RISK PREMIA ON CORPORATE SECURITIES

BY

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Abstract

This thesis focuses on the pricing of risk in equity, credit and corporate bond markets. The first chapter investigates whether financial leverage matter for the cross-section of expected equity returns, above and beyond firm size and book-to-market equity. Using a structural model of capital accumulation and leverage choice I argue that it does. Expected equity returns vary across firms because of differences in productivity, capital and leverage. Firm size and book-to-market equity cannot map all these variables. Everything else equal, leveraged firms are riskier because they disinvest less when capital is less productive. The model can generate qualitatively and, sometimes, quantitatively the cross-sectional relations between equity returns and book-to-market equity, firm size, market leverage, book leverage and debt/equity ratio.

The second chapter explores the source for common variation in the portion of returns observed in U.S. credit markets that is not related to changes in risk-free rates or expected default losses. We extract a latent common component from firm-specific changes in default risk premia that is orthogonal to known systematic risk factors during our sample period from 2001 to 2004. Asset pricing tests suggest that our factor is priced in the corporate bond market and the equity options market but not in the equity market. We develop a theoretical framework supporting our empirical findings. This framework also shows that our factor captures the jump-to-default risk associated with market-wide credit events.

Finally, the third chapter studies the determinants of the default risk premia embedded in the European credit default swap spreads. Using a modified version of the intertemporal capital asset pricing model, we show that default risk premia rep-
resent compensation for bearing exposure to systematic risk and to a new common factor capturing the proneness of the asset returns to extreme events. This new factor arises naturally because the returns on defaultable securities are more likely to have fat tails. The pricing implications of this new factor are not limited to credit markets only. We find that this common factor is priced consistently across a broad spectrum of corporate bond portfolios.
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My life so far has been a sequence of episodes, with almost no connection between each other. Looking back, I see them as little stories casting a handful of interesting characters in places identified by only one or two timeless charming features. Ten years from now, this episode will host my Mikki, the restless Buster, Gooski’s with Stephan, Claudia, Jaime, Ana, Kent, Espen, Emily, Ozge, Zumrut, Jose and Janet, my sister Ela, and a timeless spring setting in Shadyside where the only good things that have not yet happened are the summer evenings at Jitters with Wally and the yet-to-be-finished 312 project.
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CHAPTER I

Financial Leverage and
the Cross-section of Stock Returns
1 Introduction

For the past two decades, the financial economics literature has documented a number of empirical facts relating the cross-sectional distribution of equity returns to firm variables such as firm size, book-to-market equity and debt/equity.\footnote{Some of the most relevant studies in this literature include Stattman (1980), Banz (1981), Rosenberg, Reid and Lanstein (1985), Bhandari (1988), Chan, Hamao and Lakonishok (1991) and Fama and French (1992).} Ever since, academics have tried to assign economic connotations to these firm variables, but no consensus has emerged. One aspect of this debate is whether firm size and book-to-market equity, on one hand, and debt/equity ratio, on the other hand, are indicative of different sources of variation in equity returns.

The empirical evidence on this issue is mixed. For instance, Bandari (1988) argues that firm size and debt/equity proxy for different sources of variation in equity returns. However, Chan and Chen (1991), Fama and French (1992) and Chen and Zhang (1998) argue that book-to-market equity and firm size capture the informational content of debt/equity, while Ferguson and Shockley (2003) and Vassalou and Xing (2004) argue that financial leverage is at the root of why book-to-market equity and firm size matter for equity returns.

Existing structural models of risk premia in the cross section, along the lines of Berk, Green and Naik (1999), assume firms are all equity financed. Therefore they cannot tell us about the role of leverage in the cross section of equity returns. Existing structural models of leverage choice, such as Brennan and Schwartz (1984), Fischer, Heinkel and Zechner (1989), Leland (1994), Moyen (2004) and others, assume risk neutrality. Therefore, they cannot speak to differences in average equity return across firms. I include leverage choice in a structural model of the cross section of returns. With this model we can study simultaneously the role of leverage and firm characteristics in determining risk premia, and how time-varying market risk premia determine leverage choice. Equivalently, I address the following questions:
1) What are the roles of book-to-market equity, firm size and debt/equity relative to equity risk? 2) How does financial leverage impact the cross-sectional relations between equity returns and firm size or book-to-market equity? 3) Do equity returns reflect a premium for financial leverage, not related to either book-to-market equity or firm size? 4) Can this model be consistent with the empirical facts relating the cross-sectional distribution of equity returns to these firm variables?

My model develops a production economy where firms maximize the wealth of their shareholders by making investment and capital structure decisions. Firm optimizing behavior is a key element in generating an endogenous relation between firm characteristics and equity risk premia. Berk, Green and Naik (1999) show that, for an all-equity-financed firm, the fact that book-to-market equity and firm size relate to the firm’s equity risk premium follows naturally from the firm’s optimal investment behavior. My approach expands upon their ideas by modeling the firm as making both investment and capital structure decisions. My model also builds upon the work of Moyen (2004) who models the joint investment/capital structure decision of a firm. However, Moyen’s model cannot address asset pricing issues because firm owners are assumed to be risk-neutral. My setup admits time varying risk premiums by exogenously specifying a time-varying stochastic discount factor with a countercyclical price of risk.2

In my model firms employ technologies that require one input, namely capital. The output of these technologies depends not only on the stock of capital (or assets in place) but also on the realization of the aggregate and firm-specific productivity shocks. At every point in time, firms have access to a range of capital expansion options (or growth options) and investment means exercising one of these options. When internal funds are insufficient, firms can finance part of their growth with both equity and debt. In the model, debt emerges as an attractive financing alternative

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2This is also the approach of Berk et al. Gomes, Kogan and Zhang (2003) use a general equilibrium model, but without a capital structure decision
because of the classic trade-off between the tax benefits of debt and the bankruptcy costs and because of the assumption that debt is cheaper to issue than equity.

In my model, equity returns reflect the exposure to aggregate productivity risk (systematic risk) of the assets in place and growth options. The relative distribution of assets in place and growth options becomes the main determinant of equity risk premia. For all-equity-financed firms, this distribution can be summarized in terms of firm-specific productivity and two firm variables, namely book-to-market equity and firm size. These relations are particularly strong for those firms with high firm-specific productivity, because these firms value capital the most, *ceteris paribus*. For firms financed with both equity and debt, this distribution depends also on financial leverage. The market value of debt responds indirectly to innovations in productivity, because the recovery value depends on the distribution of assets in place and growth options, at the time of default. As before, book-to-market equity and firm size relate to equity returns and these relations are stronger for more productive firms.

I can now address the first question. Book-to-market equity captures *entirely* or *partly* the systematic exposure of the assets in place, depending on whether firms are financed with all equity or with both equity and debt. Similarly, firm size captures *entirely* or *partly* the systematic exposure of the growth options, depending on whether firms are financed with all equity or with both equity and debt. Financial leverage is also related to the risk profile of equity, but this relationship is more complex. First, leveraged firms trade at a discount relative to similar all-equity-financed firms, because the states where leverage binds the most are discounted less heavily. Second, leveraged firms tend to maintain elevated stocks of capital, making them riskier when capital becomes unproductive. During times of low productivity, leveraged firms tend to disinvest less, because they cannot reduce the scale of production without increasing the likelihood of default.

The two channels underlying the relationship between financial leverage and equity
returns support also the observation that leveraged firms tend to have higher equity returns and higher book-to-market equity ratios. Thus, financial leverage can lead to stronger cross-sectional relations between equity returns and firm size or book-to-market equity. This answers the second question.

To answer the third question, the model predicts that the cross-sectional variation in equity returns reflects the cross-sectional variation in capital, firm-specific productivity or financial leverage. Firm size and book-to-market equity can collectively capture the variation in equity returns due to capital and the variation due to either firm-specific productivity or financial leverage, but not both. Firm size and book-to-market equity alone cannot disentangle the variation in equity returns due to firm-specific productivity from the variation due to financial leverage. Thus, firm variables proxying for a firm’s financial leverage - such as debt/equity - are informative about the cross-sectional distribution of equity returns, above and beyond what is contained in firm size and book-to-market equity.

Finally, to answer the last question, I test the ability of the model to replicate some of the cross-sectional properties of equity returns, by using simulated analysis à la Kydland and Prescott (1996) and Berk et al. The benchmark empirical studies are Fama and French (1992) and Bhandari (1988). Both studies investigate the cross-section of equity returns, but the former focuses more on the role of firm size and book-to-market equity, while the later focuses more on the role of financial leverage. My model develops a natural laboratory in which both these perspectives on the cross-section of equity returns can be evaluated simultaneously. The findings of the model in this direction can be summarized as follows: 1) the univariate and bivariate cross-sectional relations between equity returns and variables such as market betas, book-to-market equity, firm size, market leverage and book leverage are captured qualitatively and, sometimes, quantitatively relative to the empirical evidence of Fama and French 2) the univariate and bivariate cross-sectional relations between equity
returns and market betas, firm size and debt/equity ratio are captured qualitatively and, sometimes, quantitatively relative to the empirical evidence of Bhandari 3) the multivariate cross-sectional relations resemble their empirical counterparts on most dimensions.

In addition, the model can replicate qualitatively some of the evidence in Ferguson and Shockley (2003). Specifically, the loadings on the mimicking factor associated with the leverage ratio have explanatory power for the cross-section of stock returns, even after controlling for the loadings on the market and the mimicking factors associated with book-to-market equity and firm size.

This paper is related to two strands of the financial economics literature. On one hand, it shares with a series of recent papers such as Berk (1995), Berk, Green and Naik (1999), Gomes, Kogan and Zhang (2003), Carlson, Fisher and Giammarino (2004), and Zhang (2005), the objective of explaining the cross-sectional properties of equity returns, associated with firm variables such as book-to-market equity and firm size. However, while these models focus exclusively on firms financed entirely with equity, mine focuses mostly on firms financed with both equity and debt. On the other hand, my theoretical model shares with the corporate finance literature on the optimal capital structure and investment. Important contributions to this literature include: Brennan and Schwartz (1984), Fischer, Heinkel and Zechner (1989), Goldstein, Leland, and Ju (2001), Gomes (2001), Cooley and Quadrini (2001), Cooper (2003), Moyen (2004), Hennessy and Whited (2005), Strebulaev (2005), and Hackbarth, Miao and Morellec (2005). From this perspective, the novelty in my model stems from the fact that both the capital structure and the investment decisions are related to a time-varying price of risk.

The paper is organized as follows. Section 2 presents the model, outlining the technology, the ownership structure, the objectives, and the decisions of the firms. Section 3 presents the main results of the paper, and Section 10 concludes. Appendix
A provides details on my numerical methodology as well as several supporting results. The proofs for all the results are provided in Appendix B.

2 Model

In order to investigate the impact of financial leverage on the cross-sectional properties of stock returns associated with firm characteristics such as book-to-market equity and firm size, we need a model of firm dynamics with capital accumulation, endogenous investment and optimal capital structure. Below I propose such a model, that builds on the framework of the neoclassical model of production, in a partial equilibrium setting.

In a typical production cycle, firms perform the following three functions: they produce, pay liabilities (taxes and interest expenses) and make decisions. The firms decide on whether to continue or default (henceforth referred to as the ”continue/default decision”), and, conditional on being solvent, they decide on the optimal investment, capital structure and dividend payout. All these decisions are made simultaneously, but for ease of explanation, I assume that the continue/default decision is separated from the remaining ones. For the rest of the paper I will refer to these later decisions, collectively, as the investment-capital structure-dividend payout decision.

The timeline of events for a typical firm can be described as follows: At the beginning of period $t$, the firm’s owners decide whether to honor their period $t$ liabilities (interest expenses) and continue to operate the firm, or default. If they choose to continue, they produce, pay their obligations (taxes and interest expenses), and, finally, make an investment-capital structure-dividend payout decision. If they choose to default, the creditors inherit the assets and the technology of the old firm. I assume that the new owners are better-off reorganizing the firm rather than liquidating the assets.\(^3\) After the reorganization, the new owners commence production, pay taxes

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\(^3\)This assumption prohibits the existence of a market for ”used assets”. If firms were allowed to
on profits and decide on the optimal investment - capital restructure - divided payout decision. At time $t + 1$, the process starts over, in a recursive fashion. A pictorial description of the timeline of events is presented in Figure 1.

2.1 Production Technology

Firms own identical production technologies that use one input, capital, and exhibit decreasing-returns-to-scale. The result of the technological process, $y$, is uncertain, depending on the realization of both an aggregate shock, $x$, and an idiosyncratic shock $z$. Different firms experience different idiosyncratic shocks, leading to heterogeneity in the production economy.

If $k_t$ denotes the stock of capital available at time $t$, then the level of output is described as follows:

$$y_t = k_t^\alpha \exp(x_t + z_t),$$

where $0 < \alpha < 1$ represents the capital share.

In the spirit of the traditional neo-classical growth model with uncertainty, I purchase used capital, I would have to be specific about the benefits and the limitations of installing this type of capital. However, keeping track of the capital composition, as it enters a production cycle, complicates the investment decision problem in a way that does not seem directly related to the questions that this paper tries to address.

$^4$There are several ways to rationalize the decreasing-returns-to-scale feature of the production function. For a short summary see Cooley and Quadrini (2001).
assume that the productivity shocks $x_t$ and $z_t$ follow stationary Markov processes with monotone transition probabilities. For the exact dynamics of these shocks see Appendix A.

An important assumption of this paper is that the price of output is constant (and normalized to 1). Firms compete for capital, taking this price as given. In a fully specified general equilibrium model, the price of the consumption good clears the product market, in the sense that the aggregate output of the productive sector meets the aggregate demand of the consumers. While the general equilibrium approach captures the important feedback effects of firms’ and consumers’ decision problems on prices, it complicates considerably the firm valuation problem. the approach here parallels that in Berk, Green and Naik (1999), Carlson, Fisher and Giammarino (2004) and Zhang (2005), in specifying directly the pricing kernel necessary in the valuation of future cash flows.\footnote{For a production economy where the demand side of the product market is modeled explicitly, see Gomes, Kogan and Zhang (2003).}

The law of motion for capital is driven by depreciation (expressed as a fraction, $\delta$, of the level of capital) and the rate of investment, $i_{t+1}$, and is given by the following equation:

$$k_{t+1} = (1 - \delta)k_t + i_{t+1}k_t$$

(2)

I assume that investment is reversible, but, that in order to adjust the level of capital, firms incur convex adjustment costs. Following the literature on investment with adjustment costs,\footnote{See, for instance, Lucas (1967) or Lucas and Prescott (1971).} I assume that the cost function has the following quadratic form:

$$h(i_{t+1}, k_t) = \theta \frac{i_{t+1}^2k_t}{2}$$

(3)

where $\theta > 0$ represents the number of periods it takes to install an additional unit of
capital. Notice that firms are subject to adjustment costs not only when they invest but also when they disinvest. In addition, according to Equation 3, I assume that these costs are symmetric.\(^7\)

Let \( s_t = (x_t, z_t, k_t) \) denote the vector of state variables characterizing the state of the technological process.

### 2.2 The Continue/Default Decision

A typical firm enters a new production cycle, \( t \), with a base of productive assets, \( k_t \) and a base of financial liabilities. The productivity of the assets is revealed through the realization of the aggregate and idiosyncratic productivity shocks, \( x_t \) and \( z_t \). The financial liabilities of the firm are associated with firm’s previous contractual commitments of corporate debt.

In my model, the entire corporate debt of a firm is structured as a unique console bond. This bond specifies the dollar amount of the borrowed principal and the dollar amount of the periodic interest payment. Most importantly, the bond indenture limits the extent to which the bond issuer can issue additional debt or retire existing debt. A firm with an outstanding issue of bonds can retire some of the existing debt or issue additional debt only if it repays the outstanding console bond, at the market value. This bond covenant limits the number of outstanding bond issues, which a firm can carry at any time, and, therefore, it simplifies considerably my analysis.

The bond covenant outlined previously allows us to recast the structure of the corporate debt in simpler terms. Every period, the effective financial liability of a firm is composed of the period coupon payment and the market value of the remaining interest payments. If the firm decides that the current interest payment is optimal, it will issue back a console bond with the same interest payment. The borrowed principal will offset exactly the market value of the remaining interest payments.

\(^7\)Hall (2001) and Zhang (2005) study the impact of asymmetric adjustment cost on market valuations.
which the firm was liable for. As a consequence, the firm appears as if it only makes a periodic interest payment. If the firm chooses optimally the same interest payment for several consecutive periods, the firm will appear as if it carries long-term debt on its books. This perspective on the structure of the corporate debt will become clearer in the later sections. For now, let $D_t$ denote the effective financial liabilities of the firm, at the beginning of period $t$.

Upon entering the period $t$, a firm knows the state variables $s_t = (x_t, z_t, k_t)$, associated with its technological process, and the level of its financial liabilities, $D_t$. The information contained in these state variables is sufficient to allow the firm to decide whether to continue (stay solvent) or default on its financial obligations.

If the firm decides to continue, the firm commences the production process and it produces an output $y_t = y(s_t)$. Further, it pays the taxes on realized profits, net of depreciation, $\tau(y_t - \delta k_t)$ and it services the debt $D_t$. Finally, the firm decides on the optimal investment, new capital structure and dividend payout. Here, $\tau$ denotes the flat tax rate on corporate profits, while $\delta$ denotes the depreciation rate on capital.

Let $v(s_t, D_t)$ denote the cum-dividend net worth of the shareholders and $w(s_t)$ denote the continuation value of equity, right before the firm makes the investment-capital structure-payout decision (the fact that the continuation value is not a function of $D_t$ will become clear in the following section). Then, the net worth of the shareholders of a solvent firm is given by:

$$v(s_t, D_t) = (1 - \tau)(y_t - \delta k_t) - D_t + \delta k_t + w(s_t)$$ (4)

Due to the limited liability of the shareholders, the firm defaults when the net worth of the shareholders falls below zero. That is:

$$v(s_t, D_t) \leq 0$$ (5)

For the rest of the paper, I track the default decision through the indicator function
\( \chi(s_t, D_t) \). This function takes the value 1, when the inequality (5) does not hold, and 0, otherwise.

The next two sections analyze in more detail the continue and default options.

### 2.3 The Continuation Function

Suppose that, at the beginning of period \( t \), the firm decides to stay solvent and, therefore, not to default on its financial obligations. Then, the firm produces, pays the taxes on realized profits, services the debt and, finally, makes a simultaneous investment - capital structure - dividend distribution decision. The firm accounts for the impact of this decision on the future value of equity, through the continuation function, \( w(s_t) \).

As mentioned in the previous section, one way to implement the bond covenant embedded in the outstanding issue of debt is to have the firm repay the outstanding issue of debt, at the market value, and re-issue a new console bond with optimal face value and coupon payment. If the new issue has the same coupon payment as the previous issue (that was just repaid), the face value (or the principal) of the new issue will offset exactly the market value of the previous outstanding bond issue. Thus, for this given period, the net cash outflow of the firm, due to financing activities is exactly the coupon payment.

Let \( i_{t+1} \) denote the optimal investment rate. Let \( F_{t+1} \) denote the principal and \( b_{t+1} \) denote the coupon payment, of the new issue of debt. Then the continuation value of equity, \( w(s_t) \) can be expressed as:

\[
 w(s_t) = \max_{i_{t+1}, b_{t+1}, F_{t+1}} \left\{ - [i_{t+1} k_t + h(i_{t+1}, k_t)] \\ + F_{t+1} \\ + \mathbb{E}_t [M_{t+1}(s_{t+1}, D_{t+1}) v(s_{t+1}, D_{t+1})] \right\},
\]

(6)
subject to the constraints:

\[ F_{t+1} \leq B(s_t, b_{t+1}) \]  \hspace{1cm} (7)

where \( D_{t+1} = (1 - \tau)b_{t+1} + B(s_{t+1}, b_{t+1}) \), \( B(s_t, b_{t+1}) \) is the market value of the new issue of debt, \( x_{t+1} \) and \( z_{t+1} \) are given by Equation (A.3), and \( s_{t+1} = (x_{t+1}, z_{t+1}, (1 - \delta + i_{t+1})k_t) \).

The formula in Equation (6) is the combined net present value of both an investment and a financing opportunity. In order to take advantage of the investment opportunity, the firm has to pay the initial cost \( i_{t+1}k_t \), plus an adjustment cost \( h(i_{t+1}, k_t) \). In exchange, the firm will receive a stream of future cash flows, generated from the newly installed capital, \( k_{t+1} = (1 - \delta + i_{t+1})k_t \). Similarly, in order to take advantage of the financing opportunity, the firm pre-commits (subject to the limited liability of the shareholders) to pay a periodic coupon \( b_{t+1} \), from this point onwards, in exchange for the principal, \( F_{t+1} \). This principal cannot exceed \( B(s_t, b_{t+1}) \), the market value of the future coupon payments, if the firm commits to the coupon payment \( b_{t+1} \), from now on.

The exact derivation of \( B(s_t, b_{t+1}) \) will be carry out in a later section bellow. For now, we only need to know that since the firm can default on its outstanding debt, \( B(s_t, b_{t+1}) \) will depend on the net worth of the shareholders \( v \), through the default function \( \chi \). The firm accounts for the full impact of its decisions on the market value of the new issue of debt. This point will be fully explored when I will derive the dynamics of equity.

The continuation value of equity in (6) accounts explicitly for the investment and recapitalization decision. The dividend payout decision, \( d_t \) results as a residual. Specifically, if \( i_{t+1} \) is the optimal investment rate and \( F_{t+1} \) and \( b_{t+1} \) are the optimal principal and coupon payment, the payout distribution to the shareholders of the firm is the sum of the cash flows from operating activities, investing activities and
financing activities:

\[ d_t = [(1 - \tau)(y_t - \delta k_t) + \delta k_t] + [-i_{t+1} k_t - h(i_{t+1}, k_t)] + [F_{t+1} - D_t] \] (8)

Depending on whether \( d_t \) is positive or negative, I interpret the payout as either a dividend distribution or a capital outflow, for the firm’s shareholders. The continue/default decision, discussed in the previous section, effectively limits the amount of capital that shareholders are willing to contribute. Specifically, the shareholders are willing to contribute additional capital (i.e. the payout is negative), as long as their net worth is strictly positive.

2.4 Default

Suppose now that the firms decide to default on its obligations. If this is the case, the firm ceases production and enters the bankruptcy process, in which two things happen: 1) the current ownership structure is dissolved; 2) the creditors enter into possession of both the assets and the production technology. The bankruptcy process is assumed to be instantaneous, but costly. I assume that the (bankruptcy) costs are expressed as a fraction, \( \xi \), of the book value of the assets, in default, \( k_t \), and that they are born by the creditors.\(^8\)

I further assume that the bankrupt firm is not liquidated but rather reorganized. The creditors become the new owners of the reorganized firm and they commence production, pay taxes and decide on the optimal investment - capital structure - dividend payout decision.

In order to formalize these assumptions, note first that the state of the reorganized

\(^8\)While this specification seems just as reasonable as the ones based on firm value or book value of liabilities, it offers the advantage of being easily related to estimates, uncovered in previous empirical studies, such as: Weiss (1990) and Welch (2005). For instance, Weiss estimates the legal and other professional fees incurred by firms in the bankruptcy process, as a fraction of the book value of the assets in default.
firm is summarized by the vector: $s_t^D = (x_t, z_t, (1 - \xi)k_t)$. Using the notation in the previous section, it follows that the net worth of the new owners of the firm is given by $v(s_t^D, 0)$.

2.5 The Valuation of Corporate Debt

This section deals with the valuation of the outstanding or the new issues of debt. Suppose a firm wants to issue a new perpetual bond that promises to pay a fixed coupon, $b$, until default. Also, suppose that the current state of the firm is summarized by $s_t = (x_t, z_t, k_t)$. In order to understand the cash flows to the bondholders, we need to understand the default event and the recovery, given default. Both have been already described in the previous sections, so I only summarize them, briefly: On one hand, the default event coincides with the shareholders’ net worth falling below zero. This happens precisely when the default function $\chi$ equals zero. On the other hand, in the event of default, bondholders recover $v(s_t^D, 0)$ - the value of the reorganized firm.

In terms of the previous notation, $\xi k_t$ captures the bankruptcy costs, while $(1 - \tau_0)b + B(s_t, b) - v(s_t^D, 0)$ captures the actual loss to the bondholders.

The market value of the risky, perpetual, bond, $B(s_t, b_t)$ can be formally defined as the solution to the following dynamic program:

$$B(s_t, b_t) = \mathbb{E}_t [\mathbb{M}_{t+1} \chi(s_{t+1}, D_{t+1}) \{ (1 - \tau_0)b + B(s_{t+1}, b) \}]$$

$$+ \mathbb{E}_t [\mathbb{M}_{t+1} [1 - \chi(s_{t+1}, D_{t+1})] v(s_{t+1}^D, 0)]$$

where $D_{t+1} = (1 - \tau)b + B(s_{t+1}, b)$ and $s_{t+1} = (x_{t+1}, z_{t+1}, (1 - \delta + i_{t+1})k_t)$.

Note that both the default function, $\chi$, and the recovery in the event of default depend on the net worth of the shareholders, $v$, and the optimal investment rate, $i_{t+1}$. This shows, in particular, that the firm’s investment decision can affect directly the amount of principal that the firm can borrow from the bond market. As a
consequence, the firm will choose the optimal investment rate by taking this effect into account.

2.6 The Dynamics of Equity

The results of the previous sections allow me to finally define the dynamics of \( v(s_t, D_t) \) - the net worth of the shareholders of a solvent firm.

\[
v(s_t, D_t) = (1 - \tau)(y_t - \delta k_t) + \delta k_t - D_t + \max_{i_{t+1}, b_{t+1}, F_{t+1}} \left\{ -[i_{t+1}k_t + h(i_{t+1}, k_t)] + F_{t+1} + \mathbb{E}_t[M_{t,t+1}v(s_{t+1}, D_{t+1})^+] \right\},
\]

subject to the constraint:

\[
F_{t+1} \leq B(s_t, b_{t+1})
\]

where \( B(s_t, b_{t+1}) \) is defined recursively in Equation (9), given \( v \) and \( i_{t+1} \), and where \( D_{t+1} = (1 - \tau)b_{t+1} + B(s_{t+1}, b_{t+1}) \), \( s_{t+1} = (x_{t+1}, z_{t+1}, k_{t+1}) \) and \( k_{t+1} = (1 - \delta + i_{t+1})k_t \).

2.7 General Model

This section presents the most general version of the model, which accounts for two important frictions: issuance costs on cash inflows (new equity or debt) and taxes on distributions. The issuance costs induce a slight delay in adjusting to the optimal capital level or capital structure, and help stabilize the stationary distribution of leverage ratios. The tax on distributions help increase the tax advantage of debt, and this is especially useful given the calibrated values for the corporate and personal tax rates (see the calibration section, bellow).

The dynamics of the net worth of the shareholders, conditional on the firm being
solvent, can now be described as follows:

\[ v(s_t, D_t, b_t) = (1 - \tau)(y_t - \delta k_t) + \delta k_t - D_t \\
- c^E d_t^E - T(d_t^E) \\
+ \max_{i_{t+1}, b_{t+1}} \{ -[i_{t+1}k_t + h(i_{t+1}, k_t)] \\
+ F_{t+1} - c^D [F_{t+1} - (D_t - (1 - \tau) b_t)]^+ \\
+ \mathbb{E}_t [M_{t+1} v(s_{t+1}, D_{t+1}, b_{t+1})] \}, \tag{12} \]

subject to the constraint:

\[ F_{t+1} \leq B(s_t, b_{t+1}) \quad \text{and} \quad (13) \]

where \( B(s_t, b_{t+1}) \) is defined recursively in Equation (9), given \( v \) and \( i_{t+1} \), and

\[ D_{t+1} = (1 - \tau)b_{t+1} + B(s_{t+1}, b_{t+1}) \]

\[ d_t = [(1 - \tau)(y_t - \delta k_t) + \delta k_t] + [-i_{t+1}k_t - h(i_{t+1}, k_t)] + [F_{t+1} - D_t] \\
- c^D [F_{t+1} - (D_t - (1 - \tau) b_t)]^+ \tag{14} \]

Note that the issuance costs, \( c^E \) and \( c^D \), are expressed as percentages of the amount of external financing raised. The total tax liability on distributions, \( T(d_t^E) \), can be nonlinear, reflecting a variable tax rate (see the calibration section below).

The model is solved numerically using a unique computational method. The method and the supporting results are presented in Appendix A.

### 3 Results

This section presents the main findings of my model. I introduce them in several steps. First, I describe the calibrated values of the parameters of the model and report statistics for the relevant quantities, in the simulated economy. Second, I provide a
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. PRODUCTIVITY SHOCKS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence of aggregate productivity</td>
<td>$\rho_x$</td>
<td>0.983</td>
</tr>
<tr>
<td>Conditional volatility of aggregate productivity</td>
<td>$\sigma_x$</td>
<td>0.00233</td>
</tr>
<tr>
<td>Long-run average of the aggregate productivity</td>
<td>$\bar{x}$</td>
<td>-3.47</td>
</tr>
<tr>
<td>Persistence of idiosyncratic productivity</td>
<td>$\rho_z$</td>
<td>0.97</td>
</tr>
<tr>
<td>Conditional volatility of idiosyncratic productivity</td>
<td>$\sigma_z$</td>
<td>0.1</td>
</tr>
<tr>
<td>II. CAPITAL DYNAMICS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share</td>
<td>$\alpha$</td>
<td>0.3</td>
</tr>
<tr>
<td>Monthly depreciation rate</td>
<td>$\delta$</td>
<td>0.01</td>
</tr>
<tr>
<td>Adjustment cost of investment</td>
<td>$\theta$</td>
<td>15</td>
</tr>
<tr>
<td>III. PRICING KERNEL</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>0.994</td>
</tr>
<tr>
<td></td>
<td>$\gamma_0$</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>$\gamma_1$</td>
<td>-1000</td>
</tr>
<tr>
<td>IV. TAXES</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax rate on corporate income</td>
<td>$\tau$</td>
<td>0.35</td>
</tr>
<tr>
<td>Tax rate on interest income</td>
<td>$\tau_0$</td>
<td>0.349</td>
</tr>
<tr>
<td>Limiting tax rate on dividends</td>
<td>$\overline{\tau}_d$</td>
<td>0.12</td>
</tr>
<tr>
<td>Marginal tax rate on dividends</td>
<td>$\phi$</td>
<td>0.02</td>
</tr>
<tr>
<td>V. ISSUANCE COSTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seasonal equity</td>
<td>$c^E$</td>
<td>0.03</td>
</tr>
<tr>
<td>Seasonal debt</td>
<td>$c^D$</td>
<td>0.01</td>
</tr>
<tr>
<td>VI. BANKRUPTCY PROCESS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bankruptcy costs</td>
<td>$\xi$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 1: Calibrated values for the parameters of the model

numeric analysis of the impact of the financial leverage on the cross-sectional properties of stock returns associated with book-to-market equity and firm size. Third, I present the results of the replicated empirical experiments on simulated panels of data. Fourth, and final, I provide a thorough analysis of the driving forces behind the results.

3.1 Calibration

The parameters of the model and their calibrated values are summarized in Ta-
Table 2: Simulated moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average annual interest rate</td>
<td>0.0173</td>
<td>0.018</td>
</tr>
<tr>
<td>Annual volatility of interest rate</td>
<td>0.024</td>
<td>0.03</td>
</tr>
<tr>
<td>Average annual market return</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>Annual market Sharpe ratio</td>
<td>0.42</td>
<td>0.45</td>
</tr>
<tr>
<td>Average book-to-market ratio</td>
<td>0.39</td>
<td>0.67</td>
</tr>
<tr>
<td>Annual leverage ratio</td>
<td>0.36</td>
<td>0.32</td>
</tr>
<tr>
<td>Average coverage ratio</td>
<td>3.48</td>
<td>4.05</td>
</tr>
<tr>
<td>Average annual rate of investment</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>Average annual rate of disinvestment</td>
<td>0.014</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The choice of the calibrated values is generally consistent with observable moments in the data, which relate directly or indirectly with the actual parameters. The set of parameters driving the persistence ($\rho_x$) and the conditional volatility ($\sigma_x$) of the aggregate productivity shocks is calibrated to be in line with the quarterly estimates of Cooley and Prescott (1995).\(^9\) The long-run average, $\bar{x}$, is chosen so that the rate of return on a unit of invested capital, for an average firm, is about 30 percent per year, consistent with the choice in Berk et al.. It is important to notice that the long-run average of the aggregate productivity shock does not affect firms’ equity returns, since the market price of risk relates to the actual productivity shock through deviations from $\bar{x}$ rather than levels.\(^10\) Nevertheless, the long-run average of the productivity shock determines the long-run value of one unit of installed capital.

The set of parameters driving the dynamics of the idiosyncratic productivity shock ($\rho_z$ and $\sigma_z$) is chosen such that the first two moments of the model-implied cross-

---

\(^9\)Cooley and Prescott estimate the total factor of productivity from the Sollow residual, implied by the neoclassical growth model, on a balanced growth path. The fact that my model does not have labor creates a slight discrepancy between the parameters driving the dynamics of the aggregate shock in Cooley and Prescott and the corresponding set of parameters in my model.

\(^10\)The actual specification of the market price of risk and the pricing kernel is provided in Appendix A.
sectional distribution of stock returns are close to their empirical counterparts. This calibration approach is similar to the one in Gomes (2001) and Zhang (2005).

The set of parameters driving the capital flow is calibrated as follows: the capital share $\alpha$ is similar to the choice in Kydland and Prescott (1982), the monthly depreciation rate, $\delta$, is consistent with the estimates in Cooper and Haltiwanger (2000), and, finally, the adjustment cost parameter, $\theta$, is similar to the choice in Zhang (2005) and is consistent with the empirical estimates of Whited (1992).\footnote{The choice for the value of $\theta$ is close to the lower end of the range of estimates proposed in the macroeconomic literature. For instance, Hall (2001) argue that the appropriate range of values for $\theta$ is 8 to 32 quarters. Hall also provides a careful review of the values proposed previously in the literature.}

The parameters driving the dynamics of the pricing kernel, $\beta$, $\gamma_0$ and $\gamma_1$ are chosen so that the model-implied average Sharp ratio, average real interest rate and volatility of the interest rates matches closely the ones in the data. An exact comparison between the model-implied values and the values in the data is provided in Table 2.

The corporate tax rate, $\tau$, corresponds to the tax rate of the U.S. corporations in the highest tax bracket (35 percent), while the personal tax rate on interest income, $\tau_0$, is consistent with the empirical estimate of 35.1 percent, reported by Graham (1999). Since the model does not distinguish between dividend distributions and equity repurchases, I choose to treat shareholder payout distributions as dividends. However, in general, the tax on interest income differs from the tax on distributions, and consequently I need to model the later, separately. I assume that the marginal distribution tax rate is specified as in Hennessy and Whited (2005). Namely, the tax liability on dividends is given by:

$$T(q) = \int_0^q \tau_d \left(1 - e^{-\phi s}\right) ds$$

I further follow these authors in calibrating $\phi = 0.02$ and $\tau_d = 0.12$, consistent with empirical estimate of 12 percent, reported in Graham (2000).

The issuance costs for both equity and debt are modeled simply as a percentage
of the dollar value of the issue. The parameter value for the debt issuance cost is consistent with the empirical estimates of Mikkelson and Partch (1986), who report underwriting cost for seasonal debt issues of 1.3 percent. The parameter value for the seasonal equity issuance cost is consistent with the lower end of the estimates reported by Corwin (2003) and with the estimates of Gomes (2001) and Hennessy and Whited (2005).

Finally, the bankruptcy parameter, $\xi$, which captures the amount of assets sold by the creditors to cover for the direct costs of bankruptcy process, is consistent with the empirical estimates of Weiss (1990), of approximatively 2.8 percent.

In order to investigate the extent to which the model can reproduce the first moments of some of the cross-sectional distributions of interest, I simulate 20 samples of 4000 firms, over 1000 months and compare the model-implied moments with the ones in the data. Note that the large number of months, in each sample, ensures that the cross-sample averages approach their population moments.

Since the focus of this paper is on the cross-sectional relation between average stock returns and firm characteristics such as size, book-to-market equity and the leverage ratio, the model should do a reasonable job in matching the first moments of the cross-sectional distributions of both the average equity returns and the firm characteristics. Table 2 reports the model-implied moments of these distributions and their empirical counterparts. The data for the level and volatility of real interest rates as well as the return of the equally weighted market portfolio comes from Campbell, Lo and MacKinlay (1997). The average Sharpe’s ratio is provided in Campbell and Cochrane (1999). The average book-to-market equity comes from Pontiff and Schall (1999). The mean coverage ratio is provided in Rajan and Zingales (1995). Finally, the annual average rates of investment and disinvestment are provided in Abel and Eberly (2001).
3.2 The Size and Book-to-Market Effects

In this section, I use simulated analysis to investigate the impact of financial leverage on the cross-sectional properties of stock returns, associated with book-to-market equity and firm size. Specifically, I investigate whether financial leverage affects the simulated distributions of the slope estimates of the univariate cross-sectional relations between realized equity returns and firm size or book-to-market equity.

For ease of comparison, I first investigate the shape of these distributions for an economy populated with all-equity-financed firms. The goal is to see whether the empirical estimates for these slopes (provided for instance by Fama and French (1992)) could come from the simulated distributions produced with this model.

Using the parameter values reported in Table 1, I solve the version of the model that allows firms to finance themselves with equity only. Then, I follow Berk et al. in generating panels of data and further replicating the univariate regressions performed in Fama and French (1992). I simulate 200 panels of data containing $2000 \times 330$ firm-month data points, and I run the cross-sectional univariate regressions of realized equity returns on firm size or book-to-market equity every month, for each sample. The size of each data panel is chosen to be consistent with the size of the data panel employed by Fama and French. Figure 2 plots the simulated distribution of the implied slope coefficient for each of the two effects.

In both cases, the slope coefficients estimated by Fama and French fall well within the body of the simulated distribution, suggesting that these cross-sectional effects could be generated with this version of the model. This is a reassuring result, since we already know that models of firm dynamics for all-equity-financed firms - such as Berk et al. and Gomes et al. - can generate the directions and the magnitudes of these cross-sectional effects.

Next, I investigate the impact of financial leverage on the distributions of slope estimates associated with these cross-sectional effects. As before, I use the parameter
values of Table 1, with some exceptions (listed below), to solve the general version of the model and then to simulate panels of data. To see whether financial leverage has a meaningful impact on these distributions, I simulate two distinct economies: one populated with firms financed with moderate levels of debt and another populated with firms holding high leverage ratios. The first economy consists of both leveraged and all-equity-financed firms, where leveraged firms hold moderate levels of debt on their books, throughout the entire sample. The set of parameters used in this simulation is precisely the one in Table 1. The second economy consists of mostly leveraged firms holding large amounts of debt on their books throughout the sample. To induce such a scenario, I use alternative values for some of the parameters. Specifically, I assume bankruptcy costs of $\xi = 3\%$, a tax-advantage of debt of $\tau - \tau_0 = 15\%$ and no issuance costs or tax charges on dividend distributions.

Figure 3 plots the simulated distribution of the slopes of the univariate cross-sectional regressions between equity returns and book-to-market equity or firm size, for these two economies. The top two plots correspond to the first economy, while the bottom two plots correspond to the second economy. It can be noticed that in both
Figure 3: The impact of financial leverage on the size and book-to-market effects: The plots show the simulated distributions of the slope coefficient of each of the following cross-sectional regressions: \( R_{i,t+1} = \alpha_{t+1} + \beta_{t+1} \log \frac{BE_i}{ME_i} + \epsilon_{i,t+1} \) (the two plots on the left) and \( R_{i,t+1} = \alpha_{t+1} + \beta_{t+1} \log ME_i + \epsilon_{i,t+1} \) (the two plots on the right). The top two pictures correspond to an economy with moderate levels of debt, while the bottom two correspond to an economy with high levels of debt. "FF Estimate" stands for the Fama-French estimate of the corresponding coefficient.

In cases, the simulated distributions for the size and book-to-market coefficients contain the corresponding estimates of Fama and French. However, these distributions are quite different in shape, across these economies. For the book-to-market effect, for instance, the simulated distribution of the slope coefficient shifts to the right (towards a stronger effect) for the economy with more leverage. A similar shift can be observed also for the other cross-sectional effect due to firm size. These plots restate pictorially an important implication of my model, namely, that in a leveraged economy, the size and the book-to-market effects become stronger as the economy becomes more prone...
to the risk of default. This implication will be discussed in more details in the following sections.

3.3 Cross-sectional Regressions

In this section, I concentrate on the cross-sectional properties of the model. I focus mainly on the cross-sectional relations between realized equity returns and firms characteristics such as book-to-market equity, firm size and debt/equity ratio. To allow for a fair comparison between the model implied magnitudes and the ones in the data, I replicate closely the cross-sectional experiments in two studies, namely Fama and French (1992) and Bhandari (1988).

Since the objective of this paper is to understand the impact of financial leverage on the cross section of equity returns, the choice for these two studies seems appropriate. Fama and French investigate the impact of book-to-market equity and firm size on equity returns, relative to other measures of risk including market beta, market leverage - the ratio of book assets to market equity - and book leverage - the ratio of book assets to book equity. Bhandari investigates the impact of debt/equity ratio on equity returns relative to market beta and firm size.

To replicate the empirical experiments in these studies, I construct 200 panels of data consisting of $2000 \times 1000$ firm-month datapoints. The first 100 observations are dropped to minimize the impact of potentially suboptimal starting values for the state variables. These samples have similar cross-sectional properties with the samples used in the above empirical studies. Table 2 summarizes the first and, in some cases, the second moments of several key variables such as the equally-weighted market return, the book-to-market equity ratio, the leverage ratio and the coverage ratio.

Both studies use pre-ranking betas for portfolio formation purposes. These betas are usually obtained by regressing a firm’s stock returns in excess of the risk-free rate on the market excess returns for a window of time that precedes the portfolio
formation period. All stocks entering the portfolio formation period have to be solvent for the entire time horizon spanned by the pre-formation window.

I first replicate the empirical exercise in Fama and French, Table III. The explanatory variables considered here are: the market betas, $\beta$, the firm size, $\log ME$, the "book-to-market ratio" or the ratio of book equity to market equity, $\log \frac{BE}{ME}$, the "market leverage ratio" or the ratio of book assets to market equity, $\log \frac{BA}{ME}$, and finally, the "book leverage ratio" or the ratio of book assets to book equity, $\log \frac{BA}{BE}$. All these firm characteristics are computed using their accounting definitions. Table 3, Panel B reports my results. For ease of comparison, Panel A presents the corresponding empirical regressions of Fama and French. The slope coefficients (t-statistics) are the time series averages (divided by the time series standard deviations), averaged across samples. The last three columns report the 20%, 50% and 80% percentiles of either the slope or the t-Statistic of the month-by-month cross-sectional regressions, for the univariate regressions on firm size and book-to-market equity.

The first regression investigate the univariate linear relation between the realized equity returns and the market betas. We notice that the model can generate the direction of this relation, yet it overestimates, considerably, the magnitude of the slope coefficient. This could be due to the fact that both the realized equity returns and the market beta are sensitive to financial leverage, especially during downturns. In particular, one might suspect that either the firm size or the leverage ratio should capture a fraction of this magnitude. In fact, as the following regressions will show, this turns out to be exactly the case. In particular, the third regression reporting the impact of firm size on the univariate relation between equity returns and market betas shows that firm size can actually drive out completely market beta and even induce a change in the direction of the relation (fact which is consistent with the empirical result of Fama and French, reported in Panel A). The impact of financial leverage on this relation will be reported as part of the results associated with the
empirical experiment in Bhandari.

The second and fourth regression report the univariate cross-sectional relation between realized equity returns and firm size or book-to-market equity, respectively. As it can be noticed, the model does an excellent job in capturing both the direction and the magnitude of the first relation. However, while the model still captures the direction of the second relation, it underestimates almost four times the magnitude of the slope coefficient.

The sixth regression reports the bivariate relation between realized equity returns, firm size and book-to-market equity. We notice that the results from the simulated data resemble closely those reported by Fama and French. Specifically, each regressor enters with the correct sign and the correlation between regressors leads to a slight decrease in the magnitude of the slope coefficients, relative to the univariate case. More importantly, the magnitude of the slope coefficient associated with firm size is very close to the empirical magnitude (reported in Panel A), while the drop in magnitude of the other slope coefficient (of about 40%), relative to the univariate case, is comparable to the one observed in Panel A (about 30%).

The fifth and the seventh regressions investigate the cross-sectional relation between realized equity returns and financial leverage. The measure of financial leverage employed by Fama and French relies on the joint effect of two firm characteristics, namely: the "market leverage" and the "book leverage". The former is defined as the ratio of book assets to market equity, while the later as the ratio of book assets to book equity. The first regression investigates the sole impact of financial leverage on the cross-section of equity returns. We notice that both market leverage and book leverage enter with the correct sign and, furthermore, that the slope coefficient associated with market leverage is close in magnitude to the slope coefficient associated with book-to-market equity (regression four). This fact is consistent with the corresponding empirical results, reported in Panel A. The small difference in magnitude
between the slope coefficients of regression five, in Panel A, led Fama and French to the conclusion that the joint impact of market and book leverage on the cross-section of equity returns is equivalent to the impact of the book-to-market equity. In my model, however, the magnitude of the slope coefficient associated with book leverage is about 75% higher than the magnitude of the slope coefficient associated with market beta (as opposed to only 14% in Fama and French). This clearly indicates that, in my model, book-to-market equity cannot fully account for the impact of financial leverage, when the later is measured a la Fama and French. The fact that book leverage contains more information about the cross-section of equity returns than whatever is captured through book-to-market equity can also be noted from the second regression that evaluates the impact of financial leverage. Here, while the sign and the magnitudes of the slope coefficients associated with both firm size and market equity are close to their empirical counterparts, the sign and the magnitude of the slope coefficient associated with book leverage depart substantially from their empirical benchmarks. This fact is the result of a strong financial leverage effect as shown further in Table 4, associated with the empirical experiment in Bhandari.

I now focus on the empirical exercise in Bhandari (1988). In each sample, every 24 months, stocks are ranked on the basis of their firm size and divided into three groups containing approximately equal numbers of stocks. Within each of these groups, stocks are ranked on their pre-ranking betas and divided into three groups with similar numbers of stocks. Each of the resulting nine portfolios, is further subdivided into three other portfolios depending on the leverage ratio. I obtain twenty seven equally-weighted portfolios which will become the basis for my tests. For each portfolio I compute the returns for the following 24 months. This yields a time series of returns for each portfolio, for the entire sample period. Pre-ranking betas are computed 24 months before the formation period, from the previous 24-month window. Portfolio betas are computed from the 24-month window prior to the formation period. These
portfolio betas will be further used in my cross-sectional tests.\textsuperscript{12}

For the computation of all the other firm characteristics, I use the most recent available information. The empirical exercise consists of regressing each month the cross-section of realized equity returns on the cross-section of market betas, log of the firm size and the debt/equity ratio. For each sample the estimate (t-statistic) of the slope coefficients are the time-series averages (divided by the time-series standard deviation), and averaged across samples. The results are reported in Table 4, Panel B. For comparison, Table 4, Panel A reports the results in Bhadari (1988), Table II.

The first three regressions of Table 4 are univariate regressions of the realized equity returns on market betas, firm size and debt/equity ratio, respectively. Similarly with the results in Table 4, the model does very well in matching the direction and strength of the linear relation between equity returns and firm size but it overestimates the magnitude of the slope coefficient in the linear relation between equity returns and market betas. As mentioned previously, this large coefficient may simply reflect the fact that both market betas and equity returns react similarly to financial leverage, especially during low productivity times. The fourth and the fifth regressions confirm this hypothesys, as the market beta quickly loses both economic and statistical significance, once we account for either firm size or debt/equity ratio.

The model underestimates the magnitude of the slope coefficient associated with the debt/equity, relative to its empirical counterpart. This fact can be noticed in all regressions involving this firm characteristics. The reason behind the underestimation is not the fact that the leverage effect (as measured by the debt/equity ratio) is not strong enough, but rather that, over time, the cross-sectional distribution of debt/equity ratio grows slowly in skieweness, towards higher values. This fact can be further exemplified with the last two regressions that show clearly that debt/equity

\textsuperscript{12}The procedure outlined here is nothing more than a two-stage instrumental variable approach. This methodology is usually efficient when the regressor is poorly observed, which is the case for the market betas. All other firm characteristics are observed without error, and this methodology need not be used in their case.
To summarize, the model can account qualitatively and, sometimes, quantitatively for most of the cross-sectional properties of equity returns associated with market betas and firm characteristics such as firm size, book-to-market equity, market leverage, book leverage and debt/equity ratio. The cross-sectional effect associated with financial leverage is slightly stronger than the one reported in Fama and French as well as in Bhandari, while the cross-sectional effect associated with book-to-market equity is slightly weaker. In all instances, the model captures the fact that many of these firm characteristics reduce the explanatory power of market betas, consistent with the results reported by the above-cited empirical studies.

### 3.4 Causality

The results of the previous two sections suggest that my model can capture some of the properties of the cross-section of equity returns. The next step is to understand what features of the model are behind these results. I proceed in several steps: First, I derive an approximate relation between the market value of a firm's equity and its corresponding book-to-market equity ratio and firm size, and then I investigate the extend to which financial leverage changes these relations. Second, I use the relations developed in the first step to understand the determinants of equity risk premia. Third, and final, I present the role of investment and debt financing relative to equity risk. For ease of exposition, all the formulas in the first two steps are derived within the framework of the simpler version of the model (without issuance costs or taxes on dividend distributions).

#### 3.4.1 How Do Firms Derive Value?

Suppose that at time $t$, the technological process of a solvent firm is summarized by the vector of state variables $s_t$, while its effective financial leverage is summarized
by $D_t$. Let $V$ denote the market value of the firm. Following the notation in Section 2, I can write:

$$
V(s_t, D_t, b_t) = \chi(s_t, D_t) [v(s_t, D_t) + (\tau - \tau_0)b_t + D_t] + (1 - \chi(s_t, D_t)) v(s^D_t, 0)
$$

$$
= V^u(s_t) + Z(s_t) + (\tau - \tau_0)b_t \chi(s_t, D_t) - \Delta v(s_t) (1 - \chi(s_t, D_t)).
$$

(15)

$V^u$ is defined recursively by:

$$
V^u(s_t) = (1 - \tau)(y_t - \delta k_t) + \delta k_t - [i_{t+1}k_t + h(i_{t+1}, k_t)] + \mathbb{E}_t [M_{t,t+1}V^u(s_{t+1})]
$$

(16)

and $Z$ is defined as follows:

$$
Z(s_t) = \sum_{t' \geq t+1} \mathbb{E}_t [M_{t,t'} \{ (\tau - \tau_0)b_{t'} \chi(s_{t'}, D_{t'}) - \Delta v(s_{t'}) [1 - \chi(s_{t'}, D_{t'})] \}]
$$

(17)

where $\Delta v(s_t) = v(s_t, 0) - v(s^D_t, 0)$ and $v$ is the value of equity defined in (10). The investment policy, $i_{t+1}$, in the above dynamics, is assumed to be the optimal investment policy defined in (10)-(11).

The three components of $V$ have natural interpretations: On one hand, $V^u$ can be interpreted as the market value of a similar, but unleveraged firm, following (sub-optimally) the investment policy of the leveraged firm, and, on the other hand, the last two components capture the contemporaneous and the future net contributions to the value of the firm from using debt financing.

Since $V^u$ incorporates the output generated by the firm’s technology, this component has a major impact on the value of the firm. For the rest of this section, I focus on relating $V^u$ to the value of installed capital (or assets in place) and the value of capital expansion options (growth options).

I start by linearizing the production function around the depreciated value of capital. Thus, the first order Taylor decomposition of $k^\alpha_{t+1}$ around $(1 - \delta)k_t$ yields:

$$
k^\alpha_{t+1} = (1 - \delta)^{(\alpha-1)}k_t^\alpha [1 - \delta + \alpha i_{t+1}] + o(i_{t+1}k_t)
$$
where \( o(i_{t+1}k_t) \) is the approximation error, which depends on the actual investment at time \( t + 1 \), namely \( i_{t+1}k_t \).

If the errors \( o, \) are small enough, I can decompose the firm’s output, in each of the subsequent periods as follows:

\[
y_{t+N} = e^{\theta_{t+N}k_{t+N}} = e^{\theta_{t+N}} \left[ (1 - \delta)^{(t+N)\alpha} k_t^\alpha + \alpha \sum_{s=0}^{N-1} (1 - \delta)^{(t+N-s)\alpha-1} i_{t+s+1}k_{t+s}^\alpha \right]
\]  

(18)

where \( \theta_{t+N} = x_{t+N} + z_{t+N} \) is the combined technology shock at time \( t + N \).

Using the recursive definition of \( V^u \), in (16), and the approximation for the firm’s output, from the previous equation, I can express the contribution of \( V^u \) to the market value of the firm as the combined contribution of the assets in place and the growth options.

**Proposition 1**

The contribution of \( V^u \) to the value of the firm can be approximated with the following formula:

\[
V^u_{t+1} = k_{t+1}^\alpha A_{t+1} + I_{t+1} + \delta \tau k_{t+1} K_{t+1}
\]  

(19)

where the terms of the summation are given by:

\[
A_{t+1} = [1 - \tau] \sum_{s=t+1}^{\infty} [1 - \delta]^{\alpha(s-t-1)} \mathbb{E}_{t+1} \left[ M_{t+1,s} e^{\theta_s} \right]
\]

\[
I_{t+1} = \sum_{s=t+1}^{\infty} \mathbb{E}_{t+1} \left[ M_{t+1,s} NPV_s \right]
\]

\[
K_{t+1} = \sum_{s=t+2}^{\infty} \mathbb{E}_{t+1} \left[ M_{t+1,s} \Pi_{t+1}^s [1 - \delta + i_s] \right],
\]

(20)

and where

\[
NPV_s = - [i_{s+1} k_s + h(i_{s+1}, k_s)] + [1 - \tau] \alpha i_{s+1} k_s^\alpha \sum_{n=1}^{\infty} [1 - \delta]^{\alpha n-1} \mathbb{E}_s \left[ M_{s,s+n} e^{\theta_{s+n}} \right]
\]  

(21)
The intuition behind the summation terms of the previous decomposition result is straightforward. On one hand, $A_{t+1}$ captures the present value of the future cash flows generated by one unit of capital installed at time $t + 1$, while, on the other hand, $I_{t+1}$ captures the combined net present value of the current and future investment opportunities. In addition, firms also create value through the tax savings on depreciated capital, $K_{t+1}$, but the contribution through this channel is marginal due to the small scaling factor $\tau \delta$.

In this model, the relative importance of the component associated with the assets in place (i.e. $A_{t+1}$) versus the component associated with the growth options (i.e. $I_{t+1}$) is driven by the scale of the installed assets, $k_{t+1}^\alpha$. Thus, younger firms, with fewer assets in place, derive most of their value through the latter component, not only because the scaling factor in front of $A_{t+1}$ is small, but also because the net present values of the investment opportunities, the $NPV_s$, are higher when the capital stock is lower. This later prediction of the model follows from the fact that the decreasing returns-to-scale feature of technologies and the monotonicity in capital of the adjustment cost function increase the marginal cost per unit of investment as firms accumulate more and more capital. In addition, the value derived through the later component is further enhanced when the business cycle is favorable. This is a direct consequence of one of the model’s assumptions, namely that the market price of aggregate (systematic) productivity risk is counter-cyclical. However, as firms accumulate more capital, the value of their growth options decreases the value of their assets in place increases.

The following section investigates the risk dynamics within a firm, as the distribution of relative values of the assets in place and growth options changes through time.
3.4.2 Risk Dynamics and Firm Characteristics

In order to compute the firm’s expected return on equity, at time $t$, I can assume, without loss of generality, that the firm is solvent at time $t$. In particular, let $ME_t$ denote the ex-dividend market value of equity (or the purchasing price of equity) and $R_{t+1}$ denote the realized gross return on equity. Then

\[ ME_t = V^u(s_t) + Z(s_t) - D_t - d_t \]
\[ R_{t+1} = \frac{\chi(s_{t+1}, D_{t+1})v(s_{t+1}, D_{t+1})}{ME_t} \]

(22)

where $d_t$ is the payout distribution towards the shareholders.

**Proposition 2** Suppose $B(s_t, b_{t+1})$, $b_{t+1}$ and $i_{t+1}$ are defined as in (11) – (12). Then, the expected gross return on the firm’s equity is given by the following formula:

\[ \mathbb{E}_t [R_{t+1}] = \pi_t^u + \frac{B(s_t, b_{t+1})}{ME_t} \mathbb{E}_t \left[ \{ R_{t+1}^u - R_{t+1}^B \} \chi(s_{t+1}, D_{t+1}) \right] + O_t \]

(23)

where $R_{t+1}^u = \frac{V^u(s_{t+1})}{V^u(s_t) - d_t}$, $\pi_t^u = \mathbb{E}_t \left[ R_{t+1}^u \chi(s_{t+1}, D_{t+1}) \right]$, $R_{t+1}^B = \frac{(1-\tau_0)b_{t+1} + B(s_{t+1}, b_{t+1})}{B(s_t, b_{t+1})}$, $d_t^u = d_t - [B(s_t, b_{t+1}) - D_t]$, and where the error term $O_t$ is described in Appendix B.

This proposition shows that, in my model, the wedge between expected equity returns and expected asset returns, $R_{t+1}^u$ is driven by two terms: one which depends on the debt to equity ratio and the return on the firm’s assets in excess of the returns on the firm’s debt, and another, $O_t$, which captures the marginal impact of the future tax shields and potential bankruptcy costs on the value of the firm, until default. In particular, one can easily notice that if the tax advantage of debt is zero and the bankruptcy is costless, the last term drops out and the result reduces to Proposition II of Modigliani and Miller (1958).

I can also read the previous result in terms of the debt capacity of the firm, by accounting directly for the dynamics of $B$ and $Z$ in the previous decomposition result. For more details on the derivation of the next result see Appendix B.
Proposition 3

The expected equity returns can be computed as:

\[ E_t[R_{t+1}] = \pi_t^u + \gamma_t E_t \left[ \frac{V^u(s_{t+1})}{ME_t} \chi(s_{t+1}, D_{t+1}) \right] + E_t \left[ \nu_{t+1} \frac{D_{t+1} - Z(s_{t+1})}{ME_t} \chi(s_{t+1}, D_{t+1}) \right] \]

(24)

where \( \gamma_t = E_t[M_{t,t+1} R^u_t (1 - \chi(s_{t+1}, D_{t+1}))], \nu_{t+1} = \pi_t^u M_{t,t+1} - 1 \) and \( Z \) is defined in (17).

This result shows that the spread between expected equity returns and expected asset returns is driven by two components: one capturing the impact of the assets in place and growth options, \( \frac{V^u(s_{t+1})}{ME_t} \), and the other capturing the impact of the debt capacity, \( \frac{D_{t+1} - Z(s_{t+1})}{ME_t} \).

The decomposition result of the previous section allows me to outline the impact on equity returns of the assets in place and growth options through the first component:

\[ \frac{V^u(s_{t+1})}{ME_t} = k^\alpha_{t+1} A_{t+1} + \frac{1}{ME_t} I_{t+1} + \delta \tau k^\alpha_{t+1} K_{t+1} \]

(25)

At this time, however, it is worth noticing that this component is not the only one reflecting the impact of assets in place and growth options on equity returns. A great deal of this impact is captured through \( \pi_t^u \), as well. The next result combines these two observations to allow us to obtain a cleaner view of the sources of variation in equity returns:

Proposition 4

The contribution to expected equity returns of the assets in place, the growth op-
tions and the tax-advantage of depreciation is given, respectively by:

\[ c^a_t = \frac{k^{\alpha}_{t+1}}{MA_t} + \gamma_t \frac{k^{\alpha}_{t+1}}{ME_t} \]
\[ c^o_t = \frac{1}{MA_t} + \gamma_t \frac{1}{ME_t} \]
\[ c^\delta_t = \frac{k^{\delta}_{t+1}}{MA_t} + \gamma_t \frac{k^{\delta}_{t+1}}{ME_t} \]  

(26)

Expected equity returns can now be decomposed as follows:

\[ E_t [R_{t+1}] = c^a_t E_t [A_{t+1} \chi(s_{t+1}, D_{t+1})] + c^o_t E_t [I_{t+1} \chi(s_{t+1}, D_{t+1})] + \delta \tau c^\delta_t E_t [K_{t+1} \chi(s_{t+1}, D_{t+1})] + E_t \left[ \nu_{t+1} \frac{D_{t+1} - Z(s_{t+1})}{ME_t} \chi(s_{t+1}, D_{t+1}) \right] \]  

(27)

This result shows that the distribution of relative values of the assets in place and growth options is an important determinant of equity risk premia. In fact, since the multiplicative factors in front of the third and the fourth terms, namely \( \tau \delta \) and \( \nu_{t+1} \) are positive\(^{13}\) but small in magnitude, this distribution becomes the main determinant.

\(^{13}\)To make this point, I use the dynamics of \( V^u \) to rewrite \( \pi^u_t M_{t,t+1} \) as:

\[ \pi^u_t M_{t,t+1} = q_t \left[ 1 - \text{cov}_t(M_{t,t+1}, R^u_{t+1}) \right] \frac{M_{t,t+1}}{E_t[M_{t,t+1}]} \]  

(28)

where \( q_t = \frac{E_t[R^u_{t+1} \chi(s_{t+1}, D_{t+1})]}{E_t[R^u_{t+1}]} \).

In my model, the covariance term, \( \text{cov}_t(M_{t,t+1}, R^u_{t+1}) \) is negative and its magnitude increases with the magnitude of the market price of risk. In particular, due to the counter-cyclicality of the market price of risk, the magnitude of the covariance term decreases with the level of the aggregate productivity shock. In a similar manner, the scaling term \( q_t \) is likely to be higher when the economy is further from a recession, because default is less likely (due to the persistence of the productivity shock). In the model, even though the two factors change in opposite directions in response to changes in the aggregate productivity shock, the product \( q_t \left[ 1 - \text{cov}_t(M_{t,t+1}, R^u_{t+1}) \right] \) is in general larger than one. This is due to the fact that the shape of the no-default region changes too slowly relative to the covariance factor, inducing the later to dominate the product. Moreover, due to the persistence of the aggregate productivity shock, the deviations of the pricing kernel from its conditional mean are small, relative to the magnitude of the covariance factor, especially when the state space is reduced to the no-default region.

To evaluate the impact of the pricing kernel on the product \( q_t \left[ 1 - \text{cov}_t(M_{t,t+1}, R^u_{t+1}) \right] \), notice first that, I only have to worry about the no-default states (due to the indicator function in Equation (24).
Figure 4: The Optimal Investment Policy: The left picture represents the optimal investment rate of a leveraged firm, as plotted against capital. The right picture compares the optimal disinvestment policy of a leveraged vs. an all-equity-financed firm.

of the equity risk premia.

More importantly, since both \( A_{t+1} \) and \( I_{t+1} \) depend on the firm-specific productivity shocks, the distribution of the relative values of the assets in place and growth options can be summarized in terms of the realization of the firm-specific productivity shock and a set of firm variables, including book-to-market equity \( \frac{k_{t+1}}{ME_t} \), firm size \( \frac{1}{ME_t} \) and financial leverage (through \( \gamma_t \)). In particular, for an all-equity-financed firm this distribution can be summarized in terms of the firm-specific productivity and only two firm variables, namely book-to-market equity and firm size.

### 3.4.3 Investment and the Role of Debt Financing

In this model, innovations to the aggregate productivity shock are the only source of systematic risk. These shocks impact the firms through two channels: the productivity level of their technologies and the change in the their owners’ perception of risk (the price of risk). The first channel allows firms to respond to changes in their productivity level, by adjusting the production scale. Given the persistance of the

\[
\text{in which the stochastic discount factor falls bellow its conditional mean. The restriction to the no-default region and the persistance of the aggregate shock renders the deviations of the pricing kernel from its conditional mean to be small relative to the magnitude of the covariance factor, in the above product. Thus, the covariance factor dominates, in general, the adverse impact of both } q_t \text{ and } \frac{M_t}{M_{t,t+1}E_t[M_{t,t+1}]} \text{, inducing the multiplicative factor } \pi_t^\dagger M_{t,t+1} - 1 \text{ to be non-negative.}
\]
Figure 5: **The Equity Function:** The firm’s market value of equity is plotted against capital and aggregate productivity shocks. The left picture corresponds to a less leveraged firm.

aggregate productivity shock, good news signal more good news, in the near future, and firms, acting in the best interest of their owners, rescale production in order to extract more value for their shareholders. This mechanism works with or without risk-averse shareholders, and with or without debt financing.\(^{14}\)

The second channel has an important impact on a firm’s decision process because it amplifies (de-amplifies) the marginal value of future cash flows, when the current realization of the aggregate productivity shock is high (low). Thus, when the aggregate productivity shock is high, the market price of risk is low and the discount factor is high, inducing firms to reinvest the output, rather than distribute it to their owners. In fact, at times, the marginal value of future cash flows can be so high, that firms with few to moderate number of assets are willing to invest more than their current output. The gap between optimal investment and output is financed with a mix of equity and debt. This mix depends on the firm’s capital stock and level of debt: firms with fewer assets use equity if they reached their debt capacity and a mix of equity and debt if they did not. Firms with many assets use debt when they have not reached their debt capacity and equity otherwise. Figure 4 depicts the optimal

\(^{14}\)To see this mechanism at work in a risk-neutral environment see the models of Hennessy and Whited (2005) (with debt financing) and Berk, Green and Naik (1999), Gomes, Kogan, Zhang (2003), Carlson, Fisher and Gianmarino (2004), Zhang (2005), and others (without debt financing).
investment policy as a function of capital. The left plot depicts the optimal investment rates of firms when the aggregate productivity shock is either high or low. We notice a similar pattern. Depending on the realization of the aggregate productivity shock, firms either invest or disinvest. In addition, firms with more capital invest less in good times but disinvest more in bad times. The right plot shows how financial leverage impact the optimal investment policy. When the aggregate productivity is low, firms disinvest. The amount of capital that they disinvest depends on whether firms are financed partly with debt. The plot shows that leveraged firms tend to disinvest less than all-equity-financed firms. Bellow I show why corporate investment follows the patterns in these plots.

To analyze deeper the optimal investment decision, I investigate the objective function of a firm when its capital structure decision is held fixed. Suppose therefore that $b_{t+1} = b^*$ is fixed. Then, the firm chooses the optimal rate of investment such that:

$$\max_{i_{t+1}} E_t \left[ d_t^- T(d_t^+) - [i_{t+1}k_t + h(i_{t+1}, k_t)] \right] + E_t \left[ v(s_{t+1}, D_{t+1}, b^*)^+ \right] + cov_t \left[ \Delta M_{t,t+1}, v(s_{t+1}, D_{t+1}, b^*)^+ \right]$$

(29)

where $d_t$ is the shareholders’ dividend defined in (8), $D_{t+1} = (1 - \tau)b^* + B(s_{t+1}, b^*)$ and $\Delta M_{t,t+1} = \frac{1}{E_t[M_{t,t+1}]}M_{t,t+1}$.

To formulate the optimal investment decision more precisely, I analyze the impact of the state variables on the discount rates, $v$ and the covariance term.

First, notice that in my model $v$ is an increasing function of the aggregate shock.\footnote{In fact, $v$ is a fixed point in the firm’s optimization problem (10) and it is chosen from the space of functions continuous in all arguments and increasing in the aggregate shock.} This together with the log-normality of the pricing kernel (see Appendix A for the exact functional form) imply that the covariance term in the above optimization program is negative and increasing in the market price of risk. In particular, since the market price of risk is counter-cyclical, this covariance term is more negative when
the aggregate productivity shock is low and less negative otherwise. The covariance term is also sensitive to the level of capital stock. As it is apparent from Figure 5, the larger the capital stock, the larger the sensitivity of equity to systematic risk and therefore the larger the magnitude of the covariance term.

Second, the decreasing returns-to-scale feature of the technologies induce \( v \) to have a steeper slope for lower values of capital (growth options are more valuable for firms with fewer assets).

Finally, notice that when the aggregate productivity shock is low, discount rates are high and shareholders value more dividend distributions.

We can now see that the optimal investment policy is shaped by the size of the discount factors, the size of the covariance term and the concavity of the equity function \( v \). In particular, when the aggregate productivity is low, firms with more assets tend to disinvest more (despite the higher adjustment costs) because the negative impact of the covariance term is stronger for larger levels of capital. Similarly, when the aggregate productivity is high, firms with fewer assets face a steeper \( v \) (valuable growth options), low discount rates and low negative impact from the covariance term. As a consequence these firms invest more.

To understand the role of debt relative to equity risk, I first investigate how innovations in the aggregate productivity shock impact the capital structure decision. Then I investigate how the decision to take more or less debt impacts the systematic exposure of a firm’s equity.

In my model the investment and capital structure decisions happen simultaneously and they are not independent of each other. While the impact of one decision on the other is present in the model, and important in itself, I concentrate first on understanding how firms adjust their capital structure in response to changes in their productivity shocks. For this scope I isolate the capital structure decision by holding fixed the investment decision. For simplicity, I focus on the case with no costly external financing. Suppose therefore that at time \( t \), the investment decision of a
firm is fixed to $i_{t+1} = i^*$. Then the firm’s optimal level of corporate debt, $b_{t+1}$ is determined such that:

$$\max_{b_{t+1}} \mathbb{E}_t [M_{t,t+1} \{(\tau - \tau_0)b_{t+1}\chi(s_{t+1}, D_{t+1}) - \Delta v(s_{t+1})[1 - \chi(s_{t+1}, D_{t+1})]\}]$$

(30)

where $\Delta v(s_{t+1}) = v(s_{t+1}, 0) - v(s_{t+1}^D, 0), s_{t+1} = (x_{t+1}, z_{t+1}, (1 - \delta + i^*)k_t)$ and $s_{t+1}^D = (x_{t+1}, z_{t+1}, (1 - \xi)(1 - \delta + i^*)k_t)$.

We notice that firms choose their optimal level of debt as a result of the trade off, across states, between the tax advantage of debt and the loss in firm value value due to costly bankruptcy. However, unlike the risk neutral models of optimal capital structure\(^{16}\) where the positive and negative sides of the classic trade-off have equal weights and matter only on average (across states), in my model these two opposing sides have different weights and matter state by state. The key element that differentiate my model from the previous models of optimal capital structure is the time-varying stochastic discount factor.

To exemplify, consider the case when the economy goes through a recession (the realization of the aggregate productivity is low). Then, future bad states are more likely (due to the persistence of the aggregate productivity shock) and firms assign them higher weights (the market price of risk is high and the innovation in the aggregate productivity is negative). In this context, a slightly over-leveraged firm could find itself in financial distress, in the near future, if both its productivity level and the future state of the economy show no signs of recovery. Since future bad states are weighted more heavily, a commitment by the firm to increase the level of coupon payments with one dollar leads to a marginal increase in firm value, in terms of the tax advantage of debt, at the expense of a substantial loss in firm value, due to bankruptcy costs. Thus, for this firm, reducing the level of debt seems more attrac-

---

tive than increasing the level of debt. In my model, however, the extent to which this firm can implement the debt reduction policy depends heavily on the firm’s internal resources. That is because when the market price of risk is high, shareholders value more current income than future income and they are reluctant to invest additional money in the firm, at this time.\footnote{This example also outlines the difference between the predictions of my model, relative to the optimal capital structure, and the predictions of a similar model but with constant discount factors. When the economy goes through a recession, my model predicts that average firms (leveraged with some assets) reduce their level of debt and avoid using equity financing to implement this policy. In the hypothetical model, average firms have a less stringent desire to reduce their level of debt, but when they do, they tend to finance part of the debt retirement with new equity (in this economy, shareholders are indifferent between current and future dividend distributions). Thus, during market downturns, the firms in the hypothetical model access the equity market more often, on average, than the firms in the original model. While in a model with constant discount factors the low frequency of equity issuance observed in the data is hard to reconcile with the low issuance costs (see for instance Hennessy and Whited (2005)), in my model, these two quantities are easier to reconcile because the counter-cyclical market price of risk takes away some of the pressure off the issuance costs.}

This observation is more than just an isolated case. Suppose that the investment decision is no longer fixed and that the aggregate productivity shock is low. Then, as seen before, firms tend to disinvest their unproductive capital. For firms that employ no debt, the extent to which they can disinvest depends only on the curvature of their technologies and the size of the adjustment costs. For firms that do employ debt, the extent to which they can disinvest depends also on whether they become more prone to default. In other words, a leveraged firm cannot increase the size of its dividend distributions without reducing the size of its debt burden.

I can now formulate the role of financial leverage relative to equity risk. First, according to the previous argument, leveraged firms tend to disinvest less, in recessions, (as can be seen in the right plot of Figure 4) and consequently they become riskier due to their elevated levels of unproductive capital. Second, leveraged firms have larger equity returns because they tend to trade at lower prices relative to similar, but all-equity-financed firms, especially during times of low productivity. This mechanic effect follows, for instance, from the decomposition result in Proposition 2.
To summarize, financially leveraged firms have higher equity returns because they tend to trade at lower prices and because they maintain elevated levels of capital, when capital is the least productive.

I conclude this section with the observation that due to the effect of financial leverage on prices and disinvestment, leveraged firms tend to be associated with high returns, high book-to-market equity ratios and low market values. Thus, a sample containing a sufficiently large number of leveraged firms should generate stronger cross-sectional relations between equity returns and book-to-market equity or firm size than a sample containing few or no leveraged firms. Figure (2) and (3), in the previous section, show the extent to which financial leverage impacts the strength of these cross-sectional relations.

4 Conclusion

I propose a model of firm dynamics to study the relation between financial leverage and equity risk premia, in a context where variables such as book-to-market equity and firm size arise naturally as proxy for equity risk.

Firms maximize the market value of their equity by making the appropriate investment, capital structure and payout distribution decisions, in response to changes in both aggregate and firm-specific productivity shocks. Over time, firms accumulate assets in place and they contemplate new growth opportunities. The sensitivity of these two components to aggregate productivity risk - a systematic risk - leads to variability in equity returns. Consequently, the relative distribution of assets in place and growth options becomes the main determinant of equity risk premia. For all-equity-financed firms, this distribution can be summarized in terms of firm-specific productivity and two firm variables, namely book-to-market equity and firm size. For firms financed with both equity and debt, this distribution depends also on financial
leverage.

In the model, financial leverage affects equity risk premia in two ways. Leveraged firms trade at a discount relative to similar all-equity-financed firms, because, the states where leverage binds the most are discounted less heavily. At the same time, leveraged firms tend to maintain higher stocks of capital during times of low productivity, because they cannot scale down productivity without increasing the likelihood of default.

The cross-sectional variation in equity returns reflects cross-sectional variation in capital, firm-specific productivity or financial leverage. Firm size and book-to-market equity can collectively capture the variation due to capital and the variation due to either firm-specific productivity or financial leverage, but not both. Firm size and book-to-market alone cannot disentangle the variation in equity returns due to firm-specific productivity from the variation due to financial leverage. Thus, firm variables proxying for a firm’s financial leverage - such as debt/equity - are informative about the cross-sectional distribution of equity returns, above and beyond what is contained in firm size and book-to-market equity.

Finally, the model can generate qualitatively and, sometimes, quantitatively the cross-sectional properties of equity returns associated with firm characteristics such as book-to-market equity, firm size, market leverage, book leverage and debt/equity ratio.
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<td>(2.77)</td>
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<td>(0.95)</td>
<td>(1.25)</td>
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Panel A: Historical Data

Panel B: Simulated Data

Table 3: **Cross-sectional regressions** This table reports the results of the cross-sectional regressions of the monthly realized returns, $R_{t+1}$, on the market beta, $\beta$, the firm size, $\log ME$, the ratio of book equity to market equity, $\log \frac{BE}{ME}$, the ratio of book assets to market equity, $\log \frac{BA}{ME}$, and the ratio of book assets to book equity, $\log \frac{BA}{BE}$. Panel A presents the results from Fama and French (1992), Table III, while Panel B presents the results from simulated data. The slope and the t-Statistic coefficients are Fama-Macbeth (time-series) estimates. The last three column present the 20%, 50% and 80% percentile for the corresponding coefficient.
<table>
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<th>Regressors</th>
<th>Panel A: Historical Data</th>
<th>Panel B: Simulated Data</th>
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<td>1, 2, 3</td>
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<td>−0.11</td>
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</table>

Table 4: **Cross-sectional regressions** This table reports the results of the cross-sectional regressions of the monthly realized returns, $R_{t+1}$, on the market betas, $\beta$, the firm size, log $ME$ and the book debt to market equity ratio $BD_{ME}$. Panel A reports the historical results, corresponding to Table II in Bhandari (1988). Panel B reports the Fama-Macbeth estimates and t-statistics of the coefficients, for the simulated sample.
A Computational Details

I start by simplifying the dynamic program in (10). The fact that $D_t$ enters additively in the right-hand side of the program in (10) implies that whenever this dynamic program admits a solution, $v$, it has to be the case that $v(s_t, D_t) = v(s_t, 0) - D_t$. If I denote with $u(s_t) = v(s_t, 0)$, this separation result allows me to restate the dynamic program in (10) in simpler terms:

$$u(s_t) = (1 - \tau)(y_t - \delta_k) + \delta_k$$

$$+ \max_{i_{t+1}, b_{t+1}} \left\{ -[i_{t+1}k_t + h(i_{t+1}, k_t)] ight. + \mathbb{E}_t[M_{t,t+1}\chi(s_{t+1}, D_{t+1})[u(s_{t+1}) + (\tau - \tau_0)b_{t+1}]]$$

$$+ \mathbb{E}_t[M_{t,t+1}[1 - \chi(s_{t+1}, D_{t+1})]u(s_{t+1})^D] \right\}$$

(A. 1)

where $\chi(s_{t+1}, D_{t+1}) = 1_{\{u(s_{t+1}, D_{t+1}) \geq D_{t+1}\}}$, $D_{t+1} = (1 - \tau)b_{t+1} + B(s_{t+1}, b_{t+1})$ and

$$B(s_t, b_{t+1}) = \mathbb{E}_t[M_{t,t+1}\chi(s_{t+1}, D_{t+1})\{(1 - \tau_0)b_{t+1} + B(s_{t+1}, b_{t+1})\}]$$

$$+ \mathbb{E}_t[M_{t,t+1}[1 - \chi(s_{t+1}, D_{t+1})]u(s_{t+1})^D]$$

(A. 2)

I will now focus on solving this dynamic program. Solving the model is equivalent to finding a limiting solution to the dynamic programs (A. 1)-(A. 2), in the following sense:

**Definition 1** A limiting solution to the firm’s problem is a quadruple of continuous and bounded functions, $(u^*, i^*, b^*, B^*)$, such that:

i. $B^*$ solves the dynamic program in (A. 2), given $(u^*, i^*)$

ii. $(u^*, i^*, b^*)$ solves the dynamic program in (A. 1)

In order to identify a limiting solution I implement a numerical approach that exploits the fact that the two dynamic programs in (A. 1) and (A. 2) are related only through the default function $\chi$. If I shut down for the moment this channel, I obtain the following result:
Proposition 5 Suppose that the default function can be expressed as $\chi(s_t, D_t) = \chi(s_t, (1 - \tau)b_t + B(s_t, b_t)) = 1_{\{\phi(s_t, b_t) \geq 0\}}$, for some continuous function $\phi$, that does not depend on either $u$ or $B$. Also, suppose that $\mathbb{E}[M|x] < 1$, for any $x$ in the range of the aggregate productivity shock. Then:

1. The dynamic program in (A. 1) admits a unique, bounded and continuous solution $u$.

2. Assuming that the functions $u(s_t)$ and $i_{t+1} = i(s_t)$ are continuous and bounded, the dynamic program in (A. 2) admits a unique, continuous and bounded solution.

This result is the basis of my numerical approach. The generic step of my numerical algorithm is computed as follows: Suppose the outcome of step $n - 1$ is the quadruple $(u^{n-1}, i^{n-1}, b^{n-1}, B^{n-1})$. Let $\phi^n(s_t, b) = u^{n-1}(s_t) - (1 - \tau)b - B^{n-1}(s_t, b)$ and define $\chi^n(s_t, D^{n-1}_t) = \chi^n(s_t, (1 - \tau)b - B^{n-1}(s_t, b)) = 1_{\{\phi^n(s_t, b) \geq 0\}}$. If I replace $\chi$ with $\chi^n$ in the dynamic program (A. 1), then according to [1.] of Proposition 5, (A. 1) admits a unique, continuous and bounded solution, $u^n(s_t)$. Let $i^n(s_t)$ and $b^n(s_t)$ denote the optimal policies of (A. 1). Similarly, if I replace $\chi$ with $\chi^n$ in the dynamic program (A. 2), then according to [2.] of Proposition 5, (A. 2) admits a unique, continuous and bounded solution, $B^n(s_t, b)$. To summarize, the outcome of step $n$ is the quadruple $(u^n, i^n, b^n, B^n)$.

The next proposition establishes the relationship between the outcomes of this algorithm and the concept of limiting solution.

Proposition 6 Let $\{(u^n, i^n, b^n, B^n)\}_n$ denote the sequence of iterations in the above algorithm. If the sequence $\{(u^n, i^n, b^n, B^n)\}_n$ converges, and $(u^*, i^*, b^*, B^*)$ denotes its limit, then $(u^*, i^*, b^*, B^*)$ is a limiting solution.
In order to implement the model I assume that the productivity shocks \( x \) and \( z \) follow stationary autonomous dynamics of the following form:

\[
x_{t+1} - x_t = (1 - \rho_x) (\bar{x} - x_t) + \sigma_x \epsilon_{x,t+1}^x \\
z_{t+1} - z_t = (1 - \rho_z) (0 - z_t) + \sigma_z \epsilon_{z,t+1}^x,
\]

where \( \epsilon^x \) and \( \epsilon^z \) are i.i.d. standard normals. The central tendency of the aggregate shock is captured by \( \bar{x} \), while the speed of reversion (or the persistence) and the conditional volatility are captured by \( \rho_x \) and \( \sigma_x \), respectively. The parameters \( \rho_z \) and \( \sigma_z \) for the idiosyncratic process, carry similar connotations.

I also assume that the process governing the evolution of the pricing kernel follows the specifications in Zhang (2005). That is

\[
M_{t+1} = \xi_{t+1} \quad \text{and}
\]

\[
\log \xi_{t+1} - \log \xi_t = \log \beta + \Gamma_t [x_t - x_{t+1}]
\]

\[
\Gamma_t = \gamma_0 + \gamma_1 [x_t - \bar{x}],
\]

where \( \gamma_0 > 0 \) and \( \gamma_1 < 0 \). Notice that \( \Gamma_t \) can be naturally interpreted as the market price of aggregate (systematic) risk, while the constraint on \( \gamma_1 \) ensures that this price of risk is countercyclical.

I can define the short interest rates as

\[
r_t = -\log \beta - \Gamma_t (1 - \rho_x) [\bar{x} - x_t] - \frac{1}{2} \Gamma_t^2 \sigma_x^2
\]

and the pricing kernel can be rewritten as

\[
\log \xi_{t+1} - \log \xi_t = -r_t - \frac{1}{2} \Gamma_t^2 \sigma_x^2 - \Gamma_t \sigma_x \epsilon_{x,t+1}^x
\]

The dynamics of this pricing kernel are reminiscent of those in Berk, Green and Naik (1999) with two important differences: On one hand, both the pricing kernel and the short rate are driven by one shock only (the aggregate shock \( x_t \)). On the
other hand, the market price of systematic risk is time-varying.

B Proofs

Proof of Proposition 1

Follows immediately from the definition of $V$ in Equation (16), after taking into account the decomposition formula for output in (1) and the law of motion for capital in (2).

Proof of Proposition 2 For notational tractability let $\chi_{t+1}$ denote the default indicator function $\chi(s_{t+1}, D_{t+1})$. Notice first that $Z$ can be described in a recursive fashion as follows:

$$Z(s_t) = \mathbb{E}_t[M_{t,t+1} \{ (\tau - \tau_0) b_{t+1} + Z(s_{t+1}) \} \chi_{t+1}]$$

$$+ \mathbb{E}_t[M_{t,t+1} \{ V^u(s_{t+1}) - V^u(s_{t+1}) + Z(s_{t+1}) \} (1 - \chi_{t+1})]$$

Following the definitions in (22), the realized gross return on the firm’s equity can be expressed as follows:

$$R_{t+1} = \frac{V^u(s_t) - d^u_t}{ME_t} \frac{V^u(s_{t+1})}{V^u(s_t)} \frac{\tau - \tau_0}{b_{t+1}} \chi_{t+1} + \frac{B(s_t, b_{t+1})}{ME_t} \frac{D_{t+1} \chi_{t+1}}{B(s_t, b_{t+1})} + \frac{Z(s_{t+1})}{ME_t} \chi_{t+1}$$

$$= R^u_{t+1} \left[ 1 - \frac{Z(s_t)}{ME_t} + \frac{B(s_t, b_{t+1})}{ME_t} \right] \chi_{t+1} + \frac{B(s_t, b_{t+1})}{ME_t} \frac{D_{t+1} \chi_{t+1}}{B(s_t, b_{t+1})} + \frac{Z(s_{t+1})}{ME_t} \chi_{t+1}$$

$$= R^u_{t+1} \chi_{t+1} + \frac{B(s_t, b_{t+1})}{ME_t} [R^u_{t+1} - R^B_{t+1}] \chi_{t+1}$$

$$+ \frac{(\tau - \tau_0) b_{t+1}}{ME_t} \chi_{t+1} + \frac{Z(s_{t+1})}{ME_t} \chi_{t+1} - \frac{Z(s_t)}{ME_t} R^u_{t+1} \chi_{t+1}$$

Using the dynamics of $Z$ in (B. 1), I can compute the following conditional expec-
\[
\mathbb{E}_t \left[ \frac{Z(s_{t+1})}{ME_t} \chi_{t+1} \right] - \frac{Z(s_t)}{ME_t} \pi^u_t = -\frac{(\tau - \tau_0)b_{t+1}}{ME_t} \pi^u_t \mathbb{E}_t \left[ M_{t,t+1} \chi_{t+1} \right]
+ \mathbb{E}_t \left[ (1 - \pi^u_t M_{t,t+1}) \left\{ \frac{Z(s_{t+1})}{ME_t} \chi_{t+1} \right\} \right]
+ \pi^u_t \mathbb{E}_t \left[ M_{t,t+1} \frac{V^u(s_{t+1}) - V^u(s^D_{t+1})}{ME_t} (1 - \chi_{t+1}) \right]
- \pi^u_t \mathbb{E}_t \left[ M_{t,t+1} \frac{Z(s^D_{t+1})}{ME_t} (1 - \chi_{t+1}) \right] \tag{B.3}
\]

Taking the conditional expectation of both sides of the former equation and making use of the latter equation, we obtain the desired result. The error term \( O_t \) is given by:

\[
O_t = \mathbb{E}_t \left[ \frac{V^u(s_{t+1}) - V^u(s^D_{t+1})}{ME_t} (1 - \chi(s_{t+1}, D_{t+1})) \right]
- \mathbb{E}_t \left[ \frac{Z(s^D_{t+1})}{ME_t} (1 - \chi(s_{t+1}, D_{t+1})) \right]
- \mathbb{E}_t \left[ \nu_{t+1} \frac{(\tau - \tau_0)b_{t+1} \chi(b_{t+1}, D_{t+1}) - \Delta v(s_{t+1}) [1 - \chi(b_{t+1}, D_{t+1})]}{ME_t} + Z(s_{t+1}) \right] \tag{B.4}
\]

where \( \nu_{t+1} = \pi^u_t M_{t,t+1} - 1 \).

**Proof of Proposition 3**

To arrive at the decomposition result of Proposition 3, I start by taking expectations in the decomposition result of Equation (B.2). I obtain:

\[
\mathbb{E}_t [R_{t+1}] = \pi^u_t + \frac{B(s_t, b_{t+1})}{ME_t} \mathbb{E}_t \left[ (R^u_{t+1} - R^B_{t+1}) \chi_{t+1} \right]
\frac{(\tau - \tau_0)b_{t+1}}{ME_t} \mathbb{E}_t [\chi_{t+1}] + \mathbb{E}_t \left[ \frac{Z(s_{t+1})}{ME_t} \chi_{t+1} \right] - \frac{Z(s_t)}{ME_t} \pi^u_t \tag{B.5}
\]

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From the dynamics of $Z$ in Equation (B. 1). I obtain:

$$\mathbb{E}_t \left[ \frac{Z(st_{t+1})}{ME_t} \chi_{t+1} \right] - \frac{Z(st)}{ME_t} \pi^u_t =$$

$$= \mathbb{E}_t \left[ \frac{Z(st_{t+1})}{ME_t} \chi(st_{t+1}, D_{t+1}) \right] - \mathbb{E}_t \left[ M_{t,t+1} \pi^u_t \left( \frac{(\tau - \tau_0)b_{t+1}}{ME_t} \right) \chi_{t+1} \right]$$

$$- \mathbb{E}_t \left[ M_{t,t+1} \pi^u_t \left( \frac{V^u(st_{t+1}) + Z(s^D_{t+1})}{ME_t} \right) \left[ 1 - \chi_{t+1} \right] \right]$$

$$+ \mathbb{E}_t \left[ M_{t,t+1} \pi^u_t \left( \frac{V^u(st_{t+1})}{ME_t} \right) \left[ 1 - \chi_{t+1} \right] \right]$$

$$= -\mathbb{E}_t \left[ \nu_{t+1} \frac{Z(st_{t+1})}{ME_t} \chi(st_{t+1}, D_{t+1}) \right] - \mathbb{E}_t \left[ M_{t,t+1} \pi^u_t \left( \frac{(\tau - \tau_0)b_{t+1}}{ME_t} \right) \chi_{t+1} \right]$$

$$+ \mathbb{E}_t \left[ M_{t,t+1} \pi^u_t \left( \frac{V^u(st_{t+1})}{ME_t} \right) \left[ 1 - \chi_{t+1} \right] \right]$$

$$- \mathbb{E}_t \left[ M_{t,t+1} \pi^u_t \left( \frac{V^u(st_{t+1}) + Z(s^D_{t+1})}{ME_t} \right) \left[ 1 - \chi_{t+1} \right] \right]$$

(B. 6)

Substituting the last formula into Equation (B. 5), I obtain:

$$\mathbb{E}_t \left[ R_{t+1} \right] = \pi^u_t + \frac{B(st, b_{t+1})}{ME_t} \mathbb{E}_t \left[ (R^u_{t+1} - R^B_{t+1}) \chi_{t+1} \right]$$

$$- \frac{(\tau - \tau_0)b_{t+1}}{ME_t} \mathbb{E}_t \left[ \nu_{t+1} \chi_{t+1} \right] - \mathbb{E}_t \left[ \nu_{t+1} \frac{Z(st_{t+1})}{ME_t} \chi_{t+1} \right]$$

$$+ \mathbb{E}_t \left[ M_{t,t+1} \pi^u_t \left( \frac{V^u(st_{t+1})}{ME_t} \right) \left[ 1 - \chi_{t+1} \right] \right] - \mathbb{E}_t \left[ M_{t,t+1} \pi^u_t \left( \frac{V^u(st_{t+1}) + Z(s^D_{t+1})}{ME_t} \right) \left[ 1 - \chi_{t+1} \right] \right]$$

$$= \pi^u_t + \gamma_t \mathbb{E}_t \left[ \frac{V^u(st_{t+1})}{ME_t} \chi_{t+1} \right]$$

$$+ \frac{B(st, b_{t+1})}{ME_t} \pi^u_t - \mathbb{E}_t \left[ \left( 1 - \tau \right)b_{t+1} + B(st_{t+1}, b_{t+1}) \right] \chi_{t+1}$$

$$- \frac{(\tau - \tau_0)b_{t+1}}{ME_t} \mathbb{E}_t \left[ \nu_{t+1} \chi_{t+1} \right] - \mathbb{E}_t \left[ \nu_{t+1} \frac{Z(st_{t+1})}{ME_t} \chi_{t+1} \right]$$

$$- \mathbb{E}_t \left[ M_{t,t+1} \pi^u_t \left( \frac{V^u(st_{t+1}) + Z(s^D_{t+1})}{ME_t} \right) \left[ 1 - \chi_{t+1} \right] \right]$$

(B. 7)
The recursive definition of $B$ in Equation (9) yields:

$$
\frac{B(s_t, b_{t+1})}{\pi_t^u} - \mathbb{E}_t \left[ \frac{(1 - \tau)b_{t+1} + B(s_{t+1}, b_{t+1})}{\pi_t} \chi_{t+1} \right]
= \mathbb{E}_t \left[ \nu_{t+1} \frac{(1 - \tau)b_{t+1} + B(s_{t+1}, b_{t+1})}{\pi_t} \chi_{t+1} \right]
+ \mathbb{E}_t \left[ \frac{M_{t, t+1} \nu_t(s_{t+1}, 0)\nu_t}{\pi_t} (1 - \chi_{t+1}) \right]
= \mathbb{E}_t \left[ \nu_{t+1} \frac{(1 - \tau)b_{t+1} + B(s_{t+1}, b_{t+1})}{\pi_t} \chi_{t+1} \right]
+ \mathbb{E}_t \left[ \frac{M_{t, t+1} \nu_t V^u(s_{t+1}) + Z(s_{t+1})}{\pi_t} [1 - \chi_{t+1}] \right].
$$

Substituting this formula into Equation (B. 7), I obtain the desired result:

$$
\mathbb{E}_t [R_{t+1}] = \pi_t^u + \gamma_t \mathbb{E}_t \left[ \frac{V^u(s_{t+1})}{\pi_t} \chi_{t+1} \right] + \mathbb{E}_t \left[ \nu_{t+1} \frac{D_{t+1} - Z(s_{t+1})}{\pi_t} \chi_{t+1} \right].
$$

Proof of Proposition 4 The result follows immediately from the decomposition results in Proposition 1 and Proposition 3.

The proof of Proposition 5 is based on the following Lemma:

Lemma 1

Suppose $e$ and $h$ are two continuous functions, defined on $\mathbb{R}^{3+1}$ and $\mathbb{R}$, respectively. For any $c \in [c_-, c_+] \subset \mathbb{R}$, let $\nu(c) = \int h(z)1_{[e(z,c) \leq 0]} dz$ and let $B[c_-, c_+]$ be the space of functions with common domain $\mathbb{R}$ and range $[c_-, c_+]$. Then, for any sequence of functions $c_n \in B[c_-, c_+]$ converging pointwise to $c \in B[c_-, c_+]$, I have that $\nu(c_n)$ converges pointwise to $\nu(c)$.

Proof of Lemma 1

Let $c_n$ be a sequence of functions converging pointwise to $c$. I want to show that $\nu(c_n)$ converges pointwise to $\nu(c)$. Fix $x_0$ in the common domain of $\{c_n\}_n$ and $c$, denote, for simplicity $c_n = c_n(x_0)$ and $c = c(x_0)$.
Note that:

\[ |\nu(c_n) - \nu(c)| = \int h(z) \mathbf{1}_{[e(z,c_n) \leq 0 < e(z,c)] \cup [e(z,c) \leq 0 < e(z,c_n)]} \, dz \]

Let \( g_n(z) = h(z) \mathbf{1}_{[e(z,c_n) \leq 0 < e(z,c)] \cup [e(z,c) \leq 0 < e(z,c_n)]} \). I want to show that \( g_n \to 0 \). This result is clear for those values of \( z \) that reside in the support of a finite number of \( g_n \)s, only.

For the more general case, suppose there exists \( z \) such that

\[ z \in \bigcap_{i.o.} \{ [e(z,c_n) \leq 0 < e(z,c)] \cup [e(z,c) \leq 0 < e(z,c_n)] \} \]

Then, it has to be the case that either

\[ z \in \bigcap_{i.o.} \{ [e(z,c_n) \leq 0 < e(z,c)] \} \quad \text{or} \quad z \in \bigcap_{i.o.} \{ [e(z,c) \leq 0 < e(z,c_n)] \} \]

Suppose the first case holds. Then there exists a subsequence \( c_{k_n} \) such that

\[ z \in \bigcap_{n=0}^{\infty} \{ e(z,c_{k_n}) \leq 0 < e(z,c) \} \]

Since \( \lim c_{k_n} = c \), and \( e(z,c_{k_n}) \leq 0 < e(z,c) \), for all \( n \), I have that in the limit: \( e(z,c) \leq 0 < e(z,c) \). A similar result can be derived when the second case is assumed to hold. I can conclude at this point that

\[ \bigcap_{i.o.} \{ [e(z,c_n) \leq 0 < e(z,c)] \cup [e(z,c) \leq 0 < e(z,c_n)] \} = \emptyset \]

This completes the proof of the claim that \( g_n \to 0 \). Finally, a simple argument based on the Lebesque Dominated Convergence Theorem yields that \( \lim_n \nu(c_n) = \nu(c) \). This concludes the proof of the Lemma.

Suppose the default function is characterized by the continuous function \( \phi \). Let \( T \)
be the functional operator associated with the dynamic program in (A. 1). That is:

\[
T(u)(s_t) = (1 - \tau)(y(s_t) - \delta k_t) + \delta k_t \\
+ \max_{i_t+1,b_{t+1}} \left\{ - [i_{t+1}k_t + h(i_{t+1}, k_t)] \\
+ E_t [M_{t,t+1}1_{\{\phi(s_{t+1}, b_{t+1}) \geq 0\}} u(s_{t+1}) + (\tau - \tau_0)b_{t+1}] \\
+ E_t [M_{t,t+1}1_{\{\phi(s_{t+1}, b_{t+1}) < 0\}} u(s_{t+1})] \right\}
\]

(B. 10)

where \( s_t = (x_t, z_t, k_t) \), \( s_{t+1} = (x_{t+1}, z_{t+1}, k_{t+1}) \), \( s_{D,t+1} = (x_{t+1}, z_{t+1}, (1 - \xi)k_{t+1}) \), \( k_{t+1} = (1 - \delta + i_{t+1})k_t \) and \( x_{t+1}, z_{t+1} \) are given by the dynamics in (A. 3).

**Proof of Proposition 5**

First notice that the operator \( T \) maps the space of continuous and bounded functions into itself. This is a direct application of the previous Lemma with \( e = \pm \phi \).

To see whether the operator \( T \) has a unique solution, it is sufficient to check the Blackwell’s sufficient conditions for a contraction mapping. The monotonicity condition follows immediately from the linearity of the operator \( T \) on \( u \). The discounting condition follows from the fact that

\[
T(u + a)(s_t) = (1 - \tau)(y(s_t) - \delta k_t) + \delta k_t \\
+ \max_{i_{t+1},b_{t+1}} \left\{ - [i_{t+1}k_t + h(i_{t+1}, k_t)] \\
+ E_t [M_{t,t+1}1_{\{\phi(s_{t+1}, b_{t+1}) \geq 0\}} u(s_{t+1}) + (\tau - \tau_0)b_{t+1} + a] \\
+ E_t [M_{t,t+1}1_{\{\phi(s_{t+1}, b_{t+1}) < 0\}} u(s_{t+1}) + a] \right\}
\]

(B. 11)

\[
\leq (1 - \tau)(y(s_t) - \delta k_t) + \delta k_t + E_t [M_{t,t+1}1_{\{\phi(s_{t+1}, b_{t+1}) \geq 0\}} u(s_{t+1}) + (\tau - \tau_0)b_{t+1}] \\
+ E_t [M_{t,t+1}1_{\{\phi(s_{t+1}, b_{t+1}) < 0\}} u(s_{t+1})] \\
+ E_t [M_{t,t+1}] a
\]
for any real \( a > 0 \). Therefore, as long as \( E_t[M_{t,t+1}] < 1 \), the discounting property is satisfied and \( T \) is a contraction mapping of modulus \( E_t[M_{t,t+1}] \). This ensures that the operator \( T \) has a unique fixed point in the space of continuous and bounded functions.

The second part of the Proposition is proven in exactly the same manner.

**Proof of Proposition 6**

To prove the first part of the proposition, suppose \( (u^*, i^*, b^*, B^*) = \lim_n(u^n, i^n, b^n, B^n) \).

By definition I have:

\[
\begin{align*}
u^n(s_t) &= (1 - \tau) [y(s_t) - \delta k_t] + \delta k_t \\
&\quad - \left[ i^n_{t+1} + h(i^n_{t+1}, k_t) \right] \\
&\quad + E_t \left[ M_{t,t+1} \{ \phi^n(s_{t+1}, b^n_{t+1}) \geq 0 \} \left[ u^n(s_{t+1}) + (\tau - \tau_0)b^n_{t+1} \right] \right] \\
&\quad + E_t \left[ M_{t,t+1} \{ \phi^n(s_{t+1}, b^n_{t+1}) < 0 \} u^n(s_{t+1}^D) \right],
\end{align*}
\]

(B. 12)

and

\[
\begin{align*}B^n(s_t, b_{t+1}) &= E \left[ M I \{ \phi^n(s_{t+1}, b^n_{t+1}) \geq 0 \} \left[ (1 - \tau_0)b^n_{t+1} + B^n(s_{t+1}, b^n_{t+1}) \right] \right] \\
&\quad + E \left[ M I \{ \phi^n(s_{t+1}, b^n_{t+1}) \geq 0 \} u^{n-1}(s_{t+1}^D, 0) \right],
\end{align*}
\]

(B. 13)

where \( \phi^n(s_t, b^n_{t+1}) = u^{n-1}(s_t) - (1 - \tau)b^n_{t+1} - B^{n-1}(s_{t+1}, b^n_{t+1}) \), \( s_t = (x_t, z_t, k_t) \), \( s_{t+1} = (x_{t+1}, z_{t+1}, k_{t+1}) \), \( s_{t+1}^D = (x_{t+1}, z_{t+1}, \xi k_{t+1}) \) and \( k_{t+1} = (1 - \delta + \bar{i}^n_{t+1})k_t \).

It is straightforward to notice that for a given \( s_t \), \( B^n(s_t, b) \) is constant for large \( b \).

Therefore I can safely assume that \( b^n_{t+1}(s_t) \leq \bar{b}(s_t) \). It now follows from the sequential version of the dynamic program for \( u \) and the convexity of the adjustment costs, that \( u^n(s_t) \leq \bar{u}(s_t) \).
I now take both relations under the limit. I obtain:

\[ u^*(s_t) = (1 - \tau) \left[ y(s_t) - \delta k_t \right] + \delta k_t \]

\[ - \left[ i_{t+1}^* + h \left( i_{t+1}^*, k_t \right) \right] + \mathbb{E}_t \left[ M_{t,t+1} \mathbb{1}\{ \phi^*(s_{t+1}, b_{t+1}^*) \geq 0 \} \left[ u^*(s_{t+1}) + (\tau - \tau_0) b_{t+1}^* \right] \right] \]

\[ + \mathbb{E}_t \left[ M_{t,t+1} \mathbb{1}\{ \phi^*(s_{t+1}, b_{t+1}^*) < 0 \} u^*(s_{t+1}) \right] \tag{B. 14} \]

and

\[ B^*(s_t, b_{t+1}) = \mathbb{E} \left[ M \mathbb{1}\{ \phi^*(s_{t+1}, b_{t+1}^*) \geq 0 \} \left\{ (1 - \tau_0) b_{t+1}^* + B^*(s_{t+1}, b_{t+1}^*) \right\} \right] \]

\[ + \mathbb{E} \left[ M \mathbb{1}\{ \phi^*(s_{t+1}, b_{t+1}^*) < 0 \} u^*(s_{t+1}, 0) \right] \], \tag{B. 15} \]

where \( \phi^*(s_t, b_{t+1}^*) = u^*(s_t) - (1 - \tau) b_{t+1}^* - B^*(s_t, b_{t+1}^*) \), \( s_t = (x_t, z_t, k_t) \), \( s_{t+1} = (x_{t+1}, z_{t+1}, k_{t+1}) \), \( s_{t+1}^D = (x_{t+1}, z_{t+1}, (1 - \xi) k_{t+1}) \) and \( k_{t+1} = (1 - \delta + i_{t+1}^*) k_t \). Thus, \( (B^*, v^*, k^*, b^*) \) is a limiting point.

C Additional Tests

In this section I perform one last experiment meant to further investigate the interaction between the leverage effect and the book-to-market and the size effect. This experiment is based on the Fama and French (1993) methodology of constructing mimicking portfolio factors on various firm characteristics, that proved important for the cross-section of stock returns. The idea is fairly simple: if a firm characteristic is associated with a priced effect, then it has to be the case that the loading on the corresponding mimicking portfolio has to matter for the cross-section of stock returns.

Following Ferguson and Shockley (2003), I construct mimicking portfolios for four firm characteristics, namely: the firm size, the book-to-market ratio, the debt/equity ratio and the Z-score.\(^{18}\)

\(^{18}\) Altman’s Z-score is an accounting-based measure that arguably predicts financial distress. This
Every 12 months stocks are sorted independently based on these four firm characteristics. The sorts based on firm size and Z-score, respectively, are divided into two groups with approximately equal numbers of stocks. The sorts based on book-to-market and debt/equity ratio, respectively, are divided into three groups of similar size. The intersection of the sorts on firm size and book-to-market produces six equally weighted portfolios. The mimicking portfolio for size, the ”small minus big”, or SMB portfolio invests 0.3 in each portfolio small within size bin and shorts 0.3 of each portfolio within the big size bin. The mimicking portfolio for book-to-market, the ”high minus low”, or HML portfolio invests 0.5 in the each of the two portfolios in the top book-to-market bin and shorts 0.5 of each portfolio in the lowest book-to-market bin.

In a similar fashion, the intersection of the sorts on Z-score and book/equity ratio produces six equally weighted portfolios. The mimicking portfolio for Z-score, the ”relative distress” portfolio invests 0.3 in each portfolio within high Z-score bin and shorts 0.3 of each portfolio within the low Z-score bin. The mimicking portfolio for debt/equity ratio, the ”relative leverage” portfolio invests 0.5 in the each of the two portfolios in the top debt/equity bin and shorts 0.5 of each portfolio in the lowest debt/equity bin. This procedure yields four mimicking portfolios on which I compute monthly returns for the following 12 months from formation. In the end, I obtain four time series of monthly returns, which I denote correspondingly $R_{t}^{SMB}$, $R_{t}^{HML}$, $R_{t}^{Z}$, and $R_{t}^{DE}$.

I use the six portfolios obtained from the independent sort on firm size and book-to-market ratio as my testing portfolios. In order to test whether the relative leverage measure is defined as follows:

$$Z = 1.2 \frac{WC}{BA} + 1.4 \frac{RE}{BA} + 3.3 \frac{EBIT}{BA} + 0.6 \frac{ME}{BD} + 1.0 \frac{S}{BA},$$

where $WC$ is net working capital, $BA$ is book value of assets, $RE$ is retained earnings, $EBIT$ is earnings before interest and taxes, $ME$ is market equity, $BD$ is book value of debt and $S$ is total sales revenues. I compute all these measures using their accounting definition. I compute these accounting measures using their accounting definition.
factor $R^{DE}$ is priced, I use two expected return - beta representation models, that incorporate $R^{DE}$ as one of the factors and I test whether the loadings on this factor are important for the cross-section of stock returns. Ferguson and Shockley (2003) show that, in their sample, two expected return - beta representation models outperform the Fama and French 3-factor model. The first model is a 3-factor model based on market excess return $R^{Me}$, relative leverage $R^{DE}$ and relative distress $R^{Z}$. The second model adds to these three factors the orthogonal components of the SMB and HML factors on the relative leverage and the relative distress factors. I denote this orthogonal components with $R^{SMB} \perp$ and $R^{HML} \perp$, respectively.\(^{19}\)

For the first model, for each of the six testing portfolios, the first stage of the two-pass methodology of Fama-Macbeth yields the factor loadings $\beta^{Me}$, $\beta^{SMB}$ and $\beta^{HML}$, on the market, the SMB factor and the HML factor, respectively. Each month a stock in portfolio $p$ is assigned the loadings corresponding to that portfolio. The second stage of the Fama-Macbeth procedure runs a monthly cross-sectional regression of the realized stock returns on the corresponding factor loadings. The same procedure applies for the second model as well. Table 5 reports the results of summary statistics of these cross-sectional regressions.

We notice that according to the first model, the factor loadings on the relative leverage factor seem to be important for the cross-section of stock return, above and beyond the market betas. The same cannot be said about the factor loadings on the relative distress factor. If I add the factor loadings on the orthogonal components of SMB and HML, we notice that the slope of factor loading on the relative leverage

\(^{19}\)Specifically, $R^{SMB} \perp$ is the sum of the intercept and the error in the following regression:

\[ R^{SMB}_t = \alpha^{SMB} + \beta_1 R^{DE}_t + \beta_2 R^{Z}_t + \epsilon_t, \]  

(C. 1)

while $R^{HML} \perp$ is the sum of the intercept and the error in the following regression:

\[ R^{HML}_t = \alpha^{HML} + \beta_3 R^{DE}_t + \beta_4 R^{Z}_t + \epsilon_t \]  

(C. 2)
factor suffers a drop in magnitude, in the favor of the factor loading on the orthogonal component of SMB. The factor loadings on the relative leverage factor seems to be important relative to the loadings on the relative distress factor or the orthogonal component of HML factor, but not as important as the factor loadings on the market betas and the orthogonal component of the SMB factor.
Table 5: Cross-sectional regressions. This table reports the results of the cross-sectional regressions of the monthly realized returns, \( R_{t+1} \), on the factor loading on the market, \( \beta_M \), the factor loading on the Ferguson-Shockley relative leverage factor, \( \beta_{DE} \), the factor loading on the Ferguson-Shockley relative distress factor, \( \beta_Z \), factor loading on the orthogonal component of Fama-French SMB factor, \( \beta_{SMB^\perp} \) and the factor loading on the orthogonal component of Fama-French HML factor, \( \beta_{HML^\perp} \). The orthogonal component of a factor is the error of the projection of the original factor onto the space generated by the relative leverage and relative distress factors. I report the Fama-Macbeth estimates and t-statistics.
CHAPTER II

Default Risk Premia and Asset Returns
1 Introduction

Recent empirical studies in financial economics by Berndt, Douglas, Duffie, Ferguson and Schranz (2005) and Saita (2006) suggest that the compensation demanded by investors for being exposed to credit risk, above and beyond expected default losses, is substantial, and that it varies dramatically over short horizons of time. Bendt et al. (2005) report that the size of these so called “default risk premia,” when measured as a multiple of $1 of expected default loss, ranges from 1.5 to 4, while Saita (2006) reports a range of 1 to 3.5. In terms of variation over time, the first paper shows that default risk premia peaked in the third quarter of 2002, and dropped by roughly 50% until late 2003.

If credit markets are close to being in equilibrium most of the time, any preference-based asset pricing theory will predict that investors demand risk premia on traded assets to compensate for bearing systematic risk. While investor preferences might change over time, it is quite unlikely that they would change dramatically enough over short horizons to induce a time variation in observed default risk premia of the magnitude reported in the above-mentioned study. Alternatively, investors might demand higher compensation for being more exposed to certain systematic factors, which suddenly become more important relative to other systematic factors.

This paper studies to what extent the portion of returns observed in U.S. credit markets that is not related to changes in risk-free rates or expected default losses is a compensation for bearing systematic risk. Towards this goal, we decompose these firm-specific returns into (i) a part that is explained by changes in risk-free rates and changes in expected default losses, plus (ii) a part that is due to changes in default risk premia. Elton, Gruber, Agrawal and Mann (2001) show that changes in expected default losses or risk-free rates do not contain information about systematic risk beyond what is already captured by the Fama-French factors (see, Fama and French (1993)).
Motivated by this observation, we focus on that portion of firm-specific returns in credit markets that is due to changes in default risk premia only. We investigate whether there is common time-series variation across the observed firm-specific changes in default risk premia, and test to what extend this common variation can be attributed to a systematic risk factor. Specifically, we aim at answering four main questions:

1. Do firm-specific changes in default risk premia exhibit common time-series variation?

2. If so, how much of this common time-series variation can be attributed to factors that are known to be priced in either equity or corporate debt markets, such as the Fama and French (1993) equity, treasury and corporate debt factors. How much of it is left unexplained? The latter is identified as the common latent component in changes in firm-specific default risk premia, after controlling for other sources of systematic risk. We will refer to it as the changes in default risk premia factor, or short the DRP factor.

3. Is the unexplained common time-series variation due to co-movement in time-varying firm characteristics or does it stem from exposure to a common source of risk?

4. If there is support for the later alternative, to what extent is our discovered DRP factor systematic?

Using data from 2001 to 2004 on default swap rates provided by the Markit Group and for Moody’s KMV estimates of actual default probabilities for 108 U.S. firms in nine industry groups, we find compelling support for common time-series variation in firm-specific changes in default risk premia. While up to 42% of this co-movement can be due to exposure to other known sources of common variation, a maximum of
35% of the residual is explained by a common latent component, that is, by our DRP factor. Firm characteristics such as the weekly survival probabilities, recovery rates or leverage ratios are mostly unrelated to this common time-series variation. Finally, while we find that our DRP factor is priced in the market for corporate bonds, we find very limited support for a similar conclusion in the equity markets. The test assets employed in the asset pricing tests comprise a wide rage of equity and corporate bond portfolios, formed on various firm characteristics.

Measuring (changes in) default risk premia is not a straightforward task, in part because no pure credit-contingent claims that pay one dollar in the event of no default (survival) and zero otherwise trade in the credit market. Instead, one has to find a way to imply this information from available pricing information on actively traded credit derivatives, such as credit default swaps. This process can be cumbersome, especially because the payoff structure stipulated in these contracts can interact with the default risk itself. In this paper, we use the reduced-form approach of Berndt et al. (2005) to measure default risk premia, using as pricing information credit default swap (CDS) and recovery rates obtained from Markit and estimates for actual default probabilities provided by Moody’s KMV.

To study whether changes in default risk premia exhibit common time-series variation, we first compute firm-specific model-implied returns on constant-maturity credit-sensitive securities that pay one unit of account if no default occurs before maturity and zero otherwise. We then regress, firm by firm, the portion of these returns that is not due to changes in risk-free rates or expected default losses on known systematic factors and time dummies for each week in our sample period. Among the common factors that we account for are those in Fama and French (1993), including their default and term factor, as well as the momentum factor introduced in Jagadeesh and Titman (1993). We measure our changes in default risk premia factor, at any given time, as the least-squares estimate of the contemporaneous dummy multiplier.
Using the time series of the latent common factor identified in the previous step, we then apply the Fama-MacBeth methodology to test whether our DRP factor is priced in asset returns. The test assets comprise well-diversified portfolios of stocks, index options, and corporate bonds, with mean returns spanning a wide range of values. It is important to stress that the DRP factor is extracted from credit market information only. The asset pricing tests will reveal whether it is a risk factor specific to that market, or to what extent it is priced in other markets as well.

Results using Bloomberg-NASD corporate bond indices generated from actual transaction prices of actively traded issues suggest that our discovered latent factor is priced in the corporate bond market. A cross-sectional analysis of the Merrill Lynch corporate bond portfolios, sorted on industry, maturity or rating, supports these findings. For equity markets results are mixed. The DRP factor captures some of the time-series variation in the 100 Fama-French portfolios, sorted on size and book-to-market equity, even after controlling for other potential sources of common variation. The time-series loadings of these portfolios on the DRP factor, however, do not seem to align, cross-sectionally, with the average returns of these portfolio. Finally, we form portfolios using put options written on the S&P 500 index, sorted on moneyness and maturity. We find that, for far-out-of-the-money index put options both average returns and the beta estimate for our DRP factor increase with increasing time to maturity. The same holds true for out-of-the-money and at-the-money index put options.

In order to cope with the possibility that some of the co-movement in changes in default risk premia could be due to reasons other than the common variation in covariances, we also test for firm characteristics such as the firm’s default probability and credit rating, the leverage ratio, and recovery rates. We find that the common variation in changes in default risk premia is not likely to be due to these firm characteristics, supporting our main theme that most of the common variation in changes
in default risk premia, unaccounted for by other known sources of common variation, is due to firms’ exposure to the DRP factor.

We then develop a theoretical framework in which the DRP factor arises naturally in the pricing kernel, and we show that it captures the jump-to-default risk associated with a market-wide credit events. Within this framework, we show that, unlike risk premia on corporate bonds, equity risk premia are only marginally affected by our DRP factor. This results is based on the observation that the DRP factor has a much stronger impact on the returns of assets with a non-degenerate payoff structure in the default states.

The remainder of this paper is structured as follows. Section 2 describes our data, comprised of credit default swap rates for Markit, Moody’s KMV EDF estimates for actual default probabilities and other accounting and market price data. Section 3 describes our measure of model-implied returns for constant-maturity zero-coupon corporate bonds, and Section 4 presents our methodology for extracting a latent common factor from the observed firm-specific changes in default risk premia. Section 9 presents our results from the asset pricing tests, and Section 6 proposes a theoretical framework of the relevant pricing kernel that is consistent with our empirical findings. Finally, Section 7 concludes.

2 Data

This section discusses our data sources for default swap rates, conditional default probabilities, equity and corporate bond returns, and other accounting and balance sheet information.
2.1 Credit Default Swaps

Credit default swaps (CDS) are single-name over-the-counter credit derivatives that provide bond insurance. The payoff to the buyer of protection covers losses up to notional in the event of default of a reference entity. Default events are triggered by bankruptcy, failure to pay, or, for some CDS contracts, a debt restructuring event. The buyer of protection pays a quarterly premium, quoted as an annualized percentage of the notional value, and in return receives the payoff from the seller of protection should a credit event occur. Fueled by participation from commercial banks, insurance companies, and hedge funds, the CDS market has been doubling in size each year for the past decade, reaching $12.43 trillion in notional amount outstanding by mid-2005.\(^1\)

In this paper, we use CDS spreads instead of corporate bond yield spreads as our primitive source for prices of default risk because default swap spreads are less confounded by illiquidity, tax and various market microstructure effects that are known to have a marked effect on corporate bond yield spreads.\(^2\) In particular, we use default swap spreads for five-year CDS contracts with modified restructuring (MR) for U.S.-dollar denominated senior unsecured debt. The data is provided by the Markit Group. It contains daily composite CDS spreads calculated from quotes contributed by several banks and default-swap brokers for approximately 2,000 reference entities, incorporated in the U.S. and abroad. One limitation of the Markit data is that it does not report the actual trading volume. Therefore, one concern with these CDS spreads is that no trades might have been actually transacted at the quoted spreads, especially for thinly traded reference entities. In an effort to ensure that our analysis is based on quotes that are representative of actual transaction prices,

\(^1\)See, for example, the International Swaps and Derivatives Association mid-2005 market survey. The CDS market is still undergoing rapid growth. The notional amount of default swaps grew by almost 48% during the first six months of 2005 to $12.43 trillion from $8.42 trillion. This represents a year-on-year growth rate of 128% from $5.44 trillion at mid-year 2004.

\(^2\)Recent papers that analyze the contribution of non-credit factors to bond yields include Zhou (2005), Longstaff, Mithal and Neis (2004), and Ericsson and Renault (2001).
we exclude from our analysis firms for which we have less than 1000 daily five-year CDS observations. We restrict ourselves to firms incorporated in the US so that our results are not confounded by cross-country differences in bankruptcy laws. As the capital structure of financial institutions is very different from non-financials, we also restrict our sample to non-financial corporate entities. Finally, to minimize market microstructure effects, we only use weekly data.

The sample of default swap rates used in this study consists of 108 entities from nine different industries, based on two-digit SIC codes. The sample period ranges from January 2001 to June 2005. The median firm in our sample has 7 contributors for the five-year CDS spread quote, and has 215 (of a maximum possible 231) valid weekly CDS observations. Figure 1 and Table 1 show the distribution of the 108 firms in our sample by median credit rating during 2001 through 2004, and across rating industries. Different from Berndt et al. (2005), who use industry-specific but constant loss-given-default values, we rely on contemporaneous recovery rate information from Markit.\footnote{Our understanding from conversations with Markit is that the reported recovery rates are indicative of the values used by their sources when valuing CDS rates. In that sense, we treat the recovery information as risk-neutral recoveries.} Table 2 shows the time series of median recovery rates by industry, for each week in our sample period.

### 2.2 EDF Data

We use the one-year Expected Default Frequency (EDF) data provided by Moody’s KMV as our measure of actual default probabilities. We will discuss this measure only briefly, referring the reader to Berndt et al. (2005) for a more detailed description. The concept of the EDF measure is based on structural credit risk framework of Black and Scholes (1973) and Merton (1974). In these models, the equity of a firm is viewed as a call option on the firm’s assets, with the strike price equal to the firm’s liabilities. The “distance-to-default” (DD), defined as the number of standard deviations of asset
growth by which its assets exceed a measure of book liabilities, is a sufficient statistic of the likelihood of default. In the current implementation of the EDF model, to the best of our knowledge, the liability measure is equal to the firms short-term book liabilities plus one half of its long-term book liabilities. Estimates of current assets and the current standard deviation of asset growth (volatility) are calibrated from historical observations of the firms equity-market capitalization and of the liability measure. For a detailed discussion, see, for example, Appendix A in Duffie, Saita and Wang (2005).

Crosbie and Bohn (2001) and Kealhofer (2003) provide more details on the KMV model and the fitting procedures for distance to default and EDF. Unlike the Merton model, where the likelihood of default is the inverse of the normal cumulative distribution function of DD, Moody’s KMV EDF measure uses a non-parametric mapping from DD to EDF that is based on a rich history of actual defaults. Therefore, the EDF measure is somewhat less sensitive to model mis-specification. The accuracy of the EDF measure as a predictor of default, and its superior performance compared to
Table 1: **Distribution of Firms Across Industries:** Firms are grouped into industries according to their two-digit SIC codes.

<table>
<thead>
<tr>
<th>Industry Name</th>
<th>SIC 2-digit code</th>
<th>No. of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business Services</td>
<td>73</td>
<td>8</td>
</tr>
<tr>
<td>Chemicals and Allied Products</td>
<td>28</td>
<td>20</td>
</tr>
<tr>
<td>Communication</td>
<td>48</td>
<td>14</td>
</tr>
<tr>
<td>Electric, Gas and Sanitary Services</td>
<td>49</td>
<td>19</td>
</tr>
<tr>
<td>Food and Kindred Products</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Industrial Machinery and Equipment</td>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>Instruments and Related Products</td>
<td>38</td>
<td>7</td>
</tr>
<tr>
<td>Paper and Allied Products</td>
<td>26</td>
<td>7</td>
</tr>
<tr>
<td>Transportation Equipment</td>
<td>37</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>108</td>
</tr>
</tbody>
</table>

rating-based default prediction, is documented in Bohn, Arora and Korbalev (2005). Duffie, Saita and Wang (2005) construct a more elaborate default prediction model, using distance to default as well as other covariates. Their model achieves accuracy that is only slightly higher than that of the EDF, suggesting that EDF is a useful proxy for the physical probability of default. Furthermore, the Moodys KMV EDF measure is extensively used in the financial services industry. As noted in Berndt et al. (2005), 40 of the worlds 50 largest financial institutions are subscribers.

We obtain monthly one-year EDF values from Moody’s KMV for the time period July 1993 through June 2005, as well as daily observations starting in 2001, for the same set of 108 firms described in Section 2.1. As discussed in Section 2.1, our CDS data only start in January 2001. In order to achieve sufficient power for our asset pricing tests we use weekly (Wednesday) observations of default swap rates, together with EDF values at a weekly frequency. We list the CDS and EDF coverage for each firm in our sample in Table 2 in Appendix A.

Figure 3 shows the median five-year CDS spreads and one-year EDFs across all firms in our sample. Both, CDS spreads and EDF vary considerably through our
Figure 2: Time series of median recovery rate, by industry. Source: Markit

sample period, peaking during the credit crunch of late 2002 to early 2003.

2.3 Return and Accounting Data

We obtain data on the Fama-French portfolios and factors from Ken French’s website. We also use return information for the investment-grade and high-yield Bloomberg-NASD corporate bond indices. These can be downloaded from the NASD website at http://www.nasdbondinfo.com. In addition, total index returns for the Merrill Lynch corporate bond portfolios are from Datastream. Finally, data used to compute firm-specific distance-to-default measures is from COMPUSTAT, and prices of common equity and riskless bond returns are from CRSP.
Figure 3: Time series of median five-year CDS rates and median one-year EDFs (in percent) across firms for our sample period. Sources: Markit and Moody’s KMV.

3 Measuring Returns on Risky Debt

We first describe how we estimate the excess reward investors in the corporate bond market demand for taking on credit risk, after accounting for expected default losses. Our approach to measuring actual and risk-neutral default probabilities is similar in spirit to Berndt et al. (2005). The main difference is that instead of using industry-specific but constant loss-given-default values as in their study, we rely on contemporaneous recovery rate information from Markit.\footnote{Our understanding from conversations with Markit is that the reported recovery rates are indicative of the values used by their sources when valuing CDS rates. In that sense, we treat the recovery information as risk-neutral recoveries.} Figure 2 in Section 2.1 shows the time series of median recovery rates by industry.

Given a probability space $(\Omega, \mathcal{F}, P)$ and information filtration $\{\mathcal{F}_t : t \geq 0\}$, the default intensity of a firm is the instantaneous mean arrival rate of default, conditional on all current information. More precisely, we suppose that default for a given firm
occurs at the first event time of a (non-explosive) counting process $N$ with intensity process $\lambda^P$, relative to a given probability space $(\Omega, \mathcal{F}, P)$ and information filtration $\{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions. In this case, so long as the firm survives, we say that its default intensity at time $t$ is $\lambda^P_t$. Under mild technical conditions, this implies that, conditional on survival to time $t$ and all information available at time $t$, the probability of default between times $t$ and $t + \Delta$ is approximately $\lambda^P_t \Delta$ for small $\Delta$. We also adopt the relatively standard simplifying doubly-stochastic, or Cox-process, assumption, under which the conditional probability at time $t$, for a currently surviving obligor, that the obligor survives for some time $T$, is

$$p(t, T) = E_t \left( e^{-\int_t^T \lambda^P_s \, ds} \right).$$  

(1)

Here, $E_t$ denotes expectation conditional on information available up to and including time $t$.

Under the absence of arbitrage and market frictions, and under mild technical conditions, there exists a “risk-neutral” probability measure, also known as an “equivalent martingale” measure, as shown by Harrison and Kreps (1979) and Delbaen and Schachermayer (1999). In our setting, markets should not be assumed to be complete, so the martingale measure is not unique. This pricing approach nevertheless allows us, under its conditions, to express the price at time $t$ of a security paying some amount, say $Z$, at some bounded stopping time $\tau > t$, as

$$S_t = E_t^Q \left( e^{-\int_t^\tau r_s \, ds} Z \right),$$  

(2)

where $r$ is the short-term interest-rate process\(^5\) and $E_t^Q$ denotes expectation conditional on information available up to and including time $t$ with respect to an equiva-

\(^5\)Here, $r$ is a progressively measurable process with $\int_0^t |r_s| \, ds < \infty$ for all $t$, such that there exists a “money-market” trading strategy, allowing investment at any time $t$ of one unit of account, with continual re-investment until any future time $T$, with a final value of $e^{\int_0^T r_s \, ds}$.
lent martingale measure \( Q \), that we fix. One may view (4) as the definition of such a measure \( Q \). The idea is that the actual (or physical) measure \( P \) and the risk-neutral measure \( Q \) differ by an adjustment for risk premia. For the following sections it will be useful to compute the conditional surviving probability at time \( t \), under the risk-neutral measure \( Q \). A currently surviving obligor will survives for some time \( T \) with probability

\[
p^Q(t, T) = E_t^Q \left( e^{-\int_t^T \lambda^Q_s \, ds} \right).
\]  

(3)

For a given firm, Elton, Gruber, Agrawal and Mann (2001) measure returns in corporate bonds by comparing prices for constant maturity zero-coupon bonds at which they would trade under no-arbitrage assumptions when actually issued by the firm. Prices of these fictive debt securities can be derived, for example, from firm-specific time-series information on corporate bonds or credit default swaps. In (4) we derive the price of a risky bond at time \( t \) that pays one unit of account if the firm does survive until time \( t + \Delta t \), for some \( \Delta t > 0 \), as \( P_t = E_t^Q \exp(-\int_t^{t+\Delta t} r_s + \lambda^Q_s \, ds). \)

One length-\( h \) time period earlier,\(^6\) that price was \( P_{t-h} = E_{t-h}^Q \exp(-\int_{t-h}^{t-h+\Delta t} r_s + \lambda^Q_s \, ds). \) Thus, the realized return \( R_t \) for constant-maturity \( \Delta t \)-period zero-coupon bonds issued by the firm is given by

\[
R_t = \frac{P_t}{P_{t-h}} = \frac{E_t^Q \left( e^{-\int_t^{t+\Delta t} r_s + \lambda^Q_s \, ds} \right)}{E_{t-h}^Q \left( e^{-\int_{t-h}^{t-h+\Delta t} r_s + \lambda^Q_s \, ds} \right)}.
\]

If we assume that \( \Delta t \) is small and that \( r_s \approx r_t \) and \( \lambda^Q_s \approx \lambda^Q_t \), for \( t \leq s < t + \Delta t \), then we can express \( R_t \) as:

\[
R_t = \frac{e^{-[r_t + \lambda^Q_t]\Delta t}}{e^{-[r_{t-h} + \lambda^Q_{t-h}]\Delta t}} = e^{-[r_t - r_{t-h}]\Delta t - [\lambda^Q_t - \lambda^Q_{t-h}]\Delta t}
\]

(4)

\(^6\)For the time series estimation in Section 4 and the asset pricing tests in Section 9 we will use a time step \( h \) of one week.
Since $\Delta t$ is small one can use the first-order Taylor approximation\(^7\) in $\Delta t$ to simplify further the above exponential to:

$$R_t = 1 - \left[ r_t - r_{t-h} \right] \Delta t - \left[ \lambda_t^Q - \lambda_{t-h}^Q \right] \Delta t$$  \hspace{1cm} (5)

Empirical studies of predictability of changes in credit spreads as measured by (4) have shown that structural model variables that should in theory have large explanatory power perform rather poorly (see, for example, Collin-Dufresne, Goldstein and Martin (2001)), and that changes in expected default losses do not contain information about systematic risk beyond the information already captured by Fama-French factors (see, for example, Elton et al. (2001)). Based on the later observation, we will focus on that portion $R_t^u$ of the return $R_t$ in (4) that is not due to changes in expected default losses or changes in risk-free rates. In particular, if investors where risk-neutral, the realized return on the constant-maturity $\Delta t$-period zero-coupon bonds would equal the expected loss:

$$R_t^L = \frac{E_t \left[ e^{-\int_{t-h}^{t+\Delta t} r_s + \lambda_s ds} \right]}{E_{t-h} \left[ e^{-\int_{t-h}^{t-h+\Delta t} r_s + \lambda_s ds} \right]}$$  \hspace{1cm} (6)

where $\lambda_s$ is the actual default intensity of firm $i$. When $\Delta t$ is small, we can use the approach above to simplify $R_t^L$ to:

$$R_t^L = 1 - \left[ r_t - r_{t-h} \right] \Delta t - \left[ \lambda_t - \lambda_{t-h} \right] \Delta t$$  \hspace{1cm} (7)

Thus, the unexplained component of the return $R_t$ can now be computed as:

$$R_t^u = R_t - R_t^L = - \left\{ \left[ \lambda_t^Q - \lambda_t \right] - \left[ \lambda_{t-h}^Q - \lambda_{t-h} \right] \right\} \Delta t$$  \hspace{1cm} (8)

\(^7\)More precisely, we use the approximation $e^{-ax} = 1 - ax$ with $a = \left[ r_t - r_{t-h} \right] + \left[ \lambda_t^Q - \lambda_{t-h}^Q \right]$ and $x = \Delta t$
In addition, \((\lambda^Q_t - \lambda^P_t)\) measures the difference between instantaneous risk-neutral and actual default probabilities, and can be interpreted as a measure of default risk premia. One may think, therefore, of the annualized unexplained returns \(R^a_t / h\) as minus the changes in default risk premia.

If constant-maturity zero-coupon bonds as described above were actively traded, we could observe prices \(P_{t,h}\) directly, and it would be possible to compute returns on corporate debt using (4). As this is not the case, however, we proceed by estimating time-series models for \(\lambda^P\) and \(\lambda^Q\) from different sources, and then compute model-implied actual and risk-neutral survival probabilities \(p(t, h)\) and \(p^Q(t, h)\). We follow Berndt et al. (2005) and identify the default intensity \(\lambda^P\) under the physical measure from the information contained in the Moody’s KMV EDFs, while \(\lambda^Q\) will be estimated from default swap data. Details on the choice of the time-series models for \(\lambda^P\) and \(\lambda^Q\) and our estimation technique are discussed in Section 4.

Afterwards, we will extract the latent common component, that is, our default risk premia factor, from these firm-specific unexplained returns \(R^a_t\). We describe our approach in Section 4. One may think of our DRP factor as a measure of realized excess returns of a common risk factor embedded in the default events across firms.

4 Extracting the Default Risk Premia Factor

In this section we first describe the time-series models for both actual and risk-neutral default intensities. Similar to Berndt et al. (2005), we specify a model under which the logarithm of the actual default intensities \(\lambda^P_t\) satisfies the Ornstein-Uhlenbeck equation

\[
d\log(\lambda^P_t) = \kappa(\theta - \log(\lambda^P_t)) \, dt + \sigma \, dB_t, \tag{9}
\]
where \( B_t \) is a standard Brownian motion, and \( \theta, \kappa, \) and \( \sigma \) are firm-specific constants to be estimated. The behavior for \( \lambda^P \) is called a Black-Karasinski model. (See Black and Karasinski (1991).) This leaves us with a three-dimensional vector \( \Theta = (\theta, \kappa, \sigma) \) of unknown parameters to be estimated from available firm-by-firm EDF observations of a given firm. For the majority of the 108 firms in our sample, we have 144 months of one-year EDF observations, from July 1993 to June 2005.

Given the log-autoregressive form (28) of the default intensity, in general there is no closed-form solution available for the one-year EDF, \( 1 - p(t, 1) \), from (2). We therefore rely on numerical lattice-based calculations of \( p(t, 1) \), and have implemented the two-stage procedure for constructing trinomial trees proposed by Hull and White (1994).

With regard to risk-neutral default intensities, we assume that

\[
d \log \lambda^Q_t = \kappa^Q (\theta^Q - \log(\lambda^Q_t)) \, dt + \sigma^Q \, dB^Q_t,
\]

where \( B^Q_t \) is a standard Brownian motion with regard to the physical measure \( P \), and \( \kappa^Q, \theta^Q, \) and \( \sigma^Q \) are scalars to be estimated. The risk-neutral distribution of \( \lambda^Q \) is specified by assuming that

\[
d \log \lambda^Q_t = \tilde{\kappa}^Q (\tilde{\theta}^Q - \log(\lambda^Q_t)) \, dt + \tilde{\sigma}^Q \, d\tilde{B}^Q_t,
\]

where \( \tilde{\kappa}^Q \) and \( \tilde{\theta}^Q \) are constants and \( \tilde{B}^Q_t \) is a standard Brownian motion with regard to \( Q \). Given a set of parameters \( (\tilde{\theta}^Q, \tilde{\kappa}^Q, \sigma^Q) \), we can compute model-implied values for \( \lambda^Q \) using data on five-year CDS rates and risk-neutral loss given default. For details we refer the reader to Section 5.1 in Berndt et al. (2005).

Using maximum likelihood estimation (MLE), we obtain firm-by-firm estimates for the parameters that govern the processes for \( \lambda^P \) and \( \lambda^Q \). These parameter estimates are listed in Table 3 and 4 in Appendix B. The estimation techniques employed here are similar to those used in Berndt et al. (2005), except for the fact that Markit
provides us with information on contemporaneous recovery rates, for each firm and each date, that we will use in place of an assumption of constant risk-neutral recovery in the event of default. As can be seen in Figure 2 in Section 2.1, median (risk-neutral) recovery rates range between 37% and 45%.

Figure 4 plots the time series of the median differences between estimated instantaneous risk-neutral default probabilities, $\lambda^Q_t$, and estimates for instantaneous actual default probabilities, $\lambda^P_t$, for each industry identified by its two-digit SIC code. Interpreting $\lambda^Q_t - \lambda^P_t$ as a measure of default risk premia in the corporate bond market, we find that it peaked quite dramatically for the Communication industry in the third quarter of 2002, and that it surged for both the Utilities and the Paper sector later that year.

Using our estimates for $\lambda^Q_t$ and $\lambda^P_t$, we can now compute estimates for the unexplained part $R^u_t$ of realized returns on constant-maturity zero-coupon bonds as given in (8) in Section 3. We will denote by $F^D_t$ the time-$t$ level of the latent common
component to be extracted from firm-specific unexplained returns $R^u_t$. We refer to $F^D$ as the DRP factor. As explained in Section 3, $F^D$ captures the common variation in changes in default risk premia, and one may think of it as a measure of realized excess returns of a common risk factor embedded in the default events across firms. In addition, let $F^S_t$ denote the vector of $h$-period returns on known systematic factors. Among the factors we account for are those in Fama and French (1993), including their default and term factor, and the momentum factor introduced in Jagadeesh and Titman (1993).\footnote{We run all tests with and without the default factor. Results do not change substantially, and we only report them for the scenario where the default factor is included as a known systematic factor.} Using superscript $i$ to indicate returns specific to firm $i$, for $i = 1, \ldots, N$, we run the following least-squares regression model with firm- and time-specific effects on the panel data of unexplained excess returns:

$$R^u_{it} = \alpha^i + \beta^{S,i} \cdot F^S_t + \sum_{\text{weeks } t} \delta_t 1_{\{t=t\}} + \epsilon^i_t. \tag{11}$$

For each firm $i$, the errors $\epsilon^i_t$ have a sample mean of zero across time. They will absorb any variation in default risk premia that cannot be explained by linear combinations of systematic factors $F^S_t$ and a single latent common component. In order to identify all unknown parameters $\{\alpha^i\}, \{\beta^{S,i}\}$, and $\{\delta_t\}$ we have to impose two normalizing restrictions: (i) the sample mean of $\delta_t$ is zero, and (ii) the sample correlations between $\delta_t$ and each of the systematic factors in $F^S_t$ are zero. We measure our default risk premia (or DRP) factor $F^D_t$ as

$$F^D_t = \frac{1}{N} \sum_{i=1}^{N} \hat{\alpha}^i + \hat{\delta}_t,$$

where $\hat{\delta}_t$ denotes the least-squares estimate of the time-$t$ dummy multiplier in (11). Figure 5 shows the time series of the extracted latent factor $F^D$. Note that most of the time variation in the DRP factor $F^D_t$ occurs during the second part of 2002.
5 Asset Pricing Tests

In this section we investigate whether the common variation in changes in default risk premia induced by our DRP factor, $F_t^D$, is systematic in nature. In other words, we test whether this factor is priced in the cross-section of stock and corporate bond returns. Our asset pricing test is a variant of the Fama-MacBeth methodology (see Fama and MacBeth (1973)). Specifically, we consider a set of test assets and inves-
tigate whether their loadings on known systematic factors and our DRP factor have explanatory power for the cross-section of average returns. Among the factors we account for are those in Fama and French (1993), including their default and term factor, and the momentum factor introduced in Jagadeesh and Titman (1993).

The ideal test assets should have two important features: (i) they should span the entire spectrum the capital markets, and (ii) the test assets should exhibit a high degree of variation in average returns. The first condition is important in defining the generality of our test, while the second feature ensures that the cross-section of expected returns is sufficiently rich. As a compromise between meeting these conditions and data availability, we decided on a set of test assets which consists of the 100 Fama-French portfolios formed on size and book-to-market equity, the two Bloomberg-NASD investment-grade and high-yield corporate bond portfolios, and several Merrill Lynch corporate bond portfolios, sorted by rating (7 portfolios ranging from AAA to C), by maturity (6 portfolios ranging from 1 to 3 years (1-3Y) to more than 15 years (15Y+), and by industry (30 high-yield portfolios, and 4 investment-grade portfolios).

Due to the fact that the last three Fama-French equity portfolio have missing data for the first part of the sample, we drop them from our set of test assets and, for the rest of the paper, we run our asset pricing test on 97 equity portfolios and 49 corporate bond portfolios.

Our asset pricing tests proceed in two stages. In the first step we determine the loadings of each of the test assets on explanatory factors such as the Fama and French’s stock market factors $RMO$, $SMB$ and $HML$, the momentum factor $UMD$, the corporate debt market factor $DEF$, the treasury bond market factor $TERM$, as well as our default risk premia factor $DRP$. (The DRP factor is equal to $F^D$ as
defined in (12).) More formally, we estimate the linear model

\[ R_i(t) - RF(t) = \alpha_i + \beta_{RMO}^i RMO(t) + \beta_{SMB}^i SMB(t) + \beta_{HML}^i HML(t) + \beta_{UMD}^i UMD(t) + \beta_{DEF}^i DEF(t) + \beta_{TERM}^i TERM(t) + \beta_{DRP}^i DRP(t) + \epsilon_i(t) \]  

for each test asset \( i \). Here, \( R_i(t) \) denotes the return on asset \( i \) over the time period \([t - h, t]\), where \( h \) is one week, and \( RF(t) \) measures the risk-free rate, compounded weekly from the Fama-French T-bill daily returns.

Second, we investigate the explanatory power of the estimated loadings for the cross-section of returns of the test assets. Specifically, for every week \( t \), we run a cross-sectional regression

\[ R(t)^i - RF(t) = \gamma_0(t) + \gamma_{RMO}(t)\beta_{RMO}^i + \gamma_{SMB}(t)\beta_{SMB}^i + \gamma_{HML}(t)\beta_{HML}^i + \gamma_{UMD}(t)\beta_{UMD}^i + \gamma_{DEF}(t)\beta_{DEF}^i + \gamma_{TERM}(t)\beta_{TERM}^i + \gamma_{DRP}(t)\beta_{DRP}^i + \epsilon_i(t). \]  

(13)

Notice that, under the usual OLS assumption, the unconditional version of this regression gives the very asset pricing restriction which we investigate here. In particular, we have

\[ E[R(t)^i - RF(t)] = E[\gamma_0(t)] + E[\gamma_{RMO}(t)]\beta_{RMO}^i + E[\gamma_{SMB}(t)]\beta_{SMB}^i + E[\gamma_{HML}(t)]\beta_{HML}^i + E[\gamma_{UMD}(t)]\beta_{UMD}^i + E[\gamma_{DEF}(t)]\beta_{DEF}^i + E[\gamma_{TERM}(t)]\beta_{TERM}^i + E[\gamma_{DRP}(t)]\beta_{DRP}^i. \]

5.1 Corporate Bonds

We test first whether the default risk premia factor \( DRP \) is priced in the market for corporate bonds. As mentioned above, whenever possible, we implement the two-step Fama-MacBeth procedure, which involves estimating regressions of type (12)
and (13).

Table 5 summarizes the results of the time-series regression (12), for the two NASD corporate bond portfolios. For comparison we provide the results of a similar regression, but without the DRP factor (the standard Fama and French (1993) regression, adapted for our sample period). We notice that, after controlling for other known systematic effects, both the high-yield and the investment-grade portfolios load economically and statistically significant on our default risk premia factor. Moreover, the loading for the high-yield portfolio is several times larger than the corresponding loading for the investment-grade portfolio. Given that the average annual return on the high-yield portfolio (17.46 percent) is larger than the average annual return on the investment-grade portfolio (2.93 percent), the results of Table 3 suggest that, in the market for corporate bonds, higher expected return could be compensation for bearing default risk, as proxied by our default risk premia factor. It is of interest to note that inclusion of the DRP factor in the time series regression leads to a decrease in $\alpha$ by roughly 1% a year for the high-yield corporate bond portfolio. It has a much smaller effect on the intercept for investment-grade debt. So even though DRP helps explaining the unconditional mean excess returns of the two bond portfolios, the intercepts might still appear relatively high. A potential explanation can be due to the fact that none of the explanatory variables considered here captures the high illiquidity effects in the corporate bond markets, as documented for example by Driessen (2005).\footnote{We are in the process of acquiring data which will allows us to implement some of the liquidity measures used in the literature.}

We further test the strength of this relation, by investigating the time-series behavior of corporate bond portfolios, sorted on specific characteristics or on sector and industry. Table 6 reports the estimates of the time-series regression (12), when the test assets are corporate bond portfolios, sorted on rating. Here, we notice that the loading on the DRP factor is highly correlated with the rating, which in turn is
highly correlated with the average return of the portfolios. Thus, even after controlling for other potential systematic factors, the expected returns of the portfolios and the loadings on the DRP factor are still strongly positively related. Table 7 repeats the exercise for the case where the test assets are corporate bond portfolios sorted on time to maturity. The results here support our findings in the previous two tables, namely that higher loadings on the DRP factor translate into higher expected returns, even after controlling for other potential systematic factors.

The conjectured relation between average returns and the loadings on the default risk premia factor is further tested in Table 8 and 9, where the test assets consist of high-yield corporate bond portfolios, sorted by sector. (Table 10 repeats the analysis of industry portfolios for investment-grade debt.) Table 8 shows that, on average, the default risk premia factor is significant for the time-series variation of expected bond returns. More importantly, Table 9 shows that measures of risk based on our DRP factor (that is, the time-series loadings on the factor) are relevant for the cross-section of corporate bond returns. Once again, our results suggest that the default risk premia factor is priced in the market for corporate bonds.

5.2 Equity

We now turn our focus to the equity market. Table 11 shows that our DRP factor contributes only little to the time-series variation of the equity portfolios, while other potential systematic factors account for about 82 percent, on average. In addition, the loadings on the Fama-French factors are very significant and economically consistent with the findings of Fama and French (1993), for their sample. The loadings on our DRP factor are significant, on average, but as it is apparent from Table 12 they do not matter for the cross-section of stock returns. We replicated this experiment for a wide range of test assets originating from the equity market, including the Fama-French 49 industry portfolios, the decile portfolios formed on the book-to-market equity, and
others. Up to this point, none of these tests were successful in detecting a significant relation between average stock returns and the loadings on the default risk premia factor.

Overall, our results so far suggest that the common variation in firm-specific changes in default risk premia is priced in the corporate bond market. It is important to stress the fact that the risk behind the common default component DRP is completely uncorrelated with the risk behind previously known systematic factors. (Recall that our default risk premia factor defined in (12) in Section 4 is by design orthogonal to all components in $F^S$. This orthogonality statement holds in sample. To generalize, one needs to assume that the cross-section of firms from which we extract the default risk premia factor is sufficiently large.)

### 5.3 Options

In this section, we test whether our DRP factor is priced in the options market. Motivated by the findings in Coval and Shumway (2001) and Jones (2006) that a significant portion of returns on short-term out-of-the-money index put options cannot be explained by known systematic risk factors, we form test portfolios using put options written on the S&P 500 index, sorted on moneyness and maturity. Moneyness is defined as the present value of the strike price, computed using the maturity-matched risk-free rate, divided by the current value of the S&P 500 index. We then use these portfolios to examine whether the DRP factor makes a contribution to explaining the cross-sectional variation in index put options returns.

The options data is obtained from OptionMetrics. In what follows we describe in more detail how we form the options portfolios. First, we classify the options into four maturity bins, with times to maturity of 10 to 30 days, 31 to 60 days, 61 to 150 days, and more than 150 days. We don’t use options with fewer than 10 days to maturity since reported prices of these options are more likely to be erroneous.
Next, we split each maturity bin into three sub-bins based on moneyness. For these sub-bins, we choose the moneyness cutoff points so that within each maturity bin, the moneyness bins have approximately the same number of observations. This results in 12 portfolios sorted first on time to maturity and then on moneyness. Each week (Wednesday) $t$, we assign each option to a particular bin based on its maturity and moneyness as that time, and compute its returns to week $t+1$. That is, returns calculated are weekly returns. The return of any particular maturity-moneyness portfolio at time $t+1$ is then the average of the buy-and-hold returns between from week $t$ to week $t+1$ of all options that were in this particular maturity-moneyness bin as of time $t$. We compute both, equally-weighted and value weighted returns. Prices are computed as the average of the best bid and best offer price on a given day. The value-weighted returns use prices as of time $t$ for the weights. Summary statistics for the 12 put option portfolios are reported in Table 13 in the appendix.

We restrict ourselves to options with standard settlement. To eliminate prices with large errors, we only use observations that satisfy the following criteria. The trading volume is positive, both bid and offer prices are positive, offer prices are at least as high as bid prices, the open interest is positive, the sum of the mid price plus the bid-ask spread is at least as high as the intrinsic value of the option, and the reported implied volatility is at least 1%.

The intrinsic value is calculated as the larger of the present value of dividends plus the present value of the strike price minus the S&P index closing value, and zero. Under no-arbitrage assumptions, the price of the put option should exceed its intrinsic value. To allow for non-synchronous reporting of the value of the underlying and the option, we use a somewhat looser constraint, enforcing only that the price plus spread exceeds the intrinsic value. Following Jones (2006), we also use an implied volatility cutoff to remove options prices that are suspect.

We then conduct asset pricing tests using these 12 options portfolios. We run
time-series regressions of the returns of these portfolios, in excess of the risk-free rate, on the excess returns on the market, the SMB, HML, UMD, DEF and TERM factors, as well as the volatility index factor VIX and our DRP factor. We account for VIX to capture systematic volatility risk premia. The results are shown in Table 14 in Appendix D. We find that, for far-out-off-the-money index put options (moneyness bin 1), both average returns and the beta estimate for our DRP factor increase with increasing time to maturity. The same holds true for out-of-the-money (moneyness bin 2) and at-the-money (moneyness bin 1) put options.

5.4 Test for Firm Characteristics

We conclude this section with a test based on firm characteristics. Following the argument in Daniel and Titman (1997), we study the extend to which the common component in changes in default risk premia is due to firm characteristics, which may behave very similarly, across firms, over time. Specifically, we investigate the possibility that the time variation in changes in default risk premia may be driven solely by certain firm characteristics, say \( \theta(t) \). Formally, we test whether a linear model of the form

\[
R_{u,i}(t) = \alpha_i + \beta_{RMO}^i RMO(t) + \beta_{SMB}^i SMB(t) + \beta_{HML}^i HML(t) \\
+ \beta_{UMD}^i UMD(t) + \beta_{DEF}^i DEF(t) + \beta_{TERM}^i TERM(t) \\
+ \beta_{DRP}^i DRP(t) + \beta_{Char}^i \theta^i(t - 1) + \epsilon_i(t)
\]  

(14)
can be justified in this context. On the left-hand side of (14) we have the unexplained returns on constant-maturity zero-coupon bonds of firm \( i \), \( R_{u,i}^a \), as defined in (8) in Section 3, while on the right-hand side we have the usual factors plus a time-varying firm characteristics. Notice that the time-varying (or conditional) changes in default risk premia depend on the time-varying characteristic \( \theta(t - 1) \). Thus, if the firm
characteristics move together, it would appear as if unexplained returns $R_{t}^{u,i}$ move together.

Table 15 reports the results of these regressions for firm characteristics such as the firm’s default probability (or credit rating), the leverage ratio and the recovery rate. We notice that the common variation in changes in default risk premia is very unlikely to be due to these firm characteristics. Moreover, in each of the tests, the loading on the DRP factor is always very significant, both economically and statistically.

6 A Model Framework Explaining Our Results

In this section, we propose a theoretical framework of corporate default that is consistent with our findings. We consider an economy with $N$ firms, in which the fundamentals are captured by a vector of $d$-dimensional state variables, $X_t$, with dynamics given by

$$dX_t = \mu(X_t, t)dt + \Sigma(X_t, t)dW_t,$$

where $\mu(\cdot, t)$ is a $d$-dimensional column vector of drifts and $\Sigma(\cdot, t)$ is a $d \times d$ state-dependent volatility matrix. Here, $W_t$ is a $d$-dimensional standard Brownian motion on some probabilistic space $(\Omega, \mathbb{P})$ with informational filtration $\{\mathcal{F}_t\}_{t \geq 0}$ generated by this process. The innovations $dW_t$ in $W_t$ describe the diffusive systematic risk in our economy.

We define $N+1$ stopping times

$$\bar{\tau}^i = \inf_{t \geq 0} \int_0^t \bar{\lambda}^{P,i}(X_s) ds \geq \theta^i, \text{ for } i = 0, 1, \ldots, N,$$

where, for all $i$, $\bar{\lambda}_t^{P,i} = \bar{\lambda}^{P,i}(X_t)$ is a non-negative random variable, and $\theta^i$ is an exponentially distributed random variable with mean one. One can interpret $\bar{\tau}^0$ as the arrival time of a market-wide credit event that affects all firms in the economy,
whereas \( \bar{\tau}^i \) denotes the event time of an idiosyncratic default event for firm \( i \). Let \( \{G_t\}_{t \geq 0} \) denote the extended filtration, generated by the state vector \( X_t \) and the random stopping times \( \{\bar{\tau}^i\}_{0 \leq i \leq N} \). Finally, let \( \tilde{N}_i^i = 1_{\{\bar{\tau}^i < t\}} \) denote the counting processes associated with stopping time \( \bar{\tau}^i \).

The following assumptions are key ingredients to the tractability of our model.

**Assumption 1**

(i) The joint informational content of the random variables \( \{\theta^i\}_{0 \leq i \leq N} \) is independent from the information contained in \( F_\infty \).

(ii) The functions \( \{\lambda^i\}_{0 \leq i \leq N} \) are chosen such that \( N_i^i = \tilde{N}_i^i - \int_0^t \lambda^{P,i}(X_s) \, ds \) are martingales with respect to the filtration \( G_t \), for any \( i = 0, 1, \ldots, N \).

(iii) The default event of each firm \( i \), \( i = 1, \ldots, N \), is triggered by an exogenous random variable \( \tau^i \) given by

\[
\tau^i = \min\{\bar{\tau}^0, \bar{\tau}^i\}.
\]

(15)

Define \( \bar{\lambda}_t^P = \bar{\lambda}_t^{P,1} + \ldots + \bar{\lambda}_t^{P,N} \), and let

\[
\bar{\Gamma}_t = \frac{1}{\bar{\lambda}_t^P} \sum_{i=1}^N \bar{\lambda}_t^{P,i} \Gamma^i(X_t),
\]

where \( \Gamma^i_t = \Gamma^i(X_t) \) is the market price of jump-to-default risk associated with the idiosyncratic default event \( \bar{\tau}^i \), for all \( i \). Suppose that the relevant pricing kernel \( M_t \) for this economy is given by

\[
\begin{align*}
\frac{dM_t}{M_t} &= -r(X_t) \, dt - \Lambda(X_t) \, dW_t - \sum_{i=0}^N \Gamma^i(X_{t^-}) \left( dN_i^i - \bar{\lambda}_t^{P,i} \, dt \right) \\
&= - \left( r(X_t) + \Gamma^0_t \bar{\lambda}_t^{P,0} + \bar{\Gamma}_t \bar{\lambda}_t^p \right) \, dt - \Lambda(X_t) \, dW_t - \sum_{i=0}^N \Gamma^i(X_{t^-}) \, dN_i^i \quad (16)
\end{align*}
\]
where \( r_t = r(X_t) \) is the instantaneous risk-free interest rate, \( \Lambda_t = \Lambda(X_t) \) denotes the market price of diffusive risk, and \( \Gamma^0_t = \Gamma^0(X_t) \) is the market price of jump-to-default risk associated with the market-wide credit event time \( \bar{\tau}^0 \).

As long as the functionals \( \Lambda, \Gamma^0, \{\Gamma^i\}_{1 \leq i \leq N} \) are well-behaved (e.g. bounded), the above equation admits a unique solution\(^\text{10}\)

\[
M_t = \exp \left\{ -\int_0^t r_s ds + \int_0^t \Gamma^0_s \lambda^0 ds - \frac{1}{2} \int_0^t \Lambda^2_s ds - \int_0^t \Lambda_s dW_s \right\} \times \text{exp} \left\{ \int_0^t \Gamma^0_s \lambda^0 ds \right\} \times \prod_{0 \leq s < t} \left[ 1 - \sum_{i=0}^N \Gamma^i_s \Delta \bar{N}^i_s \right], \tag{17}
\]

where \( \Delta \bar{N}^i_s = \bar{N}^i_s - \bar{N}^i_{s-} \), for all \( i = 0, 1, \ldots, N \). Note that \( \text{exp}(\int_0^t r_s ds)M_t \) is a martingale with respect to \( \mathcal{G}_t \).

Let \( \Omega_0 \) denote the subset of \( \Omega \) for which at least two of the counting processes \( \bar{N}^i_t \), \( i = 0, 1, \ldots, N \), jump at the same time (that is, \( \Delta \bar{N}^i \Delta \bar{N}^j = 1 \) for some \( 0 \leq i \neq j \leq N \)). Consider the probability space \((\Omega - \Omega_0, \mathcal{G}_t, P|_{\Omega - \Omega_0})\). The following assumption allows us to generalize the results in Dai and Singleton (2003), who consider a similar environment with one firm only, within the framework of our model.

**Assumption 2** The random variables \( \theta_i, i = 0, 1, \ldots, N \), are approximatively exponentially distributed with mean one, on the reduced space \( \Omega - \Omega_0 \).

While this assumption might seem somewhat restrictive, it is not unjustified, given that for our specifications of \( \lambda^{P,i} \), the probability that two or more of the count processes \( \bar{N}^i \) will jump at the same time is very small. For the rest of the paper we will focus only on the reduced space \((\Omega - \Omega_0, \mathcal{G}_t, P|_{\Omega - \Omega_0})\). To simplify notation, we re-denote this space as the new \((\Omega, \mathcal{G}_t, P)\). Ruling our simultaneous event times implies that the actual default intensity \( \lambda^{P,i} \) for firm \( i \) introduced in (28) in Section 4 can be though of as \( \lambda^{P,i} = \bar{\lambda}^{P,0} + \bar{\lambda}^{P,i} \).

\(^{10}\)For details, see Protter (2005), page 84, Theorem 37.
Under Assumption 2, (17) can be rewritten in a convenient way, by noticing that \( \Delta \tilde{N}^i \Delta \tilde{N}^j = 0 \) for any \( 0 \leq i \neq j \leq N \), as

\[
M_t = \exp \left\{ - \int_0^t r_s ds + \int_0^t \Gamma_s \bar{\lambda}^P_s ds - \frac{1}{2} \int_0^t \Lambda^2_s ds - \int_0^t \Lambda_s dW_s \right\} \\
\times \exp \left\{ \int_0^t \Gamma^0_s \bar{\lambda}^{P,0}_s ds \right\} \times \prod_{i=0}^N \prod_{0 \leq s < t} \left[ 1 - \Gamma^i_s \Delta \tilde{N}^i_s \right].
\]

The fact that \( \exp(\int_0^t r_s ds) M_t \) is a martingale allows us to construct an equivalent (risk-neutral) martingale measure \( Q \) on \( \Omega \) from the Radon-Nikodým density \( \frac{dQ}{dP} |_{t} = \exp(\int_0^t r_s ds) M_t \). Note that under this risk-neutral measure, \( N^Q,i_t = \tilde{N}^Q,i_t - \int_0^t \bar{\lambda}^Q,i(X_s) ds \) become martingales. Here, \( \tilde{N}^Q,i_t \) is defined as before after replacing \( \bar{\lambda}^P,i_t \) by \( \tilde{\lambda}^Q,i = (1 - \Gamma^i_t) \bar{\lambda}^P,i_t \), for all \( i = 0, 1, \ldots, N \).

Following the definition in Section 3, the unexplained return \( R^i_{t,u} \) for firm \( i \) is approximately equal to

\[
R^i_{t,u} = \left[ \Gamma^0_t \bar{\lambda}^{P,0}_t - \Gamma^0_{t-h} \bar{\lambda}^{P,0}_{t-h} \right] h + \left[ \Gamma^i_t \tilde{\lambda}^{P,i}_t - \Gamma^i_{t-h} \tilde{\lambda}^{P,i}_{t-h} \right] h.
\] (18)

Equation (18) suggests that the common component of the returns \( R^i_{t,u} \) is to a large extent driven by the changes in the market price of jump-to-default risk associated with the market-wide default event \( \bar{\tau}^0 \). Thus, our default risk premia factor \( F^D \) is likely to capture the impact on returns due to this market-wide source for jump-to-default risk.

We now investigate the effect of the diffusive risk and of the jump-to-default risk on the expected returns of a firm’s equity and debt claims. If the markets for both equity and debt are competitive, the pricing equation is the Euler equation. That is,

\[
E_t \left[ \frac{M_{t+h} \tilde{R}^i_{t+h}}{M_t} \right] = 1
\] (19)

\(^{11}\)When this is the case, we have \( 1 - \sum_{i=0}^N \Gamma^i_s \Delta \tilde{N}^i_s = \prod_{i=0}^N \left[ 1 - \Gamma^i_s \Delta \tilde{N}^i_s \right]. \)
where \( \tilde{R}_{t+h}^i \) denotes the gross return on either the equity or the debt of firm \( i \).

As long as a firm is solvent, the gross return on equity claims is non-zero. We will assume a zero-recovery value to equity holders in the event of default. This implies \( \tilde{R}_{t+h}^{E,i} = \tilde{R}_{t+h}^{S,i} 1_{\{\tau^i > t+h\}} \). Here, \( \tilde{R}_{t+h}^{S,i} \) stands for the total return on firm \( i \)'s equity if the company does not default prior to or at time \( t+h \). With this in mind, we have

\[
E_t \left[ \frac{M_{t+h}}{M_t} \tilde{R}_{t+h}^{E,i} \right] = E_t \left[ \mathcal{E}_{t+h} \prod_{i=0}^{N} \prod_{t \leq s < t+h} \left[ 1 - \Gamma_s^i \Delta N_s^i \right] \tilde{R}_{t+h}^{S,i} 1_{\{\tau^i > t+h\}} \right] \\
= E_t \left[ \mathcal{E}_{t+h} \prod_{j \neq i} \prod_{t \leq s < t+h} \left[ 1 - \Gamma_s^j \Delta N_s^j \right] \tilde{R}_{t+h}^{E,i} \right] \\
- E_tE_t \left[ \mathcal{E}_{t+h} \prod_{j \neq i} \prod_{t \leq s < t+h} \left[ 1 - \Gamma_s^j \Delta N_s^j \right] \tilde{R}_{t+h}^{E,i} \right] E_t \left[ \tilde{R}_{t+h}^{E,i} \right] \\
+ E_tE_t \left[ \mathcal{E}_{t+h} \prod_{j \neq i} \prod_{t \leq s < t+h} \left[ 1 - \Gamma_s^j \Delta N_s^j \right] \tilde{R}_{t+h}^{E,i} \right] \\
= \text{cov}_t \left[ \mathcal{E}_{t+h} \prod_{j \neq i} \prod_{t \leq s < t+h} \left[ 1 - \Gamma_s^j \Delta N_s^j \right] , \tilde{R}_{t+h}^{E,i} \right] + \frac{1}{q_t R_t^f} E_t \left[ \tilde{R}_{t+h}^{E,i} \right] , \quad (20)
\]

where and with \( R_t^f, \mathcal{E}_{t+h} \) and \( q_t \) are given by

\[
R_t^f = \left( E_t \left[ \frac{M_{t+h}}{M_t} \right] \right)^{-1} ,
\]

\[
\mathcal{E}_{t+h} = \exp \left\{ - \int_t^{t+h} r_s ds + \int_t^{t+h} \bar{\Gamma}_s \bar{\lambda}_s^P ds - \frac{1}{2} \int_t^{t+h} \Lambda_s^2 ds - \int_t^{t+h} \Lambda_s dW_s \right\} \\
\times \exp \left\{ \int_t^{t+h} \Gamma_s^0 \bar{\lambda}_s^{P,0} ds \right\} , \quad (21)
\]
and

$$q_t^i = \frac{E_t \left[ \mathcal{E}_{t+h} \Pi_j \Pi_{t \leq s < t+h} \left[ 1 - \Gamma_j^s \Delta \tilde{N}_j^s \right] \right]}{E_t \left[ \mathcal{E}_{t+h} \Pi_j \Pi_{t \neq 0, i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma_j^s \Delta \tilde{N}_j^s \right] \right]}.$$

Note that the second equality follows from the fact that $1_{\{\tau^i > t+h\}} = 1_{\{\bar{\tau}^0 > t+h\}} 1_{\{\bar{\tau}^i > t+h\}}$.

The expected gross return on the firm $i$’s equity claim can now be computed from (19) and (20) as

$$E_t \left[ \tilde{R}_{t+h}^{E,i} \right] = q_t^i \left\{ R_t^f - R_t^f \text{cov}_t \left[ \mathcal{E}_{t+h} \Pi_{j \neq 0, i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma_j^s \Delta \tilde{N}_j^s \right], \tilde{R}_{t+h}^{E,i} \right] \right\}. \quad (22)$$

According to (18) and (21), our DRP factor enters the return equation (22) mainly through $\mathcal{E}_{t+h}$. If the covariance term on the right-hand side is negligible, we will find no or only little evidence of the DRP factor being priced in equity markets. At the same time, the scaling term $q_t^i$ captures the effect of the innovations in the counting process associated with the default event $\tau^i$ on the return on firm $i$’s equity. It is driven by jump to default risk premia, that is, the ratios of risk-neutral to actual default intensities, for the market-wide and the firm-specific event times $\bar{\tau}^0$ and $\bar{\tau}^i$.

We postpone a more detailed discussion of this point until Section 7.

The fact that corporate bonds yield non-zero payoffs in the event of default substantially changes the relation between bond returns and jump-to-default risk premia. The gross returns on corporate bonds, $\tilde{R}_{t+h}^{B,i}$, can be written as

$$\tilde{R}_{t+h}^{B,i} = \tilde{R}_{t+h}^{ND,i} 1_{\{\tau^i > t+h\}} + \tilde{R}_{t+h}^{D,i} 1_{\{t < \tau^i \leq t+h\}},$$

where $\tilde{R}_{t+h}^{ND,i}$ ($\tilde{R}_{t+h}^{D,i}$) stand for the total return on firm $i$’s debt if the company does
not (does) default prior to or at time $t + h$. We can use this fact to show that

$$E_t \left[ \frac{M_{t+h}}{M_t} \tilde{R}_{t+h}^{ND,i} 1_{\{\tau^i > t+h\}} \right] = E_t \left[ \mathcal{E}_{t+h}\Pi_{j \neq 0,i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma^j_s \Delta \tilde{N}_s^j \right] \tilde{R}_{t+h}^{ND,i} 1_{\{\tau^i > t+h\}} \right]$$

$$= E_t \left[ \mathcal{E}_{t+h}\Pi_{j \neq 0,i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma^j_s \Delta \tilde{N}_s^j \right] \tilde{R}_{t+h}^{B,i} \right]$$

$$- E_t \left[ \mathcal{E}_{t+h}\Pi_{j \neq 0,i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma^j_s \Delta \tilde{N}_s^j \right] \tilde{R}_{t+h}^{D,i} 1_{\{\tau^i \leq t+h\}} \right]$$

$$= \text{cov}_t \left[ \mathcal{E}_{t+h}\Pi_{j \neq 0,i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma^j_s \Delta \tilde{N}_s^j \right] \tilde{R}_{t+h}^{B,i} \right]$$

$$+ \frac{1}{q_i^t R^f_t} E_t \left[ \tilde{R}_{t+h}^{B,i} \right]$$

$$- \text{cov}_t \left[ \mathcal{E}_{t+h}\Pi_{j \neq 0,i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma^j_s \Delta \tilde{N}_s^j \right] \tilde{R}_{t+h}^{B,i} 1_{\{\tau^i \leq t+h\}} \right]$$

$$- \frac{1}{q_i^t R^f_t} E_t \left[ \tilde{R}_{t+h}^{D,i} 1_{\{\tau^i \leq t+h\}} \right].$$

This yields that the expected return on a firm $i$'s debt claim is given by

$$E_t \left[ \tilde{R}_{t+h}^{B,i} \right] = q_i^t \left\{ R^f_t - R^f_t \text{cov}_t \left[ \mathcal{E}_{t+h}\Pi_{j \neq 0,i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma^j_s \Delta \tilde{N}_s^j \right] \tilde{R}_{t+h}^{B,i} \right] \right.$$

$$+ \left( \frac{1}{q_i^t} - R^f_t \right) E_t \left[ \tilde{R}_{t+h}^{B,i} 1_{\{\tau^i \leq t+h\}} \right]$$

$$\left. - R^f_t \text{cov}_t \left[ \mathcal{E}_{t+h}\Pi_{j \neq 0,i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma^j_s \Delta \tilde{N}_s^j \right] \tilde{R}_{t+h}^{B,i} 1_{\{\tau^i \leq t+h\}} \right] \right\}. \quad (23)$$

It is important to note that for corporate bonds of firm $i$, the DRP factor enters the return equation (23) through a new term, that is, via

$$-q_i^t R^f_t \text{cov}_t \left[ \mathcal{E}_{t+h}\Pi_{j \neq 0,i} \Pi_{t \leq s < t+h} \left[ 1 - \Gamma^j_s \Delta \tilde{N}_s^j \right] \tilde{R}_{t+h}^{B,i} 1_{\{\tau^i \leq t+h\}} \right].$$

Since the first component of the covariance term now includes the jump to default risk premia for the market-wide and the firm-specific event times $\tilde{\pi}^0$ and $\tilde{\tau}^i$, we expect the contribution of this term, and hence our DRP factor, to explaining corporate bond returns to be significant.
7 Discussion

Default risk premia display time-series properties that could stem from a systemic response to innovations in a common risk factor. This paper tests this hypothesis, by investigating the source for common variation in the portion of returns on credit default swaps that is not related to changes in risk-free rates or expected default losses. We extract a latent common component from firm specific changes in default risk premia that is orthogonal to known systematic risk factors during our sample period from 2001 to 2004. Asset pricing tests using returns on Bloomberg-NASD corporate bond indices generated from actual transaction prices of actively traded issues suggest that our discovered latent default risk premia factor (DRP) is priced in the corporate bond market. A cross-sectional analysis of Merrill Lynch corporate bond portfolios sorted on either industry, maturity or rating supports these findings. In our tests we control for firm characteristics such as contemporaneous default probabilities, leverage ratios, and recovery rates and we find that the common variation in changes in default risk premia is not likely to be due to these firm characteristics.

We also form portfolios using put options written on the S&P 500 index, sorted on moneyness and maturity. We find that for far-out-of-the-money index put options, both average returns and the beta estimate for our DRP factor increase with increasing time to maturity. The same holds true for out-of-the-money and at-the-money index put options. However, there is little to no evidence of the DRP factor being priced in the equity markets.

We develop a theoretical framework that shows that the DRP factor captures the jump-to-default risk associated with market-wide credit events. Within this framework, we show that, unlike risk premia on corporate bonds, equity risk premia are only marginally affected by our DRP factor. This results is based on the observation that the DRP factor has a much stronger impact on the returns of assets with a non-degenerate payoff structure in the default states. It is of interest to note that
Collin-Dufresne, Goldstein and Helwege (2003) also develop a reduced-form model where jump-to-default risk is priced. Their framework, however, can be interpreted as updating of beliefs due to an unexpected credit event. It is closely related to the concept of information-driven default contagion using frailty models introduced to the credit risk literature by Schonbucher (2003). In future work, we plan to extend the specification of the pricing kernel in (16), and of our assumptions regarding the physical and risk-neutral default intensity processes in (28) and (29), to allow for an updating of investor’s beliefs upon observed default events.

As a final remark we would like to point out that the result for the expected return on firm $i$’s equity claim in (22) suggests that equity risk premia depend on the ratios of risk-neutral to actual default intensities associated with the default event time $\tau_i$, as captured by the scaling factor $q_i$. For the remainder of this section, let us define jump to default risk premia for firm $i$ as $\lambda_{Q,i}^t/\lambda_{P,i}^t$. We investigate the extent to which the common component extracted from these instantaneous firm-specific jump to default risk premia, after controlling for other sources of common variation, can be associated with an alternative credit market risk factor that is more likely to be priced in equity markets.

Appendix C describes our approach to construct a new jump to default risk premia (JDRP) factor from $\{\lambda_{Q,i}^t/\lambda_{P,i}^t\}$. In order to see whether the JDRP factor is priced in equity markets we employ two sets of test assets, namely the 49 Fama-French industry portfolios and the 25 Fama-French equity portfolios sorted on size and book-to-market equity. We implement the Fama-MacBeth two-step procedure to estimate the impact of $JDRP_t$ on the average returns of these portfolios. The results of the cross-sectional regressions are reported in Table 16 and Table 17, respectively. We notice that the relation between the average excess returns on the test assets and the time-series loadings on $JDRP$ is positive and statistically significant, even after controlling for other known sources of systematic risk. These results are quite encouraging, and the
subject of future investigation.
## A  CDS and EDF Coverage

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Table 2: Firm Summary Characteristics: \( N_{CDS} \) is the number of valid observations for the five-year CDS available for each firm. Rating is the median rating. 5-yr CDS Spread is the median spread, in percent, of the 5-year CDS for senior unsecured debt for the firm, with a default event defined as modified restructuring. EDF is the median Moody’s KMV EDF in percent. The variables \( N_{CDS} \) through EDF are computed over the period January 2001 through June 2005. \( N_{EDF} \) is the valid number of monthly observations of the EDF values for the period July 1993 - June 2005. We use this longer time period to interpolate weekly EDF values from monthly EDF values.
B  Time Series Estimation of Default Intensities

Table 3: Summary statistics for firm-by-firm EDF-implied actual default intensity parameters.

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Table 4: Summary statistics for firm-by-firm MLE parameter estimates, using Markit five-year CDS rates with modified-restructuring and contemporaneous recovery rates.

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</table>

C  An Alternative Default Risk Premia Factor

This appendix describes our approach to construct the alternative jump to default risk premia (JDRP) factor, as motivated in Section 7. Let $\pi^i_t = \lambda^Q_t / \lambda^P_t$ denote the time series of jump to default risk premia for firm $i$. Also, let $F^D_t$ denote the time series of the levels of the latent common component to be extracted\textsuperscript{12}. Assume that these levels follow a following VAR process:

$$F^D_{t+1} - F^D_t = \rho(\bar{F} - F^D_t) + \sigma \xi_{t+1},$$

\textsuperscript{12}One should think of these levels as realized excess returns on a common risk factor embedded in the default events across firms.
where $\bar{F}$ can be interpreted as the long-term level that $F_t^D$ mean-reverts to, and $\rho$ and $\sigma$ are scalars.

Further, let $\pi_t^D$ denote the conditional expectation of $F_{t+1}^D$, relative to the information at time $t$. Then,

$$\pi_t^D = (1 - \rho) F_t^D + \rho \bar{F}. \quad (C.1)$$

As before, let $F_t^S$ denote the vector of returns on other known systematic factors, and let $\pi_t^S$ denote the corresponding conditional risk premia.

The process of extracting the levels of the latent component, $F_t^D$, is performed in several steps. First we estimate, firm by firm, the OLS model

$$\pi_t^i = \beta^i \cdot \pi_t^S + \epsilon_t^i,$$

and construct the time series of implied errors $\hat{\epsilon}_t^i = \pi_t^i - \hat{\beta}^i \cdot \pi_t^S$. Second, we model the errors $\epsilon_t^i$ in the previous regression as

$$\epsilon_t^i = \alpha^i + \gamma^i \left[ \pi_t^D - \bar{F} \right] + \nu_t^i,$$

where $\nu_t^i$ are standard normal variables, independent of $\pi_t^D$ and independent across firms and time.

Notice that $\alpha^i = E \epsilon_t^i$. Due to the fact that $\gamma^i$ can not be simultaneously identified, we choose the normalization

$$1 = \frac{1}{N} \sum_{i}^{N} \gamma^i.$$

This normalization, together with the assumption that the errors $\nu_t^i$ are i.i.d. across
firms, allows us to obtain an estimate for $\pi_t^D$, that is,

$$\pi_t^D - \bar{F} = \frac{1}{N} \sum_{i=1}^{N} (\hat{\epsilon}_t^i - \alpha^i). \quad (C.2)$$

We can now use (C.2) and compute the OLS estimates for $\gamma^i$ as

$$\gamma^i = \frac{COVAR(\hat{\epsilon}_t^i, \pi_t^D)}{VAR(\pi_t^D)},$$

where $COVAR$ and $VAR$ are the unconditional covariance and variance operators, respectively.

Third, note that, on one hand, the following formulas hold

$$VAR_t[\pi_{t+1}^D] = (1 - \rho)^2 VAR_t[F_{t+1}^D] = (1 - \rho)^2 \sigma^2$$

$$COVAR_t[\pi_{t+1}^D, \pi_{t+2}^D] = (1 - \rho)^3 VAR_t[F_{t+1}^D] = (1 - \rho)^3 \sigma^2 \quad (C.3)$$

On the other hand, the left hand side of the previous equations can be expressed as

$$VAR_t[\pi_{t+1}^D] = \mathbb{E}_t \left[ (\pi_{t+1}^D)^2 \right] - ((1 - \rho)^2 \left[ F_{t+1}^D - \bar{F} \right] + \bar{F})^2$$

$$COVAR_t[\pi_{t+1}^D, \pi_{t+2}^D] = \mathbb{E}_t \left[ \pi_{t+1}^D \pi_{t+2}^D \right] - ((1 - \rho)^2 \left[ F_{t+1}^D - \bar{F} \right] + \bar{F}) \left( (1 - \rho)^3 \left[ F_{t+1}^D - \bar{F} \right] + \bar{F} \right) \quad (C.4)$$

Applying the unconditional expectation operator in both (C.3) and (C.4), and comparing the right hand sides of these two equations while using the fact that the unconditional variance of $F_t^D$ is $\frac{\sigma^2}{1 - (1 - \rho)^2}$, we obtain

$$VAR[\pi_{t+1}^D] = \frac{(1 - \rho)^2}{1 - (1 - \rho)^2} \sigma^2; \quad \text{and}$$

$$COVAR[\pi_{t+1}^D, \pi_{t+2}^D] = \frac{(1 - \rho)^3}{1 - (1 - \rho)^2} \sigma^2.$$
This yields

\[ 1 - \rho = \frac{COVAR[\pi_{t+1}^D, \pi_{t+2}^D]}{VAR[\pi_{t+1}^D]}, \]

which together with (C.1) yields the level of the latent common factor, \( F_t^D \), as

\[ F_t^D = \bar{F} + \frac{VAR[\pi_{t+1}^D]}{COVAR[\pi_{t+1}^D, \pi_{t+2}^D]} [\pi_t^D - \bar{F}], \]

where \( \pi_t^D - \bar{F} \) is computed using (C.2).

Notice that our results depend additively on the free parameter \( \bar{F} \), which is not identifiable in this context. Nevertheless, for the assets pricing tests we are mainly interested in covariances, and knowledge of \( \bar{F} \) is not necessary.

**D Asset Pricing Test Results**
The NASD High-yield and Investment-grade Corporate Bond Portfolios: Time-series regressions

This table reports the results of the regressions of the excess realized returns of portfolios of investment and high grade corporate bonds on the stock market returns, $RMO$, $SMB$, $HML$, $UMD$, the corporate bond market return $DEF$, the treasury bond market return $TERM$ and the default risk premia factor returns, $DRP$: October 2002 to December 2004, 120 weeks. Specifically, for each portfolio we estimate the following regressions (with or without the last regressor): $R(t) - RF(t) = \alpha + \beta_{RMO}RMO(t) + \beta_{SMB}SMB(t) + \beta_{HML}HML(t) + \beta_{UMD}UMD(t) + \beta_{DEF}DEF(t) + \beta_{TERM}TERM(t) + \beta_{DRP}DRP(t) + \epsilon(t)$. The first part of the table reports the results for the regression without the default risk premia factor $DRP(t)$, while the second panel reports the results for the full regression. The reported intercept, slopes and $R^2$ are the mean values of the intercept, slopes and $R^2$ of each of the 97 portfolios. The t-statistics are reported in parentheses, and they are computed as the ratio of the cross-sectional mean to the cross-sectional standard deviation.

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<th>Portfolio</th>
<th>$\alpha$</th>
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<th>$\beta_{HML}$</th>
<th>$\beta_{UMD}$</th>
<th>$\beta_{DEF}$</th>
<th>$\beta_{TERM}$</th>
<th>$\beta_{DRP}$</th>
<th>$R^2$</th>
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<td>(2.8325)</td>
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Table 6: The Merrill Lynch Corporate Bond Portfolios, by Ratings: Time-series regressions This table reports the results of the regressions of the excess realized returns of 7 Merrill Lynch corporate bond portfolios, by ratings, on the stock market returns, $RMO, SMB, HML, UMD$, the corporate bond market return $DEF$, the treasury bond market return $TERM$ and the default risk premia factor returns, $DRP$: October 2002 to December 2004, 120 weeks. Specifically, for each portfolio we estimate the following regressions: $R(t) - RF(t) = \alpha + \beta_{RMO}RMO(t) + \beta_{SMB}SMB(t) + \beta_{HML}HML(t) + \beta_{UMD}UMD(t) + \beta_{DEF}DEF(t) + \beta_{TERM}TERM(t) + \beta_{DRP}DRP(t) + \epsilon(t)$. The first regression corresponds to the $AAA$ portfolio, while the last regression corresponds to $C$ portfolio. The t-statistics are reported in parentheses.
The Merrill Lynch Corporate Bond Portfolios, by Maturity: Time-series regressions

This table reports the results of the regressions of the excess realized returns of 6 Merrill Lynch corporate bond portfolios, by maturity, on the stock market returns, \( RMO \), \( SMB \), \( HML \), \( UMD \), the corporate bond market return \( DEF \), the treasury bond market return \( TERM \) and the default risk premia factor returns, \( DRP \): October 2002 to December 2004, 120 weeks. Specifically, for each portfolio we estimate the following regressions: 

\[
R(t) - RF(t) = \alpha + \beta_{RMO} RMO(t) + \beta_{SMB} SMB(t) + \beta_{HML} HML(t) + \beta_{UMD} UMD(t) + \beta_{DEF} DEF(t) + \beta_{TERM} TERM(t) + \beta_{DRP} DRP(t) + \epsilon(t) 
\]

The first regression corresponds to the 1-3\( Y \) portfolio, while the last regression corresponds to 15\( Y + \) portfolio. The t-statistics are reported in parentheses.

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<th>( \alpha )</th>
<th>( \beta_{RMO} )</th>
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<th>( \beta_{HML} )</th>
<th>( \beta_{UMD} )</th>
<th>( \beta_{DEF} )</th>
<th>( \beta_{TERM} )</th>
<th>( \beta_{DRP} )</th>
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Table 8: The Merrill Lynch High-yield Corporate Bond Portfolios, by Sectors: Time-series regressions

This table reports the results of the regressions of the excess realized returns of 30 Merrill Lynch high-yield corporate bond portfolios, by sector, on the stock market returns, $RMO$, $SMB$, $HML$, $UMD$, the corporate bond market return $DEF$, the treasury bond market return $TERM$ and the default risk premia factor returns, $DRP$: October 2002 to December 2004, 120 weeks. Specifically, for each portfolio we estimate the following regressions (with or without the last regressor): $R(t) - RF(t) = \alpha + \beta_{RMO}RMO(t) + \beta_{SMB}SMB(t) + \beta_{HML}HML(t) + \beta_{UMD}UMD(t) + \beta_{DEF}DEF(t) + \beta_{TERM}TERM(t) + \beta_{DRP}DRP(t) + \epsilon(t)$. The first part of the table reports the results for the regression without the default risk premia factor $DRP(t)$, while the second panel reports the results for the full regression. The reported intercept, slopes and $R^2$ are the mean values of the intercept, slopes and $R^2$ of each of the portfolios. The t-statistics are reported in parentheses, and they are computed as the ratio of the cross-sectional mean to the cross-sectional standard deviation.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta_{RMO}$</th>
<th>$\beta_{SMB}$</th>
<th>$\beta_{HML}$</th>
<th>$\beta_{UMD}$</th>
<th>$\beta_{DEF}$</th>
<th>$\beta_{TERM}$</th>
<th>$\beta_{DRP}$</th>
<th>$R^2$</th>
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Table 9: The Merrill Lynch High-yield Corporate Bond Portfolios, by Sectors: Cross-sectional regressions

This table reports the results of the weekly cross-sectional regressions of the excess realized returns of 30 Merrill Lynch high-yield corporate bond portfolios, by sector, on the loadings on the stock market returns, $RMO$, $SMB$, $HML$, $UMD$, the corporate bond market return $DEF$, the treasury bond market return $TERM$ and the default factor returns, $DRP$: January 2001 to December 2004, 206 weeks. Specifically, we estimate the coefficients of the following regressions (with or without the last regressor): $E[R(t)^i - RF(t)] = E[\gamma_0(t)] + E[\gamma^{RMO}(t)] \beta^{RMO}_i + E[\gamma^{SMB}(t)] \beta^{SMB}_i + E[\gamma^{HML}(t)] \beta^{HML}_i + E[\gamma^{UMD}(t)] \beta^{UMD}_i + E[\gamma^{DEF}(t)] \beta^{DEF}_i + E[\gamma^{TERM}(t)] \beta^{TERM}_i + + E[\gamma^{DRP}(t)] \beta^{DRP}_i + \epsilon^i(t)$. The coefficients are estimated as the averages of the weekly coefficients in the following cross-sectional regressions: $R(t)^i - RF(t) = \gamma_0 + \gamma^{RMO}(t) \beta^{RMO}_i + \gamma^{SMB}(t) \beta^{SMB}_i + \gamma^{HML}(t) \beta^{HML}_i + \gamma^{UMD}(t) \beta^{UMD}_i + \gamma^{DEF}(t) \beta^{DEF}_i + \gamma^{TERM}(t) \beta^{TERM}_i + + \gamma^{DRP}(t) \beta^{DRP}_i + \epsilon^i(t)$. The first half of the table reports the results of the regressions without the loading on the default risk premia factor, while the second half reports the results for the full regressions. The reported intercept, slopes and $R^2$ are the mean values of the intercept, slopes and $R^2$, across weeks. The t-statistics, reported in parentheses, are the average intercepts or slopes divided by their time series standard error, after accounting for autocorrelations.

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Table 10: The Merrill Lynch Investment-grade Corporate Bond Portfolios, by Industry: Time-series regressions

This table reports the results of the regressions of the excess realized returns of 4 Merrill Lynch investment-grade corporate bond portfolios, by industry, on the stock market returns, RMO, SMB, HML, UMD, the corporate bond market return DEF, the treasury bond market return TERM and the default risk premia factor returns, DRP: October 2002 to December 2004, 120 weeks. Specifically, for each portfolio we estimate the following regressions (with or without the last regressor): $R(t) - RF(t) = \alpha + \beta_{RMO}RMO(t) + \beta_{SMB}SMB(t) + \beta_{HML}HML(t) + \beta_{UMD}UMD(t) + \beta_{DEF}DEF(t) + \beta_{TERM}TERM(t) + \beta_{DRP}DRP(t) + \epsilon(t)$. The t-statistics are reported in parentheses.
The Size and Book-to-market Equity Portfolios: Time-series regressions

This table reports the results of the regressions of the excess realized returns of 97 portfolios formed on size and book-to-market equity on the stock market returns, RMO, SMB, HML, UMD, the corporate bond market return DEF, the treasury bond market return TERM and the default risk premia factor returns, DRP: January 2001 to December 2004, 206 weeks. Specifically, for each portfolio we estimate the following regressions (with or without the last regressor): \( R(t) - RF(t) = \alpha + \beta_{RMO}RMO(t) + \beta_{SMB}SMB(t) + \beta_{HML}HML(t) + \beta_{UMD}UMD(t) + \beta_{DEF}DEF(t) + \beta_{TERM}TERM(t) + \beta_{DRP}DRP(t) + \epsilon(t) \). The first part of the table reports the results for the regression without the default factor DRP(t), while the second panel reports the results for the full regression. The reported intercept, slopes and \( R^2 \) are the mean values of the intercept, slopes and \( R^2 \) of each of the 97 portfolios. The t-statistics are reported in parentheses, and they are computed as the ratio of the cross-sectional mean to the cross-sectional standard deviation.

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<tr>
<th>( \alpha )</th>
<th>( \beta_{RMO} )</th>
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<th>( \beta_{HML} )</th>
<th>( \beta_{UMD} )</th>
<th>( \beta_{DEF} )</th>
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<th>( R^2 )</th>
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<tr>
<td>0.0000</td>
<td>1.0246</td>
<td>0.3613</td>
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<td>-0.5106</td>
<td>0.1309</td>
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<td>0.8186</td>
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</tr>
<tr>
<td>(0.8673)</td>
<td>(42.0155)</td>
<td>(7.6857)</td>
<td>(7.7704)</td>
<td>(41.3457)</td>
<td>(2.0734)</td>
<td>(3.6345)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0000</td>
<td>1.0246</td>
<td>0.3613</td>
<td>-0.3541</td>
<td>-0.5106</td>
<td>0.1309</td>
<td>-0.0150</td>
<td>-11.2811</td>
<td>0.8201</td>
</tr>
<tr>
<td>(0.0517)</td>
<td>(57.4215)</td>
<td>(8.0426)</td>
<td>(7.9811)</td>
<td>(32.0391)</td>
<td>(2.7223)</td>
<td>(3.6662)</td>
<td>(3.2307)</td>
<td></td>
</tr>
</tbody>
</table>

Table 11: The Size and Book-to-market Equity Portfolios: Time-series regressions
Table 12: The Size and Book-to-market Equity Portfolios: Cross-sectional regressions

This table reports the results of the weekly cross-sectional regressions of the excess realized returns of 97 portfolios formed on size and book-to-market equity on the loadings on the stock market returns, $RMO$, $SMB$, $HML$, $UMD$, the corporate bond market return $DEF$, the treasury bond market return $TERM$ and the default risk premia factor returns, $DRP$: January 2001 to December 2004, 206 weeks. Specifically, we estimate the coefficients of the following regressions (with or without the last regressor): 

$$E[R(t)_i - RF(t)] = E[\gamma_0(t)] + E[\gamma^{RMO}(t)] \beta^i_{RMO} + E[\gamma^{SMB}(t)] \beta^i_{SMB} + E[\gamma^{HML}(t)] \beta^i_{HML} + E[\gamma^{DRP}(t)] \beta^i_{DRP}.$$  

The coefficient are estimated as the averages of the weekly coefficients in the following cross-sectional regressions: 

$$R(t)_i - RF(t) = \gamma_0 + \gamma^{RMO}(t) \beta^i_{RMO} + \gamma^{SMB}(t) \beta^i_{SMB} + \gamma^{HML}(t) \beta^i_{HML} + \gamma^{UMD}(t) \beta^i_{UMD} + \gamma^{DEF}(t) \beta^i_{DEF} + \gamma^{TERM}(t) \beta^i_{TERM} + \epsilon^i(t).$$  

The first half of the table reports the results of the regressions without the loading on the default risk premia factor, while the second half reports the results for the full regressions. The reported intercept, slopes and $R^2$ are the mean values of the intercept, slopes and $R^2$, across weeks. The t-statistics, reported in parentheses, are the average intercepts or slopes divided by their time series standard error, after accounting for autocorrelations.

<table>
<thead>
<tr>
<th>$E[\gamma_0]$</th>
<th>$E[\gamma^{RMO}]$</th>
<th>$E[\gamma^{SMB}]$</th>
<th>$E[\gamma^{HML}]$</th>
<th>$E[\gamma^{UMD}]$</th>
<th>$E[\gamma^{DEF}]$</th>
<th>$E[\gamma^{TERM}]$</th>
<th>$E[\gamma^{DRP}]$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0037</td>
<td>-0.0004</td>
<td>0.0017</td>
<td>0.0019</td>
<td>0.0005</td>
<td>0.0000</td>
<td>0.0044</td>
<td></td>
<td>0.3715</td>
</tr>
<tr>
<td>(3.8398)</td>
<td>(0.2701)</td>
<td>(1.8621)</td>
<td>(1.8223)</td>
<td>(0.2419)</td>
<td>(0.0457)</td>
<td>(1.4879)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0037</td>
<td>-0.0006</td>
<td>0.0017</td>
<td>0.0019</td>
<td>0.0003</td>
<td>-0.0000</td>
<td>0.0045</td>
<td>-0.0000</td>
<td>0.3847</td>
</tr>
<tr>
<td>(3.8392)</td>
<td>(0.3568)</td>
<td>(1.8960)</td>
<td>(1.8740)</td>
<td>(0.1355)</td>
<td>(0.0145)</td>
<td>(1.5482)</td>
<td>(1.2256)</td>
<td></td>
</tr>
<tr>
<td>Maturity Bin</td>
<td>10 ≤ T ≤ 30</td>
<td>31 ≤ T ≤ 60</td>
<td>61 ≤ T ≤ 150</td>
<td>151 ≤ T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
<td>-------------</td>
<td>--------------</td>
<td>---------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moneyness</td>
<td>0.90</td>
<td>0.86</td>
<td>0.83</td>
<td>0.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>Moneyness Bin</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
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<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maturity (days)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moneyness Bin</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>21.3</td>
<td>46.8</td>
<td>104.8</td>
<td>368.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20.9</td>
<td>45.0</td>
<td>100.9</td>
<td>353.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20.7</td>
<td>43.6</td>
<td>98.0</td>
<td>312.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Valid Returns Observations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moneyness Bin</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>768</td>
<td>1555</td>
<td>1295</td>
<td>1154</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>2</td>
<td>769</td>
<td>1556</td>
<td>1296</td>
<td>1155</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>768</td>
<td>1555</td>
<td>1296</td>
<td>1154</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 13: **Summary Statistics for the Index Put Option Portfolios** This table summarizes the characteristics of the options in the different maturity and moneyness bins that are used to form the put option test portfolios. *Moneyness* is the present value of the strike price divided by the current value of the S&P 500 index. *Maturity* (*T*) is the time to option expiration in days. *Returns* are the weekly buy-and-hold returns. Values for moneyness, maturity and returns are value-weighted averages for the options in each bin. $\beta_{DRP}$ is the factor loading on our default risk premia factor for the returns of the options portfolios.
<table>
<thead>
<tr>
<th>Maturity Bin</th>
<th>10 ≤ T ≤ 30</th>
<th>31 ≤ T ≤ 60</th>
<th>61 ≤ T ≤ 150</th>
<th>151 ≤ T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moneyness Bin</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-30.6</td>
<td>-12.0</td>
<td>-4.85</td>
<td>-0.680</td>
</tr>
<tr>
<td>2</td>
<td>-21.4</td>
<td>-8.26</td>
<td>-3.23</td>
<td>0.256</td>
</tr>
<tr>
<td>3</td>
<td>-10.7</td>
<td>-6.51</td>
<td>-2.35</td>
<td>-1.35</td>
</tr>
</tbody>
</table>

### Loadings on the DRP Factor

<table>
<thead>
<tr>
<th>Moneyness Bin</th>
<th>1 -949.587</th>
<th>-322.355</th>
<th>-130.370</th>
<th>-2.685</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-1.3299)</td>
<td>(-0.6511)</td>
<td>(-0.4604)</td>
<td>(-0.0185)</td>
</tr>
<tr>
<td>2</td>
<td>-1384.944</td>
<td>-416.051</td>
<td>-233.972</td>
<td>-51.447</td>
</tr>
<tr>
<td></td>
<td>(-1.6121)</td>
<td>(-1.0182)</td>
<td>(-1.0397)</td>
<td>(-0.4739)</td>
</tr>
<tr>
<td>3</td>
<td>-911.947</td>
<td>-351.320</td>
<td>-178.894</td>
<td>-104.671</td>
</tr>
<tr>
<td></td>
<td>(-1.6349)</td>
<td>(-1.2811)</td>
<td>(-0.9536)</td>
<td>(-1.0393)</td>
</tr>
</tbody>
</table>

### Loadings on the VIX Factor

<table>
<thead>
<tr>
<th>Moneyness Bin</th>
<th>1 3.5172</th>
<th>2.4808</th>
<th>1.6232</th>
<th>0.7957</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(7.6207)</td>
<td>(9.7348)</td>
<td>(11.1266)</td>
<td>(10.4411)</td>
</tr>
<tr>
<td>2</td>
<td>3.1595</td>
<td>1.8663</td>
<td>1.1921</td>
<td>0.5911</td>
</tr>
<tr>
<td></td>
<td>(5.6895)</td>
<td>(8.8736)</td>
<td>(10.2821)</td>
<td>(10.5946)</td>
</tr>
<tr>
<td>3</td>
<td>1.0469</td>
<td>0.9039</td>
<td>0.6728</td>
<td>0.4164</td>
</tr>
<tr>
<td></td>
<td>(2.9037)</td>
<td>(6.4027)</td>
<td>(6.9616)</td>
<td>(8.0548)</td>
</tr>
</tbody>
</table>

Table 14: **Index Put Options Portfolios: Time-series Regressions** This table reports the results of the regressions of the excess realized returns of 12 index put options portfolios sorted on moneyness and maturity on the stock market returns, RMO, SMB, HML, UMD, the corporate bond market return DEF, the treasury bond market return TERM, the volatility index return, VIX and the default risk premia factor returns, DRP: January 2001 to December 2004, 206 weeks. Specifically, for each portfolio we estimate the following regressions: $R(t) - RF(t) = \alpha + \beta_{RMO}RMO(t) + \beta_{SMB}SMB(t) + \beta_{HML}HML(t) + \beta_{UMD}UMD(t) + \beta_{DEF}DEF(t) + \beta_{TERM}TERM(t) + \beta_{VIX}VIX(t) + \beta_{DRP}DRP(t) + \epsilon(t)$. We only report the estimates of the loadings on the VIX and DRP factors, and their corresponding t-statistics in parentheses.
<table>
<thead>
<tr>
<th>Characteristic</th>
<th>$\Delta \alpha$ Med</th>
<th>$b_{Char}$ Mean Med tStat</th>
<th>$b_{DRP}$ Mean Med tStat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^P$</td>
<td>$2.1e^{-006}$</td>
<td>$-0.0268$ $-0.0295$ $-0.7362$</td>
<td>$1.3844$ $0.0631$ $2.0675$</td>
</tr>
<tr>
<td>RecRate</td>
<td>$6.1e^{-006}$</td>
<td>$-6.4e^{-007}$ $-1.7e^{-007}$ $-0.1485$</td>
<td>$1.3806$ $0.0700$ $2.0481$</td>
</tr>
<tr>
<td>LevRatio</td>
<td>$4.0e^{-006}$</td>
<td>$-6.6e^{-006}$ $-1.6e^{-005}$ $-0.4593$</td>
<td>$1.3882$ $0.0625$ $2.0566$</td>
</tr>
<tr>
<td>Volatility</td>
<td>$8.7e^{-008}$</td>
<td>$-1.7e^{-006}$ $-7.7e^{-006}$ $-0.4480$</td>
<td>$1.3831$ $0.0667$ $2.0500$</td>
</tr>
</tbody>
</table>

Table 15: **The Effect of the Characteristics: Time-series regressions** This table reports the results of the regressions of the changes in default risk premia of 71 firms on the stock market returns, $RMO$, $SMB$, $HML$, $UMD$, the corporate bond market return $DEF$, the treasury bond market return $TERM$, the default factor returns, $DRP$, and a time-varying characteristic: January 2001 to December 2004, 209 weeks. Specifically, for each firm we estimate the following regressions: $R_u(t) = \alpha + \beta_{RMO}R_{MO}(t) + \beta_{SMB}S_{MB}(t) + \beta_{HML}H_{ML}(t) + \beta_{UMD}U_{MD}(t) + \beta_{DEF}DEF(t) + \beta_{TERM}TERM(t) + \beta_{DRP}DRP(t) + \beta_{Char}(t-1) + \epsilon(t)$. The first column lists the name of the characteristic, while the second column reports the change in the intercept due to the characteristic. The following columns report summary statistics (across firms) of the loadings on the characteristic and the default factor. The results for the loadings on the other systematic factors are not reported.
The Industry Equity Portfolios: Cross-sectional regressions

This table reports the results of the weekly cross-sectional regressions of the excess realized returns of 49 industry portfolios on the loadings on the stock market returns, $RMO$, $SMB$, $HML$, $UMD$, the corporate bond market return $DEF$, the treasury bond market return $TERM$ and the default risk premia factor returns, $JDRP$: January 2001 to December 2004, 206 weeks. Specifically, we estimate the coefficients of the following regressions (with or without the last regressor):

$$E[R(t)^i - RF(t)] = E[\gamma_0(t)] + E[\gamma_{RMO}(t)] \beta_{iRMO} + E[\gamma_{SMB}(t)] \beta_{iSMB} + E[\gamma_{HML}(t)] \beta_{iHML} + E[\gamma_{JDRP}(t)] \beta_{iJDRP}.$$ 

The coefficient are estimated as the averages of the weekly coefficients in the following cross-sectional regressions:

$$R(t)^i - RF(t) = \gamma_0 + \gamma_{RMO}(t)\beta_{iRMO} + \gamma_{SMB}(t)\beta_{iSMB} + \gamma_{HML}(t)\beta_{iHML} + \gamma_{JDRP}(t)\beta_{iJDRP} + \epsilon(t).$$

The first half of the table reports the results of the regressions without the loading on the default risk premia factor, while the second half reports the results for the full regressions. The reported intercept, slopes and $R^2$ are the mean values of the intercept, slopes and $R^2$, across weeks. The t-statistics, reported in parentheses, are the average intercepts or slopes divided by their time series standard error, after accounting for autocorrelations.

<table>
<thead>
<tr>
<th>$E[\gamma_0]$</th>
<th>$E[\gamma_{RMO}]$</th>
<th>$E[\gamma_{SMB}]$</th>
<th>$E[\gamma_{HML}]$</th>
<th>$E[\gamma_{UMD}]$</th>
<th>$E[\gamma_{DEF}]$</th>
<th>$E[\gamma_{TERM}]$</th>
<th>$E[\gamma_{JDRP}]$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0047</td>
<td>-0.0000</td>
<td>0.0040</td>
<td>0.0006</td>
<td>0.0023</td>
<td>0.0008</td>
<td>0.0038</td>
<td>0.0001</td>
<td>0.4106</td>
</tr>
<tr>
<td>(5.0182)</td>
<td>(0.0000)</td>
<td>(2.2203)</td>
<td>(0.4229)</td>
<td>(0.8491)</td>
<td>(1.2516)</td>
<td>(0.9584)</td>
<td>(2.6012)</td>
<td></td>
</tr>
</tbody>
</table>
### Table 17: The Size and Book-to-Market Equity Portfolios: Cross-sectional regressions

This table reports the results of the weekly cross-sectional regressions of the excess realized returns of 25 portfolios formed on size and book-to-market equity on the loadings on the stock market returns, RMO, SMB, HML, UMD, the corporate bond market return, DEF, the treasury bond market return, TERM, and the default risk premia factor returns, DRP: January 2001 to December 2004, 206 weeks. Specifically, we estimate the coefficients of the following regressions (with or without the last regressor): $E[R(t) - RF(t)] = E[\gamma_0(t)] + E[\gamma_{RMO}(t)] \beta_{iRMO} + E[\gamma_{SMB}(t)] \beta_{iSMB} + E[\gamma_{HML}(t)] \beta_{iHML} + E[\gamma_{JDRP}(t)] \beta_{iJDRP} + \epsilon(t)$. The coefficient are estimated as the averages of the weekly coefficients in the following regressions:

$E[R(t) - RF(t)] = \gamma_0 + \gamma_{RMO}(t) \beta_{iRMO} + \gamma_{SMB}(t) \beta_{iSMB} + \gamma_{HML}(t) \beta_{iHML} + \gamma_{JDRP}(t) \beta_{iJDRP} + \epsilon(t)$. The first half of the table reports the results of the cross-sectional regressions without the loading on the default risk premia factor, while the second half reports the results for the full regressions. The reported intercept, slopes and $R^2$ are the mean values of the intercept, slopes and $R^2$, across weeks. The t-statistics, reported in parentheses, are the average intercepts or slopes divided by their time series standard error, after accounting for autocorrelations.

<table>
<thead>
<tr>
<th></th>
<th>$E[\gamma_{RMO}]$</th>
<th>$E[\gamma_{SMB}]$</th>
<th>$E[\gamma_{HML}]$</th>
<th>$E[\gamma_{JDRP}]$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\gamma_0]$</td>
<td>0.0037 (1.5081)</td>
<td>0.0025 (2.8334)</td>
<td>0.0039 (2.7830)</td>
<td>0.0010 (1.4794)</td>
<td>0.6916</td>
</tr>
</tbody>
</table>
CHAPTER III

THE PRICING OF RISK IN

EUROPEAN CREDIT AND CORPORATE BOND MARKETS
1 Introduction

In the recent years, academics have been studying the behavior of the credit and corporate bond markets through the lense of various measures of default risk premia. Most of the previous studies however focused on the US markets due to data availability reasons. In this paper, we focus exclusively on understanding the behavior of the European credit markets.

Default risk premia represent compensation for bearing the risk embedded in assets whose payoffs are contingent on whether a given firm defaults in a certain period of time. Intuitively, one can think of default risk premia as the difference between the market rate of a credit default swap (CDS) and the expected loss on the same CDS contract. From a traditional asset pricing perspective, one can also think of default risk premia as the expected return on a defaultable corporate bond in excess of the risk-free rate.

The goal of this paper is to understand what drives default risk premia. To this end, we propose a theoretical framework based on the intertemporal capital asset pricing model (ICAPM) of Campbell (1993) to analyze the interaction between systematic risk and returns on zero-coupon defaultable bonds with zero recovery. We chose to analyze the default risk premia of these particular defaultable assets because of their simple payoff structure and because their market values can be inferred relatively easy from the price information of tradable, but more sophisticated, defaultable assets. It should be noted that the original framework of Campbell’s ICAPM can not be applied directly here as a conditionally normal model for instantaneous returns is more likely to be mis-specified for defaultable assets than for other assets, such as stocks. Berndt, Douglas, Duffie, Ferguson and Schranz (2005) document that the instantaneous returns on zero-coupon defaultable bonds with zero recovery are more likely to follow conditionally log-normal dynamics as opposed to conditionally normal dynamics.
Our theoretical framework suggests that default risk premia arise as compensation for exposure to systematic risk and to a common factor that captures the proneness of these assets to extreme events. This common factor, which we call the credit market factor (CMF), is the common component of the deviations of the defaultable assets returns from the equivalent returns obtained under an alternative specification which assumes conditional log-normality. The model also suggests that the returns on defaultable assets are impacted by systematic risk through their covariance with two zero-cost portfolios: one that longs the market and shorts the risk-free rate and another that longs a riskless consol bond and shorts the risk-free rate.

Next, we turn to the estimation of these components of the default risk premia. First, we establish a link between the returns on defaultable zero-coupon bonds with zero recovery and the default intensities. Then, we estimate the dynamics of the actual and risk-neutral default intensities for the firms with the most liquid CDS market in Europe for the period 2003-2006. This estimation follows closely the methodology developed in Berndt et al. (2005) by exploiting the relation between the CDS spreads and the risk-neutral default probabilities. Finally, we use the main prediction of our theoretical framework to relate the returns of defaultable assets to the returns on the two zero-cost portfolios proxying for systematic risk and to identify the CMF factor.

We find that the two zero-cost portfolios can explain on average 21% of the time-variation in returns on defaultable assets while the CMF factor can explain on average 63% of the residual. These results suggest that CDS spreads of the firms in our sample incorporate compensation for bearing exposure not only to systematic risk but also to the CMF factor. To understand better the nature of the CMF factor we further investigate the pricing implications of this common factor for the corporate bond markets. We run asset pricing tests on a rich set of test assets consisting of corporate bond portfolios sorted on maturity, rating, maturity/rating and sectors. These portfolios are constructed from the non-financial/industrial sector or the entire universe of the
traded European corporate bonds. Our asset pricing tests support overwhelmingly the hypothesis the CMF factor is priced in the corporate bond markets.

We also document another interesting pattern. Most of the corporate bond portfolios load negatively on the excess returns on the market. These loadings become more negative as the maturity of the assets in the portfolio increases and less negative (sometimes even positive) as the rating of the assets decreases. In the asset pricing literature this behavior is referred to as the "flight to quality" effect. As the economy goes through a recession period investors’ appetite for risk decreases and they invest in safer assets with longer maturities. As the economy goes through an expansion period investors’ appetite for risk increases and they invest in riskier high-yield bonds.

The results in this paper complement and extend the results of Berndt, Lookman and Obreja (2006) who find that the U.S. credit and corporate markets as well as the U.S. equity options market price a common factor that is also extracted from the CDS spreads of the U.S. firms with the most liquid CDS markets for the period 2002-2006. There are some important differences however. First, this study focuses not only on a different time period, but also on a different market. Second, the CMF factor in this study is extracted from default risk premia that are measured as the expected excess holding returns of zero-coupon defaultable bonds with zero recovery. The default risk premia in Berndt et al. (2006) are measured in terms of the expected loss\(^1\) in a manner similar to the one proposed by Elton, Gruber, Agrawal and Mann (2001). Finally, the theoretical framework proposed in this paper suggests that the CMF factor captures the proneness of the defaultable securities to extreme events. In the other study, the corresponding common factor is shown to capture the jump-to-default risk associated with market-wide credit events.

This study also contributes to the growing financial economics literature concerned with the measurement of the default risk premia. Noticeable contribution to this

\(^1\)In Section 4 of this paper, we provide an extended discussion on various ways to measure default risk premia
literature include Elton et al. (2001), Amato and Remolona (2005), Longstaff, Mithal and Neis (2004), Saita (2005), Berndt et al. (2005). This paper distinguishes from all these studies on several dimensions including the choice of capital markets and the methodology. We concentrate exclusively on European credit and corporate bond markets and our theoretical approach shares with the intertemporal capital asset pricing models which previously have not been adapted to accommodate returns of defaultable securities.

The remainder of this paper is structured as follows. Section 2 describes our data and presents a thorough discussion of the general terms of the credit default swap contract and an overview of the Moody’s KMV EDF measure of default probability. Section 3 uses a simple measure of default risk premia to present some preliminary evidence supporting the common time-variation in European default risk premia across industries for the period 2003-2006. Section 4 measures default risk premia in terms of the actual and the risk-neutral default intensities. Section 5 present the theoretical determinants of default risk premia and constructs an expected returns-beta representation for defaultable assets. Section 6 estimates the dynamics of default intensities using the information embedded in the CDS spreads and the actual default probabilities as measured by Moody’s KMV measure of default. Section 7 estimates the components of default risk premia. The following two sections investigate the nature of the CMF factor. Section 8 tests whether the time-variation in CMF is due to time-varying firm characteristics such as actual default probability, firm size or market-to-book ratio, while Section 9 tests whether the CMF factor is priced in the European corporate bond markets. Finally, Section 10 concludes.

2 Data

This section discusses our data sources for default swap rates and conditional default probabilities in Europe.
2.1 Credit Default Swaps

Credit default swaps (CDS) are single-name over-the-counter credit derivatives that provide default insurance. The payoff to the buyer of protection covers losses up to notional in the event of default of a reference entity. Default events are triggered by bankruptcy, failure to pay, or, for some CDS contracts, a debt restructuring event. The buyer of protection pays a quarterly premium, quoted as an annualized percentage of the notional value, and in return receives the payoff from the seller of protection should a credit event occur. Fueled by participation from commercial banks, insurance companies, and hedge funds, the CDS market has been doubling in size each year for the past decade, reaching $12.43 trillion in notional amount outstanding by mid-2005.\(^2\) In this paper, we use CDS spreads instead of corporate bond yield spreads as our primitive source for prices of default risk because default swap spreads are less confounded by illiquidity, tax and various market microstructure effects that are known to have a marked effect on corporate bond yield spreads.\(^3\)

In particular, we use default swap spreads for five-year CDS contracts for Euro-denominated senior unsecured debt. The data is provided by Credit Market Analysis (CMA) Thomson through Datastream.

It contains daily CDS bid/ask quotes contributed by active market participants including banks, hedge funds and active managers. CMA assures full transparency for its clients by providing a qualifier (Veracity Score) for each data point of any time-series of CDS prices. The Veracity Score indicates the liquidity or if applicable, the extent to which a value has been model-derived. We focus exclusively on firms with very liquid 5-year CDS market for the sample period between January 2003

\(^2\)See, for example, the International Swaps and Derivatives Association mid-2005 market survey. The CDS market is still undergoing rapid growth. The notional amount of default swaps grew by almost 48% during the first six months of 2005 to $12.43 trillion from $8.42 trillion. This represents a year-on-year growth rate of 128% from $5.44 trillion at mid-year 2004.

\(^3\)Recent papers that analyze the contribution of non-credit factors to bond yields include Zhou (2005), Longstaff, Mithal and Neis (2004), and Ericsson and Renault (2001).
and November 2006. The CDS contracts of these firms typically make up the iTraxx CDS Europe index of 150 most liquid non-financial 5-year CDS contracts. To mitigate optimally the tradeoff between the microstructure effects of high frequency quotes and the statistical power of our tests, we focus on weekly CDS quotes. Most of the quotes have a Veracity Score of 3 or better. This indicates that the quote is associated with an actual trade or that the quote is an indication provided by a market participant. We do not consider quotes with a Veracity Score higher than 3.5. The final sample of default swap rates used in this study consists of 55 firms from eleven European countries and sixteen different industries, based on Moody’s industry classification (see Table 1). A typical firm in our sample has 150 (of 196 maximum possible weekly quotes) valid weekly CDS observations. No firm in our sample has fewer than 95 weekly observations.

The fact that our sample has only 55 firms is an important caveat of this paper. The typical major concern with small samples - such as ours - is whether the sample is representative enough to support unbiased results. We believe that despite its small size, our sample is quite diverse given that the distribution of firms in our sample spans 16 different industries. In addition, since the goal of this paper is to extract information about the compensation rewarding investors for bearing risk, we believe that this information can be extracted more precisely\footnote{In order to extract this information we use the approach in Berndt et al. (2005) which requires relatively long time-series of prices (or quotes, in our case).} from the quotes on the CDS contracts of those firms with very liquid 5-year CDS markets. To this extent, we are confident that the results in the paper are not biased by the size of our sample.

2.2 Actual Default Probabilities

We use the one-year Expected Default Frequency (EDF) data provided by Moody’s KMV as our measure of actual default probabilities. We will discuss this measure only briefly, referring the reader to Berndt et al. (2005) for a more detailed description.
Figure 1: Distribution of firms by median credit rating during the sample period.

The concept of the EDF measure is based on structural credit risk framework of Black and Scholes (1973) and Merton (1974). In these models, the equity of a firm is viewed as a call option on the firm’s assets, with the strike price equal to the firm’s liabilities. The “distance-to-default” (DD), defined as the number of standard deviations of asset growth by which its assets exceed a measure of book liabilities, is a sufficient statistic of the likelihood of default. In the current implementation of the EDF model, to the best of our knowledge, the liability measure is equal to the firms short-term book liabilities plus one half of its long-term book liabilities. Estimates of current assets and the current standard deviation of asset growth (volatility) are calibrated from historical observations of the firms equity-market capitalization and of the liability measure. For a detailed discussion, see, for example, Appendix A in Duffie, Saita and Wang (2005).

Crosbie and Bohn (2001) and Kealhofer (2003) provide more details on the KMV model and the fitting procedures for distance to default and EDF. Unlike the Merton model, where the likelihood of default is the inverse of the normal cumulative distri-
bution function of DD, Moody’s KMV EDF measure uses a non-parametric mapping from DD to EDF that is based on a rich history of actual defaults. Therefore, the EDF measure is somewhat less sensitive to model mis-specification. The accuracy of the EDF measure as a predictor of default, and its superior performance compared to rating-based default prediction, is documented in Bohn, Arora and Korbalev (2005). Duffie, Saita and Wang (2005) construct a more elaborate default prediction model, using distance to default as well as other covariates. Their model achieves accuracy that is only slightly higher than that of the EDF, suggesting that EDF is a useful proxy for the physical probability of default. Furthermore, the Moody’s KMV EDF measure is extensively used in the financial services industry. As noted in Berndt et al. (2005), 40 of the world’s 50 largest financial institutions are subscribers.

We obtained daily one-year EDF values from Moody’s KMV for the time period January 2001 through October 2006, for the same set of 55 firms described in Section 2.1. Figure 1 plots the distribution of the credit quality of the firms in our sample. As discussed in Section 2.1, our CDS data only start in January 2003. In order to achieve sufficient power for our asset pricing tests we use weekly (Wednesdays) observations of default swap rates, together with EDF values at weekly frequency.

2.3 Interest Rates, Systematic Factors and Test Assets

In Sections 3, 7 and 9 we compute expected loss for CDS contracts and realized excess returns on defaultable securities and corporate bond portfolios, and we form zero-cost portfolios to proxy for systematic risk. In all these instances we need to use information about the Euro term structure of riskless bonds. This data is obtained from Datastream from the Euro zero curves constructed relative to Euribor.\textsuperscript{5} All the excess returns and the zero-cost portfolios are computed relative to the 1-month zero

\textsuperscript{5}The mnemonics for the yield of a zero-coupon Euro bond with time-to-maturity of $n$ years and $m$ months is EM$n$Y$m$. For instance the mnemonic corresponding to the maturity of 1 year and 4 months is EM01Y04.
yield. Also, the discount factors used to compute the expected loss for CDS contracts in Section 3 are computed using the same Euro zero curves.

For the purpose of Sections 7 and 9 we need to compute zero-cost portfolios that are long the market portfolio and short the 1-month zero yield or long the 30-year zero yield and short the 1-month zero yield. For the later zero-cost portfolio we use the data in the Euro zero curves with the corresponding maturities. For the former zero-cost portfolio, we construct two types of market portfolios: one that incorporates the entire universe of European stocks and one the incorporates only the stocks from a specific country. To maintain consistency with the previous studies on the capital markets integration, we use whenever possible portfolios constructed from the data disseminated in the electronic version of Morgan Stanley’s *Capital International Perspectives* (MSCI). For those countries where MSCI data is not available we use the local portfolios constructed by FTSE. All these portfolios are available through Datastream.\(^6\)

Finally, for the purpose of Section 9 we need to compute realized returns on a range of test assets in excess of the 1-month zero yield. We consider the following test assets: the Merrill Lynch non-financial corporate bond portfolios sorted on rating or time-to-maturity, the Merrill Lynch AAA-, AA-, A- and BBB-rated corporate bond portfolios sorted on maturity, and the Lehman Brothers Euro-aggregate industrial corporate bond portfolios sorted on ratings, maturity or sectors. The time-series data for all these portfolios comes from Datastream.\(^7\)

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\(^6\)The mnemonic for the MSCI European market portfolio is MSEURIL. The mnemonics for the country-specific market portfolios are MSFRNCL (France), MSNETHL (Netherlands), MSGERML (Germany), WISWDNE (Sweden), MSITALL (Italy), MSSPANL (Spain), WIDNMKE (Denmark), WINWAYE (Norway), FTSE10E (UK), MSFINDL (Finland) and MSGDEEL (Greece).

\(^7\)These portfolios have respectively the following mnemonics: MLNF3AE, MLNF1AE, MLNF3BE, MLENFAE, MLENFCE, MLENFDE, MLENFGE, MLEC3AE, MLEC3EE, MLEC3GE, MLEC3KE, MLEC2CE, MLEC2GE, MLEC2JE, MLEC1CE, MLEC1GE, MLEC1JE, MLEC1KE, MLEC8CE, MLEC8GE, MLEC8JE, LHA13AE, LHA12AE, LHA11AE, LHA1BAE, LHEHYBA, LHAC1YE, LHAC3YE, LHAC5YE, LHAC7YE, LHAC10E, LHEAEDE, LHEBANK, LHEBMAT, LHECAPG, LHECHEM, LHECOMM, LHACCYE, LHACNCE, LHEDEMAN, LHAFBVE, LHALODE, LHAREFE, LHATLPE, LHATBCE, LHAWRSE, and LHAMNCE.
3 Preliminary Regression Analysis

In this section we provide a preliminary analysis of the time-series properties of the European default risk premia extracted from the CDS spreads. This analysis is meant to motivate the more thorough analysis of Sections 5-9. As Section 4 will show more clearly, one way to measure the risk premia embedded in the CDS spreads is to compare the market spread with the spread obtained by setting the expected loss of the CDS to zero at the time of the issuance. Let $S_t$ denote the actual CDS spread. We can therefore measure the default
Figure 2: The time-series variation in default risk premia across industries

Specifically, we compute this "expected loss spread" as follows: Let \( D(t, n) \) denote the discount factor for the period \([t, t+n]\) and \( p(t, n) \) the actual survival probability of an obligor over the same period of time. Let \( L \) denote the recovery rate (as a percent of the principal) in the event of default. Then we define the expected loss spread as the premium \( S \) that solves:

\[
\sum_{n=1}^{N} D(t, n) p(t, n) S = \sum_{n=1}^{N} D(t, n) [p(t, n-1) - p(t, n)] \left[ L - \frac{1}{8} S \right]
\]

where \( N \) is the number of payments stipulated in the original CDS contract (\( N \) corresponds to the number of quarters, which for a 5-year contract amounts to 20).

The left-hand side in the above equation is the present value of the future payments by the buyer of the protection, while the right-hand side is the present value of the recovery in the event of default. This later quantity is not straightforward to compute as it requires information about the time of default. We chose to model it in the manner suggested by Berndt et al (2005). In particular, we assume that if the default occurs in the time period \([t+n, t+n+1]\), the protection seller returns to the buyer the fraction \( L \) of the principal, less any accrued interest. The actual survival probabilities for maturities longer than one year are estimated as simple products of the one-year survival probabilities obtained from Moody’s KMV. The discount factors are computed from the term structure of Euro zero-coupon yields. We use the time-series of Euro riskless term structures relative to Euribor. For more information see Section 2.3. Following Berndt et al. (2005) we assume that \( L \) is relatively stable over the sample period at around 75%. This value corresponds to the medium recovery rate in the US for the period 2002-2006 as documented for instance in Berndt, Lookman and Obreja (2006). We do not have data on recovery rates for the current sample
To capture the time variation of these risk premia across industries, for a given level of credit worthiness, we run the following panel regression:

$$\log(S_i^t) - \log(ELS_i^t) = \alpha + \beta \log EDF_i^t + \sum_{m} \sum_{p} \delta_{m}^p d_i^t(m, p) + \epsilon_i^t$$  \hspace{1cm} (1)$$

where $d_i^t(m, p)$ is a dummy variable which equals 1 if week $t$ is in month $m$ and firm $i$ is in industry $p$.

Figure 2 plots the time-series variation of the monthly estimates for $\exp \delta_{m}^p$, relative to the last month in our sample. The plot shows that for a given level of credit worthiness, there is substantial variation in risk premia over time. More importantly, the risk premia of the firms in different industries seem to move together. This co-movement is typically indicative of exposure to common risk factors or to the fact that the firms in our sample have similar characteristics (in the spirit of Daniel and Titman (1997)). To the extent that the firms in our sample are different enough from each other, the goal of this paper is to disentangle how much of the co-movement in these firms’ risk premia is due to "likely" systematic factors, and how much is due to other, potentially new, priced factors.

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9We use the log specification rather than the simple difference of the two measures because the relation between the CDS spreads and the EDF rates (the actual default probabilities) is more likely to be multiplicative rather than linear, as the following regressions show:

$$S_i^t = 52.1577 + 0.4993 EDF_i^t + \epsilon_i^t$$  \hspace{1cm} (53.6031) (62.1077)$$

$$\log S_i^t = 3.1715 + 0.2777 \log EDF_i^t + \epsilon_i^t$$  \hspace{1cm} (208.9279) (59.8403)$$

10Specifically, we plot $\exp (\delta_{m}^p - \delta_{m_0}^p)$, where $m_0$ is the last month in our sample.

11Section 5 will provide some theoretical guideness in determining the systematic factors that should affect capital markets.
4 Measuring Default Risk Premia

This section describes different ways of measuring default risk premia and it provides a simple characterization of default risk premia in terms of default intensities.

Given a probability space $(\Omega, \mathcal{F}, P)$ and information filtration $\{\mathcal{F}_t : t \geq 0\}$, the default intensity of a firm $\lambda_t^P$ is the instantaneous mean arrival rate of default, conditional on all current information. Intuitively, conditional on survival to time $t$ and all information available at time $t$, the probability of default between times $t$ and $t + \Delta$ is approximately $\lambda_t^P \Delta$ for small $\Delta$. In this setting the conditional probability of surviving between $t$ and $T > t$ can be expressed as:

$$p(t, T - t) = E_t [1_{\{\tau > T\}}] = E_t \left[ e^{-\int_t^T \lambda_s^P \, ds} \right].$$  \hspace{1cm} (2)

where $\tau$ denotes the default time and $E_t$ denotes the expectation operator conditional on the information available up to and including time $t$.

Under the absence of arbitrage and market frictions there exists a stochastic discount factor, $M$.\(^{12}\) Moreover, under mild technical conditions, Harrison and Kreps (1979) and Delbaen and Schervsmayer (1999) show that there exists a "risk-neutral" probability measure associated with $M$. Let $Q$ denote this measure. Note that in our setting, markets are not necessarily complete, so the stochastic discount factor and the associated risk-neutral measure might not be unique. This pricing approach nevertheless allows us to express the price at time $t$ of a security paying $Z$ at time $T > t$, as $E_t [M_{t,T} Z] = E_t^Q \left[ e^{-\int_t^T r_s \, ds} Z \right]$, where $r$ is the short-term interest rate and $E_t^Q$ denotes the expectation operator conditional on the information available up to and including time $t$, with respect to the equivalent martingale measure $Q$. In particular, the market value of a defaultable zero-coupon bond that pays one unit of account in the event that a given firm does not default before time $T$ and 0 otherwise is given

\(^{12}\)See for instance Duffie (2001).
by:

$$P(t, T - t) = E_t \left[ M_{t,T} 1_{\{\tau > T\}} \right] = E_t^Q \left[ e^{-\int_t^T r_s ds} 1_{\{\tau^Q > T\}} \right],$$  (3)

where $\tau^Q$ denotes the default time of the firm under the measure $Q$.

We can make the simplifying assumption that the default time $\tau^Q$ can be fully described by a doubly-stochastic exponentially-distributed random variable with intensity $\lambda^Q$. In this case the above formula reduces to:

$$P(t, T - t) = E^Q_t \left[ e^{-\int_t^T r_s ds + \lambda^Q_s ds} \right]$$  (4)

If investors are risk-neutral (i.e. $M_{t,T}$ degenerates to $e^{-\int_t^T r_s ds}$) or the default event of this firm is idiosyncratic, the market value of such a defaultable bond should only reflect the expected loss, namely:

$$P^L(t, T - t) = E_t \left[ M_{t,T} \right] E_t \left[ 1_{\{\tau > T\}} \right] = E_t \left[ e^{-\int_t^T r_s ds} \right] p(t, T - t)$$  (5)

If investors are not risk-neutral or if the default event is not diversifiable then the market value of the defaultable bond reflects a risk adjustment relative to the expected loss. This risk adjustment is given by:

$$P^L(t, T - t) - P(t, T - t) = -cov_t \left[ M_{t,T}, 1_{\{\tau > T\}} \right]$$  (6)

This measure of risk compensation is particularly appealing since both terms in the left-hand side of this equation can be computed relatively easy, once the dynamics of the default intensities are known. However, for the purpose of this paper, we are more interested in relating the dynamics of the default intensities to a more traditional measure of risk, namely the risk premium. Let $R_{t+1} = P(t+1, T - t - 1)/P(t, T - t)$ denote the gross holding return on the defaultable zero-coupon bond. Then the risk
premium of this asset is defined through the Euler equation as follows:

\[ E_t R_{t+1} - R^f_{t+1} = -R^f_{t+1} \text{cov}_t [M_{t,t+1}, R_{t+1}] \] (7)

where \( R^f \) denotes the gross return on the risk-free bond. Unless we make relatively strong assumptions about the dynamics of the default intensities,\(^\text{13}\) neither side of the above equation are easy to relate to the dynamics of the default intensities. Nevertheless, for the rest of this section we present a special case which allows us to establish this relation in a relatively straightforward manner.

When, \( t = T - 1 \), it can be easily shown that the risk premium and the risk adjustment relative to expected loss are identical:\(^\text{14}\)

\[ \frac{P^L_t - P_t}{P_t} = \frac{E_t R_{t+1} - R^f_{t+1}}{R^f_{t+1}} \] (9)

Moreover, when the length \( dt \) of the time interval \([t - 1, t]\) is sufficiently small, the left-hand side becomes:

\[ e^{[\lambda^Q_t - \lambda^P_t]dt} - 1 \approx \left[ \lambda^Q_t - \lambda^P_t \right] dt. \] (10)

Thus, for defaultable bonds with very short maturities, the risk premium per unit of time equals the difference between the risk-neutral and the actual default intensity times the gross return on the risk-free rate. However, this need not be the case if

\(^\text{13}\)For instance, if the default intensities follow Gaussian processes under both the physical and the risk-neutral measure, the risk premium can be computed in closed form.

\(^\text{14}\)This identity can be stated in a slightly more general version, for any \( t < T \):

\[ \frac{P^L_t - P_t}{P_t} = \frac{E_t R_{t,T} - R^f_{t,T}}{R^f_{t,T}} \] (8)

where \( R_{t,T} \) is the holding return between \( t \) and \( T \), while \( R^f_{t,T} \) is the yield of a riskless zero-coupon bond that matures at \( T \). This is merely a consequence of the fact that for zero-coupon bonds (riskless or defaultable), \( R_{t,T} = R_{t,t+1} R_{t+1,T} \) and thus the Euler equation holds at larger horizons.
either $t < T - 1$ or the time to maturity of the defaultable bonds is large. The next section, presents a simple way to deal with the potentially complex relation between risk premia and default intensities, for the general case. It also addresses the more general question of the likely determinants of the risk premia on defaultable bonds.

5 Theoretical Determinants of Default Risk Premia

In this section we use the discrete intertemporal capital asset pricing model of Campbell (1993) to identify likely sources of macroeconomic risk and to understand the impact of these sources of risk on the prices of defaultable bonds.

Suppose the economy is populated with identical agents with non-expected-utility preferences of the following form:

$$U_t = \{(1 - \beta)C_t^{\frac{1-\gamma}{\sigma}} + \beta \left( E_t \left[ U_{t+1}^{1-\gamma} \right] \right)^\frac{1}{\theta}\}^{\frac{\theta}{1-\gamma}} \tag{11}$$

where $\gamma$ is the coefficient of relative risk aversion, $\sigma$ is the elasticity of intertemporal substitution and $\theta = \sigma^{-\frac{1-\gamma}{\sigma-1}}$.

As Epstein and Zin (1989, 1991) show, the first order condition of the representative agent in this economy can be stated as:

$$1 = E_t \left[ \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\theta}} \right\}^\theta \left\{ \frac{1}{R_{t+1}^m} \right\}^{1-\theta} R_{t+1}^i \right] \tag{12}$$

where $C$ is the aggregate consumption, $R_{t+1}^m$ is the return on the market portfolio and $R_{t+1}^i$ is the return on a security $i$.

Campbell (1993) shows that under the assumption that asset returns and consumption growth are jointly conditionally homoskedastic and log-normally distributed.

\[\text{For more details on the parameters see Campbell (1993).}\]
the aggregate budget constraint can be exploited to substitute out consumption and to simplify the Euler equation to:

$$E_t r^i_{t+1} - r^f_{t+1} = -\frac{1}{2} V_{ii} + \gamma V_{im} + (\gamma - 1)V_{ih}$$

(13)

where \( r^* (\ast = i, f) \) denotes log returns, \( V_{ii} = \text{Cov}_t(r^i_{t+1}, r^i_{t+1}) \), \( V_{im} = \text{Cov}_t(r^i_{t+1}, r^m_{t+1}) \) and \( V_{ih} = \text{Cov}_t(r^i_{t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^m_{t+1+j}) \). The second argument of the last covariate captures the news about the future returns on the market. \( \rho \) is the steady-state ratio of invested wealth to total wealth.\(^{16}\)

Furthermore, if \( r^b_{t+1} \) denotes the return on a riskless consol bond that pays one unit of account every period, Campbell (1993) shows that the above equation can be further simplified to:

$$E_t r^i_{t+1} - r^f_{t+1} = -\frac{1}{2} V_{ii} + \gamma V_{im} + (1 - \gamma)V_{ib}$$

(14)

where \( V_{ib} = \text{Cov}_t(r^i_{t+1}, r^b_{t+1}) \).

Let \( r^b_{t+1} = r^b_{t+1} - \beta^b_{t,m} r^m_{t+1} \) with \( \beta^b_{t,m} = \frac{\text{cov}_t(r^b_{t+1}, r^m_{t+1})}{V_{mm}} \). Substituting \( r^b_{t+1} \) in the above equation yields:

$$E_t r^i_{t+1} - r^f_{t+1} = -\frac{1}{2} V_{ii} + \left[ \gamma + \beta^b_{t,m} (1 - \gamma) \right] V_{im} + (1 - \gamma)V_{ib}$$

(15)

where \( V_{ib} = \text{Cov}_t(r^i_{t+1}, r^b_{t+1}) \). If we further assume that \( r^b_{t+1} \) and the consumption growth are both jointly conditionally homoskedastik and log-normally distributed, we can apply the above relation to both \( r^m_{t+1} \) and \( r^b_{t+1} \). Using the unconditional versions

\(^{16}\)See Campbell (1993) for the exact definition.
of these relations we obtain:

\[
\begin{align*}
[\gamma + \bar{\beta}^{h,m}(1 - \gamma)] &= \frac{Er_t^{m,e}}{V_{mm}} - \frac{1}{2} \\
1 - \gamma &= \frac{Er_t^{b,\perp,e}}{V_{bb}} - \frac{1}{2}
\end{align*}
\] (16)

where \( E \) denotes the unconditional expectation operator, \( \bar{\beta}^{h,m} = E\beta^{h,m} \), \( r_t^{m,e} = r_t^m - r_t \) and \( r_t^{b,\perp,e} = r_t^{b,\perp} - r_t^f \). Substituting these formulas back into (15) and taking expectations yields the following expected returns - beta representation:

\[
Er_{i,t} + \frac{1}{2}V_{ii} = \beta_{im}\left[Er_t^{m,e} + \frac{1}{2}V_{mm}\right] + \beta_{ib}\left[Er_t^{b,\perp,e} + \frac{1}{2}V_{bb}\right]
\] (17)

where \( \beta_{im} = V_{im}/V_{mm} \) and \( \beta_{ib} = V_{ib}/V_{bb} \).\footnote{Notice that \( \beta_{i,m} \) and \( \beta_{i,b} \) are in fact the conditional betas, which happen to be constant under the homoskedasticity assumption. Thus they can be different from the unconditional betas.}

The expected returns-beta representation in equation (17) suggests that the time variation in returns is mainly due to time variation in the returns on the market portfolio in excess of the riskless short rate and the time variation in the returns of a portfolio that longs a riskless console bond and shorts the riskless short-rate. Close relatives of this later portfolio have been previously used in the financial economic literature. One of the best known is the spread between long- and short-term treasury bonds, or TERM, for short. For the exact definition see Fama and French (1993).

The representation in equation (17) applies to any returns that are both jointly homoskedastik and conditionally log-normally distributed with the consumption growth and the market return. However, returns on certain assets are less likely to satisfy the later condition. For instance, Berndt et al. (2005) document that the instantaneous excess returns on defaultable zero-coupon bonds are more likely to be log-normally distributed rather than normally distributed (recall that the instantaneous returns are natural logs of the gross returns). Thus, the above pricing equation might not work...
as well for this type of returns. Under certain conditions, the expected return-beta representation model in (17) can be slightly generalized to accommodate returns that are not necessarily conditionally log-normally distributed. We describe this modified model below.

Suppose the returns on a defaultable bond $r^D_t$ can be decomposed into a component, $r^{D,c}_t$, that is jointly homoskedastic and log-normally distributed with the consumption growth and the market return and another component, $r^{D,n}_t$, that is orthogonal on the information contained on both the consumption-growth and the market.\(^\text{18}\) This later component is going to capture the impact of the departure from the conditional log-normality assumption on prices. Under these assumptions it can be easily shown that the expected returns - beta representation in equation (17) becomes:

$$E \left[ r^{D,c,e}_t + 1 \right] = \beta^{c}_D \left[ E \left[ r^{m,e}_t + 1 \right] + \frac{1}{2} V^{mm}_m \right] + \beta^{b\perp}_D \left[ E \left[ r^{b\perp,e}_t + 1 \right] + \frac{1}{2} V^{bb}_b \right] + E \Delta z_t \quad (18)$$

where $r^{D,c,e}_t = r^{D,c}_t - r^f_t$, $V^{DD}_D = \text{var}(r^{D,c}_t)$, $\beta^{c}_D = \text{cov}(r^{D,c}_t, r^{m}_t) / V^{mm}_m$, $\beta^{b\perp}_D = \text{cov}(r^{D,c}_t, r^{b\perp}_t) / V^{bb}_b$, and $z_t = -\log E \left[ e^{D,n}_t \right]$. Making use of the fact that $r^{D,n}$ is orthogonal on the information contained in the market returns and the long-short treasury portfolio\(^\text{19}\), we can rewrite the above as:

$$E \left[ r^{D,c}_t + 1 \right] = \beta^{c}_D \left[ E \left[ r^{m,c}_t + 1 \right] + \frac{1}{2} V^{mm}_m \right] + \beta^{b\perp}_D \left[ E \left[ r^{b\perp,c}_t + 1 \right] + \frac{1}{2} V^{bb}_b \right] + E \Delta z_t \quad (19)$$

\(^{18}\)One way to implement such a decomposition is as follows: Let $\mu = E r^D_t$ and $k_r = \text{cov}(r_{t+1}, r_t) / \text{var}(r_t)$. Define $\nu_{t+1} = \left[ r^{D,c}_t + 1 - \mu - k_r (r^{D,c}_t - \mu) \right]$. Let $\nu^{c\perp}_{t+1}$ denote the linear projection of $\nu_{t+1}$ onto the space generated by the consumption growth and the market return. Let $\nu^{c\perp}_{t+1} = \nu_{t+1} - \nu^{c\perp}_{t+1}$ denote the orthogonal residual. Since both the consumption growth and the market return are conditionally normally distributed, $\nu^{c\perp}_{t+1}$ will be also conditionally normally distributed. In addition since $\nu_{t+1}$ has zero mean, both $\nu^{c\perp}_{t+1}$ and $\nu^{c\perp}_{t+1}$ can be normalized to have zero mean. Define $r^{D,c}_t$ recursively as follows: $r^{D,c}_t = \nu^{c\perp}_{t+1}$, with $r^{D,c}_0 = r^{D,c}_0$. Also, define $r^{D,n}_t$ recursively as follows: $r^{D,n}_t = k_r (r^{D,c}_t + 1) + \nu^{n\perp}_{t+1}$, with $r^{D,n}_0 = 0$. Then $r^D_t = r^{D,c}_t + r^{D,n}_t$ and $r^{D,c}_t$ and $r^{D,n}_t$ satisfy the desired properties.

\(^{19}\)Campbell (1993) shows that the informational content of this portfolio overlaps with that of the market returns and the consumption growth.
where $\Delta z_t = E_{t} r_{t+1}^{D,n} - \log E_{t} e^{r_{t+1}}$.

Thus, just like conditionally log-normal returns, the returns of defaultable bonds vary over time in response to changes in excess market returns and the returns on the long-short treasury portfolio. However, unlike conditionally log-normal returns, the returns of defaultable bonds also move because of changes in the shape of the conditional distribution relative to a normal distribution (captured by $\Delta z_t$). This later source of time variation could host both a time-varying common component as well as undiversifiable firm-specific components. Both these types of components affect the level of expected returns directly rather than through covariances.

We next focus on computing the returns on defaultable zero-coupon bonds using the methodology developed in Section 4. Following the notation in Section 4, the holding returns between $t$ and $t+1$ for a defaultable zero-coupon bond with maturity $T > t + 1$ is given by

$$r_{t+1} = \log P(t + 1, T - t - 1) - \log P(t, T - t) \quad (20)$$

where $P(t, T - t)$ is defined in Section 4. The holding returns for the period $[T - 1, T]$ can be computed with

$$r_T = -\log P(T - 1, T) \quad (21)$$

It is important to notice that these returns cannot be computed directly since we do not have data on defaultable zero-coupon corporate bonds. However, we can use the apparatus developed in Section 4 to compute the returns on these hypothetical assets in terms of quantities that can be measured directly or indirectly from the CDS and EDF data that we have available.

We start with the formula in equation (4). Suppose the risk-neutral default intensity $\lambda_t^Q = \lambda_t^{Q,c} + \lambda_t^{Q,n}$ such that $\lambda_t^{Q,c}$ and $r_s$ are correlated Gaussian processes (in
Thus, log
$$\text{in footnote } (18)$$
with the initial conditions and the market returns. Then,
$$P(t, T - t) = E_t \left[ M_{t,T} e^{-\int_t^T \lambda_Q^Q d\lambda} \right] = E \left[ M_{t,T} e^{-\int_t^T \lambda_Q^c} \right] E_t \left[ e^{-\int_t^T \lambda_Q^a} \right]$$
$$= E_t^Q \left[ e^{-\int_t^T r_t^f + \lambda_Q^c} \right] E_t \left[ e^{-\int_t^T \lambda_Q^a} \right]$$
(22)

Since \( r_t \) and \( \lambda_Q^c \) are correlated Gaussian processes, it can be easily established that
$$\log E_t^Q \left[ e^{-\int_t^T r_t^f + \lambda_Q^c} \right] = A(T - t) - B(T - t)r_t^f - C(T - t)\lambda_t^c$$
(23)

where \( A(T - t) \), \( B(T - t) \) and \( C(T - t) \) depend on \( T - t \) only. Thus, \( P(t, T - t) \) can be rewritten as:
$$\log P(t, T - t) = A(T - t) - B(T - t)r_t^f - C(T - t)\lambda_t^c + \log E_t \left[ e^{-\int_t^T \lambda_Q^a} \right]$$
(24)

---

**Footnote (18):** See footnote (18) for a way to construct such a decomposition.

**Footnote (21):** The coefficients \( A(T - t) \), \( B(T - t) \) and \( C(T - t) \) can be derived in a recursive fashion as it is typically done in the affine term-structure literature. Suppose \( r_t^f \) and \( \lambda_t^c \) follow jointly Gaussian dynamics of the following form:
$$r_{t+1}^f = k_r \tilde{r}^f + (1 - k_r)r_t^f + \sigma_r \tilde{\xi}_{t+1}$$
$$\lambda_{t+1}^c = k_\lambda \tilde{\lambda}_t^c + (1 - k_\lambda)\lambda_t^c + \sigma_\lambda \tilde{\xi}_{t+1} + \sigma_r \sigma_r \tilde{\xi}_{t+1} + \sigma_\lambda \tilde{\xi}_{t+1}$$

Then, for any \( t < T \) we have
$$A(T - t) = A(T - t - 1) - [B(T - t - 1) + 1]k_r \tilde{r}^f - [C(T - t - 1) + 1]k_\lambda \tilde{\lambda}_t^c$$
$$+ \frac{1}{2} \left( [C(T - t - 1) + 1] \sigma_\lambda \tilde{\xi}^2 + \frac{1}{2} [C(T - t - 1) + 1] \sigma_\lambda^2 \right)$$
$$B(T - t) = [B(T - t - 1) + 1] (1 - k_r)$$
$$C(T - t) = [C(T - t - 1) + 1] (1 - k_\lambda)$$

with the initial conditions \( A(0) = B(0) = C(0) = 0 \). Notice that under the decomposition suggested in footnote (18), \( k_\lambda \) can be computed as follows:
$$1 - k_\lambda = \frac{\text{cov}[\lambda_t^Q, \lambda_{t+1}^Q]}{\text{var}[\lambda_t^Q]} = \frac{\text{cov}[\lambda_t^Q, \lambda_{t+1}^Q]}{\text{var}[\lambda_t^Q]}.$$
Combining we finally obtain the following expression for $r_{t+1}$:

$$
\begin{align*}
    r_{t+1} &= \left[ B(T-t)r_t^f - B(T-t-1)r_{t+1}^f \right] \\
    &\quad + \left[ C(T-t)\lambda_{t+1}^Q - C(T-t-1)\lambda_{t+1}^{Q,c} \right] + \Delta\tilde{z}_{t+1} \\
    &= \left[ B(T-t)r_t^f - B(T-t-1)r_{t+1}^f \right] \\
    &\quad + \left[ C(T-t)\lambda_t^Q - C(T-t-1)\lambda_{t+1}^Q \right] + \Delta z_{t+1}
\end{align*}
$$

where

$$
\begin{align*}
\Delta\tilde{z}_t &= A(T-t) - A(T-t-1) + \log E_{t+1} \left[ e^{-\int_{t+1}^T \lambda_{n}^Q} \right] - \log E_t \left[ e^{-\int_{t}^T \lambda_{n}^Q} \right] \\
\Delta z_t &= \Delta\tilde{z}_{t+1} - \left[ C(T-t)\lambda_{t+1}^{Q,n} - C(T-t-1)\lambda_{t+1}^{Q,n} \right].
\end{align*}
$$

Note that $\Delta z_{t+1}$ measures (up to a constant) the departure from the normal distribution of the conditional distribution of $\lambda_{s}^{Q,n}$. Given the orthogonality assumptions on $\lambda_t^{Q,c}$ and $\lambda_t^{Q,n}$, we can substitute

$$
\begin{align*}
    r_{t+1}^D &= \left[ B(T-t)r_t^f - B(T-t-1)r_{t+1}^f \right] + \left[ C(T-t)\lambda_t^Q - C(T-t-1)\lambda_{t+1}^Q \right] \\
    r_{t+1}^{D,c} &= \left[ B(T-t)r_t^f - B(T-t-1)r_{t+1}^f \right] + \left[ C(T-t)\lambda_t^{Q,c} - C(T-t-1)\lambda_{t+1}^{Q,c} \right]
\end{align*}
$$

in equation (15) to obtain the following expected return-beta representation for returns on defaultable bonds:

$$
\begin{align*}
    E[B(T-t)r_t^f - B(T-t-1)r_{t+1}^f] &= E \left[ C(T-t)\lambda_t^Q - C(T-t-1)\lambda_{t+1}^Q \right] - Er_{t+1}^f \\
    &\quad + \frac{1}{2}V_{DD}^e \beta_{Dm} \left[ Er_{t+1}^{m,e} + \frac{1}{2}V_{mm} \right] + \beta_{Db} \left[ Er_{t+1}^{b,\perp,e} + \frac{1}{2}V_{bb} \right] + E\Delta z_{t+1}
\end{align*}
$$

The pricing equation (26) can be tested on a panel dataset of defaultable bonds with constant maturity $d = T - t$:

$$
\begin{align*}
    r_{t+1}^{i,e} &= \alpha^i + \beta_{im} r_{t+1}^{m,e} + \beta_{ib} r_{t+1}^{b,\perp,e} + \sum_s f_s 1_{\{t+1 = s\}} + \epsilon_{t+1}^i
\end{align*}
$$

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where $r_{t+1}^{i,e} = B^i(d)r_t^f - B^i(d-1)r_{t+1}^f + C^i(d)\lambda_{t}^{i,Q} - C^i(d-1)\lambda_{t+1}^{i,Q} - r_{t+1}^f$ is the excess realized return on firm $i$'s defaultable zero-coupon bond maturing in $d-1$ periods, $f_s$ captures the value of the potential common component at time $s$ while $\epsilon_{t}^{i}$ captures the undiversifiable firm-specific component of firm $i$.

The following section focuses on the estimation of the default intensities from the CDS and EDF data.

6 Estimating the Default Intensities

In this section we first describe the time-series models for both actual and risk-neutral default intensities. Similar to Berndt et al. (2005), we specify a model under which the logarithm of the actual default intensities $\lambda^P_t$ satisfies the Ornstein-Uhlenbeck equation

$$d\log(\lambda^P_t) = \kappa(\theta - \log(\lambda^P_t)) dt + \sigma dB_t,$$

(28)

where $B_t$ is a standard Brownian motion, and $\theta$, $\kappa$, and $\sigma$ are firm-specific constants to be estimated. The behavior for $\lambda^P$ is called a Black-Karasinski model. (See Black and Karasinski (1991).) This leaves us with a three-dimensional vector $\Theta = (\theta, \kappa, \sigma)$ of unknown parameters to be estimated from available firm-by-firm EDF observations of a given firm. For the 55 firms in our sample we have daily observations of one-year EDFs, from January 2001 to October 2006. However, for the estimation procedure we only use weekly quotes (Wednesdays).

Given the log-autoregressive form (28) of the default intensity, in general there is no closed-form solution available for the one-year EDF, $1 - p(t, 1)$, from (2). We therefore rely on numerical lattice-based calculations of $p(t, 1)$, and have implemented the two-stage procedure for constructing trinomial trees proposed by Hull and White (1994).
With regard to risk-neutral default intensities, we assume that

\[ d \log \lambda_t^Q = \kappa^Q(\theta^Q - \log(\lambda_t^Q)) \, dt + \sigma^Q \, dB_t, \tag{29} \]

where \( B_t^Q \) is a standard Brownian motion with regard to the physical measure \( P \), and \( \kappa^Q, \theta^Q, \) and \( \sigma^Q \) are scalars to be estimated. The risk-neutral distribution of \( \lambda^Q \) is specified by assuming that

\[ d \log \lambda_t^Q = \tilde{\kappa}^Q(\tilde{\theta}^Q - \log(\lambda_t^Q)) \, dt + \tilde{\sigma}^Q \, dB_t^Q, \]

where \( \tilde{\kappa}^Q \) and \( \tilde{\theta}^Q \) are constants and \( B_t^Q \) is a standard Brownian motion with regard to \( Q \). Given a set of parameters \((\tilde{\theta}^Q, \tilde{\kappa}^Q, \sigma^Q)\), we can compute model-implied values for \( \lambda^Q \) using data on five-year CDS rates and risk-neutral loss given default. For details we refer the reader to Section 5.1 in Berndt et al. (2005). We estimate the parameters driving the dynamics of the risk-neutral default intensities under both phisycal and risk-neutral measure using the over-identifying restriction \( \kappa^Q = \tilde{\kappa}^Q. \)

Using maximum likelihood estimation (MLE), we obtain firm-by-firm estimates for the parameters that govern the processes for \( \lambda^P \) and \( \lambda^Q \). The estimated values of these parameters are shown in Table 2.

As shown in Section 4, the difference between the risk neutral and the actual default intensity can be interpreted as a measure of instantaneous risk premia, embedded in the CDS spreads. It would be informative to see whether the time-variation in these measures of risk premia resembles the patterns in Figure 2.

To see this we employ a panel regression approach similar to the one in Section 3 and extract the time-series patterns of the loadings on the dummy variables controlling for month and industry.\(^{22}\)

\(^{22}\)This over-identifying restriction improves considerably the reliability of our estimates.

\(^{23}\)Specifically, we run the following regression:

\[ \lambda_t^{i,Q} - \lambda_t^{i,P} = \alpha + \sum_m \sum_p \delta_{m,p}^i d_t(m,p) + \epsilon_t^i \]
Figure 3 plots the estimates of the slope coefficients on the dummy variables controlling for month and industry. We notice that the patterns are relatively similar with the ones in Figure 2.

Given the estimated time-series for the actual and the risk-neutral default intensities, we can now compute returns of defaultable zero-coupon bonds and we can formally test the expected returns - beta representation derived in Section 5.

## 7 The Components of Default Risk Premia

In this section we separate the components of the default risk premia by estimating the returns model described in equation (27). We first discuss our choices for the

where $d_{1}^{t}(m, p)$ is a dummy variable which equals 1 if week $t$ is in month $m$ and if firm $i$ is in industry $p$. 

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empirical implementation of the theoretical portfolios proxying for systematic risk, namely the excess market portfolio and the portfolio that is long a riskless perpetuity and short the riskless short rate.

The reference entities behind the CDS contracts in our dataset are from various countries within Europe. Most of these countries are also part of the European Monetary Union\textsuperscript{24} but there are few countries that are not (UK, Denmark, Norway and Sweden). Since capital markets throughout Europe are more or less integrated,\textsuperscript{25} we proxy for the market portfolio with both a portfolio tracking the largest stocks throughout Europe as well as local portfolios tracking the largest most liquid stocks within a specific country. To maintain consistency with the previous studies on the capital markets integration, we use whenever possible portfolios constructed from the data disseminated in the electronic version of Morgan Stanley’s *Capital International Perspectives* (MSCI). For those countries where MSCI data is not available we use the local portfolios constructed by FTSE. For more information on these portfolios see Section 2.3. Since the CDS spreads in our dataset are reported relative to the Euro term structure it is important that the returns on these portfolios are extracted from prices reported in Euros. We denote with $r_{t}^{EMKT,e}$ the weekly returns on the European market portfolio in excess of the riskless short rate and with $r_{t}^{CMKT,e}$ the weekly returns on the local market portfolio in excess of the riskless short rate. The riskless short rate corresponds to the yield of the one-month zero-coupon Euro bond. For more information on the Euro term structure curves see Section 2.3.

The other portfolio that we have to worry about is the portfolio that longs a riskless console bond paying one unit of account every week and shorts the short interest rate. We proxy for this portfolio with a portfolio that longs the 30-years zero-coupon riskless Euro bond and shorts the 1-month Euro bond. We denote the

\textsuperscript{24}See Table 1 for more details
\textsuperscript{25}There is quite a bit of literature on this topic. Some of the most well known studies include Fama and French (1998), Griffin (2002), Ferson and Harvey (1993), Bekaert and Harvey (1995), and Karolyi and Stulz (2003).
weekly returns of this portfolio with $r_{i}^{TERM}$.

The return model in equation (27) becomes:

$$r_{i,t+1}^{i,e} = \alpha_{i} + \beta_{EMKT} r_{EMKT,t+1}^{EMKT} + \sum_{c} \beta_{CMKT}^{c} 1_{\{C=c\}} r_{CMKT,t+1}^{CMKT} + \beta_{TERM}^{i} r_{TERM,t+1}^{TERM}$$

$$+ \sum_{s} f_{s} 1_{\{t+1=s\}} + \epsilon_{i,t+1}^{i}$$

(30)

where $c$ is an index for countries and

$$r_{i,t+1}^{i,e} = B^{i}(d) r_{t}^{f} - B^{i}(d-1) r_{t+1}^{f} + C^{i}(d) \lambda_{t}^{i,Q} - C^{i}(d-1) \lambda_{t+1}^{i,Q} - r_{t+1}^{f}$$

(31)

Using the time series of estimates for the actual and the risk-neutral default intensities derived in the previous section we can compute the excess returns on the left-hand side of the above equation for various times to expiration $d$. Table 3 shows the averages of the estimated coefficients across firms when the time to maturity takes various values. We noticed some interesting patterns. First, except for the special case when the time to maturity is the shortest possible, namely 1 week, all the pricing errors are large, on average, and statistically significant. The systematic factors suggested by the theory capture only some of time-variation of the excess returns on the defaultable bonds. The loadings for each factor display an increasing pattern as we increase the time to maturity of the defaultable bonds. The most significant, economically and statistically, of the three factors seems to be the $TERM$ factor capturing the spread between the 30-year and the 1-month riskless Euro bonds. Nevertheless, these systematic factors have relatively little explanatory power, as indicated by the $R^{2}$s in the last column of Table 3.

Of interest to us, however, is whether the pricing errors obtained from the returns model in equation (30) without time dummies move together over time. The common component of these errors is captured precisely by the loadings on the time dummies,
Figure 4: The variance of the CMF factor as a percentage of the variance of the pricing error obtained under the returns model in (30) with no time dummies. The CMF factor is extracted from excess zero-coupon defaultable bonds with time to maturity varying from 1 week to 5 years. Each histogram in the plot shows the firm-specific percentages of the pricing error explained by the CMF factor. Each histogram corresponds to a specific time to maturity, indicated on the horizontal axis.

namely $f_s$. We will refer to this common component as the default risk premia factor, or CMF for short. We notice that CMF can capture quite a large portion of the pricing errors, as indicated by the fifth column in Table 3. Figure 4 shows more descriptively the distribution of the fraction of the pricing error captured by the common component for each firm in our sample and for each time to maturity. This plot shows that the percentage of the pricing error captured by the CMF is very high when the time to maturity is the shortest possible. These percentages are several times lower for times to maturity of 1 year or longer and they display a monotonic pattern in the direction of longer maturities.

What is behind the CMF factor? According to the pricing equation (26), CMF captures the extent to which the conditional distribution of the asset returns differs from the normal distribution. In other words, CMF captures the extent to which
certain assets are more likely to have return distributions with fatter tails or non-zero skewness. These attributes are usually indicative of higher sensitivities to extreme events. This suggests that $CMF$ captures the proneness of the defaultable securities to extreme events.

For the rest of the paper we want to understand the nature of this common component. If this common component were to show up in the pricing errors of a large cross-section of defaultable bonds than it can be easily shown that this component cannot be diversified away.\footnote{This is essentially a consequence of the fact that under the assumption that the pricing errors in excess of the common component are truly firm specific and are drawn from a common distribution, the variance of a well diversified equally-weighted portfolio of defaultable bonds is given by three components: one which contains the systematic factors, $\text{var}(f_s)$ and $\text{var}(\epsilon)/N$. When $N$ is large only the last component goes away.} Since our cross-section of firms is not too large, however, there is a chance that our common component might capture some the time-variation of firm characteristic that is common for most of our firms. To rule out this possibility we need to control for various firm characteristics and see whether the common component survives. If the common component survives we can understand more about the nature of this component by investigating whether this common component is priced in large portfolios of assets. These tests are performed in the following two sections.

8 Testing for Firm Characteristics

In this section, we study whether the common component $CMF$ arises because the firms in our sample have similar firm characteristics. This is an important step in understanding the nature of $CMF$.

To understand whether the time-variation associated with our common component is actually due to a certain firm characteristic, we implement the following time-series
regressions as suggested by Daniel and Titman (1997):

\[ r_{t+1}^{i,e} = \gamma_0^i + \gamma_\phi^i \phi_t + \gamma_{EMKT}^{EMKT,e} t_{t+1} + \sum_c \gamma_{CMKT}^{CMKT,e} 1(C=c) t_{t+1} \\
+ \gamma_{TERM}^{TERM} t_{t+1} + \gamma_{CMF}^{CMF} C t_{t+1} + \epsilon_{t+1}^i \]

(32)

where \( \phi_t \) is the firm characteristic. This regression tells us that if the common variation in conditional default risk premia (obtained by conditioning on the information at time \( t \)) is in fact due to the firm characteristic rather than the common factor then we should see \( \gamma_{CMF}^{CMF} \) being close to zero and statistically insignificant. Table 4 reports the average slope coefficients and their corresponding t-Statistics across firms, for three firm characteristics: the actual default probabilities, the firm size and the market-to-book ratio.\(^{27}\) We notice that in all instances, the slope on the firm characteristic is both economically and statistically significant suggesting that part of the common variation in conditional default risk premia can be attributed to these firm characteristics. However, we are interested in understanding whether our common component arises because of the common variation in these firm characteristics. This turns out not to be the case as can be noticed from the last three columns of Table 4. In all instances, the common component remains both economically and statistically significant. These results suggest that our common component contains important information beyond what is contained in the firm characteristics considered here.\(^{28}\)

\(^{27}\)Actual default probabilities correspond to the 1-year EDF values (see Section 2 for more details). Firm size and market-to-book ratio are constructed using the firm-level data from Datastream. Whenever the market capitalization of a firm is expressed in a currency other than Euro we convert it into Euros. Firm size is then the log of the market capitalization. Market-to-book ratio is the ratio recovered the time-series variable PTBV in Datastream.

\(^{28}\)Another firm characteristic that could be a natural candidate is the leverage ratio. However, the data for the book value of long/short liabilities is only available at annual frequencies from Compustat which means that for our sample we essentially have only three or four datapoints (2003, 2004, 2005, 2006 when available) for each firm. Thus the time variation in the leverage ratio is essentially driven by the time variation in the market value of equity, which we’ve already considered as a firm characteristic.
9 Asset Pricing Tests

In this section we want to understand the nature of the common component $CMF$. We plan to do so by investigating whether this common component has any pricing implications for other classes of assets.

To see whether $CMF$ is priced by assets other than the CDS spreads used to extract the factor, we implement simple asset pricing tests in the spirit of Fama and French (1993). We use as test assets corporate bond portfolios sorted by ratings, time-to-maturity or both, as well as corporate bond portfolios sorted by sector. These portfolios are constructed by either Merrill Lynch or Lehman Brothers and they focus on either the entire universe of European corporate bonds or on the non-financial/industrial sectors. For more information on these portfolios see Section 2.3. These portfolios are particularly attractive because they are sorted on characteristics - such as ratings or time-to-maturity - which can be easily related to risk. In particular, the portfolios sorted on these characteristics have different exposures to risk which leads to different average returns.

The asset pricing test that we implement is a time-series regression of the following form:

$$r_{i,t+1}^{e} = \alpha_{i} + \beta_{EMKT}^{i}r_{t+1}^{EMKT,e} + \beta_{TERM}^{i}r_{t+1}^{TERM} + \beta_{CMF}^{i}CMF_{t+1} + \epsilon_{i,t+1}$$  \hspace{1cm} (33)

where $r_{i,t+1}^{e}$ is the excess return on the portfolio $i$ used as test asset.

The null hypothesis is that the $CMF$ factor loads up more heavily on portfolios that are more exposed to this factor. Recall that the $CMF$ factor captures the extent to which assets are prone to extreme events. As a consequence, we should expect portfolios of corporate bonds with either higher maturities or lower ratings to load up more heavily on $CMF$. In addition, if the returns model in equation (33) is the true model, then we should also see small and statistically insignificant pricing
errors.

Tables 5 to 11 report the estimated coefficients for various portfolios used as test assets. For these tests we use the CMF factor extracted from defaultable zero-coupon bonds with the shortest time to maturity (1 week). Throughout these tables, we notice a striking pattern. In almost all tests, the loadings on the CMF factors are positive and in many cases statistically significant. More importantly, these loadings are higher for portfolios that contain higher maturity or lower rating corporate bonds, just as the model predicted. Figures 5 to 7 show that most of the patterns uncovered when the CMF factor is extracted from defaultable zero-coupon bonds with 1 week until maturity also hold when the CMF factor is extracted from defaultable zero-coupon bonds with 1 year, 2 years, 3 years, 4 years or 5 years until maturity.

The asset pricing tests also indicate that almost all the pricing errors are statistically insignificant. The average size of these errors, in many of the tests, is not however very small, indicating that we should be careful in inferring whether the evidence in Tables 5 to 11 supports the hypothesis that the returns model in (33) is the true model.

Tables 5 to 11 also depict another interesting pattern. Most of the corporate bond portfolios load negatively on the market. These loadings become more negative as the maturity of the assets in the portfolios increases and less negative (and even positive) as the rating of the assets deteriorates. This fact seems to confirm the so called ”flight to quality”. As the economy goes through an expansion, investors’ appetite for risk increases and they’re more likely to invest in riskier assets such as high yield (lower rating) corporate bonds. As the economy goes through a recession, investors’ appetite for risk turns sour and they prefer to invest in safer assets with longer maturity (such as highly-rated long-term corporate bonds).
10 Conclusion

In this paper we use quotes on CDS contracts of the firms with the most liquid CDS market in Europe to extract the components of default risk premia, measured as the average excess return of a zero-coupon defaultable bond with zero recovery. Extending the theoretical framework of Campbell’s ICAPM to accommodate returns with fat-tailed distributions - such as the returns of a defaultable bond - we find that default risk premia have two major components: one associated with systematic risk and another associated with a new common factor that captures the proneness of the asset returns to extreme events. This theoretical framework yields a model of returns for defaultable securities which recognizes as main sources of time variation the returns on the market and a riskless consol bond in excess of the risk-free rate - as proxy for systematic risk - and the returns on the new common factor. To identify the new common factor we apply this model to the returns of defaultable zero-coupon bonds with zero recovery, which, despite the fact that they are not traded, we can compute using the default intensities embedded in the CDS spreads. We find that the two zero-cost portfolios proxying for systematic risk capture on average of 21% of the time-variation in the returns of the zero-coupon defaultable bonds while the new factors captures on average 63% of the residual. Moreover, we find that this new factor is also priced consistently across a broader spectrum of corporate bond portfolios, suggesting that both the European credit and the European corporate bond markets factor in the proneness of defaultable securities to extreme events. These results complement and expand the results of Berndt, Lookman and Obreja (2006) who show that a similar type of factor seems to be priced in the U.S. credit and corporate bond market, as well as the U.S. equity options market. This paper also documents a “flight to quality” effect in the European corporate bond markets.
## Distribution of Firms

<table>
<thead>
<tr>
<th>Industry Name</th>
<th>No. of Firms</th>
<th>Country</th>
<th>No. of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hotels</td>
<td>1</td>
<td>France</td>
<td>13</td>
</tr>
<tr>
<td>Airlines</td>
<td>4</td>
<td>Netherlands</td>
<td>4</td>
</tr>
<tr>
<td>Chemicals</td>
<td>5</td>
<td>UK</td>
<td>11</td>
</tr>
<tr>
<td>Telecom</td>
<td>13</td>
<td>Germany</td>
<td>10</td>
</tr>
<tr>
<td>Food/Soft Drinks</td>
<td>2</td>
<td>Sweden</td>
<td>5</td>
</tr>
<tr>
<td>Retail-grocery Chains</td>
<td>6</td>
<td>Italy</td>
<td>2</td>
</tr>
<tr>
<td>Automotives</td>
<td>6</td>
<td>Greece</td>
<td>1</td>
</tr>
<tr>
<td>Entertainment</td>
<td>1</td>
<td>Spain</td>
<td>2</td>
</tr>
<tr>
<td>Aerospace/Defence</td>
<td>2</td>
<td>Finland</td>
<td>5</td>
</tr>
<tr>
<td>Machinery</td>
<td>1</td>
<td>Denmark</td>
<td>1</td>
</tr>
<tr>
<td>Paper</td>
<td>3</td>
<td>Norway</td>
<td>1</td>
</tr>
<tr>
<td>Utilities</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Printing/Publishing</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Media</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advertising</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>55</strong></td>
<td><strong>55</strong></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: **Distribution of Firms Across Industries and Countries** Firms are grouped into industries according to the Moody’s industry classification.
### B Actual and Risk-neutral Default Intensities: Estimation Results

<table>
<thead>
<tr>
<th>Actual default intensities</th>
<th>Risk-neutral default intensities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa \theta$</td>
<td>$\theta^Q$</td>
</tr>
<tr>
<td>mean</td>
<td>1.47</td>
</tr>
<tr>
<td>median</td>
<td>0.83</td>
</tr>
<tr>
<td>std. dev.</td>
<td>1.93</td>
</tr>
</tbody>
</table>

Table 2: **Estimation of the actual and risk-neutral default intensities** Summary statistics for the firm-by-firm parameter estimates describing the dynamics of the actual and risk-neutral default intensities in equations (28) - (30).

### C Extracting the CMF Factor: Estimation Results
<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta_{EMKT}$</th>
<th>$\beta_{CMKT}$</th>
<th>$\beta_{TERM}$</th>
<th>$\text{Perc}(C)$</th>
<th>$\text{Perc}(S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>-0.0008</td>
<td>-0.0005</td>
<td>0.2345</td>
<td>62.75</td>
<td>20.78</td>
</tr>
<tr>
<td>1.0960</td>
<td>0.0117</td>
<td>0.00703</td>
<td>4.1520</td>
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<td></td>
</tr>
<tr>
<td>-0.0007</td>
<td>0.0094</td>
<td>0.0041</td>
<td>1.4754</td>
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<td>8.32</td>
</tr>
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<td>3.9568</td>
<td>0.8896</td>
<td>0.1571</td>
<td>3.5443</td>
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<td></td>
</tr>
<tr>
<td>-0.0012</td>
<td>0.0114</td>
<td>0.0078</td>
<td>2.4066</td>
<td>19.33</td>
<td>8.61</td>
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<td>4.4765</td>
<td>0.7916</td>
<td>0.1615</td>
<td>3.8588</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.0120</td>
<td>0.0097</td>
<td>3.1751</td>
<td>23.51</td>
<td>8.75</td>
</tr>
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<td>4.7590</td>
<td>0.7286</td>
<td>0.1555</td>
<td>4.0645</td>
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<td></td>
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<td>0.0106</td>
<td>3.8440</td>
<td>27.68</td>
<td>8.81</td>
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<tr>
<td>4.9392</td>
<td>0.6837</td>
<td>0.1478</td>
<td>4.2017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0024</td>
<td>0.0128</td>
<td>0.0109</td>
<td>4.4399</td>
<td>31.50</td>
<td>8.83</td>
</tr>
<tr>
<td>5.0638</td>
<td>0.6489</td>
<td>0.1409</td>
<td>4.2980</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: **Estimates for the return model in (30)** This table reports the results of the panel regression of the excess returns of defaultable zero-coupon bonds on the excess market returns $EMKT$, the excess local market return $CMKT$, the spread between long and short Euro bonds $TERM$ and the dummies controlling for specific week between January 2003 and October 2006, 197 weeks. The left-hand side excess returns correspond to defaultable bonds with the following times to expiration: 1 week, 1 year, 2 years, 3 years, 4 years and 5 years. The first line in the table corresponds to the estimates of the returns model where the left-hand side returns correspond to corporate bonds with the shortest time to expiration. The $CMF$ factor at time $t$ corresponds to the slope coefficient of the dummy controlling for time $t$. $\text{Perc}(C)$ column reports the variance of the $CMF$ factor as a percentage of the variance of the pricing error obtained under the returns model in (30) without time dummies, while $\text{Perc}(S)$ column reports the $R^2$ of the return models without time dummies. The t-statistics are reported in parentheses. The reported values for the estimates are averages across firms of the corresponding firm-specific estimates.
### Table 4: Testing for firm characteristics

This table reports the results of the regressions of excess returns of defaultable zero-coupon bonds on a time-varying characteristic, the excess market returns $EMKT$, the excess local market return $CMKT$, the spread between long and short Euro bonds $TERM$, and the CMF factor. Specifically, for each firm we estimate the following regressions:

$$r_{it+1}^{ie} = \gamma_0^i + \gamma_0^i \phi_t + \gamma_{EMKT,e}^i r_{it+1}^{EMKT,e} + \sum_{c=0}^{C} \gamma_{CMKT,e}^i r_{it+1}^{CMKT,e} + \gamma_{TERM}^i r_{it+1}^{TERM,e} + \gamma_{CMF}^i CMF_{it+1} + \epsilon_{it+1}.$$  

We consider three firm characteristics: the actual default probability measured by the 1-year EDF value, the firm size and the market-to-book ratio. The first column lists the name of the characteristic. The following columns report summary statistics (across firms) of the loadings on the characteristic and the default factor. The $t$-Statistics are the median $t$-Statistics across firms. The results for the loadings on the other systematic factors are not reported.

<table>
<thead>
<tr>
<th>Name</th>
<th>$\gamma_0$</th>
<th>$\gamma_0$ (Med)</th>
<th>$t$-Stat</th>
<th>$\gamma_{CMF}$</th>
<th>$\gamma_{CMF}$ (Med)</th>
<th>$t$-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>EDF</td>
<td>0.0001</td>
<td>0.0001</td>
<td>5.1091</td>
<td>0.1613</td>
<td>0.6424</td>
<td>7.3966</td>
</tr>
<tr>
<td>Size</td>
<td>-0.0001</td>
<td>-0.0002</td>
<td>-3.7908</td>
<td>0.2280</td>
<td>0.9964</td>
<td>9.1808</td>
</tr>
<tr>
<td>M/B</td>
<td>-0.0000</td>
<td>-0.0001</td>
<td>-2.4827</td>
<td>0.2477</td>
<td>1.1315</td>
<td>11.7675</td>
</tr>
</tbody>
</table>
### D Asset Pricing Tests: Results from the Time-series Regressions

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta_{EMKT}$</th>
<th>$\beta_{TERM}$</th>
<th>$\beta_{CMF}$</th>
<th>$R^2$</th>
<th>$E[R]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0002</td>
<td>-0.0595</td>
<td>0.5730</td>
<td>2.6583</td>
<td>0.2037</td>
<td>0.0006</td>
</tr>
<tr>
<td>(0.2011)</td>
<td>(6.2292)</td>
<td>(0.3225)</td>
<td>(1.5745)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0006</td>
<td>-0.0449</td>
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<td>2.9709</td>
<td>0.1008</td>
<td>0.0007</td>
</tr>
<tr>
<td>(0.5547)</td>
<td>(3.9228)</td>
<td>(0.7846)</td>
<td>(1.4665)</td>
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<td></td>
</tr>
<tr>
<td>-0.0010</td>
<td>-0.0182</td>
<td>2.4341</td>
<td>5.1873</td>
<td>0.0667</td>
<td>0.0009</td>
</tr>
<tr>
<td>(1.0018)</td>
<td>(1.6771)</td>
<td>(1.2074)</td>
<td>(2.7078)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: The Merrill Lynch non-financial corporate bond portfolios sorted on rating
This table reports the results of the time-series regressions of the excess realized returns of three Merrill Lynch non-financial corporate bond portfolios sorted on rating (AAA, A and BBB), on the excess market returns $EMKT$, the spread between long and short Euro bonds $TERM$ and the $CMF$ factor between January 2003 and October 2006, 197 weeks. The $CMF$ factor is extracted from returns on defaultable zero-coupon bonds maturing in one week, according to the model in equation (30). The first line corresponds to the higher rating portfolio. The t-statistics are reported in parentheses.
<table>
<thead>
<tr>
<th>α</th>
<th>β_{EMKT}</th>
<th>β_{TERM}</th>
<th>β_{CMF}</th>
<th>R²</th>
<th>E[R]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA Portfolios Sorted on Maturity</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>−0.0004</td>
<td>−0.0228</td>
<td>0.8468</td>
<td>1.3852</td>
<td>0.1849</td>
<td>0.0005</td>
</tr>
<tr>
<td>(1.1404)</td>
<td>(5.6032)</td>
<td>(1.1162)</td>
<td>(1.9214)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>−0.0005</td>
<td>−0.0561</td>
<td>1.1302</td>
<td>2.7471</td>
<td>0.1908</td>
<td>0.0005</td>
</tr>
<tr>
<td>(0.5571)</td>
<td>(5.9223)</td>
<td>(0.6408)</td>
<td>(1.6391)</td>
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<td></td>
</tr>
<tr>
<td>−0.0003</td>
<td>−0.0759</td>
<td>0.9565</td>
<td>3.6126</td>
<td>0.1802</td>
<td>0.0007</td>
</tr>
<tr>
<td>(0.2325)</td>
<td>(5.7469)</td>
<td>(0.3892)</td>
<td>(1.5468)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0004</td>
<td>−0.1068</td>
<td>0.1358</td>
<td>4.3918</td>
<td>0.1469</td>
<td>0.0010</td>
</tr>
<tr>
<td>(0.2207)</td>
<td>(5.1418)</td>
<td>(0.0352)</td>
<td>(1.1961)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AA Portfolios Sorted on Maturity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−0.0005</td>
<td>−0.0405</td>
<td>1.2071</td>
<td>2.1288</td>
<td>0.1746</td>
<td>0.0005</td>
</tr>
<tr>
<td>(0.7774)</td>
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<td>(1.6465)</td>
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<td></td>
</tr>
<tr>
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<td>−0.0735</td>
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<td>0.1739</td>
<td>0.0007</td>
</tr>
<tr>
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</tr>
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<td>4.1630</td>
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<td>0.0009</td>
</tr>
<tr>
<td>(0.0053)</td>
<td>(5.1725)</td>
<td>(0.2590)</td>
<td>(1.4300)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: The AAA-rated and AA-rated Merrill Lynch corporate bond portfolios sorted on maturity. This table reports the results of the time-series regressions of the excess realized returns of four AAA-rated Merrill Lynch corporate bond portfolios sorted on maturity (1-3 years, 3-5 years, 5-7 years and 10+ years) and three AA-rated Merrill Lynch corporate bond portfolios sorted on maturity (1-5 years, 5-7 years, and 7-10 years), on the excess market returns EMKT, the spread between long and short Euro bonds TERM and the CMF factor between January 2003 and October 2006, 197 weeks. The CMF factor is extracted from returns on defaultable zero-coupon bonds maturing in one week, according to the model in equation (30). The first line in each of the two panels corresponds to the lower maturity portfolio. The t-statistics are reported in parentheses.
Table 7: The A-rated and BBB-rated Merrill Lynch corporate bond portfolios sorted on maturity
This table reports the results of the time-series regressions of the excess realized returns of four A-rated Merrill Lynch corporate bond portfolios sorted on maturity (1-5 years, 5-7 years, 7-10 years and 10+ years) and three BBB-rated Merrill Lynch corporate bond portfolios sorted on maturity (1-5 years, 5-7 years, and 7-10 years), on the excess market returns $EMKT$, the spread between long and short Euro bonds $TERM$ and the $CMF$ factor between January 2003 and October 2006, 197 weeks. The $CMF$ factor is extracted from returns on defaultable zero-coupon bonds maturing in one week, according to the model in equation (30). The first line in each of the two panels corresponds to the lower maturity portfolio. The t-statistics are reported in parentheses.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta_{EMKT}$</th>
<th>$\beta_{TERM}$</th>
<th>$\beta_{CMF}$</th>
<th>$R^2$</th>
<th>$E[R]$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A Portfolios Sorted on Maturity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.0006$</td>
<td>$-0.0333$</td>
<td>$1.3889$</td>
<td>$2.4965$</td>
<td>$0.1454$</td>
<td>$0.0006$</td>
</tr>
<tr>
<td>$(0.8808)$</td>
<td>$(4.7334)$</td>
<td>$(1.0610)$</td>
<td>$(2.0070)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.0005$</td>
<td>$-0.0638$</td>
<td>$1.5622$</td>
<td>$4.2429$</td>
<td>$0.1452$</td>
<td>$0.0008$</td>
</tr>
<tr>
<td>$(0.3940)$</td>
<td>$(4.8715)$</td>
<td>$(0.6406)$</td>
<td>$(1.8309)$</td>
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<td></td>
</tr>
<tr>
<td>$-0.0003$</td>
<td>$-0.0683$</td>
<td>$1.5004$</td>
<td>$4.9212$</td>
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<td>$0.0010$</td>
</tr>
<tr>
<td>$(0.2072)$</td>
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<td>$(0.4928)$</td>
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<td></td>
</tr>
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<td>$-0.0607$</td>
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<td>$0.0012$</td>
</tr>
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<td>$(0.2323)$</td>
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<td>$(1.3259)$</td>
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<td></td>
</tr>
<tr>
<td><strong>BBB Portfolios Sorted on Maturity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.0008$</td>
<td>$-0.0107$</td>
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<td>$0.0679$</td>
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</tr>
<tr>
<td>$(1.2825)$</td>
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<td></td>
</tr>
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<td>$5.8806$</td>
<td>$0.0718$</td>
<td>$0.0009$</td>
</tr>
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<td>$0.0012$</td>
</tr>
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<td>$(0.9845)$</td>
<td>$(2.7117)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 8: The Merrill Lynch non-financial corporate bond portfolios sorted on maturity

This table reports the results of the time-series regressions of the excess realized returns of four Merrill Lynch non-financial corporate bond portfolios sorted on maturity (1-3 years, 3-5 years, 5-7 years and 10+ years), on the excess market returns $EMKT$, the spread between long and short Euro bonds $TERM$ and the $CMF$ factor between January 2003 and October 2006, 197 weeks. The $CMF$ factor is extracted from returns on defaultable zero-coupon bonds maturing in one week, according to the model in equation (30). The first line corresponds to the lower maturity portfolio. The t-statistics are reported in parentheses.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta_{EMKT}$</th>
<th>$\beta_{TERM}$</th>
<th>$\beta_{CMF}$</th>
<th>$R^2$</th>
<th>$E[ R ]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0006</td>
<td>-0.0145</td>
<td>1.4045</td>
<td>2.2386</td>
<td>0.1046</td>
<td>0.0006</td>
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<tr>
<td>(1.3937)</td>
<td>(3.0539)</td>
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</tr>
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<td>-0.0328</td>
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<td>0.1017</td>
<td>0.0007</td>
</tr>
<tr>
<td>(1.0168)</td>
<td>(3.5472)</td>
<td>(1.2486)</td>
<td>(2.0486)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0008</td>
<td>-0.0493</td>
<td>2.1074</td>
<td>4.7712</td>
<td>0.1023</td>
<td>0.0008</td>
</tr>
<tr>
<td>(0.6534)</td>
<td>(3.6939)</td>
<td>(0.8491)</td>
<td>(2.0229)</td>
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<td></td>
</tr>
<tr>
<td>-0.0006</td>
<td>-0.0597</td>
<td>2.7593</td>
<td>7.6113</td>
<td>0.0551</td>
<td>0.0015</td>
</tr>
<tr>
<td>(0.2597)</td>
<td>(2.4429)</td>
<td>(0.6064)</td>
<td>(1.7602)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 9: The Lehman Brothers Euro-aggregate industrial corporate bond portfolios sorted on rating

This table reports the results of the time-series regressions of the excess realized returns of five Lehman Brothers Euro-aggregate industrial corporate bond portfolios sorted on rating (AAA, AA, A, BAA and High Yield), on the excess market returns $EMKT$, the spread between long and short Euro bonds $TERM$ and the $CMF$ factor between January 2003 and October 2006, 197 weeks. The $CMF$ factor is extracted from returns on defaultable zero-coupon bonds maturing in one week, according to the model in equation (30). The first line corresponds to the higher rating portfolio. The t-statistics are reported in parentheses.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta_{EMKT}$</th>
<th>$\beta_{TERM}$</th>
<th>$\beta_{CMF}$</th>
<th>$R^2$</th>
<th>$E [R]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.0013$</td>
<td>$-0.0550$</td>
<td>$0.9363$</td>
<td>$3.1294$</td>
<td>$0.1763$</td>
<td>$-0.0003$</td>
</tr>
<tr>
<td>$0.14395$</td>
<td>$0.5805$</td>
<td>$0.5109$</td>
<td>$1.7968$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.0010$</td>
<td>$-0.0605$</td>
<td>$0.4328$</td>
<td>$3.3011$</td>
<td>$0.1445$</td>
<td>$-0.0003$</td>
</tr>
<tr>
<td>$0.9280$</td>
<td>$4.9913$</td>
<td>$0.1919$</td>
<td>$1.5405$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.0010$</td>
<td>$-0.0497$</td>
<td>$0.7245$</td>
<td>$2.1791$</td>
<td>$0.0905$</td>
<td>$-0.0002$</td>
</tr>
<tr>
<td>$0.9014$</td>
<td>$3.8842$</td>
<td>$0.3045$</td>
<td>$0.9638$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.0017$</td>
<td>$-0.0371$</td>
<td>$1.7892$</td>
<td>$4.5144$</td>
<td>$0.0751$</td>
<td>$-0.0003$</td>
</tr>
<tr>
<td>$1.5304$</td>
<td>$2.9211$</td>
<td>$0.7578$</td>
<td>$2.0122$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.0022$</td>
<td>$0.0554$</td>
<td>$4.3796$</td>
<td>$12.5528$</td>
<td>$0.0693$</td>
<td>$0.0011$</td>
</tr>
<tr>
<td>$0.9110$</td>
<td>$2.0726$</td>
<td>$0.8804$</td>
<td>$2.6555$</td>
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<td></td>
</tr>
</tbody>
</table>
Table 10: The Lehman Brothers Euro-aggregate corporate bond portfolios sorted on maturity

This table reports the results of the time-series regressions of the excess realized returns of five Lehman Brothers Euro-aggregate corporate bond portfolios sorted on maturity (1-3 years, 3-5 years, 5-7 years, 7-10 years and 10+ years), on the excess market returns $EMKT$, the spread between long and short Euro bonds $TERM$ and the $CMF$ factor between January 2003 and October 2006, 197 weeks. The $CMF$ factor is extracted from returns on defaultable zero-coupon bonds maturing in one week, according to the model in equation (30). The first line corresponds to the lower maturity portfolio. The t-statistics are reported in parentheses.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta_{EMKT}$</th>
<th>$\beta_{TERM}$</th>
<th>$\beta_{CMF}$</th>
<th>$R^2$</th>
<th>$E[R]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0013</td>
<td>-0.0188</td>
<td>1.3124</td>
<td>2.1769</td>
<td>0.1190</td>
<td>-0.0001</td>
</tr>
<tr>
<td>(-2.8902)</td>
<td>(-3.7374)</td>
<td>(1.3987)</td>
<td>(2.4415)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0013</td>
<td>-0.0386</td>
<td>1.2124</td>
<td>2.4885</td>
<td>0.0992</td>
<td>-0.0002</td>
</tr>
<tr>
<td>(-1.4862)</td>
<td>(-3.9172)</td>
<td>(0.6616)</td>
<td>(1.4290)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0018</td>
<td>-0.0693</td>
<td>2.4489</td>
<td>4.0009</td>
<td>0.1466</td>
<td>-0.0002</td>
</tr>
<tr>
<td>(-1.4634)</td>
<td>(-4.9331)</td>
<td>(0.9368)</td>
<td>(1.6107)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0013</td>
<td>-0.0763</td>
<td>0.5133</td>
<td>4.9687</td>
<td>0.1137</td>
<td>-0.0005</td>
</tr>
<tr>
<td>(-0.8228)</td>
<td>(-4.2719)</td>
<td>(0.1544)</td>
<td>(1.5727)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0019</td>
<td>-0.1013</td>
<td>3.6662</td>
<td>12.6694</td>
<td>0.0939</td>
<td>0.0007</td>
</tr>
<tr>
<td>(-0.6869)</td>
<td>(-3.3023)</td>
<td>(0.6424)</td>
<td>(2.3361)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
$$\alpha \quad \beta_{EMKT} \quad \beta_{TERM} \quad \beta_{CMF} \quad R^2 \quad E[R]$$

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional Averages of the Estimates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.0012$</td>
<td>$-0.0455$</td>
<td>$1.2214$</td>
<td>$0.2250$</td>
<td>$0.1005$</td>
<td>$-0.0002$</td>
</tr>
<tr>
<td>Cross-sectional Standard Deviations of the Estimates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.0003$</td>
<td>$0.0147$</td>
<td>$0.8873$</td>
<td>$0.2358$</td>
<td>$0.0645$</td>
<td>$0.0001$</td>
</tr>
<tr>
<td>Cross-sectional Averages of the t-Statistics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(1.0525)$</td>
<td>$(3.6354)$</td>
<td>$(0.4937)$</td>
<td>$(1.6493)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross-sectional Standard Deviations of the t-Statistics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(0.3135)$</td>
<td>$(1.1553)$</td>
<td>$(0.3352)$</td>
<td>$(1.8141)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11: The Lehman Brothers Euro-aggregate corporate bond portfolios sorted on sector

This table reports the results of the time-series regressions of the excess realized returns of sixteen Lehman Brothers Euro-aggregate corporate bond portfolios sorted on sector (Aero/Defense, Banking, Building Materials, Capital Goods, Chemicals, Communications, Consumer Non-cyclical, Consumer Cyclical, Diversified Manufacturing, Food and Beverages, Lodging, Refining, Telephone, Tobacco, Wireless and Media Non-Cable), on the excess market returns $EMKT$, the spread between long and short Euro bonds $TERM$ and the $CMF$ factor between January 2003 and October 2006, 197 weeks. The $CMF$ factor is extracted from returns on defaultable zero-coupon bonds maturing in one week, according to the model in equation (30). Each panel reports an average statistic across portfolios.
Figure 5: The estimates of the slope coefficient on the CMF factor extracted from returns on defaultable zero-coupon bonds with maturity varying from 1 to 5 years. These slopes are estimated from the time-series regressions of the excess realized returns of five Lehman Brothers Euro-aggregate industrial corporate bond portfolios sorted on rating (AAA, AA, A, BAA and High Yield), on the excess market returns $EMKT$, the spread between long and short Euro bonds $TERM$ and the $CMF$ factor between January 2003 and October 2006, 197 weeks.
Figure 6: The estimates of the slope coefficient on the CMF factor extracted from returns on defaultable zero-coupon bonds with maturity varying from 1 to 5 years. These slope coefficients are estimated from the time-series regressions of the excess realized returns of five Lehman Brothers Euro-aggregate corporate bond portfolios sorted on maturity (1-3 years, 3-5 years, 5-7 years, 7-10 years and 10+ years), on the excess market returns EMKT, the spread between long and short Euro bonds TERM and the CMF factor between January 2003 and October 2006, 197 weeks.
Figure 7: The estimates of the slope coefficient on the CMF Factor extracted from returns on defaultable zero-coupon bonds with maturity varying from 1 to 5 years. These slope coefficients are estimated from the time-series regressions of the excess realized returns of four A-rated Merrill Lynch corporate bond portfolios sorted on maturity (1-5 years, 5-7 years, 7-10 years and 10+ years) and three BBB-rated Merrill Lynch corporate bond portfolios sorted on maturity (1-5 years, 5-7 years, and 7-10 years), on the excess market returns $EMKT$, the spread between long and short Euro bonds $TERM$ and the CMF factor between January 2003 and October 2006, 197 weeks.
References


Duffie, D. *Dynamic Asset Pricing Theory*.


