The first two chapters of this thesis focus on the operation of rental businesses, which provides a novel and increasingly significant application of a supply chain and service operation. According to IBISWorld industry reports, at least fifteen different rental industries in the United States have annual revenues exceeding $1 billion. Furthermore, the consumer trend of "access over ownership" has spurred the creation of new and disruptive business models such as Rent the Runway, which rents designer dresses by mail to over 2 million users. However, existing operations management literature offers little support to rental operations subject to complex demand characteristics and the loss of rental units due to customer damage or purchase.

Motivated by new and innovative rental business models, we study the operation of a rental system with random loss of inventory due to customer use. We use a discrete-time model in which the inventory level is chosen before the start of a finite rental season, and customers not immediately served in each period are lost. Demand, rental durations, and rental unit lifetimes are stochastic, and sample path coupling allows us to derive structural results that hold under limited distributional assumptions. Considering different "recirculation" rules – i.e., which rental unit to choose to meet each demand – we prove the concavity of the expected profit function and identify the optimal recirculation rule under two different models of a rental unit's state: the number of times rented out or its condition. A numerical study clarifies when considering rental unit loss and recirculation rules matters most for the inventory decision: Accounting for rental unit loss can increase the expected profit by 7% for a single season and becomes even more important as the time horizon lengthens. We also observe that the optimal inventory level in response to increasing loss probability is non-monotonic. Finally, we show that choosing the optimal recirculation rule over a commonly used policy suggests that more rental units should be added, and the profit-maximizing service level increases by up to six percentage points.

The second chapter extends our rental model to include the problem of admission control through accepting and rejecting reservation requests. We use a stochastic model of a rental operation to study the problem of whether the firm should accept each reservation request over the course of a rental season. In our model, the firm must balance a desire to serve more customers, thereby achieving a higher utilization of its rental assets, with the risk of being unable to serve a reservation that was previously accepted. Service variability is considered through using exponentially distributed service times, and each acceptance decision is made only with knowledge of the number of rental units that are currently busy and the list of accepted reservation requests. We discuss challenges in proving the optimal policy for admitting reservations, and propose an easy-to-implement newsvendor-style heuristic for accepting reservations. We show that the heuristic and two extensions perform well for test cases motivated by three different rental businesses, and compare its performance to bounds and simple heuristics. Furthermore, numerical results reveal that increasing the notice time – i.e., the time between when customers make a reservation request and service begins – decreases the expected profit.

The third chapter of the thesis addresses both strategic and tactical policy questions for the operation of a statewide adoption network, which matches children in state custody with prospective adoptive families. The Pennsylvania Adoption Exchange (PAE) helps case workers who represent children in state custody by recommending prospective families for adoption. We describe PAE's operational challenges using case worker
surveys and analyze child outcomes through a regression analysis of data collected over multiple years. A match recommendation spreadsheet tool implemented by PAE incorporates insights from this analysis and allows PAE managers to better utilize available information. Using a discrete-event simulation of PAE, we justify the value of a statewide adoption network and demonstrate the importance of better information about family preferences for increasing the percentage of children who are successfully adopted. Finally, we detail a series of simple improvements that PAE achieved through collecting more valuable information and aligning incentives for families to provide useful preference information.