth graders.

The number of violent and property crimes in each census tract were obtained from the 2007 Annual Report of the City of Pittsburgh Bureau of Police. Violent crime includes homicide, rape, robbery, and aggravated assault; property crime includes burglary, theft, motor vehicle theft, and arson. To normalize, I use violent crime rates per capita.

Finally, I employ data on the availability of public housing in each neighborhood (and its immediate neighbors, as a proximity measure) to allow for a comparison of vouchers to publicly managed housing. Data on public housing, occupancy rates, and whether it is publicly or privately managed is obtained from the ACHA and the HACP.

Housing vouchers may be desirable since they provide recipients with greater residential and neighborhood choice than supply-side subsidies such as public housing. Since the 1990s, HACP has been increasing the number of households in its housing voucher program while reducing its stock of public housing properties. The map in Figure 3.2 compares the density of housing vouchers in each census tract to the locations of HACP’s public housing structures. The map illustrates that vouchers clearly obtain a different residential sorting outcome than public housing. However, voucher use is highest in areas neighboring public housing structures.

\(^{17}\)These data stem from a CMU-RAND study, funded by the Institute of Education Sciences (Davis, Engberg, Epple, Sieg, & Zimmer, 2010)
Figure 3.2: Housing Voucher Density and Public Housing Locations in Pittsburgh, 2006
Table 3.4 compares the mean census tract attributes weighted by HACP public housing households, HACP households with vouchers, and all Pittsburgh households eligible for vouchers, and the full Pittsburgh population. On average, the voucher households live in more desirable neighborhoods than households in Pittsburgh’s public housing. For example, voucher households tend to live in neighborhoods with lower crime, better schools, and less poverty than households in public housing.\(^\text{18}\) However, by most measures voucher households do not live in neighborhoods that are better than the typical neighborhood of a household eligible for a housing voucher. Compared to the eligible population, households with vouchers live in neighborhoods with higher crime, lower test scores, and more poverty.

Table 3.4: Mean Census Tract Statistics, Weighted by Number of Households (Pittsburgh, PA), * Indicates Significantly Different from Vouchers \((p < .01)\)

<table>
<thead>
<tr>
<th></th>
<th>Public Housing</th>
<th>Voucher</th>
<th>Eligible</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Violent Crimes per 1000</td>
<td>30.35*</td>
<td>25.94</td>
<td>21.02*</td>
<td>19.91*</td>
</tr>
<tr>
<td>Mean Test Score (in 100s)</td>
<td>24.10*</td>
<td>25.18</td>
<td>26.14*</td>
<td>26.23*</td>
</tr>
<tr>
<td>% Black</td>
<td>0.73*</td>
<td>0.45</td>
<td>0.32*</td>
<td>0.28*</td>
</tr>
<tr>
<td>% Single Mother Households</td>
<td>0.32*</td>
<td>0.13</td>
<td>0.12</td>
<td>0.10*</td>
</tr>
<tr>
<td>% Living in Poverty</td>
<td>0.49*</td>
<td>0.24</td>
<td>0.22</td>
<td>0.21*</td>
</tr>
<tr>
<td>Mean Pub Trans Time in Minutes</td>
<td>61*</td>
<td>86</td>
<td>100*</td>
<td>103*</td>
</tr>
<tr>
<td>% Commute by Pub Trans</td>
<td>0.43*</td>
<td>0.28</td>
<td>0.24</td>
<td>0.22*</td>
</tr>
<tr>
<td>Mean Str Intersections per Acre</td>
<td>0.42*</td>
<td>0.54</td>
<td>0.48*</td>
<td>0.47*</td>
</tr>
<tr>
<td>Park Land per Acre</td>
<td>0.03*</td>
<td>0.06</td>
<td>0.07*</td>
<td>0.07*</td>
</tr>
<tr>
<td>% Female, Completed College</td>
<td>0.09*</td>
<td>0.17</td>
<td>0.22*</td>
<td>.24*</td>
</tr>
<tr>
<td>% Male, Completed College</td>
<td>0.11*</td>
<td>0.20</td>
<td>0.26*</td>
<td>.28*</td>
</tr>
<tr>
<td>% Male Employed</td>
<td>0.40*</td>
<td>0.54</td>
<td>0.57</td>
<td>0.56</td>
</tr>
<tr>
<td>% Households That Rent</td>
<td>0.78*</td>
<td>0.52</td>
<td>0.49</td>
<td>.47*</td>
</tr>
<tr>
<td>Median Rent</td>
<td>225*</td>
<td>364</td>
<td>393*</td>
<td>407*</td>
</tr>
<tr>
<td>Median Income</td>
<td>31,471*</td>
<td>33,255</td>
<td>32,826</td>
<td>33,085</td>
</tr>
</tbody>
</table>

\(^{18}\)The public housing data is taken from ?) and also ?), who employ panel data on public housing households in their study of mobility of low-income households.
3.2.4 Housing Prices

Variation in prices for housing services across neighborhoods is important in explaining sorting patterns. Property values from 2004 are obtained from the Allegheny County Office of Property Assessments. Both the most recent sale values and the assessed values are available, I use the assessed property values. Of all properties, I estimate housing prices from residential one-, two-, three-, and four-family homes over 100 square feet; this leaves 93,415 residences.

Housing prices are obtained by estimating a hedonic regression model, equation (3.2).

\[ \ln h_{jn} = \ln p_j + \kappa \ln f_n + \nu_{jn} \]  (3.2)

Controlling for production costs, \( p_j \) is the price of a unit of housing services in neighborhood \( j \). The regression model is estimated using micro-level data on 93,415 residences with attributes \( f_n \), including the number of bedrooms, number of bathrooms, air conditioning, heating type, presence of central air, architectural style, furnished living area, lot square footage, overall condition, etc. I assume production costs are constant across all of the neighborhoods in the metropolitan area.

There is significant variation in housing prices across all neighborhoods. Normalizing so that the price in the least expensive community is equal to one, neighborhood price indices ranging from 1.03 (\( p=0.01 \)) to 3.78 (\( p<0.001 \)). I assume that the ratios of the price indices of assessed properties are equivalent to the ratios of price indices in unobserved lease contracts.

3.3 The Model

A priori, households have decided to live within a specific metropolitan area. Within the metropolitan area there is a finite number of neighborhoods \( J \). I model households’ simultaneously choice of

---

19 The data were made available to me by Michael Peress and Brett Gordon and were also used in Epple, Peress, Sieg (2010)
20 Sieg, Smith, Banzhaf, and Walsh (2002) discuss and evaluate this approach, finding it an appropriate method for constructing price indices for a model with separable preferences for housing and community services.
one of these neighborhoods \( j \in J \) to live in, a level of housing services \( h \), and a private composite good \( b \) representing non-housing expenses. The levels of housing services and the composite good are continuous, homogeneous variables. Choices are constrained by budgets governed by household income, housing prices, minimum acceptable housing standards, and - if applicable - terms of a housing subsidy. Each neighborhood offers a bundle of predetermined public amenities and households can move freely between the neighborhoods. A neighborhood \( j \in J \) is characterized by public amenities observed by the econometrician \( (Z_j) \), unobserved public amenity \( \xi_j \), and the price of housing \( p_j \).

I begin with a direct utility function to obtain indirect utility functions that respect various continuous and discontinuous budget constraints. Household \( i \) has utility \( u_{ij} \) for neighborhood \( j \) given by:

\[
u_{ij} = U(h, b, Z_j, \xi_j, p_j, X_i, \gamma_i, \nu_i, \epsilon_i; \theta)
\]

where \( \theta \) represents a set of common utility parameters.

The utility function allows for heterogeneity so that households with the same income may make different choices. The model permits households \( i \) to make different choices due to random characteristics \( (\gamma_i, \nu_i, \epsilon_i) \). Variations in household characteristic \( \gamma_i \) describe tastes for public services, \( \nu_i \) describes variation in private consumption patterns, and \( \epsilon_i \) a vector of household-neighborhood specific preferences. In addition to allowing variation, the \( \gamma_i \) is interacted with household type \( X_i \) so that different households may have different mean utility for certain neighborhood characteristics, for example households with children might have a strong preference for neighborhoods with good schools.

As some previous work in residential choice has found significant nonseparable preferences for neighborhood and housing choice,\(^{21}\) the model allows for non-separable preferences for housing

\(^{21}\)Rapaport (1997) finds that allowing for nonseparability increases the price elasticity of demand and reduces the differential between price elasticities of white and comparable nonwhite households. Ioannides & Zabel (2008) find that nonseparable preferences are statistically significant and increase the estimate of the elasticity of housing demand with
services and neighborhood attributes. In particular, the model allows variation in tastes for public amenities \((\gamma_i)\) and private consumption \((\nu_i)\) to be correlated, but both are independent of \(\epsilon_i\). For example, families with children might require more housing services and have a stronger preference for public school quality; however these correlated preferences are constrained to be independent of household-neighborhood specific preferences, for example, a preference for a specific nearby childcare center or grocery store. Dubin and McFadden (1984) presented the first work on a model with non-separable preferences of a simultaneous discrete-continuous choice. Their model allows the unobserved component of the continuous good’s demand elasticity to covary with the \(\epsilon_i\). For the present topic, the housing demand’s random component and the neighborhood unobservable \(\epsilon_i\) are independent but the housing demand’s random component co-varies with preferences for the neighborhoods’ observable attributes. The reason for this choice is that the interpretation will be generalizable beyond the specific neighborhoods in the estimation data set, despite a lack of detailed demographic data the model can obtain a reasonable degree of heterogeneity, and there are fewer parameters to estimate than there would be if \(\nu_i\) were correlated to the vector \(\epsilon_i\). Moreover, with many similar neighborhoods in a metropolitan area, this specification should yield more informed substitution patterns without, for example, the need for the econometrician to specify a nested logit.

Households have a Stone-Geary-like utility function for composite good \(b\) and housing services \(h\). This specification allows the possibility for minimum consumption requirements, which may best describe the choices of low-income households. Specifically, there is a minimum housing consumption level \(H\) that is constant across all households. \(H\) can be interpreted as a minimum level of shelter required for survival, or minimum standard of residential zoning compliance; in this instance, the former interpretation is more applicable because \(H\) is constant across all neighborhood choices.\(^{22}\) There is also a minimum non-housing consumption which is allowed to vary by neighborhood characteristics \(Z_j\). Transportation costs and school costs are two examples that might vary respect to mean neighbors’ demand by 17 percent.

\(^{22}\)In the model and its estimation, it is feasible for \(H\) to be interacted with household composition or other observables. However, random or unobserved components of \(H\) would create residuals in the housing demand that are correlated to price.
by location. The minimum non-housing consumption also has a household-specific component \( \nu_i \) that is constant across all neighborhoods but is unobservable by the econometrician. If the estimates suggest a negative minimum non-housing consumption level, the interpretation would be that it acts a simple demand shifter. The total utility from choices \((j, h, b)\) is then given by:

\[
\begin{align*}
    u_{i,j} &= \gamma Z_j + \gamma_i X_i Z_j + \xi_j + \alpha \log(h - H) + (1 - \alpha) \log(b - B_{ij}) + \epsilon_{ij} \\
    B_{ij} &= \beta X_i Z_j + \nu_i
\end{align*}
\] (3.4) (3.5)

The \( \alpha \) parameter is constrained to be between (0, 1). The estimate of \( H \) is constrained to be less than the least value of an observed housing consumption \( h \). The sum of the \( B_{ij} \) components are restricted to be less than \( b \), which imposes a truncation of the random components in the term \( B_{ij} \). The truncation point is endogenous to the other utility parameters and is also a function of the minimum income level.\(^{23}\)

For the estimation, there are two types of households: households without a voucher and households with a voucher.\(^{24}\) For those without a voucher, households’ decision problem is to simultaneously choose a neighborhood \( j \) and a level of housing services \( h \). Households optimize their decision constrained by their income \( y \) and face the typical linear budget constraint \( y = p_j h + b \). One advantage of the Stone-Geary function is that it results in a housing demand equation that is linear in the unobserved parameter. The housing expenditure function resulting from utility optimization is given by equation (3.7).\(^{25}\)

\[
h p_j = \alpha y + H(1 - \alpha)p_j - \alpha \beta X_i Z_j - \alpha \nu_i
\] (3.6)

Given this specification, the housing price elasticity in equation (3.8) is a function of price, income,

---

\(^{23}\)I set the truncation point to be equal to \( \min_j (y_{\text{min}} - H p - \beta x) \).

\(^{24}\)The model could easily be extended to include any finite number of household types.

\(^{25}\)For low-income rental households, Friedman and Weinberg found that a log-log housing demand function and a linear housing demand function yielded comparable price and income elasticities. See Sieg, Smith, Banzhaf, & Walsh (2002) for a general discussion on the estimation of housing prices.
and neighborhood attributes.

\[ E_{p,j} = \frac{\alpha(\beta X_i Z_j + v_i - y)}{\alpha(y - \beta X_i Z_j - v_i)} + H p_j (\alpha - 1) \] (3.7)

Now suppose a household receives a housing voucher. The voucher program participation is not modeled here; rather, the neighborhood and housing choice conditional on voucher receipt.\(^{26}\) As discussed in the Data Section, voucher households optimize their housing consumption in any neighborhood by consuming the maximum allowable voucher amount, \(h^* = \frac{FMR - m_{ph}}{p_j}\). Their decision problem is therefore simplified to a discrete choice over neighborhoods \(J\).\(^{27}\) The indirect utility function for a household with a housing voucher is equation (3.9).

\[ v_{i,j} = \gamma Z_j + \gamma_i X_i Z_j + \xi_j + \alpha \log(\frac{FMR - m_{ph}}{p_j}) + (1 - \alpha) \log(\gamma y - B_{ij}) + \epsilon_{ij} \] (3.8)

This model assumes that the voucher program participation decision is exogenous, for lack of better data. Specifically, I assume that the distribution of the random preferences \(f(\gamma_i, v_i, \epsilon_{ij})\) is identical for both the unsubsidized households and the subsidized households. In reality, it could be the case that the population of voucher recipients has a different distribution of preferences than the general, eligible population. If this is the case, the identification of parameters governing the distribution \(f(\gamma_i, v_i, \epsilon_{ij})\) will be compromised. To address this type of endogeneity problem, I appeal to the fact that there are such severe supply restrictions that access to vouchers is somewhat random. The HACP offers its limited number of vouchers in a first-in-first-out queue with some discretion, for example higher priorities for homeless families and veterans. The waitlist for HACP housing

\(^{26}\) In practice, households apply to the local housing authority, are assessed for eligibility, and put on a first-in-first-out waitlist for an available voucher. In some periods, the demand for vouchers may be so high that housing authorities close the waitlist. New vouchers become available when a participating household leaves the program or the housing authority receives additional funding.

\(^{27}\) The literature on piecewise budget constraints proposed several issues in modeling demand, for example Hausman & Wise (1980) Moffitt (1986) Hausman (1985). In the current application, the data suggest that almost all voucher participants are exercising the full amount of the voucher but not choosing to pay any additional, allowable, rent above FMR; for this reason, there is no demand equation to estimate for voucher participants. There is also no measurement error in assigning which budget constraint they are facing. The possible endogeneity of the program participation decision is discussed in the next paragraph.
vouchers is often long enough that it is closed to new applicants for months, up to two years, at a time. The wait time itself is often observed to be more than two years. Also, to address some concern about the possible endogeneity of the program participation decision, the model controls for observed heterogeneity of household composition and ethnicity by interacting these characteristics with some neighborhood characteristics.

Another type of endogeneity problem arises from household-specific preferences for housing consumption. For example, perhaps a household has a particularly strong draw for $\epsilon_{ij}$ that spending less than 30 percent of its income on [substandard] housing to live in a neighborhood $j$ is its preferred choice. Similarly, households with a particularly extreme draw of $\nu_i$ might prefer to spend more than the 40 percent of their income towards housing that the voucher program allows. Owing to the large dimensionality of the vector $\epsilon$, correcting the expectation of $(\nu_i, \gamma_i, \nu_i, \epsilon_{ij})$ for the program participation decision would be computationally burdensome.

Also, I do not consider the possible endogeneity of income, for example by including employment decisions or any income generating process. In the estimation, unsubsidized low income and very-low income households are included along with the voucher households. There are several studies suggesting that housing program participation slightly reduces participants’ employment and earnings. In terms of policy analysis, this paper focuses on interesting nominal analyses of policies involving incremental change in the voucher program that would likely not significantly change voucher participants’ employment incentives.

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28 Studying food stamps, Daponte Sanders and Taylor (1999) found that eligible households elected not to enroll in the subsidy program for reasons consistent both with choice theory and lack of information. In almost half of the cases, eligible households did not enroll in the program because the benefits of being in the program did not outweigh the administrative or time costs of program participation (for example, they were eligible for less than $10 a month in benefits). In the remaining cases, households enrolled in the program after they received detailed information about the program.

29 For example, see Jacob & Ludwig (2008) Susin (2005) and Olsen et al. (2005b)
3.4 Estimation

The paper develops a new simulated method of moments estimator for the parameters of the model that accounts for the endogeneity of housing prices and selection of households into communities.\(^\text{30}\)

There are three types of moment conditions that are used in the estimation. The first moment condition is the unobserved neighborhood amenity \(\xi_j\), which is simply the difference between the mean neighborhood quality \(d_j\) and its observable components \(\gamma Z_j\). The second set of moments is the difference between the observed and expected percent of the population in each neighborhood; this calculation is done for population sub-groups, separated by subsidy type (voucher or non-voucher), race (white and nonwhite) and presence of children, for a total of eight moments. The third set of moments stems from estimation of the housing demand of the non-voucher households and corrects for the endogeneity of neighborhood choice while instrumenting for the endogeneity of price, which enters linearly into the demand equation.\(^\text{31}\) Estimation relies on micro-level data of households with vouchers and aggregate data on households without vouchers. The household-specific preference in housing elasticity is not identified from the voucher households alone, as they do not optimize over a continuous budget constraint. Identification requires the inclusion of households without vouchers.

For the housing demand equations, the endogeneity of neighborhood choice is accounted for by using the conditional expectation of individual-specific housing budget share variation, as in equation (3.10).\(^\text{32}\) The conditional expectation correction for \(\nu_i\) is obtained using a Monte Carlo

\(^{30}\)See, for example, Pakes & Pollard (1989) for a discussion of the asymptotics of optimization estimators with simulation.

\(^{31}\)An estimation procedure similar to the one used here is also described in Fullerton, Gan, Hattori (2005) who use aggregate data to estimate a Dubin McFadden choice model for vehicle usage and emissions, with the addition of random coefficients. Imbens & Lancaster (1994) review the issues of accuracy, efficiency, and compatibility in estimating economic models with a combination of micro and macro data. Also, see 2) for estimation matching different moment conditions of a sorting equilibrium.

\(^{32}\)Dubin and McFadden (1984) compared the conditional expectation correction usage to other approaches, and found that the conditional expectation correction led to the least biased estimates in the presence of nonzero covariance of non-separable preferences.
simulation and Bayes’ rule.\textsuperscript{33}

\[
E[hp_j|d_j = 1] = \alpha y + H(1 - \alpha)p_j - \alpha\beta X_i Z_j - \alpha E[\nu_i|I(d_j = 1)]
\] (3.9)

The 2000 Census provides the joint distribution of income and rent for each census tract as well as the budget share of housing for each income group.\textsuperscript{34} For each census tract, the estimation employs three housing demand observations: specifically, the estimation utilizes the housing budget share of the first three income categories (less than $10,000, $10,000 - $19,999, $20,000 - $34,999). This exclusion means that only households with annual incomes of less than $35,000 are represented, providing an adequate comparison population to the low-income households in the voucher program. The simulated method of moments estimation minimizes the distance of the two sides of equation (3.10), with respect to instruments for price.

The housing demand is estimated simultaneously with the remainder of the model parameters, which are identified by matching predicted and estimated neighborhood choice shares ($s_j$ and $\hat{s}_j$, respectfully) using the method proposed by Berry (1994) and Berry, Levinsohn, and Pakes (1995).\textsuperscript{35} Let the indicator variable $d_{i,j}^m$ equal one if a household in housing program $m$ chooses neighborhood $j$, otherwise it is equal to zero. For this exposition, let $m = 0$ if a household is not in the voucher program, and thus faces the indirect utility obtained by substituting the housing demand in equation 3.10 and the continuous budget constraint into the direct demand in equation 3.4. Also, let $m = 1$ if a household participates in the voucher program and faces the indirect utility specified in equation

\textsuperscript{33}I use simulation methods to compute the expectation of $\nu_i$ conditional on the probability of choosing neighborhood $j$.

\textsuperscript{34}Unfortunately, in the public data this joint distribution is not conditional on household characteristics such as race or presence of children. To work around this, I weight the bins of this distribution according to the joint income, race, presence of children, and housing tenure distributions that are publicly available.

\textsuperscript{35}Berry and Haile (2010) discuss the identification criteria for BLP. The model in this paper meets the criteria set by Berry and Haile, namely, that $\nu_{i,j}$ is monotonic in mean neighborhood utility $\delta_j$ (equation (3.13)), the indirect utility is quasi-linear in elements of $Z_j$, there is perfect substitutability of $Z_j$ for neighborhood unobservable $\xi_j$ and the location of $\xi_j$ is normalized. Identification also rests on the quality of the instruments $W$[(Berry & Haile, 2010), (Newey & Powell, 2003)].
3.9. Neighborhood choices are mutually exclusive and hence:

\[
\sum_{j \in J} d_{i,j}^m(y, X_i, \gamma_i, \nu_i, \epsilon_i; \theta) = 1 \tag{3.10}
\]

The share of households in neighborhood \( j \) is the integral of the decision variable \( d_{i,j} \) over the distribution of the random preference parameters, equation (3.12).

\[
\hat{s}_j(X|\theta) = \frac{\sum_{m \in \{0, 1\}} N_m}{N_m + N_{m=1}} \int d_{i,j}^m(y, X, \gamma, \nu, \epsilon_i|\theta) P(\gamma_i, \nu_i, \epsilon_i) f(y|X) \, \partial_y \gamma_i \partial_y \nu_i \partial_y \epsilon_i \tag{3.11}
\]

Berry, Levinsohn, and Pakes (1995) ("BLP") show there is a contraction mapping that computes the best estimates of neighborhood unobservables \( \xi_j \) given the remaining parameters in the model.

Let:

\[
\delta_j = \gamma Z_j + \xi_j \tag{3.12}
\]

The BLP contraction mapping \( T(\delta_j) \) defined by

\[
T(\delta_j^{n+1}) = \delta_j^n + \ln s_j - \ln \hat{s}_j \tag{3.13}
\]

is applied until convergence. To normalize, the lowest-priced neighborhood quality is fixed at \( \delta_0 = 0 \) and the utility function of the choice \( j = 0 \) is simplified to equation 3.15.

\[
u_0 = \alpha \log(y) \tag{3.14} \]

As in typical discrete choice models, the household-neighborhood specific random preferences \( \epsilon_{ij} \) are assumed independent and identically distributed according to the Extreme Value Type 1 distribution (?). The random coefficients \( (\nu_i, \gamma_i) \) are assumed to vary according to a truncated multivariate normal distribution with covariance \( \Sigma \). I assume the \( \nu_i \) are mean zero, but the means of \( \gamma_i \) are...
 nonzero and depend on household demographics, for example blacks may have a preference for living in a neighborhood with a large minority presence. I integrate over this multivariate distribution using a simulated integral to obtain the estimated share of households in each neighborhood.\footnote{For the pure frequency simulation, I use the Marsaglia method to generate standard normal variables and I multiply them by the estimates the lower diagonal of the Cholesky decomposition matrix for $\Sigma$.}

To summarize, the moments are listed in equation (3.16):

\[
\begin{align*}
    g_j(\Theta) &= \\
    &\sum_{j=1}^{J} \left[ \tilde{s}_j^{m=0}(Z_j, \hat{\xi}_j, p_j|\theta, \text{white, kids}) - \tilde{s}_j^{m=0}(\cdot, \text{white, kids}) \right] \\
    &\sum_{j=1}^{J} \left[ \tilde{s}_j^{m=0}(Z_j, \hat{\xi}_j, p_j|\theta, \text{white, no kids}) - \tilde{s}_j^{m=0}(\cdot, \text{white, no kids}) \right] \\
    &\sum_{j=1}^{J} \left[ \tilde{s}_j^{m=0}(Z_j, \hat{\xi}_j, p_j|\theta, \text{black, kids}) - \tilde{s}_j^{m=0}(\cdot, \text{black, kids}) \right] \\
    &\sum_{j=1}^{J} \left[ \tilde{s}_j^{m=0}(Z_j, \hat{\xi}_j, p_j|\theta, \text{black, no kids}) - \tilde{s}_j^{m=0}(\cdot, \text{black, no kids}) \right] \\
    &\sum_{j=1}^{J} \left[ \tilde{s}_j^{m=0}(Z_j, \hat{\xi}_j, p_j|\theta, \text{white, kids}) - \tilde{s}_j^{m=0}(\cdot, \text{white, kids}) \right] \\
    &\sum_{j=1}^{J} \left[ \tilde{s}_j^{m=0}(Z_j, \hat{\xi}_j, p_j|\theta, \text{white, no kids}) - \tilde{s}_j^{m=0}(\cdot, \text{white, no kids}) \right] \\
    &\sum_{j=1}^{J} \left[ \tilde{s}_j^{m=0}(Z_j, \hat{\xi}_j, p_j|\theta, \text{black, kids}) - \tilde{s}_j^{m=0}(\cdot, \text{black, kids}) \right] \\
    &\sum_{j=1}^{J} \left[ \tilde{s}_j^{m=0}(Z_j, \hat{\xi}_j, p_j|\theta, \text{black, no kids}) - \tilde{s}_j^{m=0}(\cdot, \text{black, no kids}) \right]
\end{align*}
\]

\[
\hat{\Theta} = \arg \min_{\Theta} \left\{ \frac{1}{J} \sum_{j=1}^{J} w_j g_j(\Theta) \right\}' \hat{\Lambda}^{-1} \left\{ \frac{1}{J} \sum_{j=1}^{J} w_j g_j(\Theta) \right\} \tag{3.16}
\]
with the weighting matrix $\hat{\Lambda}^{-1}$ as the inverse of the covariance of the moments of the first stage estimator.\textsuperscript{37}

While there are only fifteen moments in equation (3.16), there are at most 18 parameters to estimate. To identify them, I employ 8 instruments. The set of instruments $w_j$ is chosen with the goal of a nonzero correlation with endogenous variables but independence of error in the model as estimated by $g_j(\Theta)$. There are three types of error: the estimates of the joint income and housing expenditure distribution from the cross section reported in the Summary Level 3 Census Data, error from the Monte Carlo integration methods, and unobserved household and neighborhood attributes.

Several observable neighborhood variables in the model specification are assumed to be endogenous: price, percent black, percent of women who are college graduates, and the percent of males who are employed. The set of instruments $w_j$ identifies the mean and variance of preferences for the endogenous neighborhood characteristics, and identifies the parameters embedded in the price coefficient of the housing demand equation ($\alpha$ and $H$). Following Bayer, McMillan and Reuben (2005), the main instrument for price is constructed based on housing prices in similar neighborhoods. This method recognizes that the prices of homes in similar but distant neighborhoods contain information about price variation attributable to exogenous features across similar neighborhoods. I construct the instrument by clustering the neighborhoods and computing the mean housing prices of neighborhoods within each cluster, excluding the neighborhood for which the instrument is being computed.\textsuperscript{38} For additional instruments, I assume the following characteristics are exogenous: the percent of housing units that are owner-occupied in similar neighborhoods, the ratio of the HACP voucher-friendly apartment listings to the number of rental units in similar neighborhoods, the median age of rental properties, the average commute time of commuters using public transit, the number of violent crimes per capita, the percent of acres that are designated park land, and mean test scores.\textsuperscript{39}

\textsuperscript{37}The first stage employed the identity matrix (basic model) or the weighting matrix from the estimates of the corresponding baseline model.

\textsuperscript{38}To compute the clusters, I use the k-means algorithm on the observed neighborhood attributes, with 25 clusters across the 114 neighborhoods.

\textsuperscript{39}Test scores are also used as exogenous variables in other residential choice models, for example Bayer et al. (2004).
I test several specifications of the model, even the largest (with eighteen parameters) has several exclusion restrictions. For the $Z_j$ neighborhood characteristics directly affecting housing demand, I include the percent of land area that is designated as a public park, the average commute time using public transit, and school quality multiplied by a dummy variable indicating whether a household has children. Households enjoy direct utility ($\gamma_i$) from a wider set of neighborhood services $Z_j$, specifically the number of violent crimes per capita, school quality multiplied by the with-children indicator, the percent of land area that is designated park land, intersection density, the average commute time of persons using public transit to get to their job, the percent of females that completed a college degree, and the percent of adult males that are employed. I include these variable choices as they are possible neighborhood attributes that affect outcomes such as physical and mental health, employment, and education measured by other studies such as the Moving To Opportunities study. All other neighborhood characteristics are absorbed in the estimated neighborhood-specific effect $\xi_j$. I also estimate a racial preference parameter for the percent of households that are black, interacted with householders’ own race. Preference for school is interacted with presence of children. In some models I allow variation in the preference for school quality, average length of commute by public transit, and/or race, and I estimate their covariance with variation around mean zero for randomness in housing demand $\nu_i$. Preferences for the remaining observable neighborhood attributes are constant across all households.

The 2-stage optimal GMM optimization routine is based on a simplex algorithm with simulated annealing; starting values were obtained from reduced-form estimates. The three-dimensional integral of $\nu_i$ and the $\gamma_i$ coefficients is estimated using 4000 sets of randomly, independently generated standard uniform variables multiplied by estimates of the covariance matrix $\Sigma$’s Cholesky decomposition, with the $\nu_i$ truncated.\footnote{I re-draw a set of variables if the $\nu_i$ component exceeds the truncation value. The truncation value is endogenously determined by estimates of the minimum survivable consumption levels $H$ and $B_j$.}
3.5 Estimation Results

The model is estimated on 114 census tracts in the City of Pittsburgh. Table 3.5 explains the size of each population sub-group: whites/blacks and households residing/not residing with own children under the age of 18.

<table>
<thead>
<tr>
<th>Source</th>
<th>Demographic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Black, Kid</td>
</tr>
<tr>
<td>2000 Census</td>
<td>9,693</td>
</tr>
<tr>
<td>HACP Voucher 2006</td>
<td>1,694</td>
</tr>
</tbody>
</table>

The parameter estimates of several specifications are displayed in Table 3.6. The Table displays the resulting mean price elasticity and the derivative of the price elasticity with respect to several neighborhood characteristics. Model 1 does not include random coefficients and maintains constant price elasticity across neighborhoods and it is a useful baseline model to compare the remaining models. Models 2 and 4 relax the price elasticity restrictions and attempt different inclusions of neighborhood amenities; model 4 is useful because it adds human capital variables that could influence program participants’ labor market outcomes. Models 3 and 5 offer different specifications of random coefficients and thus allow for nonseparability between the housing and neighborhood choice. The standard errors of parameter estimates were generated by bootstrapping.\(^{41}\)

Most specifications agree that positive neighborhood attributes ($\gamma$) are lower crime rates and lower street grid density, although the parameters for street density were not significant. All models suggest that households with children have a high preference for neighborhoods with high eighth grade test scores, blacks have a high preference for living in neighborhoods with other blacks. Average public transit commute times and acreage of public parks were not found to be significant in neighborhood choice, although each of these attributes were found to significantly affect housing

\(^{41}\)To bootstrap, the model was re-estimated 25 times, each time excluding a different random set of 30% of the neighborhoods.
### Table 3.6: Parameter Estimates (and Standard Errors)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td></td>
<td>.13* (.02)</td>
<td>.13* (.04)</td>
<td>.10* (.01)</td>
<td>.17* (.02)</td>
<td>.12* (.02)</td>
</tr>
<tr>
<td>H</td>
<td></td>
<td>2.10* (.17)</td>
<td>1.17* (.33)</td>
<td>1.36* (.20)</td>
<td>1.40* (.29)</td>
<td>1.59* (.26)</td>
</tr>
<tr>
<td>β</td>
<td>const</td>
<td>-2.76* (.13)</td>
<td>-16.04* (.40)</td>
<td>-15.97* (2.07)</td>
<td>-15.84* (2.00)</td>
<td>-16.14* (2.32)</td>
</tr>
<tr>
<td></td>
<td>parks</td>
<td>2.03* (.17)</td>
<td>2.13* (.40)</td>
<td>1.33* (.43)</td>
<td>2.12* (.34)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>educ Xkid</td>
<td>-21.23* (.29)</td>
<td>-21.24* (3.57)</td>
<td>-19.89* (2.56)</td>
<td>-21.25* (4.33)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bus</td>
<td>-2.61* (.28)</td>
<td>-2.55* (.51)</td>
<td>-2.71* (1.31)</td>
<td>-2.58* (.36)</td>
<td></td>
</tr>
<tr>
<td>σνᵵ</td>
<td></td>
<td>2.71* (.575)</td>
<td>.03 (.125)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>γ</td>
<td>crime</td>
<td>-14.90* (1.00)</td>
<td>-3.80* (1.87)</td>
<td>-3.37 (1.88)</td>
<td>-6.69* (3.24)</td>
<td>-3.34* (1.37)</td>
</tr>
<tr>
<td></td>
<td>educ Xkid</td>
<td>2.88* (1.09)</td>
<td>4.78 (2.88)</td>
<td>5.41* (1.38)</td>
<td>2.61* (1.16)</td>
<td>4.70* (1.50)</td>
</tr>
<tr>
<td></td>
<td>race Xblck</td>
<td>4.51* (.68)</td>
<td>4.63* (.32)</td>
<td>4.52* (.96)</td>
<td>5.64* (.67)</td>
<td>3.71* (1.12)</td>
</tr>
<tr>
<td></td>
<td>bus</td>
<td>.33* (.17)</td>
<td>.01 (.18)</td>
<td>-.20 (.50)</td>
<td>-.16 (.41)</td>
<td>.03 (.68)</td>
</tr>
<tr>
<td></td>
<td>strts</td>
<td>-.20 (.44)</td>
<td>-1.04 (.55)</td>
<td>-.86 (.77)</td>
<td>-1.01 (1.07)</td>
<td>-1.44 (.93)</td>
</tr>
<tr>
<td></td>
<td>parks</td>
<td>0.60 (.70)</td>
<td>-.70 (.83)</td>
<td>0.29 (1.32)</td>
<td>-1.11 (.71)</td>
<td>.57 (1.26)</td>
</tr>
<tr>
<td></td>
<td>college</td>
<td></td>
<td></td>
<td>.18 (.63)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>work</td>
<td></td>
<td></td>
<td>.50 (.73)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σγᵵ</td>
<td>race Xblck</td>
<td>3.40* (.56)</td>
<td>2.73* (.40)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>σγᵵ</td>
<td>educ Xkid</td>
<td>3.11* (.47)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pγᵱνᵵ</td>
<td>race Xblck, νᵵ</td>
<td>-.96* (.04)</td>
<td>.08 (.90)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pγᵱνᵵ</td>
<td>educ Xkid, νᵵ</td>
<td>-.97* (.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pγᵱνᵵ</td>
<td>race Xblck, educ Xkid</td>
<td>.99* (.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pγᵱνᵵ</td>
<td>bus, νᵵ</td>
<td></td>
<td>.08 (.65)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pγᵱνᵵ</td>
<td>race Xblck, bus</td>
<td></td>
<td>.99* (.25)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Indicates significant at .05% level

### Table 3.7: Mean Elasticities

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>-.440</td>
<td>-.760</td>
<td>-.659</td>
<td>-.784</td>
<td>-.830</td>
</tr>
<tr>
<td>educ Xkid</td>
<td>.165</td>
<td>.141</td>
<td>.163</td>
<td>.147</td>
<td></td>
</tr>
<tr>
<td>parks</td>
<td>-.002</td>
<td>-.002</td>
<td>-.001</td>
<td>-.002</td>
<td></td>
</tr>
<tr>
<td>bus</td>
<td>.045</td>
<td>.038</td>
<td>.049</td>
<td>.039</td>
<td></td>
</tr>
</tbody>
</table>

76
demand. Model 4 suggests that households value living in neighborhoods where a higher percentage of females graduated college and a higher percentage of males are currently employed, however these estimates were not statistically significant. To illustrate the relative magnitudes of these estimates, consider a black household with children weighing options according to model 3: a one standard deviation decrease in violent crime per capita (a decrease of 0.3 violent crimes per 1000 people), a .62 standard deviation increase in mean eighth grade test scores (3.5 score points), and a .02 percentage point increase in the percent of blacks in the neighborhood.

Overall, the $\beta$ terms governing private consumption sum to a negative number, so that the interpretation of this component as a minimum non-housing expenditure in a given neighborhood is not feasible. Instead, we interpret them as demand shifters. The neighborhood amenities affect housing demand by the contribution ($-\alpha\beta$). The $\beta$ contribution to the housing demand suggests that average public transit commute time is a complement to housing demand while parks are a (small) substitute. For households with children, school quality is a complement to housing demand. The estimation suggests that the price elasticity of housing demand ranges from -.44 to -.83, which is slightly lower than other estimates published in the literature which generally range from -0.6 to -1.7, but reasonable given the fact that we focus only on poor households eligible for housing subsidies.42

The ability to include random preferences for neighborhood attributes is important, as the model identified significant taste variation for school quality (model 3 and model 5) and racial mix (model 3). Model 3, but not model 5, also identified significant taste variation for housing consumption. Model 3 also found significant correlation between neighborhood attributes and housing consumption; namely, race and education were positively correlated with each other and negatively correlated with housing services consumption. These directions are plausible for urban, very low income groups because very-low-income households with children, and thus interest in public schools, tend to be racial minorities.

42See Friedman & Weinberg (1982) for an overview, literature summary, and elasticity estimates from the 1974 Housing Demand Experiment.
Although the parameters for the random preferences are statistically significant, the random preferences for neighborhood amenities only have a small effect on the endogenous housing demand, with the expectation corrections in the housing demand equation contributing up to a maximum of 0.05 percent of the housing demand. Previous literature (see Section 3) has found important homogeneities of the joint housing consumption and neighborhood selection decision, which partly motivated the model selection for this work. While the parameters estimated here cannot support these previous findings, there are some significant differences between this study and previous ones. In the present study, the narrow focus on the very-low income population might be a limitation in estimating taste variation across the whole population. In addition, I do not use micro-level data on housing consumption for unsubsidized households.

For the nominal analysis described in the next section of this paper, model 3 is the preferred specification because it contains coefficients allowing for nonseparability that are estimated to be significant and whereas model 5 finds a positive correlation between blacks and housing consumption, model 3 identifies a negative correlation which I believe to be more plausible.

Tables 3.7 and 3.8 show how well a simulation of model 3 fits the data. The simulation takes as input only income, race, and presence of children, and simulates both neighborhood choice and housing services demand for each sub-population. Starred values indicate the outcomes of \( \chi^2 \) tests of goodness of fit where we do not reject the null hypothesis that simulated values equal observed values, with a significance level of 0.05. Table 3.7 compares the simulated and observed choices of the voucher program participants. For the voucher population, most outcomes are replicated reliably by the model: for example, households with children consume more housing than those without children; blacks consume more housing services than whites; and blacks live in more violent neighborhoods. The model correctly replicates that blacks with children live in neighborhoods with higher test scores than those that do not have children, but the model misses this relationship for the white voucher holders, perhaps because of the population of whites is not as large. The main difference between the actual choices of voucher participants and the simulated choices is that racial preferences are slightly exaggerated. This issue is not a problem that appears in comparing the
unsubsidized population; there appears to be less racial sorting among voucher households than one would suspect by estimated the model only based on unsubsidized households.

Table 3.8: Simulated and Observed Choices of Voucher Program Participants

<table>
<thead>
<tr>
<th></th>
<th>Black, Kids</th>
<th>Black, No Kids</th>
<th>White, Kids</th>
<th>White, No kids</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sim ( Obs )</td>
<td>Sim ( Obs )</td>
<td>Sim ( Obs )</td>
<td>Sim ( Obs )</td>
</tr>
<tr>
<td>h</td>
<td>3.42* ( 3.53 )</td>
<td>3.33* ( 3.29 )</td>
<td>2.74* ( 2.95 )</td>
<td>2.69 ( 2.59 )</td>
</tr>
<tr>
<td>b</td>
<td>8.36^{i/a} ( 8.36 )</td>
<td>8.26^{i/a} ( 8.26 )</td>
<td>8.23^{i/a} ( 8.23 )</td>
<td>8.28^{i/a} ( 8.28 )</td>
</tr>
<tr>
<td>Violent</td>
<td>26.11* ( 25.97 )</td>
<td>26.66* ( 28.70 )</td>
<td>18.72* ( 19.14 )</td>
<td>19.73* ( 22.75 )</td>
</tr>
<tr>
<td>Black</td>
<td>0.59 ( 0.50 )</td>
<td>0.61* ( 0.55 )</td>
<td>0.17* ( 0.15 )</td>
<td>0.20* ( 0.16 )</td>
</tr>
<tr>
<td>PubTransTime</td>
<td>0.82* ( 0.87 )</td>
<td>0.81* ( 0.78 )</td>
<td>1.10* ( 1.08 )</td>
<td>1.09 ( 0.94 )</td>
</tr>
<tr>
<td>Intersection</td>
<td>0.50* ( 0.53 )</td>
<td>0.50 ( 0.56 )</td>
<td>0.49* ( 0.53 )</td>
<td>0.50* ( 0.53 )</td>
</tr>
<tr>
<td>Parks</td>
<td>0.05* ( 0.05 )</td>
<td>0.05* ( 0.05 )</td>
<td>0.08* ( 0.06 )</td>
<td>0.07* ( 0.11 )</td>
</tr>
<tr>
<td>FemaleCollege</td>
<td>0.16 ( 0.14 )</td>
<td>0.15 ( 0.17 )</td>
<td>0.24 ( 0.16 )</td>
<td>0.21 ( 0.25 )</td>
</tr>
<tr>
<td>MaleEmployed</td>
<td>0.50 ( 0.53 )</td>
<td>0.49* ( 0.52 )</td>
<td>0.61* ( 0.61 )</td>
<td>0.60* ( 0.60 )</td>
</tr>
<tr>
<td>PercentRent</td>
<td>0.57 ( 0.48 )</td>
<td>0.57* ( 0.59 )</td>
<td>0.45 ( 0.39 )</td>
<td>0.45 ( 0.52 )</td>
</tr>
<tr>
<td>MQrent</td>
<td>331 ( 348 )</td>
<td>326 ( 361 )</td>
<td>414 ( 385 )</td>
<td>402 ( 417 )</td>
</tr>
<tr>
<td>MedianInc</td>
<td>32.97* ( 32.28 )</td>
<td>32.96 ( 34.66 )</td>
<td>32.63* ( 30.41 )</td>
<td>32.58* ( 34.54 )</td>
</tr>
<tr>
<td>SingleFKid</td>
<td>0.18 ( 0.14 )</td>
<td>0.19 ( 0.14 )</td>
<td>0.08* ( 0.08 )</td>
<td>0.09 ( 0.07 )</td>
</tr>
<tr>
<td>MaleCollege</td>
<td>0.18* ( 0.17 )</td>
<td>0.17 ( 0.20 )</td>
<td>0.28 ( 0.19 )</td>
<td>0.25 ( 0.30 )</td>
</tr>
<tr>
<td>Poverty</td>
<td>0.30* ( 0.25 )</td>
<td>0.31* ( 0.27 )</td>
<td>0.17* ( 0.17 )</td>
<td>0.18 ( 0.19 )</td>
</tr>
</tbody>
</table>

*Indicates do not reject the hypothesis of equality, \( p < .05 \)

Table 3.8 compares the simulated and observed choices of the unsubsidized low-income households. Relative neighborhood outcomes between sub-populations in the simulated model remain true to the relative outcomes in the observed data; the strength of this fit is probably driven by the fact that unsubsidized households accounted for the largest portion of neighborhood shares, thus having a large influence on the estimation of the neighborhood unobservables \( \xi_j \) in the BLP contraction mapping step.
Table 3.9: Simulated and Observed Choices of Unsubsidized Households, *Indicates do not reject the hypothesis of equality, $p < .05$.

<table>
<thead>
<tr>
<th></th>
<th>Black, Kids</th>
<th>Black, No Kids</th>
<th>White, Kids</th>
<th>White, No kids</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>3.94* (3.60)</td>
<td>3.91* (3.60)</td>
<td>3.46* (3.45)</td>
<td>3.44* (3.38)</td>
</tr>
<tr>
<td>b</td>
<td>11.22* (12.07)</td>
<td>15.59* (16.38)</td>
<td>10.66* (10.57)</td>
<td>15.13* (14.81)</td>
</tr>
<tr>
<td>Violent</td>
<td>25.65* (27.33)</td>
<td>26.34* (28.74)</td>
<td>18.07* (15.60)</td>
<td>19.20* (19.43)</td>
</tr>
<tr>
<td>Grd8Alone</td>
<td>25.07* (24.47)</td>
<td>24.89* (24.38)</td>
<td>26.89* (27.01)</td>
<td>26.52* (27.01)</td>
</tr>
<tr>
<td>Black</td>
<td>0.57 (0.68)</td>
<td>0.60 (0.74)</td>
<td>0.16* (0.12)</td>
<td>0.19* (0.13)</td>
</tr>
<tr>
<td>PubTransTime</td>
<td>0.83* (0.76)</td>
<td>0.81 (0.69)</td>
<td>1.11* (1.20)</td>
<td>1.09* (1.09)</td>
</tr>
<tr>
<td>Intersection</td>
<td>0.49* (0.49)</td>
<td>0.49* (0.51)</td>
<td>0.47* (0.47)</td>
<td>0.49* (0.48)</td>
</tr>
<tr>
<td>Parks</td>
<td>0.06* (0.05)</td>
<td>0.05* (0.04)</td>
<td>0.08* (0.08)</td>
<td>0.07* (0.08)</td>
</tr>
<tr>
<td>FemaleCollege</td>
<td>0.17* (0.14)</td>
<td>0.16* (0.14)</td>
<td>0.27* (0.26)</td>
<td>0.24* (0.28)</td>
</tr>
<tr>
<td>MaleEmployed</td>
<td>0.50* (0.48)</td>
<td>0.50* (0.44)</td>
<td>0.61* (0.63)</td>
<td>0.60* (0.61)</td>
</tr>
<tr>
<td>PercentRent</td>
<td>0.56* (0.59)</td>
<td>0.57* (0.62)</td>
<td>0.45* (0.40)</td>
<td>0.46* (0.48)</td>
</tr>
<tr>
<td>MQrent</td>
<td>339* (311)</td>
<td>333* (305)</td>
<td>429* (435)</td>
<td>419* (438)</td>
</tr>
<tr>
<td>MedianInc</td>
<td>33.019* (33.77)</td>
<td>33.03* (33.77)</td>
<td>32.80* (31.99)</td>
<td>32.75* (32.70)</td>
</tr>
<tr>
<td>SingleFKid</td>
<td>0.18* (0.22)</td>
<td>0.18* (0.20)</td>
<td>0.08* (0.07)</td>
<td>0.08* (0.06)</td>
</tr>
<tr>
<td>MaleCollege</td>
<td>0.19* (0.15)</td>
<td>0.18* (0.15)</td>
<td>0.31* (0.30)</td>
<td>0.28* (0.33)</td>
</tr>
<tr>
<td>Poverty</td>
<td>0.29* (0.33)</td>
<td>0.30* (0.35)</td>
<td>0.17* (0.15)</td>
<td>0.18* (0.17)</td>
</tr>
</tbody>
</table>
3.6 Policy Simulations

I consider the question of optimal voucher policy design. I compare the cost of policy changes to the expected benefit as estimated by households’ compensating variation. It is well-known that this type of discrete choice models does not yield closed form solutions for compensating variations. I, therefore, follow McFadden (1989, 1995) and adopt a simulation based approach. There are some limitations to this analysis. First, I assume households act with full information, i.e. that the choices I observe in the data reflect full information of benefits or disadvantages they would derive from neighborhood and housing outcomes. Second, participants’ willingness to pay is of course limited by their already very-low incomes. Finally, potential neighborhood spillover effects are not included.

I run several policy experiments to compare housing consumption, neighborhood, and program cost. The policy simulations are motivated by the proposed voucher policies tested in the Housing Allowance Demand experiments (Friedman & Weinberg, 1982), the Gautreaux Program (Rosenbaum, 1994), and the Moving To Opportunities experiment (Kling et al., 2007). I compare all policies to the model’s simulation of the current housing voucher policy: participant contribution of 30% of income, an FMR of about 40% of local median rents, and a minimum housing standard.43 Table 3.9 compares neighborhood characteristics, housing consumption, income remaining for non-housing expenses, and program costs under the proposed policies to the choices predicted by the baseline model.

The first simple policy change would be to increase the amount of the voucher (column "Big" in Table 3.9). I simulate what the neighborhood outcomes might be if the HACP simply increased the voucher amount by 20%. The simulation suggests that most of the voucher increase would be dedicated to increased (perhaps, excess) housing consumption. In addition, there are slight changes

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43I do not compare models with and without a minimum housing requirement. It is difficult to calibrate the appropriate level of housing services $h$ that corresponds to the HACP’s list of housing requirements, which includes for example that electric stoves have a separate electric line, rather than be plugged into a wall outlet. (Though livable, the apartment I rented during graduate school would not have met the requirements for this reason).
in mean neighborhood attributes that are generally positively correlated with price: -0.75% lower crime, .019% better schools, 4.2% more college-educated women. White households, on average, would benefit more from the increase in the voucher amount (-2.26% lower crime, .51% better schools, 10.5% more college-educated women), especially due to their willingness to move into areas with fewer minorities. However, the increased program cost of increasing the voucher amount by 20% exceeds the sum of the compensating variations.\footnote{Not reported here, I also simulated expected outcomes if the HACP were to decrease the voucher amount by 20%. Most of the voucher decrease is felt in housing consumption, but there are also declines in mean utility from neighborhood attributes. The lower voucher results in residents living in poor neighborhoods with more crime and lower performing students, for example. The savings in program cost would be less than the cost of compensating program participants for the policy change.}

Clearly there is great price variation within a metropolitan area and setting an FMR for an entire region may be too restrictive. "ConstH" compares current outcomes to outcomes expected if the HACP set housing services (\(h\)) constant but allowed the voucher amount to vary with neighborhood price variation, rather than regional price variation. This practice would require the housing authority to have a decent estimate of price variation across communities. As black households without children are often empty-nesters enjoying the larger space for which they previously qualified, on average they would experience a -13% decrease in their level of housing consumption. Whites, however, would experience an increase. All groups would experience significant neighborhood gains including -2.1% lower crime and 0.5% better schools (-4.8% and 1%, respectfully, for whites); these gains are generally more than 2 times greater than they would be under the program to increase the voucher amount by 20%.

I consider a rental rebate ("Rebate") program that was tested in the Housing Demand Experiments. This simulation compares outcomes if the HACP were to get rid of the income contribution requirement and the voucher maximum, but instead offered a 50% rebate for rental expenses, excluding utility payments. In general, the rebate program would result in over-consumption of housing (an increase of 60%) and lower amounts of non-housing private consumption (about -25%), especially for those with children. There are gains in mean neighborhood amenities as well, exceeding the gains expected from simply increasing the voucher amount. For example, black households
would, on average, locate to neighborhoods with -1.3% less crime and 0.3% better schools, -4% and .7% respectfully for whites. However, the gains in mean neighborhood amenities are typically less than the gains expected under the policy of allowing FMR to vary based on local neighborhood prices (‘ConstH’). The cost of the program is lower than the expected benefit as measured by the sum of households’ willingness to pay for the change in policy. These results do not agree with those of Friedman & Weinberg (1982). Friedman and Weinberg found that the majority of participants in their rebate implementations (or, ”Percent of Rent”) continued to consume substandard or overcrowded housing after two years in the program. The simulations of my model do not include moving costs; perhaps moving costs are a factor. Also, the parameter estimates in my model did not yield a positive lower bound on minimum non-housing consumption. As a result, the expected decrease in non-housing consumption predicted by the model under a rebate policy may not accurately reflect minimum survivable non-housing consumption.

Finally, I consider the paternalistic policies of the Gautreaux program and the experimental group in the Moving To Opportunities program. In both of these studies, the experimental group with vouchers was required to move to a neighborhood with a low poverty level. I consider a policy that requires voucher recipients to live in neighborhoods with less than a 30 percent poverty rate (“30% Req”). This requirement would yield significant improvements in most mean neighborhood amenities, for example a -20% decrease in violence (-10% for whites), a 2.45% increase in school quality (1.3% increase for whites), and a 35% increase in females who graduated from college (16% for whites). Mean housing consumption slightly increases, consistent with higher implicit prices for minorities due to racial sorting preferences (see, for example Bayer et al. (2004)) and the possibility that landlords traditionally serving voucher-friendly neighborhoods extract the largest rents possible from the HACP. However, the restriction reduces the number of neighborhood choices by 22 percent. Naturally, compensating variations are positive because recipients lose full range of choice, in particular with regard to race-specific preferences and the individual-neighborhood specific unobservables $\epsilon_{ij}$. The expected compensating variation for black households is quite high owing to the large drop in the percent of blacks in the reduced set of neighborhoods; on average,
black participants would locate to neighborhoods with -23% fewer blacks. Even if the parameter estimates I obtained for racial preference are unbiased, paternalistic policies such as this one are motivated by the belief that households do not have enough information to properly gage the benefit of moving out of their preferred neighborhood. Despite the estimation’s use of instruments, there could be bias in the parameter estimates for racial preference due to the confounding factor of racial discrimination against prospective tenants, a factor that is very difficult to observe or capture. If the parameter estimates for racial preference are biased in this regard, the compensating variation might be reduced if paired with assistance and advocacy for tenants in their apartment search, as well as mentoring and counseling programs after placement. The HACP’s expenditure on landlord contracts would remain the same, but the simplified cost analysis presented in Table 3.9 might hide the additional need for housing counseling or relocation assistance.

Overall, while parameter estimates suggest that enjoyment of neighborhood amenities accounts for 25 percent of overall utility, the types of neighborhoods chosen by voucher participants is not greatly affected by changes to the budget constraint alone. The most effective policy change in achieving different neighborhood selection is to impose a requirement that households live in neighborhoods with poverty rates below some acceptable maximum, such as 30 percent. In analyzing the budget constraint, my analysis suggests that changing the structure of the program to be a rebate instead of a voucher would improve participants’ utility, achieve neighborhood selection most similar to a program with an unrestricted voucher amount, and would significantly lower costs. While I present expected mean neighborhood outcomes, housing outcomes, and compensating variation, a concern with the rebate policy is that it might endanger the non-housing consumption of the poorest program participants because it rewards excessive consumption of housing.

3.7 Conclusions

This paper analyses the Housing Choice Voucher Program, finding that voucher recipients are able to achieve better housing consumption with a voucher than without, but that vouchers alone do not lead
### Table 3.10: Policy Simulations

#### Expected Changes for Blacks with Children

<table>
<thead>
<tr>
<th>Policy</th>
<th>Big</th>
<th>ConstH</th>
<th>Rebate</th>
<th>30% Req</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing (h)</td>
<td>19.24%</td>
<td>3.28%</td>
<td>63.27%</td>
<td>12.63%</td>
</tr>
<tr>
<td>Non-Housing (b)</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-24.42%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Violent</td>
<td>-0.75%</td>
<td>-2.11%</td>
<td>-1.36%</td>
<td>-19.46%</td>
</tr>
<tr>
<td>School</td>
<td>0.19%</td>
<td>0.46%</td>
<td>0.31%</td>
<td>2.45%</td>
</tr>
<tr>
<td>Poverty</td>
<td>-0.52%</td>
<td>-1.36%</td>
<td>-0.90%</td>
<td>-39.70%</td>
</tr>
<tr>
<td>Black</td>
<td>0.29%</td>
<td>0.95%</td>
<td>0.68%</td>
<td>-23.11%</td>
</tr>
<tr>
<td>FemaleCollege</td>
<td>4.21%</td>
<td>9.13%</td>
<td>6.65%</td>
<td>35.55%</td>
</tr>
<tr>
<td>Household CV</td>
<td>-$111</td>
<td>-$68</td>
<td>-$72</td>
<td>$221</td>
</tr>
</tbody>
</table>

#### Expected Changes for Blacks with No Children

<table>
<thead>
<tr>
<th>Policy</th>
<th>Big</th>
<th>ConstH</th>
<th>Rebate</th>
<th>30% Req</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing (h)</td>
<td>19.18%</td>
<td>-12.92%</td>
<td>47.17%</td>
<td>12.81%</td>
</tr>
<tr>
<td>Non-Housing (b)</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-18.10%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Violent</td>
<td>-0.75%</td>
<td>-2.06%</td>
<td>-1.37%</td>
<td>-18.51%</td>
</tr>
<tr>
<td>School</td>
<td>0.20%</td>
<td>0.45%</td>
<td>0.31%</td>
<td>2.29%</td>
</tr>
<tr>
<td>Poverty</td>
<td>-0.56%</td>
<td>-1.32%</td>
<td>-0.83%</td>
<td>-40.15%</td>
</tr>
<tr>
<td>Black</td>
<td>0.28%</td>
<td>0.92%</td>
<td>0.70%</td>
<td>-22.44%</td>
</tr>
<tr>
<td>FemaleCollege</td>
<td>4.76%</td>
<td>9.92%</td>
<td>7.40%</td>
<td>36.72%</td>
</tr>
<tr>
<td>Household CV</td>
<td>-$93</td>
<td>$30</td>
<td>-$45</td>
<td>$242</td>
</tr>
</tbody>
</table>

#### Expected Changes for Whites with Children

<table>
<thead>
<tr>
<th>Policy</th>
<th>Big</th>
<th>ConstH</th>
<th>Rebate</th>
<th>30% Req</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing (h)</td>
<td>17.78%</td>
<td>28.78%</td>
<td>74.42%</td>
<td>14.82%</td>
</tr>
<tr>
<td>Non-Housing (b)</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-29.15%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Violent</td>
<td>-2.31%</td>
<td>-4.83%</td>
<td>-4.04%</td>
<td>-8.30%</td>
</tr>
<tr>
<td>School</td>
<td>0.45%</td>
<td>0.92%</td>
<td>0.70%</td>
<td>1.30%</td>
</tr>
<tr>
<td>Poverty</td>
<td>-1.31%</td>
<td>-2.90%</td>
<td>-2.07%</td>
<td>-14.06%</td>
</tr>
<tr>
<td>Black</td>
<td>0.12%</td>
<td>0.63%</td>
<td>0.61%</td>
<td>-25.53%</td>
</tr>
<tr>
<td>FemaleCollege</td>
<td>8.19%</td>
<td>15.99%</td>
<td>12.77%</td>
<td>12.79%</td>
</tr>
<tr>
<td>Household CV</td>
<td>-$146</td>
<td>-$229</td>
<td>-$140</td>
<td>$57</td>
</tr>
</tbody>
</table>

#### Expected Changes for Whites, No Children

<table>
<thead>
<tr>
<th>Policy</th>
<th>Big</th>
<th>ConstH</th>
<th>Rebate</th>
<th>30% Req</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing (h)</td>
<td>17.39%</td>
<td>7.84%</td>
<td>58.74%</td>
<td>13.60%</td>
</tr>
<tr>
<td>Non-Housing (b)</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-20.70%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Violent</td>
<td>-2.26%</td>
<td>-4.81%</td>
<td>-4.14%</td>
<td>-10.33%</td>
</tr>
<tr>
<td>School</td>
<td>0.51%</td>
<td>0.99%</td>
<td>0.77%</td>
<td>1.49%</td>
</tr>
<tr>
<td>Poverty</td>
<td>-1.55%</td>
<td>-3.34%</td>
<td>-2.47%</td>
<td>-16.56%</td>
</tr>
<tr>
<td>Black</td>
<td>0.09%</td>
<td>0.69%</td>
<td>0.62%</td>
<td>-29.33%</td>
</tr>
<tr>
<td>FemaleCollege</td>
<td>10.55%</td>
<td>19.91%</td>
<td>16.30%</td>
<td>16.23%</td>
</tr>
<tr>
<td>Household CV</td>
<td>-$122</td>
<td>-$105</td>
<td>-$123</td>
<td>$27</td>
</tr>
</tbody>
</table>

#### Expected Changes in Cost, Benefit

<table>
<thead>
<tr>
<th>Policy</th>
<th>Big</th>
<th>ConstH</th>
<th>Rebate</th>
<th>30% Req</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ Cost</td>
<td>$574,900</td>
<td>$210,600</td>
<td>-827,850</td>
<td>$0</td>
</tr>
<tr>
<td>Δ Benefit</td>
<td>$402,368</td>
<td>$192,086</td>
<td>$276,824</td>
<td>-697,823</td>
</tr>
</tbody>
</table>
to an increase in households’ access to better neighborhoods. Compared to several proposed policy specifications, a rental rebate scheme would reduce the cost of the program and serve most program participants better than the current scheme of a maximum voucher amount subject to contribution of a fixed portion of income. A program requirement to live in a neighborhood where less than 30 percent of households live below the poverty level would relocate participants to neighborhoods with much lower crime levels and improved schools. This requirement would most negatively affect minority households that have a high regard for locating in neighborhoods with high minority concentrations; perhaps relocation assistance and counseling could overcome these issues.

The equilibrium residential sorting model proposed here offers a method to study the impact of housing policy on residential sorting, as it incorporates households with different budget constraints. With its direct utility specification the model can be used to study choices derived from housing policy, discontinuous borrowing constraints for residential mortgages, or other constraints on neighborhood or housing choice that impact residential sorting equilibria. In addition, the model offers several desirable features found to be important in the literature, including horizontal demand and nonseparable random preferences of the neighborhood public services and housing demand. The error structure proposed in the model allows the research to study the covariance of housing demand elasticity and preferences for specific observable public goods. In particular, this paper finds that a public parks are a substitute for housing services, while school quality and mean public transit commute time are complements. The generalized model presented here provides ample scope for studies on policy or borrowing practices that induce discontinuous budget constraints in households’ joint discrete-continuous choice making.
Bibliography


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*Econometrica*, 75(1), 83–119.


Appendix A

The Extended SIPP Sample

In addition to the Pittsburgh sample, we also construct a larger sample adding data from 13 metropolitan areas that have similar ratios of public housing units per household as Pittsburgh. Table A.1 provides some summary statistics of these MSA’s.

Table A.1: Urban Areas Included in Sample

<table>
<thead>
<tr>
<th>City</th>
<th>Eligible for Public Housing</th>
<th>Median Income</th>
<th>Unemployment Rate</th>
<th>Minority Fair Market Rent 2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pittsburgh</td>
<td>.0546</td>
<td>37467</td>
<td>4.4%</td>
<td>10%</td>
</tr>
<tr>
<td>Columbus</td>
<td>.0384</td>
<td>44782</td>
<td>2.7%</td>
<td>19%</td>
</tr>
<tr>
<td>Allentown</td>
<td>.0375</td>
<td>43098</td>
<td>4.2%</td>
<td>10%</td>
</tr>
<tr>
<td>Albany</td>
<td>.0373</td>
<td>43250</td>
<td>3.4%</td>
<td>10%</td>
</tr>
<tr>
<td>Dayton</td>
<td>.0372</td>
<td>41550</td>
<td>4.5%</td>
<td>18%</td>
</tr>
<tr>
<td>Buffalo</td>
<td>.0339</td>
<td>38488</td>
<td>5.3%</td>
<td>16%</td>
</tr>
<tr>
<td>Scranton</td>
<td>.0607</td>
<td>34161</td>
<td>5.6%</td>
<td>3%</td>
</tr>
<tr>
<td>St. Louis</td>
<td>.0169</td>
<td>44437</td>
<td>3.5%</td>
<td>22%</td>
</tr>
<tr>
<td>Madison</td>
<td>.0124</td>
<td>49223</td>
<td>1.7%</td>
<td>11%</td>
</tr>
<tr>
<td>Detroit</td>
<td>.0159</td>
<td>49160</td>
<td>3.9%</td>
<td>27%</td>
</tr>
<tr>
<td>Cleveland</td>
<td>.0291</td>
<td>42215</td>
<td>4.2%</td>
<td>21%</td>
</tr>
<tr>
<td>Cincinnati</td>
<td>.0109</td>
<td>44914</td>
<td>3.5%</td>
<td>15%</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>.0266</td>
<td>47528</td>
<td>4.1%</td>
<td>27%</td>
</tr>
<tr>
<td>Milwaukee</td>
<td>.0193</td>
<td>46132</td>
<td>3.1%</td>
<td>22%</td>
</tr>
</tbody>
</table>
Table A.1 reports the MSA’s ratio of public housing units to households eligible for public housing. We also show the 1999 MSA median income, 1999 unemployment rate, and the HUD-determined 2001 fair market rent for a one-bedroom unit.\textsuperscript{1} Table A.1 shows that Pittsburgh is representative of many other large urban areas in the Northeast and Midwest that face similar challenges in providing affordable housing for low-income households.

\textsuperscript{1}The number of public housing units is taken from the HUD 1998 Picture of Subsidized Housing. Percent minority and median incomes are from the 2000 Census. Unemployment is from The Real Estate Center at Texas A\& M University. Fair Market Rents are published on the HUD website.
Appendix B

A Monte Carlo Study

Since our estimation procedure is non-standard, we conducted a number of Monte Carlo studies to study the properties of the estimators when the true data generating process is known. Below we report the results for one specification that we tested.\(^1\)

<table>
<thead>
<tr>
<th>Name</th>
<th>Variable</th>
<th>random sample</th>
<th>enriched sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Effect PH1</td>
<td>(\gamma_1)</td>
<td>[-0.887, 1.763]</td>
<td>[-0.947, 1.763]</td>
</tr>
<tr>
<td>Fixed Effect PH2</td>
<td>(\gamma_2)</td>
<td>[-0.8142, 1.585]</td>
<td>[-1.010, 1.585]</td>
</tr>
<tr>
<td>Fixed Effect PH3</td>
<td>(\gamma_3)</td>
<td>[-0.806, 1.744]</td>
<td>[-0.850, 1.744]</td>
</tr>
<tr>
<td>Beta</td>
<td>(\beta)</td>
<td>[-0.191, 0.079]</td>
<td>[-0.191, 0.082]</td>
</tr>
<tr>
<td>Offer Prob PH1</td>
<td>(\pi_1)</td>
<td>[-0.021, 0.019]</td>
<td>[-0.020, 0.019]</td>
</tr>
<tr>
<td>Offer Prob PH2</td>
<td>(\pi_2)</td>
<td>[-0.043, 0.050]</td>
<td>[-0.046, 0.055]</td>
</tr>
<tr>
<td>Offer Prob PH3</td>
<td>(\pi_3)</td>
<td>[-0.013, 0.010]</td>
<td>[-0.013, 0.010]</td>
</tr>
</tbody>
</table>

In our Monte Carlo there is only one observed household characteristic (‘income’). We assume that \(f(x_t, d_{t-1})\) is log-normally distributed with known mean and variance. We consider a model with three public housing communities with \(\gamma_1 = 7.6\), \(\gamma_2 = 7.0\) and \(\gamma_3 = 0.4\). We set the coefficient of income \(\beta = 0.4\). We assign 30% of the population to private housing, 24, 28, and 18 percent to

\(^1\)More results for different parametrizations, sample sizes and sampling schemes are available upon request from the authors.
the three housing communities. This implies that in equilibrium the offer probabilities are \( \pi_1 = 0.11 \), \( \pi_2 = 0.24 \) and \( \pi_3 = 0.05 \).

We consider the properties of the estimator above under two sampling designs: random sampling and enriched sampling. For each parameter vector, one hundred model simulations and estimations are completed, each with sample size 2000. Starting values are initially chosen from a uniform distribution between \((0, 1)\) for \( \beta \) and between \([0, 12]\) for the fixed effects, but any starting values that would lead to unreasonable offer probabilities (probabilities greater than 40%) are rejected. The table above summarizes the performance of the model and reports 95% confidence for the absolute error of parameter estimate and the implied offer probabilities.

In general we find that our estimator works well both under random and enriched sampling. The absolute errors are small and approximately centered around zero. Generally, we find that the estimate for the fixed effects are slightly biased upward and the coefficients on income are slightly biased downward in samples with 2000 observations. In general, larger samples help reduce the estimation bias. Imposing the equilibrium conditions seems to work well, and the estimates of the offer probabilities that are implied by the structural parameters of the model are accurate.