The Term Structure of Interest Rates and Monetary Policy

Ph.D. Thesis by:

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Abstract

This dissertation aims to contribute to our understanding of the dynamics of interest rates, monetary policy and economic activity. It consists of three chapters.

The first chapter develops a theoretical framework to analyze how changes in the credibility of monetary policy affect the dynamics of the term structure of interest rates. A robust empirical fact about U.S. nominal interest rates is that they exhibit time-varying risk premia. In the last 20 years, the dynamics of these premia has changed. This chapter provides a monetary-policy explanation for this change. Monetary policy in the United States has achieved greater credibility and, as a result, market participants require less compensation for holding financial assets exposed to inflation risk. This explanation is substantiated using a general equilibrium model with nominal rigidities and habit formation in preferences. Two monetary policy regimes are analyzed: discretion and commitment. The model implies that the inflation risk premia are always lower under commitment and, thus, expected excess returns on bonds are lower and may become negative. The real effects of monetary policy in the model have an important asset-pricing implication: if the elasticity of intertemporal substitution of consumption is lower (greater) than the elasticity of substitution across goods, the inflation risk premium is negative (positive). The model is calibrated to the U.S. economy and is found to be consistent with the recent facts on interest rate behavior and the greater macroeconomic stability observed during the period.

The second chapter studies the economic content of the term structure and how it can be useful to conduct monetary policy. Interest rates are a rich source of information and thus can be a powerful tool for policy making. However, this information is difficult to extract given its dependence on the actual policy regime and the existence of time-varying term premia. This chapter analyzes the economic content of interest rates when term premia vary over time and monetary policy is optimal. The analysis is complemented with a potentially welfare-improving application for policy making: the formulation of optimal policy rules based on term-structure information. The analysis is conducted for policies with high or low weights on inflation stabilization. It shows that a high inflation weight increases the compensation for real risks in the term structure. As a result, forward rates are less informative about expected future monetary policy, and the term spreads and short-term rate predict better real economic activity and inflation, respectively. In addition, the optimal responses in an interest-rate rule to the lagged short-term rate and term spreads decline.

The third chapter is based on joint work with Michael Gallmeyer, Burton Hollifield and Stanley Zin. It shows how a model that includes a simple monetary policy rule can help us explain
the significant volatility observed in long-term interest rates. The model has two important characteristics: (i) inflation is endogenously determined by an interest-rate policy rule and (ii) non-trivial term-premium dynamics are generated by stochastic habit formation in preferences. The model is compared to a similar one with exogenous inflation. The two models capture the average level and shape of the yield curve, the volatility of the short-term interest rate and selected descriptive statistics of consumption growth and inflation. However, the exogenous-inflation model is not able to generate the observed volatility of long-term rates. The endogenous-inflation model with a countercyclical price of consumption growth risk, highly persistent policy shocks and negative correlation between consumption growth and inflation captures long-term rate volatility. Its success relies on an equilibrium inflation process that depends on highly autocorrelated policy shocks. The endogenous-inflation model is used to analyze the effects on the yield curve of policy rules with different responses to economic conditions. It is found that a policy rule with a stronger reaction to inflation explains recent developments in the dynamics of interest rates and inflation.
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Chapter 1

Interest Rates, Bond Premia and Monetary Policy

1.1 Introduction

Fama and Bliss (1987), Campbell and Shiller (1991) and other studies\(^1\) provide robust empirical evidence against the theory of the expectations hypothesis of interest rates. This evidence is consistent with time-varying expected excess returns on bonds or, equivalently, with time-varying risk premia in long-term interest rates. Recent contributions to the term-structure literature as Dai and Singleton (2002) and Duffee (2002) provide models that successfully capture this time variation in risk premia. This achievement, however, has been obtained specifying interest rates as functions of latent variables with no evident links to economic fundamentals. As a result, the underlying sources of variation in long-term interest rates still need to be explained. What are the economic forces driving the behavior of long-term interest rates? An answer to this question requires the analysis of the sources of variation in the real and inflation components of the stochastic discount factor. Since monetary policy is an important determinant of inflation and, arguably, economic activity, it follows that monetary policy might play an important role explaining the rich dynamics of long-term interest rates.

An appealing approach to study the impact of monetary policy on long-term rates is to analyze whether a regime switch in monetary policy is accompanied by changes in the properties of interest rates. Fortunately, recent developments in the United States economy offer an exceptional ground on which to conduct this analysis. In the last twenty years the variability of interest-rate risk premia has declined by a significant economic amount. Rudebusch and Wu (2004b) and Fama (2006) analyze changes across time in the coefficients of standard regressions that test the

expectations hypothesis. They find that recent changes in the regression coefficients imply less rejections of the expectations hypothesis and thus, less variation in risk premia. At the same time, monetary policy has also changed in the last two decades. Clarida, Gali and Gertler (2000) provide empirical evidence of changes in the policy from a passive stance in the pre-Volcker years towards a stronger anti-inflationary stance during the Volcker-Greenspan era. This is consistent with a stronger commitment of the Federal Reserve to achieve its economic objectives and results in improved policy credibility.²

This chapter analyzes the effects of improvements in the credibility of monetary policy on the behavior of interest rates. More specifically, it asks whether greater credibility in monetary policy is consistent with lower variability in interest-rate risk premia. This analysis is useful to understand fundamental issues in finance and macroeconomics. It provides an additional tool to interpret movements in long-term interest rates, with direct implications on, for instance, portfolio selection, risk management and policy making.³ The portfolio choice implications of inflation, the sign of bond risk premia or the financial-market channel of transmission of monetary policy can be studied in this framework.

I develop a general equilibrium model to address the question above. This model builds on the standard framework for monetary policy analysis presented in Woodford (2003) and extends it to capture salient asset-pricing facts, i.e. time-varying risk premia. This is achieved incorporating stochastic external habit formation in preferences.⁴ An important characteristic of the model is that monetary policy affects real economic activity due to nominal rigidities in the production of multiple goods. It implies that monetary policy affects the real and nominal components of the stochastic discount factor and thus the valuation of financial assets with real and nominal payoffs. Credibility in monetary policy is captured assuming that the monetary authority tries to maximize welfare in two possible regimes: discretion or commitment. The difference between the two regimes is that under commitment the policy is perfectly credible and affects private sector’s expectations about future economic conditions. Consequently, the comparison of the equilibrium characteristics of long-term interest rates under the two regimes captures the effects of policy

²Ben Bernanke labels the pre-Volcker period and the Volcker-Greenspan era as periods of policy under discretion and commitment, respectively. On February 3, 2003, before the Money Marketeers of New York University, Bernanke says “In the United States, the heyday of discretionary monetary policy can be dated as beginning in the early 1960s.” and then he adds “… the Fed in recent years has demonstrated a commitment to keeping inflation low and stable.”

³A popular example of our limited understanding of the link between long-term interest rates and monetary policy is given by the following statement. In February 2005 Alan Greenspan declared that “Long-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. Historically, even distant forward rates have tended to rise in association with monetary policy tightening. … For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum.”

credibility on the term structure of interest rates. The model delivers an affine term structure similar to Duffie and Kan (1996). It has the advantage that interest rates are linear functions of macroeconomic variables and the coefficients of the functions depend on the monetary policy regime and deep parameters of the economy, e.g. preference and production parameters. It provides the necessary link to understand the economic sources of risk that drive the behavior of the term structure.

The asset-pricing implications of policy credibility in the model can be understood considering the differences in the perception of inflation risk under discretion and commitment. Under commitment, the stochastic discount factor is less affected by the source of inflation risk in the economy, i.e. supply shocks. The reason is that commitment in monetary policy reduces the impact of supply shocks on economic performance. A unit of inflation risk generates less inflation and less negative effects on real economic activity and, as a result, the price of this risk decreases. The relation between inflation and economic activity also provides insights into the sign of the compensation for inflation risk. It turns out that, in both regimes, the sign of the inflation risk premia depends on the tradeoff between the elasticity of intertemporal elasticity of consumption (EIS) and the elasticity of (intratemporal) substitution across goods (ESG). This tradeoff determines whether the intertemporal economic distortions caused by inflation are greater than the intratemporal distortions. This can be understood by decomposing the stochastic discount factor in its real and nominal components. A shock that increases inflation decreases consumption growth and therefore increases the intertemporal marginal utility. This effect on the real component is offset by the inflation adjustment generated by the nominal component. If the EIS is lower than the ESG, the real component effect outweighs the nominal one. In that case, assets positively correlated with inflation involve a negative risk premium. The reason is that inflation has large negative effects on consumption growth, it induces a high demand for assets that hedge inflation risk and reduces the expected excess return on the assets. The effect is the opposite if the EIS is greater than the ESG. Inflation creates significant intratemporal distortions and market participants demand high expected excess returns to hold financial assets exposed to inflation risk. Similarly, the sensitivity of bond returns to inflation risk is also affected by credibility improvements. Since risk premia in interest rates, i.e. term premia, are generated by the covariance between the nominal stochastic discount factor and bond returns, the net effect of credibility on the term premia depends on changes in the inflation-related covariance between the stochastic discount factor and bond returns. The magnitude and direction of the change are obtained from a calibration of the model.

The model is calibrated to the U.S. economy for the Pre-Volcker period 1961-1979 to replicate some equilibrium properties of the economy under discretion. This period is considered of low commitment in monetary policy in comparison to the Volcker-Greenspan era. Subsequently, a
policy experiment is conducted to observe the effects on the behavior of interest rates of a regime switch from discretion to commitment. The term premia implied under discretion are positive and highly volatile. Under commitment the term premia become negative and less volatile for long-term bonds. Policy credibility changes the sign of expected excess returns on bonds. While nominal bonds are risky assets under discretion, they become a hedge for the marginal utility of wealth under commitment. The reduced volatility of the term premia can be explained by the reduction in the persistence of inflation observed under commitment. High inflation persistence implies high predictive power of inflation about future economic conditions. As a result, the compensation for inflation risk under discretion depends considerably on the current level of inflation. In contrast, when the monetary authority is committed to stabilizing inflation, the current level of inflation is less useful to predict future economic conditions and the compensation for inflation risk depend less on the state of the economy. In summary, the model provides strong support for the claim that greater credibility in monetary policy plays a significant role in reducing the variability of risk premia in interest rates. These conclusions gain further support from the fact that the model also captures the significant decrease in macroeconomic volatility and the lower correlation between inflation and economic activity observed in the last twenty years.

The chapter is organized as follows. Section 1.2 presents the related literature. Section 1.3 shows the relevant empirical evidence. The model is presented in Section 1.4 and its term structure implications are shown in Section 1.5. The analysis of the model is presented in Section 1.6 and Section 1.7 concludes.

1.2 Related Literature

This chapter joins a growing body of work that relates the term structure of interest rates to monetary policy. In spite of the obvious connections between the two, while the financial economics literature usually analyzes the term structure of interest rate without considering the effects of monetary policy on the short-term rate, the monetary policy literature is usually developed without exploring the policy effects on long-term interest rates. Diebold, Piazzesi and Rudebusch (2005) summarize recent attempts in empirical and theoretical grounds to understand the joint dynamics of the term structure of interest rates, macroeconomic variables and monetary policy. The empirical literature has benefited from studies identifying the effects of macroeconomic variables and monetary policy on the yield curve. For instance, Ang and Piazzesi (2003) and Piazzesi (2005) show how economic information and monetary policy helps us improve the empirical fitting of the yield curve relative to successful latent factor models. Other studies, such as Rudebusch and Wu (2004a), Ang, Piazzesi and Wei (2005) or Diebold, Rudebusch and Aruoba (2005) provide macroeconomic interpretations for traditional term structure factors such as the “level”, “slope” and “curvature” of Litterman and Scheinkman (1991). Simultaneously, the mon-
etary policy literature has benefited from the term structure literature. For example, Ang, Dong and Piazzesi (2005) and Bikbov and Chernov (2005) have recognized the absence of arbitrage in bond prices as an additional restriction to estimate monetary policy rules and identify the systematic component of monetary policy.

The empirical literature is complemented by theoretical contributions that link monetary policy and the term structure through structural macro models. This work provides macro factors for no-arbitrage term structure models. Some examples of these contributions are Bekaert, Cho and Moreno (2005), Hördahl, Tristani and Vestin (2004), Wu (2005) and Rudebusch and Wu (2004a). However, these structural models are not able to capture time-variation in risk premia from first principles and, therefore, their ability to explain deviations from the expectations hypothesis is limited.

In order to understand the sources of time variation in expected excess returns on bonds, we need a model able to capture deviations from the expectations hypothesis. One of these sources should be monetary policy, as proposed by McCallum (1994). McCallum’s idea is that observed deviations from the expectations hypothesis are compatible with time-varying risk premia and a monetary authority setting a short-term rate as a response to observed term spreads and the desire to smooth interest rates over time. Gallmeyer, Hollifield and Zin (2005), following this direction, explore the kind of structural models that imply endogenous time-varying risk premia and deliver an affine term structure. They assume a very special reaction function for monetary policy.

Ravenna and Seppälä (2005), Buraschi and Jiltsov (2005) and Rudebusch and Wu (2004b) are three contributions close in spirit to this one. Ravenna and Seppälä (2005) propose a New-Keynesian model that generates time-varying term premia endogenously to show that the systematic component of monetary policy can explain the rejection of the expectations hypothesis. They attribute deviations from the expectations hypothesis to technology and preference uncertainty. Contrary to that, Buraschi and Jiltsov (2005) find that a time-varying inflation risk premium and monetary policy shocks are important to explain the rejection of the expectations hypothesis. They develop a structural monetary model with taxes and endogenous monetary policy to analyze nominal and real term premia. Rudebusch and Wu (2004b) examine the recent shift in the dynamics of the term structure of interest rates and suggest a link between this shift and changes in the perception of the dynamics of the inflation target. These papers do not incorporate a monetary policy with the explicit objective of maximizing welfare and, as a consequence, do

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5 The frameworks in Bekaert, Cho and Moreno (2005) and Wu (2005) imply that the expectations hypothesis holds. Hördahl, Tristani and Vestin (2004) impose an exogenous time-varying market price of risk, which is not consistent with the endogenous discount factor. The theoretical foundations in Rudebusch and Wu (2004a), as they mention, are tenuous, as an exogenous time-varying market price of risk is imposed.
not allow us to identify the welfare implications of the policy and the effects on the equilibrium characteristics of interest rates of changes in policy credibility.

1.3 Empirical Evidence

This section presents simple descriptive statistics to illustrate the simultaneous changes in the dynamics of interest rates, macro variables and monetary policy in the United States during the last two decades. More rigorous econometric approaches can be found in Rudebusch and Wu (2004b) for shifts in the term structure of interest rates and Kim and Nelson (1999) and McConnell and Perez-Quiroz (2000) for developments in macroeconomic variables. Bordo and Haubrich (2004) present long-run evidence relating changes in the yield curve to changes in the credibility of the monetary regime.

The data consist of United States time series for interest rates, consumption and consumer prices from 1961:Q4 to 2005:Q4. The term structure series was obtained from quarterly data on bond yields for yearly maturities from 1 to 5 years from the Fama-Bliss discount bonds database found in CRSP and the short-term nominal interest rate is the 3-month T-bill from the Fama risk-free rates database. The consumption growth series was constructed using quarterly data on real per-capita consumption of nondurables and services from the Bureau of Economic Analysis. The inflation series was constructed following the methodology used in Piazzesi and Schneider (2006) to capture inflation related to non-durables and services consumption only. ⁶

In order to understand the effects of changes in policy credibility on changes in the dynamics of interest rates we focus on two subperiods: The Pre-Volcker years from 1961 to 1979 and the Greenspan era from 1988 to 2005. While the first subperiod is associated to low credibility, the Greenspan era is seen as one of high credibility on the commitment of the Federal Reserve to stabilize inflation. The period that Paul Volcker served as Chairman of the Federal Reserve Board is excluded from the analysis for two reasons. First, from October 1979 to October 1982 the Fed conducted a policy experiment changing the instrument of monetary policy from an interest rate target to a money supply target. This change increased dramatically the volatility of interest rates and, therefore, including this period to analyze the dynamics of interest rates can be misleading. Second, the Volcker era can be seen as a transition period when the Fed was trying to restore its credibility and thus is a period of great instability on people’s expectations.

⁶The details of the construction of the series can be found in http://faculty.chicagogsb.edu/monika.piazzesi/research/macroannual/. One important difference between the series constructed in Piazzesi and Schneider (2006) and the one presented here is the additional adjustment made here to drop the effect of population growth and obtain inflation in per-capita terms. The price index series and the population series used for the adjustment were obtained from the Bureau of Economic Analysis.
Figure 1.1 presents changes in selected statistics for interest rates. Panel A shows us that the average level of interest rates of short and long maturities has declined and the average spread between the three-month rate and long-interest rates has widened. Panel B shows that interest rate volatility has increased and this increase has been greater for short-term maturities. Panel C presents the coefficients of Campbell-Shiller regressions for the two periods. Under the expectations hypothesis of interest rates these coefficients should be one. From the figure we can observe that for the Pre-Volcker years these coefficients are negative and for the Greenspan era they are closer to one. This suggests that the common variability of interest-rate spreads and term premia has reduced.

Table 1.1 presents statistics of macroeconomic variables for the two subperiods. The average level and volatility of consumption growth and inflation are lower during the 1988-2005 period. This reduction has been accompanied by a decrease in the autocorrelation of consumption growth and a significant reduction in the persistence of inflation. At the same time, the negative correlation between consumption growth and inflation has reduced.

Figure 1.2 suggests significant changes in the joint dynamics of macroeconomic variables and interest rates. The figure shows that the positive slope coefficients in the regression consumption growth vs. lagged spreads and the negative slope coefficients in the regression inflation vs. lagged spreads are much closer to zero for the Greenspan era than during the Pre-Volcker years. This suggests a declining power of the term structure of interest rates to predict future economic activity.

7The effect is more dramatic if inflation is measured using the consumer price index from the Center for Research in Security Prices (CRSP). During 1961-1979 the autocorrelation of this index was 0.79 and decreased to -0.22 during
Table 1.1: US Consumption Growth ($\Delta c$) and Inflation ($\pi$) Statistics.

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<th>1988-2005</th>
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<tr>
<td>$\mathbb{E} [\Delta c_t] \times 4$</td>
<td>2.72%</td>
<td>1.84%</td>
</tr>
<tr>
<td>$\mathbb{E} [\pi_t] \times 4$</td>
<td>4.81%</td>
<td>2.94%</td>
</tr>
<tr>
<td>$\sigma (\Delta c_t) \times 4$</td>
<td>1.79%</td>
<td>1.35%</td>
</tr>
<tr>
<td>$\sigma (\pi_t) \times 4$</td>
<td>2.87%</td>
<td>1.26%</td>
</tr>
<tr>
<td>corr($\Delta c_t, \Delta c_{t-1}$)</td>
<td>0.39</td>
<td>0.28</td>
</tr>
<tr>
<td>corr($\pi_t, \pi_{t-1}$)</td>
<td>0.88</td>
<td>0.53</td>
</tr>
<tr>
<td>corr($\Delta c_t, \pi_t$)</td>
<td>-0.52</td>
<td>-0.18</td>
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Figure 1.2: Slope coefficients of regressions of consumption growth and inflation on one-quarter lagged spreads, for yields of maturities from 1 to 5 years.

In summary, there are significant changes in the dynamics of interest rates and macro variables that chronologically coincide with changes in the U.S. monetary regime: from a low credibility regime to a high credibility one.

1.4 The Model

I model a discrete-time closed economy populated by households that derive utility from the consumption of an aggregate of differentiated goods and disutility from labor. Households provide labor to firms that maximize profits in a monopolistic competitive setting with price rigidities and a labor-only technology. In this economy, monetary policy is conducted to maximize welfare using the nominal one-period interest rate as the instrument. The policy can be credible or not depending on whether the monetary authority follows the policy under commitment or discretion, 1988-2005.
respectively. There are three sources of uncertainty: productivity, supply and preference shocks, that, for simplicity, are assumed to be uncorrelated. Throughout the chapter the subindex \( t \) denotes time and \( \Delta \) is the difference operator.

### 1.4.1 Households (Aggregate Demand)

Households exhibit preferences over an infinite horizon on consumption, \( C \), and labor, \( h(i) \), which is provided to a continuum of firms owned by the households and indexed by \( i \in [0,1] \). The consumption good is the Dixit-Stiglitz aggregate of a continuum of differentiated goods, \( C(i) \), given by

\[
C_t = \left[ \int_0^1 C_t(i) \theta^{-1} \frac{d \theta}{\theta} \right]^{\frac{\theta}{\theta-1}},
\]

where \( \theta > 1 \) is the elasticity of substitution between differentiated goods. The expected utility of the households is represented by

\[
E \left[ \infty \sum_{t=0}^\infty \beta^t \left( \frac{1}{1-\gamma} \frac{C_{t}^{1-\gamma}}{Q_t} - \frac{1}{1+\omega} \int_0^1 h_t(i)^{1+\omega} \frac{d \omega}{\omega} \right) \right],
\]

where \( \gamma^{-1} \) and \( \omega^{-1} \) are the elasticities of intertemporal substitution of consumption and labor, respectively. The utility derived from consumption is affected by the external habit \( Q \). This habit is shaped by aggregate consumption, \( \tilde{C} \) and preference shocks \( \varepsilon_{\eta} \sim N(0,1) \). Specifically, denoting the aggregate consumption growth by \( \Delta \tilde{c}_t \equiv \log \frac{\tilde{C}_t}{\tilde{C}_{t-1}} \), the process for \( q \equiv \log Q \) is

\[
q_{t+1} = q_t + \eta \Delta \tilde{c}_t + (1 + K_{\eta} \Delta \tilde{c}_t)^{1/2} \sigma_{\eta} \varepsilon_{\eta,t+1},
\]

where \( \eta \) is the average sensitivity of changes in the habit to lagged consumption growth and \( K_{\eta} \) captures the dependence of the volatility of the habit on lagged consumption growth. The second term in equation (1.2) is the aggregate disutility from the labor supplied to the production of differentiated goods.

Households choose contingent consumption and labor streams to maximize their expected utility subject to the contingent budget constraints

\[
\int_0^1 P_t(i)C_t(i)di + W_t^+ \leq W_t^- + \int_0^1 w_t(i)h_t(i)di + \int_0^1 \Pi_t(i)di,
\]

for all \( t \), where \( P(i) \) denotes the prices of the differentiated goods. According to this equation, consumption and investment in financial instruments, \( W_t^+ \), are constrained by previous period financial wealth plus return, \( W_t^- \), labor income from the production sector, \( w_t(i)h_t(i) \), and profits
from the firms supplying the differentiated goods, $\Pi_t(i)$. Financial markets are complete and nominal and real zero-net-supply default-free bonds are traded for all maturities.

Writing the price of the consumption good as $P$, the value of the consumption good is $PC = \int_0^1 P(i)C(i)di$, and we can replace the set of budget constraints with the intertemporal budget constraint

$$E \left[ \sum_{t=0}^{\infty} M_{0,t}P_tC_t \right] \leq W_0 \quad + \quad E \left[ \sum_{t=0}^{\infty} M_{0,t} \left( \int_0^1 w_t(i)h_t(i)di + \int_0^1 \Pi_t(i)di \right) \right],$$

where $M_{t,t+n} > 0$ is the nominal pricing kernel that discounts nominal cashflows at time $t + n$ to time $t$.

The solution to the household problem implies that

$$M_{t,t+n} = \beta^n \left( \frac{C_{t+n}}{C_t} \right)^{-\gamma} \left( \frac{Q_{t+n}}{Q_t} \right)^{-1} \left( \frac{P_{t+n}}{P_t} \right)^{-1}$$

and the price at time $t$ of a nominal bond with maturity at $t + n$ is given by

$$b_t^{(n)} = E_t[M_{t,t+n}].$$

In particular, households can invest in a one-period nominal bond which pays the nominal one-period rate $i_t$. This rate is the instrument of monetary policy for the economy and must satisfy

$$e^{-i_t} = E_t M_{t,t+1} = E_t \left[ \exp(\log \beta - \gamma \Delta c_{t+1} - \Delta q_{t+1} - \pi_{t+1}) \right],$$

where $\pi_{t+1} \equiv \log \left( \frac{P_{t+1}}{P_t} \right)$ is the rate of inflation. This equation shows that habit persistence makes the optimal interest rate to depend not only on expectations about future consumption growth and future inflation but also on the current level of consumption growth and preference shocks.

To complete the analysis of the household problem, the intratemporal marginal rate of substitution

$$\frac{w_t(i)}{P_t} = h_t(i)^{\omega}C_t^{\gamma}Q_t$$

provides the tradeoff between consumption and labor that must be satisfied in equilibrium.

---

Equation (1.1) can be seen as the production function of $C_t$ with inputs $C_t(i)$ for a competitive producer with optimal profits of zero. It implies $P_t = \left[ \int_0^1 P_t(i)^{1-\omega}di \right]^{\frac{1}{1-\omega}}$. 

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1.4.2 Production Sector (Aggregate Supply)

The production of differentiated goods is characterized by monopolistic competition and price rigidities and affected by productivity and supply shocks. There is a continuum of suppliers of differentiated goods $i \in [0, 1]$ who have market power to set their own prices and face demand curves

$$Y_t(i) = Y_t\left(\frac{P_t(i)}{P_t}\right)^{-\theta},$$  \hspace{1cm} (1.8)

where $Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta}{\theta - 1}} di\right]^{\frac{\theta - 1}{\theta}}$ is the aggregate output.\(^9\)

The production function of a differentiated good is

$$Y_t(i) = A_t h_t(i),$$  \hspace{1cm} (1.9)

where changes in labor productivity $A_t$ are modeled as the autoregressive process

$$\Delta \log A_{t+1} = (1 - \phi_a)g_a + \phi_a \Delta \log A_t + (1 + K_a \Delta \log A_t)^{1/2} \sigma_a \varepsilon_{a,t+1},$$  \hspace{1cm} (1.10)

with $\varepsilon_a \sim \mathcal{N}(0, 1)$. The volatility of the technology shocks is time-varying and depends on changes in productivity.

Using the production function (1.9) and the marginal rate of substitution (1.7) we can write the real marginal production cost of a differentiated good $s_t(i)$ as

$$s_t(i) = \frac{1}{Y_t(i)} \left(\frac{Y_t(i)}{A_t}\right)^{1+\omega} Y_t Q_t,$$  \hspace{1cm} (1.11)

where $\omega$ can be seen as the elasticity of the real marginal cost with respect to the supply of a differentiated good.

Now, with the purpose of facilitating the exposition of the production problem with price rigidities, it is convenient to derive the aggregate output of a hypothetical economy with full price flexibility. This output is known as the natural rate of output and is a reference point to conduct monetary policy. Denoting this output by $Y^n_t$, the profit-maximization problem is

$$\max_{P_t(i)} P_t(i)Y^n_t - w_t(i)h_t(i)$$

\(^9\)Demand curves can be obtained as the result of a profit maximization problem for a price-taking producer of good $Y$ with aggregate production costs $\int_0^1 P_t(i)Y_t(i) di$. 

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subject to (1.8) and (1.9). The solution to this problem implies that \( \frac{P_t(i)}{\bar{P}_t} = \mu s_t(i) \), with the markup \( \mu \equiv \frac{\theta}{\sigma - 1} \) arising from market power. In addition, under price flexibility \( P_t(i) = P_t, \ Y_t(i) = Y_t \) and the real marginal cost for all suppliers becomes \( s_t^n = \mu^{-1} \). Using this condition and equation (1.11), we obtain the output process

\[
y_t^n = \frac{1}{\omega + \gamma} \left[ \log \mu^{-1} - q_t^n + (1 + \omega) \log A_t \right],
\]

(1.12)

where \( y_t^n \equiv \log Y_t^n \). Besides the positive dependence on the level of technology and the negative dependence on the markup, the process for the natural rate of output depends on the natural level of habit, \( q_t^n \equiv \log Q_t^n \), that is, the external habit that characterizes an economy with price flexibility. Since \( \tilde{c} = y_t^n \) in equilibrium, the process for the natural output growth is

\[
\Delta y_{t+1}^n = \frac{1}{\omega + \gamma} \left[ -\eta \Delta y_t^n + (1 + \omega) \Delta \log A_{t+1} - (1 + K_{\eta} \Delta y_t^n)^{1/2} \sigma_{\eta, \varepsilon_t, t+1} \right].
\]

(1.13)

The habit makes current output growth to depend on lagged output growth and preference shocks.

At this point we can incorporate price rigidities following the Calvo (1983) staggered price setting, where only a fraction \( 1 - \alpha \) of suppliers is able to change prices optimally in a given period, while a fraction \( \alpha \) keeps last period prices. Within this framework, the optimal price set a time \( t \) solves the profit-maximization problem

\[
\max_{P_t(i)} E_t \left[ \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left( e^{\epsilon_{T} P_t(i) Y_{T|t}(i)} - w_{T|t}(i) h_{T|t}(i) \right) \right]
\]

subject to (1.8) and (1.9), for all \( T \). That is, suppliers that are able to reset prices choose a price that maximizes expected discounted profits taking into account the probability of not resetting that price in the future. It implies that output is \( Y_{T|t}(i) = Y_T \left( \frac{P_t(i)}{\bar{P}_T} \right)^{-\theta} \), with a similar reasoning for \( w_{T|t} \) and \( h_{T|t}(i) \). In addition, profits are affected by supply shocks \( \epsilon_T \). These shocks follow the autoregressive process

\[
\epsilon_{t+1} = \phi \epsilon_t + (1 + K_{\epsilon} \epsilon_t)^{1/2} \sigma_{\epsilon} \epsilon_{t+1},
\]

(1.14)

with \( \epsilon_t \sim \mathcal{N}(0, 1) \) and the volatility of the shocks depending on the level of \( \epsilon_t \).

From the equations above, Appendix 1.8.1 shows that the aggregate-supply condition for the
The economy can be written as

\[ \pi_t = \kappa x_t + \frac{\kappa}{\omega + \gamma} l_t + \beta \mathbb{E}_t \pi_{t+1} - \frac{\kappa}{\omega + \gamma} \epsilon_t, \]  

(1.15)

where \( \kappa \equiv \frac{(1 - \alpha \omega)(1 - \alpha)}{\alpha} \zeta \) and \( \zeta \equiv \frac{\omega + \gamma}{1 + \theta \omega}. \) The output gap, \( x_t \equiv y_t - y^*_t \), is the deviation of actual output from the natural rate of output and the habit gap, \( l_t \equiv q_t - q^*_t \), is the deviation of the actual habit from the natural habit. This forward-looking equation captures the short-term tradeoff between output and inflation induced by price rigidities. It shows us that inflation is driven by expectations on future inflation, current and lagged levels of the output gap and supply shocks. The dependence on lagged output gaps is captured by the habit gap. The sensitivity of inflation to the habit gap is the sensitivity of inflation to the output gap, \( \kappa \), adjusted by the elasticities of intertemporal substitution of labor and consumption. In addition, taking the first difference of equation (1.15) we can see that the habit gap makes changes in inflation to depend on lagged changes in the output gap.

### 1.4.3 Monetary Policy

Monetary policy is conducted to maximize households’ welfare, that is, the expected utility of households. We assume that the monetary authority uses the nominal one-period interest rate, \( i_t \), as the policy instrument and decides between following the policy under commitment or under discretion. While under discretion the monetary authority is unable to generate credibility, under commitment the policy is perfectly credible and, therefore, affects expectations of households and firms about future economic activity.

The monetary authority perfectly knows the functioning of the economy, including the functional form (1.2) of the expected utility, as well as the aggregate demand (1.6) and supply (1.15) conditions. As a result, the monetary policy problem can be written as

\[
\max_{\{\pi_t, y_t, i_t\}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1 - \gamma} Y_t^{1-\gamma} - \frac{1}{1 + \omega} \int_0^{1} \left( \frac{Y_i(i)}{A_t} \right)^{1+\omega} di \right) \right]
\]  

(1.16)

10This condition is known as the standard New-Keynesian Phillips Curve, in this case, expanded to take into account the effect on inflation of the external habit.

11The term \( \zeta \) describes the degree of strategic complementarity between the price setting decisions of the suppliers of differentiated goods. It determines the size of the distortions caused by price rigidities and, therefore, how useful monetary policy can be. An economy is said to be characterized by strategic complementarity when \( \zeta < 1 \), since it implies \( \frac{dP(i)}{di} > 0 \). In this case, optimal prices tend to follow the aggregate price level, distortions in relative prices caused by price rigidities are significant and, therefore, production decisions tend to differ considerably from the natural rate of output. Some authors refer to \( \zeta \) as the degree of real rigidity in the economy.
subject to

\[ e^{-i t} = E_t \left[ \exp(\log \beta - \gamma \Delta y_{t+1} - \Delta q_{t+1} - \pi_{t+1}) \right] \]

and

\[ \pi_t = \kappa x_t + \frac{\kappa}{\omega + \gamma} l_t + \beta E_t \pi_{t+1} - \frac{\kappa}{\omega + \gamma} \epsilon_t, \]

for all \( t \). Appendix 1.8.2 shows that maximizing welfare is equivalent to targeting inflation and output to minimize the loss function\(^{12}\)

\[
\frac{1}{2} E \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \left( x_t - x^* + \frac{1}{\omega + \gamma} (l_t - l^*) \right)^2 + \frac{\theta}{\kappa \pi_t^2} \right] \right\},
\]

where \( x^* \) and \( l^* \) are, respectively, the output gap target and the habit gap target for the monetary authority, and satisfy\(^{13}\)

\[ x^* + \frac{l^*}{\omega + \gamma} = \frac{1}{\omega + \gamma} \log \mu. \]

Thus, welfare is maximized when inflation is zero and output and habit gaps reach targets that depend on the degree of market power in the economy. The monetary authority must assign a weight of \( \frac{\theta}{\kappa + \theta} \) to inflation stabilization and \( \frac{\kappa}{\kappa + \theta} \) to stabilize output. Habit formation in preferences captures the idea that deviations from the target output in the past affect household’s welfare today.

The solution to the monetary policy problem and, therefore, the equilibrium output, inflation, and interest rates depend on whether the monetary authority is committed to the long-term objective of welfare maximization, that is, on how credible the policy is. A policy under commitment has the ability that a policy under discretion lacks to affect expectations of households and firms. The following two propositions present the equilibrium outcomes for output and inflation of policies followed under discretion and commitment, respectively. The analysis of the effects on interest rates of the two policies is the topic of the next two sections.

**Proposition 1.** Optimal monetary policy under discretion implies that output growth and inflation follow the processes

\[
\Delta y^d_t = \frac{1}{\omega + \gamma} \left[ -\Delta q^d_t + (1 + \omega) \Delta \log A_t \right] - \theta \Delta \pi^d_t \tag{1.17}
\]

\(^{12}\)There is nothing in particular about solving the monetary authority problem using the loss function instead of the (log-linearized) welfare function. It is simply a convenient change of variable from output in the welfare function to output gap in the loss function to make evident the objective of the monetary authority.

\(^{13}\)The output gap target is defined as the difference between the steady-state (efficient) output under perfect competition and the steady-state natural rate of output. The interpretation for the habit gap target is similar.
and
\[ \pi_t^d = \delta_{\pi} - \delta_{\epsilon} \epsilon_t, \] respectively, where \( \delta_{\pi} = \left( \frac{\kappa}{\kappa \theta + 1} \right) \log \frac{\mu}{\omega + \gamma} \) is the inflation bias and \( \delta_{\epsilon} = \left( \frac{\kappa}{(\omega + \gamma)(\kappa \theta + 1 - \beta \phi)} \right). \)

\textbf{Proof.} See Appendix 1.8.3.

If the monetary policy is conducted under discretion, inflation is proportional to supply shocks and has the positive bias \( \delta_{\pi} \). It implies that the persistence of inflation is equal to the persistence of the shocks. Output growth depends on lagged output growth and is negatively affected by changes in inflation, with the size of the impact determined by the elasticity of substitution of differentiated goods.

Under commitment, output growth is affected directly by the level of inflation and inflation is determined by lagged inflation and changes in supply shocks, as the following proposition shows.

\textbf{Proposition 2.} Optimal monetary policy under commitment implies that output growth and inflation follow the processes

\[ \Delta y_t^c = \frac{1}{\omega + \gamma} \left[ - \Delta q_t^c + (1 + \omega) \Delta \log A_t \right] - \theta \pi_t^c \] (1.19)

and

\[ \pi_t^c = \varphi_1 \pi_{t-1}^c - \varphi_2 \epsilon_t, \] (1.20)

respectively, where \( \varphi_1 = \frac{1}{\omega + \gamma} \left[ \kappa \theta + 1 + \beta - \sqrt{(\kappa \theta + 1 + \beta)^2 - 4 \beta} \right] \) and \( \varphi_2 = \frac{\kappa}{(\omega + \gamma)(\kappa \theta + 1 + \beta (1 - \varphi_1 / \phi))}. \)

\textbf{Proof.} See Appendix 1.8.4.

We can notice that under commitment the inflation bias of policy under discretion disappears and the persistence of inflation is not longer equal to the persistence of the supply shocks.

\subsection*{1.5 The Term Structure of Interest Rates}

Equilibrium interest rates for all maturities and their associated term premia are obtained from the equilibrium processes for output growth and inflation described in Propositions 1 and 2. Specifically, using the pricing kernels \( M_{t,t+n} \) in equation (1.4) and the bond price equation (1.5), interest rates and term premia can be written as linear functions of a small set of macroeconomic variables. As a consequence, we obtain an affine term structure with analytical solutions for coefficients that depend on deep parameters of the economy. This section starts describing the affine
framework linked to the model of the previous section, to conclude presenting the equilibrium interest rates and term premia in Propositions 3 and 4.

Macroeconomic variables follow autoregressive processes with time-varying volatility. Therefore, we can group them in a vector of state variables, \(s_t\), and write their evolution in matrix form as

\[
s_{t+1} = \psi + \Phi s_t + \Psi(s_t)\Sigma^{1/2}\varepsilon_{t+1},
\]

where

\[
\Sigma = \text{diag}\{\sigma_a^2, \sigma_e^2, \sigma_\eta^2\} \quad \text{and} \quad \varepsilon = (\varepsilon_a, \varepsilon_e, \varepsilon_\eta)^\top.
\]

The specific content and dimensions of vectors \(s\) and \(\psi\) and matrices \(\Phi\), \(\Psi\) and \(\Psi(s_t)\) depend on the equilibrium characteristics of the particular monetary policies under study. The matrix \(\Phi\) has the autoregressive coefficients of the state variables and the matrices \(\Psi\) and \(\Psi(s_t)\) contain the constant and time-varying components of the volatility of the state variables, respectively. The vector \(\varepsilon\) captures the technology, supply and preference innovations.

Using the specification in (1.21), the one-period nominal pricing kernel in (1.4) is

\[
-\log M_{t,t+1} = \Gamma_0 + \Gamma_1^\top s_t + \lambda^\top \Psi(s_t)\Sigma^{1/2}\varepsilon_{t+1},
\]

with \(\Gamma_0 = -\log \beta + \ell_0^\top \psi, \Gamma_1 = \Phi^\top \ell_0 + \ell_1\) and \(\lambda = (\Psi_c^\top \ell_0 + \ell_2)\) and vectors \(\ell_0, \ell_1\) and \(\ell_2\) depending on the monetary policy.

It is of particular interest the last term of equation (1.22). It describes the stochastic character of the price kernel and, as a result, provides a clear idea of the compensations for risk in the economy and their dependence on monetary policy. Given that this term is a function of \(\Psi(s_t)\), risk premia depend on the state of the economy. The analysis of this term and, specifically, of its constant component \(\lambda\) is fundamental to understand the effects of credibility of the policy on the term structure. This analysis is undertaken in the next section.

Denoting the one-period interest rate on the nominal bond maturing at \(t + n\) by \(i_t(n) = -\frac{1}{n} \log b_t(n)\), and noticing that the bond price equation (1.5) can be written recursively as

\[
b_t(n) = \mathbb{E}_t \left[M_{t,t+1}b_t(n-1)\right],
\]

we can conjecture the solution for interest rates of the form

\[
i_t(n) = \frac{1}{n} \left[A_n + B_n^\top s_t\right].
\]
That is, interest rates are linear functions of macroeconomic variables. Replacing this form in equation (1.23) we obtain recursive formulas for coefficients $A_n$ and $B_n$ given by

\[
A_n = \Gamma_0 + A_{n-1} + B_{n-1}^\top \psi - \frac{1}{2} \lambda_n^\top \Sigma \lambda_n,
\]

\[
B_n^\top = \Gamma_1^\top + B_{n-1}^\top \Phi - \frac{1}{2} \lambda_n^\top \Sigma \lambda_n.
\]

with initial conditions $A_0 = 0$ and $B_0 = 0^\top$, where $\Psi(s_t)^2 = \Sigma + \Sigma \text{diag} \{\lambda_n\}$ and

\[
\lambda_n^\top \equiv \lambda^\top + B_{n-1}^\top \Psi_c.
\]

The term $\lambda_n$ plays an important role in the characterization of the risk premia contained in the term structure of interest rates. It captures maturity-specific adjustments to the constant component of the pricing kernel, $\lambda$, given by the factor $B_{n-1}^\top \Psi_c$. We can refer to this factor as the $n$-maturity cashflow risk adjustment. From it, we can notice that, if interest rates do not depend on the state of the economy ($B_n = 0$), there is not adjustment for the cashflow risk involved in long-term bonds implying that the expected returns offered by long-term bonds must be the same as the return of a one-period bond, $i_t$.

To formalize the notion presented above, consider the term premia contained in the term structure of interest rates. The term premium for an $n$-period bond captures the deviation of the $n$-period interest rate from a weighted average of the one-period interest rate and expectations of future interest rates. Explicitly, it is defined by

\[
\xi_t^{(n)} = i_t^{(n)} - \frac{1}{n} \left[ i_t + (n - 1) \mathbb{E} i_t^{(n-1)} \right].
\]  

(1.25)

Replacing the affine representation for interest rates (1.24) into (1.25) we can also write the term premium as

\[
\xi_t^{(n)} = \frac{1}{2n} \left[ \lambda^\top \Psi(s_t) \Sigma \Psi(s_t) \lambda^\top - \lambda_n^\top \Psi(s_t) \Sigma \Psi(s_t) \lambda_n^\top \right].
\]  

(1.26)

While the first term in this equation contains the precautionary savings effect on the one-period interest rate induced by uncertainty in the one-period pricing kernel, the second term contains the precautionary savings effect induced by the interaction of the uncertainty of the one-period pricing kernel and the one-period cashflow risk of the particular bond. Therefore, term premia also capture expected excess returns over the one-period interest rate. Using the definitions of $\lambda_n$ and $\Psi(s_t)$ we obtain the affine form

\[
\xi_t^{(n)} = \xi_{A,n} + \xi_{B,n} \Psi(s_t)^\top s_t
\]  

(1.27)
with coefficients given by

\[ \xi_{A,n} = -\frac{1}{2n} (\lambda + \lambda_n)^\top C \Sigma (\lambda_n - \lambda) \]

and

\[ \xi_{B,n}^\top = -\frac{1}{2n} (\lambda + \lambda_n)^\top K \Sigma \text{diag} \{\lambda_n - \lambda\} \hat{A}. \]

We can notice from this set of equations that a term premium is zero if \( \lambda_n - \lambda = B \top_n - 1 \Psi_c = 0 \). In addition, term premia are sensitive to the state of the economy \( s_t \) only if \( K \neq 0 \), that is, only if the state variables are affected by time-varying volatility. If the precautionary savings effects of cashflow risk depend on the state of the economy and macroeconomic variables have time-varying volatility, the size of the term premia will also depend on the state of the economy.

We can use now the affine framework presented above to characterize the equilibrium interest rates and term premia when monetary policy is conducted under discretion or commitment, as the following two propositions show.

**Proposition 3.** The equilibrium interest rate characteristics for the optimal monetary policy under discretion are described by equations (1.21), (1.22), (1.24) and (1.27) when

\[ s_t \equiv (\Delta \log A_t, \pi_t^d, \Delta y_t^d)^\top, \]

\[ \psi = \left( (1 - \phi_a)g_a, (1 - \phi_e)\delta_\pi, \frac{1 + \omega}{\omega + \gamma}\right) (1 - \phi_a)g_a - (1 - \phi_e)\theta_\pi \right)^\top, \]

\[ \Phi = \begin{bmatrix} \phi_a & 0 & 0 \\ 0 & \phi_\epsilon & 0 \\ \frac{1 + \omega}{\omega + \gamma} \phi_a & \theta (1 - \phi_\epsilon) & -\eta \frac{1 + \omega}{\omega + \gamma} \end{bmatrix}, \Psi_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\delta_\xi & 0 \\ \frac{1 + \omega}{\omega + \gamma} \theta_\xi & -\frac{1 + \omega}{\omega + \gamma} \end{bmatrix}, \]

\[ \Psi(s_t) = \text{diag} \left\{ (1 + K_a \Delta \log A_t)^{1/2}, (c_\pi + K_\pi \pi_t)^{1/2}, (1 + K_\eta \Delta y_t^d)^{1/2} \right\}, \]

\[ \ell_0 = (0, 1, \gamma)^\top, \quad \ell_1 = (0, 0, \eta)^\top, \quad \ell_2 = (0, 0, 1)^\top, \]

\[ C = \text{diag} \{1, c_\pi, 1\}, \quad K = \text{diag} \{K_a, K_\pi, K_\eta\} \text{ and } \hat{A} = I_{(3 \times 3)}, \]

where

\[ c_\pi = 1 + K_e \frac{\delta_\pi}{\delta_\xi} \text{ and } K_\pi = -\frac{K_\xi}{\delta_\xi}. \]

Proof. The processes for \( \Delta \log A_t \) and \( \Delta y_t \) are obtained from equations (1.10) and (1.17), respectively. The process for \( \pi_t \) is obtained replacing (1.14) into (1.18). The process for the discount factor is obtained replacing equation (1.21) into (1.22). \( \square \)
This proposition tells us that, when monetary policy is conducted under discretion, the dynamics in the term structure of interest rates can be explained by the dynamics of three macroeconomic factors: changes in labor productivity, inflation and output growth. Proposition 4 shows that under commitment we require an additional factor, supply shocks, to explain interest rates.

**Proposition 4.** The equilibrium interest rate characteristics for the optimal monetary policy under commitment are described by equations (1.21), (1.22) (1.24) and (1.27) when

\[ s_t \equiv (\Delta \log A_t, \epsilon_t, \pi_t^c, \Delta y_t^c)^\top, \]

\[ \psi = (1 - \phi_a)g_a \left( 1, 0, 0, \frac{1 + \omega}{\omega + \gamma} \right)^\top, \]

\[
\Phi = \begin{bmatrix}
\phi_a & 0 & 0 & 0 \\
0 & \phi \epsilon & 0 & 0 \\
0 & \varphi_2(1 - \phi \epsilon) & \varphi_1 & 0 \\
\frac{1 + \omega}{\omega + \gamma} \phi_a & -\varphi_2(1 - \phi \epsilon) & -\theta \varphi_1 & -\frac{\eta}{\omega + \gamma}
\end{bmatrix},
\Psi_c = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & -\varphi_2 & 0
\end{bmatrix},
\Psi(s_t) = \text{diag}\left\{ (1 + K_a \Delta \log A_t)^{1/2}, (1 + K_\epsilon \epsilon_t)^{1/2}, (1 + K_\eta \Delta y_t^c)^{1/2} \right\},
\ell_0 = (0, 0, 1, \gamma)^\top, \quad \ell_1 = (0, 0, 0, \eta)^\top, \quad \ell_2 = (0, 0, 1)^\top,
\]

\[\mathbb{C} = I_{(3 \times 3)} , \quad \mathbb{K} = \text{diag}\{K_a, K_\epsilon, K_\eta\}\]

and

\[ A = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.\]

**Proof.** The processes for the state variables are obtained from equations (1.10), (1.14), (1.20) and (1.19). The process for the discount factor is obtained replacing equation (1.21) into (1.22). \(\square\)

To conclude this section, we can notice that using equation (1.24) the instrument of monetary policy, \(i_t\), can be written as

\[ i_t = \Gamma_0 - \frac{1}{2} \lambda^\top \mathbb{C} \Sigma \lambda + \left( \Gamma_1 - \frac{1}{2} \lambda^\top \mathbb{K} \Sigma \text{diag}\{\lambda\} \right)^\top s_t, \tag{1.28}\]

resembling a monetary policy rule where the monetary authority reacts to the level of macroeconomic variables. The reaction coefficients are functions of preference and production parameters.
1.6 Analysis

This section is devoted to understand the mechanism linking interest rates and monetary policies under discretion and commitment in the model. The analysis shows that there is a specific channel driving the asset-pricing implications of credibility improvements. This channel is the reduction generated by policy credibility of the effects of supply shocks on the economy. To understand this channel, this section describes first the macroeconomic and general asset-pricing implications and then focuses on the effects on the term structure of interest rates. The analysis is complemented by a quantitative exercise where the model is calibrated to match some interest rate and macroeconomic characteristics of the U.S. economy. Although this exercise evidences the difficulty to simultaneously capture (quantitatively) interest rate and macroeconomic properties, it shows that enhancements in policy credibility are helpful to explain some of the observed changes in the dynamics of interest rates, term premia and macro variables.

1.6.1 Macroeconomic Implications

The macroeconomic implications can be understood comparing the output growth and inflation processes in (1.17) to (1.20). Inflation and output growth under commitment are less affected by supply shocks than under discretion. In fact, under commitment, inflation reacts to changes in the level of supply shocks and not to the level itself as under discretion. The result is that credibility improvements decrease the vulnerability of the economy to supply shocks and therefore affect the nature of macroeconomic risk. A shift in monetary policy from discretion to commitment reduces the level and the variability of inflation and the variability of output growth. It implies that the tradeoff between inflation volatility and output volatility is improved under commitment and it translates into higher social welfare.

Inflation

A policy conducted under discretion generates the positive inflation bias $\delta_\pi$ in equation (1.18). This bias disappears under commitment. That is, if the policy is conducted under commitment the average inflation reaches the zero target. Propositions 5 and 6 summarize the volatility and persistence properties of inflation for the two policies, respectively.

**Proposition 5.** The variance of inflation is $\delta_\pi^2 \text{var}(\epsilon_t)$ under discretion and \[ \frac{2}{1+\phi_1} \frac{1-\phi_1}{1-\phi_1} \frac{1-\phi_2}{\phi_2} \text{var}(\epsilon_t) \] under commitment. Therefore, a policy conducted under commitment reduces the volatility of inflation if $\phi_2 > \frac{1}{2+\varphi_1}$.

**Proof.** The variance of inflation is obtained solving for the unconditional moments of the inflation processes in 3 and 4. The result follows from subtracting the variance under commitment from the variance under discretion. \qed
The variability of inflation is proportional to the variability of the supply shocks, but commitment in monetary policy reduces the sensitivity of inflation to these shocks.

**Proposition 6.** The first-order autocorrelation of inflation is \( \phi_e \) under discretion and \( \phi_e - \frac{1}{2}(1 - \varphi_1)(1 + \phi_e) \) under commitment. Therefore, a policy conducted under commitment always reduces the persistence of inflation.

**Proof.** The covariance of inflation is obtained solving for the unconditional moments of the inflation processes in 3 and 4. Algebraic manipulations of the implied correlations provide the results.

Under discretion, inflation inherits the autocorrelation properties of the supply shocks. Under commitment the persistence of inflation is reduced by the fact that inflation reacts to changes in the level of supply shocks instead of reacting to the level as in the discretion case. This decline in sensitivity is determinant for the asset-pricing implications of the model.

**Output Growth**

The average output growth for the economy, \( \frac{1+\omega}{\omega+\gamma+\eta}g_a \), does not depend on monetary policy because, in the long-run, there is no tradeoff between inflation and output growth. The expressions obtained for the variance of output growth do not allow to understand clearly under which conditions improvements in policy credibility reduce the variability of output. However, different calibrations, as the one presented in this section, suggest that the variability of output growth under commitment is always lower than under discretion. The effects of credibility improvements on the autocorrelation of output growth and the correlation of output growth and inflation are difficult to explore. Different calibrations suggest that the autocorrelation of output growth tend to increase with improvements in credibility. Additionally, calibrations involving highly persistent supply shocks imply that credibility improvements imply less negative correlation between output growth and inflation. However, the effect is reversed as the persistence of the shocks is reduced.

Summarizing, enhancements in policy credibility increase macroeconomic stability. The change is driven by a reduced reaction of inflation to supply shocks. Not surprisingly, this change also drives the change in the dynamics of interest rates.

**1.6.2 Risk Premia**

Improvements in policy credibility affect risk premia through a very specific channel: the reduction of the systematic component of inflation risk. Credibility enhancements reduce the effects of supply shocks on inflation and real activity and thus this component declines. As a consequence, the precautionary savings motive induced by supply shocks weakens with credibility enhancements.
The stochastic term of the pricing kernel in (1.22) provides us the time-varying risk premia $\lambda^\top \Psi(s_t)$. This term has three components since there are three shocks affecting the economy: technology, supply and preference shocks. The inflation processes in equations (1.18) and (1.20) imply that the only source of uncertainty for inflation is supply shocks, therefore, we can refer to supply shocks as inflation risk and label the risk premium for supply shocks as the inflation risk premium.

**Average Risk Premia**

In order to understand the impact of credibility on the average risk premia we need to compare the constant component $\lambda$ for the two policies.\(^{14}\) This comparison is presented in Proposition 7.

**Proposition 7.** The risk premium factor $\lambda$ for the optimal monetary policy under discretion is

$$
\left( \frac{\gamma}{\omega + \gamma}, \delta_1(\gamma \theta - 1), \frac{\omega}{\omega + \gamma} \right)^\top
$$

and for the optimal monetary policy under commitment is

$$
\left( \frac{\gamma}{\omega + \gamma}, \phi_2(\gamma \theta - 1), \frac{\omega}{\omega + \gamma} \right)^\top
$$

Therefore, the average risk premia for technology and preference shocks are not affected by the credibility of the policy and the magnitude of the premium for inflation risk is always lower under commitment than under discretion.

**Proof.** Equations (1.29) and (1.30) are found from the definition of $\lambda$ and the associated parameters in Propositions 3 and 4. Since the first and third components of $\lambda$ are the same for both policies, it follows that technology and preference risk premia are unaffected by the policy. The fact that $\delta_1 > \phi_2$ makes the magnitude of the inflation risk premia under commitment to be always lower than under discretion.

---

\(^{14}\)To see this, consider $E[\text{var}(\log M_{t+1})] = \lambda^\top \Sigma E[\Psi(s_t)^2] \lambda$. The matrix $\Psi(s_t)^2$ is a diagonal matrix with exogenous components $\Delta \log A_t$ and $\epsilon_t$ in the first two rows and an endogenous component $\Delta y_t$ in the third row. Since the unconditional expectation of output growth is independent of monetary policy, changes in the policy only affect the long-run properties of the risk premia through $\lambda$. 

---

24
Risk Premium for Productivity Shocks ($RP_a$)

The first component of $\lambda$, the risk premium for productivity shocks, is unaffected by the level of commitment in monetary policy because the effect on output of changes in the level of labor-productivity are independent from the policy. The productivity risk premium has two components: the local risk aversion coefficient, $\gamma$, and the sensitivity of output growth to changes in labor productivity, $\frac{1+\omega}{\omega+\gamma}$. Noticing that $\frac{dRP_a}{d\gamma} = \omega$, we can see that a reduction in the elasticity of intertemporal substitution of consumption (increase in local risk aversion) increases this premium, and the change in the premium is greater for low levels of intertemporal elasticity of substitution of labor. That is, the compensation for productivity shocks depends on the ability of households to smooth consumption changing their labor supply. Also, an increase in the elasticity of intertemporal substitution of labor, $w^{-1}$, increases the risk premia only if $\gamma < 1$, meaning that if people are willing to accept non-smooth consumption and labor streams intertemporally, they require high compensations for facing productivity risk.

Risk Premium for Supply Shocks ($RP_\epsilon$)

The effects of credibility on the average risk premia are concentrated on the second component of $\lambda$, the risk premium for supply shocks (inflation risk premium). Improvements on policy credibility decrease the magnitude of the inflation risk premium because the sensitivity of output growth and inflation to supply shocks is lower under commitment than under discretion. That is, the economy is less vulnerable to supply shocks under commitment than under discretion and, therefore, the compensation for facing this risk is also lower (in absolute value). It is equivalent to say that the systematic component of supply shocks decreases as monetary policy shifts from discretion to commitment.

We can also observe a close connection between the size of the reduction in the magnitude of the inflation risk premium and the reduction in the persistence of inflation. Looking at the definition of $\delta_\epsilon$ and $\varphi_2$ in Propositions 1 and 2, we can see that the reduction in the magnitude of the inflation risk premium is proportional to $1 - \varphi_1$. Since, by Proposition 6, the reduction in the persistence is also proportional to $1 - \varphi_1$, the decrease in the magnitude of the inflation risk premium is proportional to the decrease in persistence caused by gains in credibility.

Risk Premium for Preference Shocks ($RP_\eta$)

The third component of $\lambda$, the risk premium for preference shocks, is unaffected by the level of commitment in monetary policy. Since a positive preference shock decreases the habit, it decreases the marginal utility of consumption and, consequently, implies a positive risk premium. Writing the premium as $\frac{\gamma^{-1}}{\omega^{-1}+\gamma^{-1}}$, it is possible to see that its size depends on the proportion of intertemporal elasticity explained by consumption relative to that one explained by labor. It means that more willingness to smooth consumption adjusting labor supply is accompanied by a lower premium for preference shocks.
Time Variation in Risk Premia

The time-varying component $\Psi(s_t)$ of the volatilities of productivity growth, supply shocks and the habit generates time variation in risk premia. While the time variation in the premium for productivity shocks is not sensitive to monetary policy, the time variation of the premia for supply and preference shocks depend on the type of policy that is conducted. The analysis of the sources of this time variation, shown below, is simplified by the fact that $\Psi(s_t)$ is diagonal in the model.

Time variation in the premium for productivity shocks is driven entirely by the level of productivity growth of the economy, $\Delta \log A_t$. Since it is exogenous, it is not affected by monetary policy. Given that US data do not show heteroskedasticity in labor productivity, the parameter $K_a$ was set to zero in the calibration, implying a constant premium for productivity risk.

Variability in the premium for supply shocks is linked to the level of the shocks, $\epsilon_t$, as the second element in the diagonal of $\Psi(s_t)$ shows. The volatility of the premium is reduced by improvements in credibility of monetary policy through the reduction of the vulnerability of the economy to supply shocks as in the case of the average risk premium. Since supply shocks are the only source of inflation risk and variability in inflation tend to increase with the level of inflation, the parameter $K_\epsilon$ is negative in the calibration. It means that higher inflation in the economy increases the inflation risk premia but the impact is attenuated by improvements in credibility.

The time variation of the preference premium is given by $\sqrt{1 + K_n \Delta y_t}$. Since output growth is affected by credibility in monetary policy, the volatility of the premium is also affected. For a countercyclical risk premium ($K_n < 0$), when a negative supply shock affects the economy, output growth under discretion declines more than under commitment and, as a result, the preference premium under discretion increases more than under commitment. In the calibration $K_n$ is negative.

Asset-Pricing Implications of Monetary Policy

There is an important asset-pricing implication for real and nominal financial assets linked to the inflation risk premium in the model. It arises from the dependence of output growth on inflation and thus, is useful to observe the effects of nominal rigidities on asset prices.

Financial assets with real cashflows positively affected by inflation involve a negative premium for inflation risk. To see this, consider the one-period real pricing kernel for the equilibrium of the model,

$$\log M_{t+1}^{real} = \log \beta - \gamma \Delta y_{t+1} - \Delta q_{t+1},$$

(1.31)
and the output growth equations (1.17) and (1.19). We can see that the real pricing kernel depends positively on future inflation, $\pi_{t+1}$. It implies that assets with real payoffs affected positively by inflation have a positive covariance with the pricing kernel and therefore expected returns that are lower than the one-period (risk-free) interest rate. People are willing to accept low expected returns for these assets because they hedge consumption risk: these assets pay high when output growth is low.

Financial assets with nominal cashflows that are positively sensitive to inflation involve a premium for inflation risk whose sign depends on the relation between the intertemporal elasticity of substitution of consumption and the elasticity of substitution of differentiated goods. Noting that positive supply shocks reduce inflation, the sign of the embedded premium for inflation risk is given by the sign of $1 - \gamma \theta$. That is, if the elasticity of intertemporal substitution of consumption, $\gamma^{-1}$, is lower than the elasticity of substitution of differentiated goods, $\theta$, the embedded risk premium is negative. The reason is that the benefits of a higher cashflow with higher inflation are more than offset by the reduction in output growth caused by the higher inflation. If people’s needs of smooth intertemporal consumption are greater than their needs of smooth intratemporal consumption across goods, the expected returns of holding these assets are lower than the one-period interest rate. On the other hand, if smoothing consumption across goods is more important than intertemporal smoothing, investors require an incentive (positive expected excess returns) to hold assets with inflation protected cashflows.

### 1.6.3 Interest Rates and Term Premia

The affine term structure framework presented in Section 1.5 shows that interest rates are functions of macro variables. Therefore, the effects of credibility on macro variables propagate on the dynamics of interest rates. The average level of interest rates decreases and the volatility and average shape of the yield curve changes. As a result, changes in policy credibility generate changes in term premia. More specifically, term premia are affected because the correlation between the nominal pricing kernel and bond returns induced by supply shocks is affected by credibility. This shift in the dynamics of the term premia diminishes the deviations from the expectations hypothesis.

The **One-Period Interest Rate**

The effects of credibility on the short-term rate can be better understood considering first the interest rate observed in a hypothetical economy with flexible prices, no supply shocks and zero
inflation. This rate is known as the natural rate of interest and is given by\textsuperscript{15}

\[ r^* = -\log \beta + \gamma \frac{1 + \omega}{\omega + \gamma} (1 - \phi_a) g_a - \frac{1}{2} \gamma^2 \left( \frac{1 + \omega}{\omega + \gamma} \right)^2 \sigma_a^2 - \frac{1}{2} \left( \frac{\omega}{\omega + \gamma} \right)^2 \sigma_y^2. \]

Besides the positive dependence of the interest rate on the level of labor productivity, the stochastic habit makes the interest rate to depend on current output growth. Since \( \eta < 0 \), the habit reduces the natural rate of output to dissuade households from excessive savings.

Given that prices are not perfectly flexible and the economy is affected by supply shocks, the interest rate differs from the natural rate of interest and depends on monetary policy. Specifically, the equilibrium level of the interest rate depends on current changes in the output gap and inflation. If the policy is conducted under discretion the one-period interest rate is

\[ i^d_t = r^n_t + (1 - \gamma \theta)(1 - \phi_e) \delta_x + \frac{\omega}{\omega + \gamma} \left[ \eta - \frac{1}{2} K\eta \sigma_y^2 \right] \Delta x^d_i + [(1 - \gamma \theta) \phi_e + \gamma \theta] \pi^c_i - \frac{1}{2} (1 - \gamma \theta)^2 \varphi^2 (1 + K\epsilon \epsilon) \sigma_e^2. \]  

(1.33)

Under commitment, the interest rate is

\[ i^c_i = r^n_t + \frac{\omega}{\omega + \gamma} \left[ \eta - \frac{1}{2} K\eta \sigma_y^2 \right] \Delta x^c_i + (1 - \gamma \theta) \varphi_1 \pi^c_i - \frac{1}{2} (1 - \gamma \theta)^2 \varphi^2 (1 + K\epsilon \epsilon) \sigma_e^2. \]  

(1.34)

We can notice from these two equations that when output growth is lower than the growth in the natural rate of output, the external habit raises the interest rate to decrease the excess consumption demand today. Under discretion, positive inflation increases the interest rate and the response of the interest rate is more than one for one only if \( \theta > \gamma^{-1} \). In the case of commitment, the reaction of the interest rate goes in the same direction as inflation only if \( \gamma^{-1} > \theta \) and, in this case, it is always less than one for one. These two rates imply Proposition 8.

**Proposition 8.** The long-run difference in the average one-period interest rates implied by monetary policies under discretion and under commitment is

\[ \mathbb{E} \left[ i^d_t - i^c_t \right] = (1 + \gamma \theta) \delta_x - \frac{1}{2} (1 - \gamma \theta)^2 (\delta^2 - \varphi^2) \sigma_e^2. \]

The second term in the equation of the proposition is a precautionary savings effect resulting from supply shocks. Since this term is, in general, small, the level of the one-period interest rate under commitment is lower than under discretion by more than the inflation bias \( \delta_x \). It is

\textsuperscript{15}This rate is found solving the Euler equation for one-period bonds \( e^{-r^*_i} = \mathbb{E}_t [\exp(\log \beta - \gamma \Delta y^n_i - \Delta q^n_i)] \).
interesting to note that the effect of the inflation bias on the long-run interest if the policy lacks credibility depends on intertemporal and intratemporal elasticities of substitution. If needs to substitute consumption across goods exceed needs to substitute consumption intertemporally, the effect of the inflation bias is amplified.

The volatility of the interest rate implied by the model is difficult to analyze analytically. The calibration exercise below shows that the volatility of interest rates of long maturities declines with gains in credibility, while the volatility for short-term rate increases.

**Term Premia, Spreads and Deviations from the Expectations Hypothesis**

Credibility improvements in monetary policy affect the sensitivity of bond returns to inflation risk. This change, caused by the fact that credibility makes output growth and inflation less vulnerable to supply shocks, affect the joint dynamics of term premia and spreads. In particular, since the responses of spreads and term premia to the state of the economy vary with credibility, the size of the deviations from the expectations hypothesis is affected.

Consider first the average premia in the term structure of interest rates. Equation (1.26) tells us that the average effects of different monetary policies on the term premia are captured by the effects of the policies on $\lambda$, $\lambda_n$, and their interaction. Since the effects of credibility on the risk premia component $\lambda$ are analyzed above, completing the analysis of the average term premia amounts to describe the effects of discretion and commitment on $\lambda_n - \lambda = B_{n-1}^T \Psi_c$. That is, we need to analyze the sensitivity to uncertainty of the one-period return of an $n$-maturity bond. In general, the analytical solutions for these terms are complicated. However, the analysis of the two-period component and the numerical properties of the components for bonds with longer maturities provide the necessary intuition to understand the transmission channel of credibility on the premia.

**Proposition 9.** The difference in the $B_1^T \Psi_c$ component between discretion and commitment is

$$0, (\delta_e - \varphi_2) \left[ (1 - \phi_e)(1 - \gamma \theta) + \theta \frac{\omega \eta}{\omega + \gamma} - \delta_e + \varphi_1 \varphi_2 (1 - \gamma \theta) \right] + \frac{1}{2} K_\epsilon (1 - \gamma \theta)^2 (\delta_3^3 - \varphi_3^3), 0 \right).$$

Therefore, the average 2-period term premium for productivity and preference shocks is not affected by the credibility of the policy.

**Proof.** Noting that $B_1^T = \Gamma_1^T - \frac{1}{2} \lambda^T \Sigma \text{diag} \{\lambda\} \Lambda$, the equation follows from subtracting the $B_1^T \Psi_c$ implied by Proposition 3 from that one implied by Proposition 4.

The second term in the equation of the proposition above is positive and small for reasonable parameters. Neglecting this term, $\theta > \gamma^{-1}$ implies that credibility reduces the sensitivity of the
2-period bond return to supply shocks. The total effect of this reduction on the term premium depends on the interaction of this component with the reduction of the inflation risk premium as can be seen from equation (1.26).

The analysis can be complemented with numerical computations of \( B_{n-1}^\top \Psi_c \) for both policies and different maturities, \( n > 2 \), to see that the first and third elements of this sensitivity vector are not affected by the type of policy, while the second component changes. It makes clear that term premia for all maturities are affected by credibility through changes in the sensitivity of the discount factor and bond returns to supply shocks.

It is convenient now to describe the spread between the yield of a bond maturing at time \( n \) and the one-period interest rate using the affine representation

\[
i_t^{(n)} - i_t = c_n + D_n^\top s_t = \left( \frac{1}{n} A_n - A_1 \right) + \left( \frac{1}{n} B_n - B_1 \right)^\top s_t. \tag{1.35}
\]

Solving the recursive equations for the interest rates implies complicated analytical expressions for spreads that are difficult to interpret. However, different calibrations show that credibility may cause, in average, steeper, flatter or inverted yield curves, and thus, the implications of credibility on the level of spreads are not clear. For that reason, the analysis of spreads is limited to understanding how changes in their properties affect the size of the deviations from the expectations hypothesis.

Rejections of the expectations hypothesis of interest rates can be explained by the existence of time variation in term premia that is correlated to time variation in spreads. To see this, we can consider the Campbell and Shiller (1991) regressions

\[
i_{t+1}^{(n-1)} - i_t^{(n)} = \alpha^{(n)} + \frac{\beta^{(n)}}{n-1} \left( i_t^{(n)} - i_t \right) + \varepsilon_{CS,t}^{(n)}. \tag{1.36}
\]

These regressions are a standard exercise to test the expectations hypothesis of interest rates. Under the expectations hypothesis the Campbell-Shiller coefficients, \( \beta^{(n)} \), are equal to one, meaning that spreads only reflect expected changes in bond yields. We can use the definition of term premium, as shown in Appendix 1.8.5, to show that the coefficients implied by the model can be written as

\[
\beta^{(n)} = 1 - \frac{\text{var}(s_t) \xi_{B,n}}{\text{var}(s_t) D_n^\top D_n}.
\]

We can infer from this representation that deviations from the expectations hypothesis are explained by the correlation between a time-varying spread and a time-varying term premia. Moreover, in order to obtain \( \beta^{(n)} < 1 \), this correlation has to be positive. That is, according to the
model, the expectations hypothesis is rejected because spreads contain information about a term premia that depends on the state of the economy. As a consequence, arguing that the size of deviations hypothesis decline with gains in monetary policy credibility is equivalent to say that credibility reduces the positive correlation between term premia and spreads and this is motivated by changes in the joint reaction of term premia and spreads to supply shocks. In addition, if credibility makes the $\beta(n)$ coefficients closer to 1, spreads will reflect expected changes in short term interest rates and, then, expected changes in monetary policy. Consequently, credibility may improve the ability to extract expectations on future monetary policy from the term structure.

**Predictive Power of the Term Structure**

Following the same arguments as above, the ability of the term structure to predict future economic activity depends on whether monetary policy is conducted under discretion or commitment. Intuitively, under commitment macroeconomic variables are less affected by supply shocks, and this change is propagated into short and long-term interest rates. As a result, the information content of interest rates about current economic conditions varies and so does the ability to infer future conditions from interest rates.

The affine framework for macroeconomic variables and interest rates allows us to obtain coefficients that relate future output growth and inflation to the shape of the yield curve. Comparing the coefficients implied by the model under commitment with those under discretion is helpful to see how credibility affects the forecast ability of spreads. From the state variable process (1.21) and denoting by $e_i$ the column vector (of appropriate size) with a 1 in the $i$-th position and zeros in the other positions, the expected value for a macroeconomic variable $mv = \{\Delta y_t, \pi_t\}$ at time $t + k$ can be written as

$$E_t [mv_{t+k}] = e_i^T \left[ (I - \Phi)^{-1} \left( I - \Phi^k \right) \psi + \Phi^k s_t \right].$$

From it, the coefficient that explains the predictability of $mv$ at time $t + k$ using the $n$-maturity spread is

$$\beta_{mv,k}^{(n)} = \frac{e_i^T \Phi^k \text{var}(s_t) D_n}{D_n \text{var}(s_t) D_n}.$$ 

For $i$ equal 2 and 3, for discretion and commitment, respectively, the predictability coefficients of inflation are found under both policies. When $i$ takes the values 3 and 4, respectively, we find the predictability coefficients for output growth. Different calibrations show that the magnitudes of these coefficients for both, output growth and inflation, are lower under commitment than under discretion, meaning that gains in credibility reduce the power of spreads to predict future economic activity.
1.6.4 Calibration and Policy Experiment

The purpose of this exercise is to gain further insights about the effects of credibility on long-term interest rates. The model is calibrated using the U.S. quarterly data described in section 1.3 for bond yields, inflation and consumption growth from 1961:Q4 to 1969:Q4. The main assumption for this calibration is that, during this period, the Federal Reserve conducted a policy with low credibility and, therefore, the equilibrium properties of the model under discretion characterize the economy. The calibration is complemented with a policy experiment that consists on evaluating the characteristics of an economy characterized by a policy under commitment using the parameter values of the calibration exercise. This experiment allows us to compare the main long-run differences between the two types of policies and capture the effects of credibility. The details of the calibration procedure are presented in Appendix 1.8.6.

Table 1.2: Parameter Values

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<th>$\beta$</th>
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<th>$\theta$</th>
<th>$g_a$</th>
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<table>
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<th>$\sigma_a$</th>
<th>$K_\epsilon$</th>
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<tbody>
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<td>0.009</td>
<td>-77</td>
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<td>0.07</td>
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</table>

The calibration focused on capturing the slope of the yield curve, the Campbell-Shiller coefficients and the persistence of inflation. The persistence of inflation is important to identify the credibility of the monetary regime. Quantitatively, the results of the calibration evidence the difficulty to simultaneously capture interest rates and macroeconomic properties of the US economy. This is not surprising given the well-known limitations of general equilibrium models of production economies. However, as shown in Figure 1.3, the model is able to capture the slope of the yield curve and deviations from the expectations hypothesis for the 1961-1979 period. In particular, it is achieved with a relatively low value for $\gamma$ and negative values for $K_\eta$ and $K_\epsilon$. It is consistent with a countercyclical variation in the volatility of preference shocks and a volatility of inflation that increases as the level of inflation increases. The volatility of interest rates is higher than the observed in the data but relatively flat across maturities. The policy experiment implies a flat average yield curve for the policy under commitment. This curve is basically reflecting the real term structure of interest rates given that the long-run inflation under commitment is zero. The volatility of interest rates increases significantly for short-term maturities and decays quickly. Panel C shows the deviations from the expectations hypothesis. Under commitment, the Campbell-Shiller coefficients are very close to one, implying that, for this calibration, deviations from the expectations hypothesis are mainly explained by lack of credibility of monetary policy.

Figure 1.4 shows the average level and volatility of term premia implied by the model. The
positive term premia under discretion becomes negative under commitment. It implies that lack of credibility makes economic agents demand a positive compensation for holding nominal bonds. This is consistent with an upward sloping yield curve. Under commitment, bonds represent a hedge for investors and, therefore, are willing to hold them giving up excess returns. Panel B shows that the average volatility of the one-period term premia under discretion reaches a maximum around the two-year maturity and then decays slowly. The pattern is significantly different under commitment. Volatility increases fast and dies out quickly. In summary, the dynamics of the risk premia in the term structure of interest rates can be dramatically affected by improvements in credibility. In particular, the reduction of the systematic component of inflation risk observed with gains in credibility makes the term premia negative.

Figure 1.3: Model-implied interest rate properties. The asterisk denotes the corresponding data statistic for the 1961-1979 period and the circle denotes the corresponding data statistic for the 1988-2005 period.

Figure 1.4: Model-implied term premia.
In order to gain some insights about the changes in the deviations of the expectations hypothesis it is useful to analyze the impulse responses to supply shocks of spreads and term premia. Figure 1.5 shows the response of the five-year spread and term premium to a positive inflation (negative supply shock). The magnitude and sign of the responses make evident that the joint dynamics of spreads and term premia change under the two regimes. While under commitment the response of the spread to supply shocks is larger than under discretion, the opposite is true for the term premium. This differences can be explained by the different reaction of inflation under the two regimes. Under commitment, the initial increase in inflation is followed by a period of disinflation, such that in the long run, inflation is zero. That makes the term premium to remain stable. People do not need large changes in the compensation for holding long-term bonds because inflation is not persistent. Simultaneously, under commitment the equilibrium short term interest rate drops down dramatically and the spread widens. In summary, deviations of the expectations hypothesis diminish because the positive covariance between spreads and term premia is reduced as policy credibility increases and the volatility of spreads increases.

Table 3.3 shows the macroeconomic implications of the calibration. In order to match interest-rate dynamics, the model requires higher and more volatile consumption growth and lower and less volatile inflation in comparison to the empirical counterparts. However, important differences between the low and high credibility regimes are captured. The volatility of consumption growth and inflation diminish as credibility increases, the reduction in the persistence of inflation decreases and inflation and output growth are less correlated.

Appendix 1.8.6 shows other characteristics of the calibration. It contains figures of changes in the ability of interest rates to predict future consumption growth and inflation. It also contains
some impulse responses to technology, supply and preference shocks, that show that credibility only affect the impulse responses of supply shocks.

### 1.7 Conclusion

This chapter provides a structural affine term-structure model that links the dynamics of interest rates to optimal monetary policy and macroeconomic risk. The model is used to understand the effects on interest rates of improvements in the credibility of monetary policy. The main finding is that credibility improvements have significant implications on the properties of bond risk premia through a very specific channel. This channel is the unambiguous reduction of the systematic component of inflation risk. Credibility improvements makes the economy less vulnerable to inflation risk affecting the excess return required to hold long-term bonds. To gain further insights on these effects and explore the relation between deviations from the expectations hypothesis and policy credibility, the model was calibrated to U.S. data and a policy experiment was conducted. When a credible policy is implemented, the term premia become negative and less variable and that reduces the deviations from the expectations hypothesis. This is consistent with recent developments in U.S. interest rates.

This structural framework is also useful to understand general asset-pricing implications of inflation. Inflation affects consumption growth, thus the marginal utility of wealth and the price of risk of financial assets with real and nominal cashflows. The valuation effect of inflation on assets with real payoffs is the result of nominal rigidities in the economy. In particular, under optimal monetary policy the model delivers an important asset pricing implication that depends on the relation between intertemporal and intratemporal elasticity of substitution. The inflation risk premium of nominal assets whose payoffs increase with inflation is negative if smoothing consumption intertemporally is more important than smoothing consumption across goods. The
reason is that low inflation increases consumption growth and, as a result, these assets provide a hedge for marginal utility and offer negative expected excess returns. This analysis might also be useful to understand the effects of inflation on the equity premium and the intertemporal nature of the negative implications of inflation on welfare.
1.8 Appendix

1.8.1 Aggregate Supply Equation

Writing \( M_{t,T} = \beta^{T-t} \Lambda_T, \mu_t = \mu e^{-\epsilon t} \) and noting that

\[
\frac{\partial \Pi_T[i]}{\partial P_t(i)} = Y_T[i(i)] \left( 1 - \theta \right) e^{e_T} \left( P_T - \mu_T S_T[i(i)] \right),
\]

we can write the first order condition to the problem as

\[
\begin{align*}
\mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta)^T Y_T[i(i)] e^{e_T} P_T^* \right] &= \mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta)^T \Lambda_T Y_T[i(i)] e^{e_T} \mu_T S_T[i(i)] \right]. \tag{1.37}
\end{align*}
\]

Since all producers who change prices optimally at \( t \) face the same problem, \( Y_T[i(i)] = Y_T[i], P_T^*(i) = P_T^* \) and \( S_T[i(i)] = S_T[i] \). Applying the Taylor expansion \( a_t b_t = \bar{a} \bar{b} + \bar{b}(a_t - \bar{a}) + \bar{a}(b_t - \bar{b}) \) to both sides of the equation around a steady-state with \( \bar{P} = \mu \bar{S} \) and \( \epsilon = 0 \), we have for the left hand side of the equation

\[
\begin{align*}
\mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta)^T \Lambda_T Y_T[i(i)] e^{e_T} P_T^* \right] &= \Lambda \bar{Y} \bar{P} \sum_{T=t}^{\infty} (\alpha \beta)^T - \bar{P} \mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta)^T (\Lambda_T Y_T[i(i)] e^{e_T} - \Lambda \bar{Y}) \right] \\
&+ \Lambda \bar{Y} (P_T^* - \bar{P}) \sum_{T=t}^{\infty} (\alpha \beta)^T
\end{align*}
\]

and for the right hand side

\[
\begin{align*}
\mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta)^T \Lambda_T Y_T[i(i)] e^{e_T} \mu_T S_T[i(i)] \right] &= \mu \Lambda \bar{Y} \bar{S} \sum_{T=t}^{\infty} (\alpha \beta)^T - \mu \bar{S} \mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta)^T (\Lambda_T Y_T[i(i)] e^{e_T} - \Lambda \bar{Y}) \right] \\
&+ \mu \bar{Y} \mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta)^T (S_T[i] - \bar{S}) \right].
\end{align*}
\]

Noting that the first and second terms in both sides of the equation are the same, equation (1.37) becomes

\[
\begin{align*}
\frac{1}{(1 - \alpha \beta)} P_T^* &= \mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta)^T \mu e^{-\epsilon T S_T[i(i)]} \right].
\end{align*}
\]
Since \( S_{T|t} = s_{T|t} P_T \), replacing equation (1.11) in the equation above and re-arranging terms, we obtain

\[
\frac{1}{(1 - \alpha \beta)} (P^*_t)^{1+\theta \omega} = \mathbb{E}_t \left[ \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \mu e^{-\epsilon T} P_T^{1+\theta \omega} Y_T^{\omega+\gamma} Q_T A_T^{-(1+\omega)} \right].
\]

Dividing by \( \bar{P}^{1+\theta \omega} \), the equation can be written in terms of the output gap and the habit gap as

\[
\frac{1}{(1 - \alpha \beta)} \left( \frac{P^*_t}{P} \right)^{1+\theta \omega} = e^{-\epsilon T + (\omega + \gamma) x_t + l_t} \left( \frac{P_t}{P} \right)^{1+\theta \omega} + \frac{\alpha \beta}{1 - \alpha \beta} \mathbb{E}_t \left[ \left( \frac{P^*_t}{P} \right)^{1+\theta \omega} \right].
\]

Letting \( p^*_t = \log \frac{P^*_t}{P} \) and using the approximation \( e^x \approx 1 + x \), we obtain

\[
\frac{1}{(1 - \alpha \beta)} \left( 1 + (1 + \theta \omega) p^*_t \right) = 1 - \epsilon t + (\omega + \gamma) x_t + l_t + (1 + \theta \omega) p_t + \frac{\alpha \beta}{1 - \alpha \beta} \mathbb{E}_t [1 + (1 + \theta \omega) p^*_{t+1}].
\]

A first order Taylor approximation of \( P_t = \left[ (1 - \alpha) (P^*_t)^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{1/\theta} \) results in

\[
p_t = (1 - \alpha) p^*_t + \alpha p_{t-1}.
\]

Replacing this equation in equation (1.38) and noting that \( \pi_t = p_t - p_{t-1} \), the aggregate supply condition in equation (1.15) follows.

### 1.8.2 Loss Function

Define as \( \bar{Y}_t \) the steady-state output level under monopolistic competition (similar for \( \bar{Q}_t \) and \( \bar{A}_t \)), that is, its associated real marginal cost \( \bar{s}_t = \mu^{-1} \). Then, its process satisfies

\[
\frac{Y_t^{\omega+\gamma}}{Q_t^{-1} A_t^{1+\omega}} = \frac{1}{\mu}
\]

Define as \( Y_t^* \) the steady-state output level when producers are competitive (similar for \( Q_t^* \)), that is, its associated real marginal cost \( s_t^* = 1 \). Then, its process satisfies

\[
\frac{(Y_t^*)^{\omega+\gamma}}{(Q_t^*)^{-1} A_t^{1+\omega}} = 1
\]

From the equations above and the natural rate of output process, the utility from consumption...
in equation (1.16) can be written as
\[
\tilde{U}(Y_t; Q_t) = \frac{1}{1 - \gamma} \left( \frac{Y_t}{Q_t} \right)^{1-\gamma} e^{(1+\omega)\left(\hat{y}_t^n - \hat{a}_t\right)} \left(1 + (1 - \gamma)z_t - \delta_t + \frac{1}{2}(1 - \gamma)^2 z_t^2 + \frac{1}{2} \delta_t^2 - (1 - \gamma)z_t\delta_t\right) + O(||z, \delta||^3).
\]

Replacing equation (1.8), the disutility from labor in equation (1.16) is
\[
\int_0^1 \bar{v}(y_t(i); A_t) di = \frac{1}{1 + \omega} \left( \frac{Y_t}{A_t} \right)^{(1+\omega)} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\theta(1+\omega)} di.
\]

Since \(\left( \frac{Y_t}{A_t} \right)^{(1+\omega)} = e^{(1+\omega)(\hat{y}_t^n - \hat{a}_t)} e^{(1+\omega)(x_t - x^*)} \left( \frac{Y_t}{A_t} \right)^{(1+\omega)} \) and \(\left( \frac{Y_t}{A_t} \right)^{(1+\omega)} = \left( \frac{Y_t}{Q_t} \right)^{(1-\gamma)}\) then
\[
\int_0^1 \bar{v}(y_t(i); A_t) di = \frac{1}{1 + \omega} \left( \frac{Y_t}{Q_t} \right)^{1-\gamma} e^{(1+\omega)(\hat{y}_t^n - \hat{a}_t)} \int_0^1 e^{(1+\omega)(x_t - x^*) - \theta \tilde{p}_t(i)} di,
\]

where \(\tilde{p}_t(i) = \log \frac{P_t(i)}{P_t^\gamma}\). The term under the integral can be log-linearized as
\[
\int_0^1 e^{(1+\omega)[x_t - \theta \tilde{p}_t(i)]} di = 1 + (1 + \omega)z_t + \frac{1}{2}(1 + \omega)^2 z_t^2 - \theta(1 + \omega) \mathbb{E}_i \tilde{p}_t(i)
\]
\[+ \frac{1}{2} \theta^2 (1 + \omega)^2 [\text{var}_i(\tilde{p}_t(i)) + (\mathbb{E}_i \tilde{p}_t(i))^2] + O(\tilde{p}^3)
\]
\[= 1 + (1 + \omega)z_t + \frac{1}{2}(1 + \omega)^2 z_t^2
\]
\[+ \frac{1}{2} \theta(1 + \omega) [1 + \theta \omega] \text{var}_i(p_t(i)) + O(\tilde{p}^3)
\] (1.41)

where the second equality comes from the log-linearization \(p_t = \int_0^1 p_t(i) di + \frac{1}{2}(1 - \theta) \text{var}_i(p_t(i)) + O(p^3)\) and \(\tilde{p}_t(i) = p_t(i) - p_t\).
Under Calvo staggered price setting, Woodford (2003), pages 399 and 400, shows that

\[ \text{var}_i(p_t(i)) = \alpha \text{var}_i(p_{t-1}(i)) + \frac{\alpha}{1 - \alpha} \pi_t^2 + \mathcal{O}(p^3) \]

\[ = \frac{\alpha}{1 - \alpha} \sum_{s=0}^{t} \alpha^{t-s} \pi_s^2 + \alpha^{t+1} \text{var}_i(p_{t-1}(i)). \tag{1.42} \]

The term \((\omega + \gamma) \frac{(Y^*_t)^{1-s}}{Q_t} e^{(1+\omega)(\tilde{y}_t^n - \tilde{a}_t)}\) can be log-linearized to extract the time-varying component \(\tilde{y}_t^n - \tilde{a}_t\) which does not depend on monetary policy. Defining as \(\Omega \equiv (\omega + \gamma) \frac{(Y^*_t)^{1-s}}{Q_t}\) and given the log-linearizations for the first and second term in equations (1.40), (1.41) and (1.42), the loss function is obtained.

### 1.8.3 Optimal Policy under Discretion

From the first order conditions of the monetary authority problem, the optimal inflation can be written in terms of actual deviations from the target output and habit gaps as

\[ \pi_t = -\frac{1}{\theta}(x_t - x^*) - \frac{1}{\theta(\omega + \gamma)}(l_t - l^*). \tag{1.43} \]

Replacing equation (1.43) in equation (1.15), we obtain the linear rational expectations equation

\[ -\frac{1}{\theta} x_t - \frac{1}{\theta(\omega + \gamma)} l_t = -\frac{1 + \beta}{\theta} \left( x^* + \frac{l^*}{\omega + \gamma} \right) + \kappa x_t + \frac{\kappa}{\omega + \gamma} l_t - \frac{\kappa}{\omega + \gamma} \epsilon_t - \frac{\beta}{\theta} E_t x_{t+1} \]

\[ - \frac{\beta}{\theta(\omega + \gamma)} [l_t + \eta(x_t - x_{t-1})] \tag{1.44} \]

where the last term is \(\frac{\beta}{\theta(\omega + \gamma)} E_t l_{t+1}\).

Suppose that the optimal output gap can be written as \(x_t = \phi_1 + \phi_2 x_{t-1} + \phi_3 l_t + \phi_4 \epsilon_t\). Replacing this form in the equation above and matching coefficients, it can be seen that the coefficients must satisfy the following system of equations:

\[ \phi_1 = -\frac{1 - \beta}{\theta} \left( x^* + \frac{l^*}{\omega + \gamma} \right) \left( \phi_{aux} + \frac{\beta}{\theta} \right)^{-1}, \]

\[ \phi_2 = \frac{\beta}{\theta} \eta \left( \frac{1}{\omega + \gamma} + \phi_3 \right) \phi_{aux}^{-1}, \]

\[ \phi_3 = \left[ -\frac{\beta}{\theta} \left( \frac{1}{\omega + \gamma} + \phi_3 \right) + \frac{1}{\omega + \gamma} \left( \frac{1}{\theta + \kappa} \right) \right] \phi_{aux}^{-1}, \]

\[ \phi_4 = -\frac{\kappa}{\omega + \gamma} \left( \phi_{aux} + \frac{\beta}{\theta} \phi_4 \right)^{-1}. \]

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where $\phi_{aux} = \frac{\beta}{\eta} \left( \frac{1}{\omega + \gamma} + \phi_3 \right) + \frac{\beta}{\eta} \phi_2 - \frac{1}{\eta} - \kappa$.

The system of equations has two solutions. Their $\phi_2$ and $\phi_3$ coefficients are, respectively:

$$\phi_2 = 0, \quad \phi_3 = -\frac{1}{\omega + \gamma}$$

and

$$\phi_2 = \frac{\beta \eta + (\omega + \gamma)(1 + \kappa \theta)}{\beta (\omega + \gamma + \eta)}, \quad \phi_3 = \frac{1 - \beta + \kappa \theta}{\beta (\omega + \gamma + \eta)}.$$

Notice that when $\eta = 0$ (no external habit persistence), the first solution implies $\phi_2 = 0$ while the second solution does not. Therefore, the second solution for the output gap depends on the lagged output gap even when the lagged term does not appear in the rational expectations equation, and thus violates the MSV criterion. Following McCallum (1999b), the second solution is ruled out.

Therefore, the output gap is given by

$$x_t = \left( \frac{1 - \beta}{1 - \beta + \kappa \theta} \right) \left( x^* + l^* \right) - \frac{1}{\omega + \gamma} l_t + \frac{\kappa \theta}{(\omega + \gamma)(1 - \beta \psi + \kappa \theta)} \epsilon_t,$$  \hspace{1cm} (1.45)

and the growth in the output gap is then

$$\Delta x_{t+1} = -\frac{1}{\omega + \gamma} \eta_{t+1} \Delta x_t + \delta_t \Delta \epsilon_{t+1},$$

where, as for the natural rate of output growth, the persistence of the process depends on the stochastic habit. Using equations (1.12) and (1.45) and differencing, the optimal output growth in (1.17) is obtained. Replacing (1.45) in (1.43), the inflation process is obtained.

### 1.8.4 Optimal Policy under Commitment

The welfare problem for the monetary authority can be written as

$$\max_{\{\pi_t, z_t\}} -\frac{1}{2} \mathbb{E} \left\{ \sum_{t=0}^{\infty} \beta^t \Omega \left[ (z_t - z^*)^2 + \frac{\theta}{\kappa} \pi_t^2 \right] \right\},$$ \hspace{1cm} (1.46)

subject to

$$\pi_t = \kappa z_t + \beta \mathbb{E}_t \pi_{t+1} - \frac{\kappa}{\omega + \gamma} \epsilon_t, \quad \forall t \in \mathbb{N}_0,$$ \hspace{1cm} (1.47)

where $z_t = x_t + \frac{l_t}{\omega + \gamma}$ and $z^* = x^* + \frac{l_t^*}{\omega + \gamma}$. Solving for $z_t$ in (1.47) in terms of inflation and expected future inflation, we can replace it in (1.46) to obtain a maximization problem that only depends
on inflation. The first order conditions for all $t$ imply that $\pi_0 = -\frac{z_0 - z^*}{\theta}$ and $\pi_t = -\frac{z_t - z_{t-1}}{\theta}$, $\forall t > 0$.

Assuming $t > 0$ we can replace $\pi_t$ in (1.47) to obtain a rational expectations equation for the variable $z_t$. Guessing that $z_t = \varphi_0 + \varphi_1 z_{t-1} + \varphi_2 \theta \epsilon_t$ and matching coefficients, the rational expectations equation has to satisfy the system of equations:

$$\begin{align*}
\varphi_0 &= 0, \\
\varphi_1^2 - \frac{1}{\beta} (\kappa \theta + 1 + \beta) \varphi_1 + \frac{1}{\beta} &= 0, \\
\varphi_2 &= \frac{\kappa}{(\omega + \gamma)(\phi_{aux} \theta - \beta \varphi_1)}
\end{align*}$$

where $\phi_{aux} = \kappa + \frac{1}{\beta} (1 + \beta - \beta \varphi_1)$. The equation for $\varphi$ has two solutions. Discarding the solution which implies $\varphi_1 > 1$ and using the definition of $z_t$ we obtain the expressions for output and inflation in Proposition 2.

1.8.5 Term premium derivation

Assuming that the error term in equation (1.25) is negligible and given the multivariate normality of the state variables, interest rates are normally distributed. Therefore, writing bond prices in terms of bond yields and solving the expectation of a log-normal variable, equation (1.23) becomes

$$e^{-ni_t^{(n)}} = \exp \left\{ \mathbb{E}_t \left[ \log M_{t,t+1} - (n-1)i_{t+1}^{(n-1)} \right] + \frac{1}{2} \text{var}_t \left( \log M_{t,t+1} - (n-1)i_{t+1}^{(n-1)} \right) \right\},$$

then,

$$-ni_t^{(n)} = -i_t - (n-1)\mathbb{E}_t \left[ i_{t+1}^{(n-1)} \right] + \frac{1}{2} \left[ \text{var}_t \left( \log M_{t,t+1} - (n-1)i_{t+1}^{(n-1)} \right) - \text{var}_t (\log M_{t,t+1}) \right].$$

Comparing the result above with the definition of term premium in equation (1.25) and the analytical expressions for the variance terms derived from equations (1.22) and (1.25), we can write the term premium as

$$\xi_t^{(n)} = \frac{1}{2n} \left[ \lambda(s_t) \Sigma \lambda(s_t) - k_n(s_t) \Sigma k_n(s_t) \right],$$

where $k_n(s_t) = k_{0,n} + k_{1,n} s_t$. Replacing the definitions for $\lambda(s_t)$ and $k_n(s_t)$, we obtain (1.27).

To obtain the $\beta^{(n)}$ coefficients, consider the representations

$$i_{t+1}^{(n-1)} - i_t^{(n)} = \frac{A_{n-1}}{n-1} + \frac{B_{n-1}^\top}{n-1} (I - \Phi) \psi - \frac{A_n}{n} + \left( \frac{B_{n-1}^\top \Phi}{n-1} - \frac{B_n^\top}{n} \right) s_t + \frac{B_{n-1}^\top}{n-1} \Psi (s_t) \Sigma^{1/2} \epsilon_{t+1}$$

$$= V_1 + V_2^\top s_t + \nu_{t+1}$$
and

\[ i_t^{(n)} - i_t = \frac{A_n}{n} - A_1 + \left( \frac{B_n}{n} - B_1 \right) s_t \]

\[ = W_1 + W_2^\top s_t, \]

derived from (1.21) and (1.25). Noting that the \( \beta^{(n)} \) coefficient in (1.36) is the scaled projection of \( i_{t+1}^{(n-1)} - i_t^{(n)} \) on \( i_t^{(n)} - i_t \), we get

\[ \frac{\beta^{(n)}}{n-1} = \frac{\text{cov} \left( V_2^\top s_t, W_2^\top s_t \right)}{\text{var} \left( W_2^\top s_t \right)} = \frac{V_2^\top \text{var} (s_t) W_2}{W_2^\top \text{var} (s_t) W_2}. \]

Now, we can note from the recursive formulation of \( B_n, B_1 \) and the term premium, that \( V_2^\top = \frac{W_2^\top - 1}{n-1} \left( \lambda_0^\top \Sigma \lambda_1 - k_{0,n}^\top \Sigma k_{1,n} \right) = \frac{W_2^\top}{n-1} - \frac{n}{n-1} \xi_{B,n}^\top. \) Replacing and defining \( D_n = W_2 \), we obtain the formula for \( \beta^{(n)} \).

1.8.6 Calibration and Policy Experiment.

The calibration consists of matching selected long-term properties of the US economy for the 1961:Q4 to 196 period to the associated unconditional first and second moments of the equilibrium properties of the model under discretion.

The unconditional expectations of the state variables, \( s_t \), are

\[ \mathbb{E} [s_t] = (I - \Phi)^{-1} \psi \]

and an approximation to the unconditional covariance matrix is given by the solution \( \text{var}(s_t) \) to the Lyapunov equation

\[ \text{var} (s_t) - \Phi \text{var} (s_t) \Phi^\top = \Psi_c \Sigma (I + \xi \text{diag} \{ \mathbb{E} [s_t] \}) \Psi_c^\top \]

This solution is an approximation since \( s_t \) is constrained to take values that ensures that \( \Phi(s_t)^2 \) is positive definite. The solution to this equation is given by

\[ \text{var}(s_t) = \sum_{t=0}^{\infty} \Phi^t \Psi_c \Sigma \left( C + \xi \text{diag} \{ \mathbb{E} [s_t] \} \right) \Psi_c^\top \left( \Phi^\top \right)^t. \]

Using the unconditional moments above and the appropriate formulas from the affine framework in Section 1.5, it is possible to compute unconditional moments for interest rates, term premia and Campbell-Shiller coefficients. The calibration amounts then to finding parameter values that minimize a measure of deviations of the theoretical unconditional moments from their
empirical counterparts. This measure assigns considerable weight to deviations from the levels of interest rates and Campbell-Shiller coefficients. In addition, the persistence of supply shocks, $\phi_\epsilon \rho \sigma$, is set to equal the first-order autocorrelation of inflation. This particular calibration seems appropriate to understand the effects of credibility improvements on the level of interest rates and the term premia, given that, as shown in Section 1.6.1, inflation persistence is an important indicator of the level of credibility of the policy.

There are 15 parameters involved in the calibration: six preference parameters, $\beta$, $\gamma$, $\eta$, $\omega$, $K_\eta$, and $\sigma_\eta$, and nine production parameters, $\alpha$, $\theta$, $g_a$, $\phi_a$, $\phi_\epsilon$, $K_\epsilon$, $K_a$, $\sigma_a$ and $\sigma_\epsilon$. In order to obtain values for $g_a$, $\phi_a$ and $K_a$, the labor productivity series from Gomme and Rupert (2005)\(^{16}\) was used to fit an AR(1) model for growth in labor productivity as in equation (1.10). Since there is not statistical evidence of heteroscedasticity in the errors, $K_a$ is set to 0. The other 12 parameters were chosen to minimize the measure of deviations subject to the restriction that these parameters must be in a range of sensible values according to other studies.

The policy experiment consists of evaluating the characteristics of an economy characterized by a policy under commitment using the parameter values of the calibration exercise above. This experiment allows us to compare the main long-run differences between the two types of policies and capture the effects of credibility.

Figure 1.6: Model-implied slope coefficients for regressions of one-quarter ahead consumption growth and inflation on yield spreads.

\(^{16}\)The data corresponds to GDP per hour of work. The series is found in http://clevelandfed.org/research/Models/ rbc/Index.cfm and the source is Haver Analytics. This series has the inconvenient that it measures productivity of output growth instead of no durables and services consumption. Therefore, it is implicitly assumed that the productivity of consumption goods and services, and aggregate output are the same.
Figure 1.7: Impulse responses to a positive technology shock.
Figure 1.8: Impulse responses to a negative supply shock.
Figure 1.9: Impulse responses to a positive preference shock.
Chapter 2

The Economic Content of Interest Rates, Time-Varying Risk Premia and the Implementation of Optimal Monetary Policy

2.1 Introduction

The term structure of interest rates is a rich source of economic information. Forward-looking market participants implement their investment and savings decisions by trading default-free government bonds for a wide spectrum of maturities. As a result, interest rates (bond prices) must reflect these decisions and contain current and expected future economic conditions for different time horizons. Then, the term structure is potentially useful for policy makers to predict the future state of the economy and obtain market expectations about future policies.\(^1\) This task is far from trivial and demands an underlying economic theory of the term structure.\(^2\)

Most of the analysis of the economic content of interest rates has been conducted assuming the validity of the theory of the expectations hypothesis.\(^3\) This theory simplifies the interpretation of the content of long-term interest rates since all variation in long-term rates must reflect changes in expectations of future short-term rates. Unfortunately, numerous studies such as Fama and Bliss (1987) or Campbell and Shiller (1991) provide empirical evidence rejecting this theory and shed

\(^1\)See Goodfriend (1998) for potential applications of the term structure for monetary policy.

\(^2\)See the remarks of the chairman of the Federal Reserve Ben Bernanke before the Economic Club of New York, March 20, 2006. They provide an idea of some of the difficulties faced by policy makers in extracting information from the yield curve.

\(^3\)See for instance Estrella and Hardouvelis (1991) or Estrella (2005).
doubts about the validity of its implications. Alternatively, Duffee (2002) and Dai and Singleton (2002) show that time-varying term premia provide a better description of the observed properties of interest rates. This finding suggests that, in order to properly extract information from the term structure, it is important to understand the sources of time variation in risk premia. This is not a simple task in the absence of a model that simultaneously captures macroeconomic behavior and asset-pricing properties. In addition, the analysis becomes more complicated if one of the purposes is to answer policy-related questions. It is the case since different policies may have different effects on the properties and economic content of interest rates.

This chapter explores the economic content of long-term interest rates when time-varying term premia and optimal monetary policy are taken into consideration. It analyzes the extraction from interest rates of expectations about future monetary policy and current and expected future macroeconomic conditions. It also examines the implementation of monetary policy with policy rules that react to the term structure. Additionally, it allows welfare interpretations for changes in the content of interest rates that are related to changes in the objective of monetary policy.

The framework for the analysis is a general equilibrium model that links economic activity to monetary policy and the term structure of interest rates. Monetary policy has real economic effects and is conducted under discretion by a social planner that assigns weights to inflation and output stabilization. The purpose of the policy is to minimize, given these weights, deviations from output and inflation targets in a policy function. Depending on the policy, risk premia vary over time as a result of preference shocks in the households’ utility that are sensitive to lagged aggregate consumption. The equilibrium interest rates and term premia for all maturities can be expressed as affine functions of macroeconomic variables. This feature provides a direct link between economic conditions and interest rates and greatly simplifies the analysis of the economic content of the term structure. In order to understand this content, the model is calibrated and a policy experiment with welfare interpretations is performed. The experiment consists in reducing the weight of inflation in the policy function. Clarida, Galí and Gertler (1999) show that certain policies under commitment can be implemented under discretion by increasing the weight of inflation in the policy function. Given that optimal policies under commitment involve welfare benefits over policies under discretion, the reduction of the inflation weight in the policy experiment can be seen as a policy-motivated reduction in economic welfare.

The weight of inflation in the policy function has a considerable impact on the behavior of interest rates. This weight determines the stability of real activity and inflation around their targets. In particular, a policy that assigns a high weight to inflation allows for greater instability in output and thus increases the proportion of real risks to nominal risks. As a result, market participants demand higher compensations for financial assets that are exposed to real risks and then, the economic content of nominal interest rates is significantly affected.
The existence of time-varying term premia has important implications for the content of the yield curve. For instance, it is not possible to extract expectations about future short rates from forward rates directly. One-period forward rates do not only contain expectations about the one-period short rate over time. They also embed compensations for risk that depend on the state of the economy and monetary policy. The analysis shows that depending on the type of shock that hits the economy (demand or supply), the risk premia component in forward rates can vary significantly. Specifically, the impact is high when a demand shock affects an economy where the inflation weight in the policy objective is high. Assuming that monetary policy is implemented using the short-term nominal rate as instrument, this variation represents a considerable challenge for policy makers who want to extract expected paths for monetary policy from interest rates. In that case, this chapter shows that they can complement forward rates with the current level of the short-term rate and lagged macroeconomic information to obtain better forecasts of future monetary policy.

The information of interest rates about current and future macroeconomic conditions is also affected by the existence of time-varying term premia. For instance, expected inflation can not be obtained as the difference between nominal and real rates. This difference also captures a risk premium component due to the correlation between real activity and inflation. The size of this component and its responses to uncertainty are determined by monetary policy. It is shown that this component varies significantly with supply shocks if the inflation weight in the policy objective function is high. More generally, this chapter also contains an analysis of the predictive power of interest rates for future consumption growth and inflation at different horizons. It shows that while the information of interest-rate spreads about future consumption growth is low in comparison to the information of the short-term rate, spreads have an important predictive power for future inflation. In addition, this chapter shows how to extract current macroeconomic information from term-structure information.

The analysis is complemented with a potential application for the implementation of optimal monetary policy. The application is the formulation of short-term interest-rate policy rules that react to the term structure. In practice, the implementation of macroeconomic-based policy rules is limited by the availability and quality of macroeconomic information. For instance, Taylor (1993) proposes an interest-rate rule where the Fed’s operating target is a function of current inflation and deviations of real GDP from “potential output”. However, at the time the operating target is chosen, the current inflation rate and the GDP figures have not been released and must be forecast using alternative indicators. Orphanides (2001) shows that policy recommendations based on these indicators differ considerably from those obtained with the ex post revised data. As a result, these recommendations may lead to inappropriate policy outcomes. On this ground, McCallum (1999a) criticizes the Taylor rule for not being operational and proposes to focus on
rules that explicitly specify the operating target as a function of data available to the policy maker at the time the target decision is made. Since bond prices are available on real-time, long-term interest rates embed informational advantages over macroeconomic data which can be exploited through a term-structure-based policy rule.

The fact that the equilibrium term structure is affine allows a simple evaluation of several types of interest-rate rules. The short-term rate can be written as a linear combination of contemporaneous and lagged variables. These variables can be macroeconomic variables or interest rate levels and spreads. Specifically, three different types of rules are analyzed: (i) a term structure rule, (ii) a macro and term structure rule and (iii) a rule with interest rate smoothing. The term structure rule is useful to determine how the short-term rate should react to different sectors of the curve. It is found that this rule involve negative reaction to the short and long term spreads of the curve and a positive response to intermediate maturity spreads. Also, the sensitivity of the rule to term spreads increases with the inflation weight in the policy objective. The macro and term structure rules are helpful to determine which sectors of the curve provide information that is not contained in a particular macro variable. A policy with high weight on inflation imply high reactions to both macro and term structure variables. Finally, the analysis of policy rules with interest rate smoothing show that the smoothing coefficient increases with the maturity of the spread that is used in the rule and decreases with the weight of inflation in the policy objective function.

The rest of the chapter is organized as follows. Section 2.2 explains the economic model and the equilibrium affine term structure. Section 2.3 develops the analysis and presents the calibration and policy experiment. Section 2.4 concludes and the Appendix contains useful mathematical derivations and calibration details.

2.2 The Model

This section presents the economic model that is used for the joint analysis of monetary policy and the term structure of interest rates. The framework is a standard dynamic general equilibrium model for the analysis of monetary policy as presented in Clarida, Gali and Gertler (1999) or Woodford (2003), extended to capture long-term interest rate properties. In particular, monetary policy is conducted optimally in the model economy and the equilibrium term structure of interest rates is affine with time-varying term premia. This model links the behavior of interest rates to the objective of monetary policy and macroeconomic conditions. As a result, the model allows us to understand the implications of monetary policy on interest rates and thus to analyze the economic content of the term structure.
2.2.1 Optimal Monetary Policy

The monetary authority in the economy can be seen as a social planner that chooses optimal contingent paths for output, inflation and a nominal interest rate to maximize welfare. The maximization problem is constrained by conditions that characterize the optimal behavior of the private sector (households and firms) and, therefore, must be satisfied in equilibrium. Firms are monopolistic competitor suppliers of multiple goods in an environment where nominal prices are not perfectly flexible. This friction allows monetary policy to have effects on the real economy. We abstract from investment and capital accumulation and thus the equilibrium output is equal to aggregate consumption.

Let $c_t \equiv \log C_t$ be the level of consumption (output) in logarithmic terms and $\pi_t$ be the deviation of the inflation rate from its long-run level. It is convenient to define the difference between actual consumption and the consumption that would be observed if prices were perfectly flexible ($c_{nat}^t \equiv \log C_{nat}^t$). This difference is known as the “output gap”

$$x_t \equiv c_t - c_{nat}^t.$$  \hspace{1cm} (2.1)

In addition, let $i_t$ be the one-period nominal interest rate that is determined by the monetary policy. Using this notation, the monetary policy problem can be written as

$$\max_{\{\pi_t, x_t, i_t\}} \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t \left[ \rho \pi_t^2 + \kappa (x_t - x^*)^2 \right] \right\}$$ \hspace{1cm} (2.2)

subject to

$$e^{-it} = \mathbb{E}_t [M_{t,t+1}]$$  \hspace{1cm} (2.3)

and

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + \epsilon_t,$$  \hspace{1cm} (2.4)

where $\beta$ is the subjective discount factor. The objective function (2.2) describes a welfare measure where deviations of the output gap from the target $x^*$ reduce welfare as well as deviations of inflation from a zero target. The parameters $\rho$ and $\kappa$ are the relative weights assigned to inflation and output deviations, respectively. Rotemberg and Woodford (1999) show that the objective function can be obtained from a quadratic approximation of the utility function. We do not restrict here the objective function to be the utility function for the economy presented in subsection 2.2.2 below. This flexibility allows us to analyze policies where the monetary authority is more or less conservative about inflation than what the utility-based welfare measure prescribes.
Equation (2.3) is the optimality condition for the household problem that relates the price of a one-period nominal bond to the one-period marginal rate of substitution of consumption, $M_{t,t+1}$. The characterization of the marginal rate plays a fundamental role capturing interest rate properties and is presented in subsection 2.2.2.

The aggregate supply equation (2.4) describes the optimality condition for the profit maximization problem of monopolistic competitor suppliers of multiple goods. The staggered price setting generates a short-run relation between inflation and the output gap that is captured by the parameter $\kappa$. Simultaneously, given the forward-looking nature of economic agents, the current level of inflation is affected by expectations about future inflation. An important component to capture time variation in inflation is the cost-push shock $\epsilon_t$. This shock can be interpreted as a markup shock or, more general, a deviation breaking the proportionality between marginal costs and the output gap. We assume that the cost-push shock follows the process

$$
\epsilon_{t+1} = \phi \epsilon_t + \sigma \epsilon_{t,t+1},
$$

where $\epsilon_{t,t} \sim \text{IID } N(0, 1)$.

The monetary authority solves the policy problem under discretion. That is, it re-optimizes every period and takes the expectations of the private sector as given. Under this assumption the optimality condition for the problem above becomes\(^4\)

$$
x_t - x^* = -\rho \pi_t.
$$

(2.5)

This equation can be seen as a targeting rule for inflation. It tells us that the optimal policy under discretion involves a tradeoff between deviations from the target output gap and inflation. The tradeoff depends on the relative weight of inflation in the policy objective function (2.2). A higher inflation weight allows more deviations in the output gap.

From the optimality condition above and equation (2.4) we can write the output gap in terms of inflation and obtain a rational expectations equation in terms of the endogenous inflation and the exogenous cost-push shock. It can be shown that the equilibrium inflation process can be written as

$$
\pi_t = \delta \pi_t + \delta \epsilon_t,
$$

(2.6)

\(^4\) Notice that the short-term interest rate equation (2.3) does not play a role in determining the optimality condition, given that the objective function does not depend on this interest rate.
where the inflation bias and the sensitivity of inflation to the cost-push shock are, respectively,

\[ \delta_\pi = \frac{\kappa x^*}{1 - \beta + \rho \kappa} \quad \text{and} \quad \delta_\epsilon = \frac{1}{1 - \beta \phi + \rho \kappa}. \]

The inflation bias \( \delta_\pi \) arises when the output gap target is not zero.\(^5\) It is also clear that the sensitivity of inflation to cost-push shocks and the inflation bias are lower when the monetary authority is more conservative about inflation (high \( \rho \)).

Given the assumed process for the cost-push shock, the inflation process can also be written as the autoregressive process

\[ \pi_{t+1} = (1 - \phi_\epsilon)\delta_\pi + \phi_\epsilon \pi_t + \delta_\epsilon \sigma_\epsilon \varepsilon_{t,t+1}, \]

showing that equilibrium inflation inherits the persistence of the cost-push shock regardless of the relative weight \( \rho \).

By replacing the equilibrium processes for consumption and inflation into equation (2.3) we obtain the equilibrium one-period interest rate in terms of the marginal rate of substitution of consumption \( M_{t,t+1} \). This rate is characterized in the next subsection.

### 2.2.2 The Marginal Rate of Substitution of Consumption

We derive the intertemporal marginal rate of substitution of consumption following the specification in Gallmeyer, Hollifield and Zin (2005) for the utility derived by households from consumption.\(^6\) This utility is given by

\[ \sum_{t=0}^{\infty} \beta^t \frac{C_{t}^{1-\gamma}}{1-\gamma} Q_t, \]

with \( \gamma \) capturing local risk aversion. This representation differs from the standard relative risk aversion setting in that it incorporates the preference shock \( Q_t \). The process for the shock \( q_t \equiv \log Q_t \) is

\[ -\Delta q_{t+1} = \frac{1}{2} \eta^2 \Delta c_t^2 \text{Var}_t(\Delta c_{t+1}) + \eta \Delta c_t (\Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1}), \quad (2.7) \]

which potentially allows for risk aversion that changes over time. The parameter \( \eta \) is the sensitivity of the shock to unexpected consumption growth per unit of lagged consumption growth. Given the dependence on lagged aggregate consumption, the preference shock can also be interpreted as an external habit. The quadratic term on \( \Delta c_t \) is a correction term that makes \( Q_t = \mathbb{E}_t [Q_{t+1}] \) and preserves the affine framework for the equilibrium term structure as shown in subsection 2.2.3.

---

\(^5\)A utility-based welfare function implies \((x^* \neq 0)\) when the natural rate of consumption is not efficient.

\(^6\)Based on the assumption that consumption and labor are separable in the utility function, we do not need to specify preferences for labor to obtain the marginal rate of substitution of consumption.
By solving the household utility maximization problem we obtain the intertemporal marginal rate of substitution of consumption between periods $t$ and $t+n$, $M_{t,t+n}$, given by

$$M_{t,t+n} = \beta^n \left( \frac{C_{t+n}}{C_t} \right)^\gamma \left( \frac{Q_{t+n}}{Q_t} \right) \left( \frac{P_{t+n}}{P_t} \right).$$

(2.8)

Additionally, the household problem provides us with the no-arbitrage price for nominal bonds. The price of an $n$-period default free bond $b_t^{(n)}$ is

$$b_t^{(n)} = \mathbb{E}_t[M_{t,t+n}]$$

(2.9)

and the associated continuously-compounded interest rate is

$$i_t^{(n)} = -\frac{1}{n} \log b_t^{(n)}.$$ 

Therefore, the constraint (2.3) in the policy problem can be seen as a particular case of equation (2.9) when $n = 1$. This equation becomes

$$e^{-it} = \mathbb{E}_t [\exp(\log \beta - \gamma \Delta c_{t+1} + \Delta q_{t+1} - \pi_{t+1} - \theta)]$$

(2.10)

where $\theta = \log (P_{t+1}/P_t) - \pi_{t+1}$ is the inflation trend.

Equation (2.10) tells us that the equilibrium one-period interest rate $i_t$ depends on the processes for consumption growth and inflation. Differencing equation (2.1), consumption growth is given by the natural rate of consumption growth and changes in the output gap. From the target rule (2.5) we find that changes in the output gap are given by $\Delta x_t = -\rho \Delta \pi_t$ and thus the consumption growth process is

$$\Delta c_t = \Delta c_{t}^{nat} - \rho \Delta \pi_t,$$

where the equilibrium inflation is found from equation (2.6). Thus, the consumption growth process under the optimal discretionary policy is negatively affected by positive changes in inflation and the effect is amplified when the monetary authority is more conservative about inflation.

In order to obtain a complete characterization of consumption growth, we assume that the natural rate of consumption growth $\Delta c_{t}^{nat}$ is not affected by monetary policy and define the exogenous process

$$\Delta c_{t+1}^{nat} = (1 - \phi_c)\theta_c + \phi_c \Delta c_{t}^{nat} + \sigma_c \varepsilon_{c,t+1},$$

where $\varepsilon_{c,t} \sim \text{IID} \mathcal{N}(0, 1)$. 

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2.2.3 The Term Structure of Interest Rates

Bond prices and long-term interest rates for different maturities are obtained from the equilibrium equations (2.8) and (2.9) in terms of deep economic parameters such as preference, production and policy parameters. This representation is useful to understand the link between macroeconomic activity, monetary policy and long-term interest rates. This subsection shows that the equilibrium interest rates can be described by using a discrete-time version of the Duffie and Kan (1996) affine class of arbitrage-free term-structure models, where interest rates can be expressed as linear combinations of some state variables.\(^7\) One of the advantages of this framework is the possibility to compare interest rates for different maturities in terms of exposures to a reduced set of state variables. We describe first the general affine framework and then we present the restrictions on the parameters imposed by the equilibrium term structure in the economic model.

Consider the \(k\)-dimensional vector of state variables \(s_t\), following the first-order vector autoregression

\[
s_{t+1} = (\mathbb{I} - \Phi)\theta + \Phi s_t + \Psi^\top \Sigma^{1/2} \varepsilon_{t+1}, \tag{2.11}
\]

where \(\mathbb{I}\) is the \(k \times k\) identity matrix and \(\varepsilon_t \sim \text{IID} N(0, \mathbb{I})\) is an \(m \times 1\) vector that contains \(m\) sources of uncertainty. The \(k \times k\) matrix \(\Phi\) is the autoregressive matrix for the set of state variables, \(\theta\) is a \(k \times 1\) vector with the unconditional means of the state variables and \(\Psi\) is an \(m \times k\) vector that determines the sensitivity of the state variables to the uncertainty. Therefore, the constant matrix \(\Psi^\top \Sigma \Psi\) describes the conditional covariance structure of the state variables.

The one-period stochastic discount factor \(M_{t,t+1}\) is described by

\[
-\log M_{t+1} = \Gamma_0 + \Gamma_1^\top s_t + \frac{1}{2} \lambda(s_t)^\top \Psi^\top \Sigma \Psi \lambda(s_t) + \lambda(s_t)^\top \Psi^\top \Sigma^{1/2} \varepsilon_{t+1}. \tag{2.12}
\]

where the \(k \times 1\) vector \(\Gamma_1\) represents the “factor loadings” for the pricing kernel and the \(k \times 1\) vector \(\lambda(s_t)\) contains the potentially time-varying prices of risk. The prices of risk admit the affine representation

\[
\lambda(s_t) = \Lambda_{k,m} (\lambda_0 + \lambda_1 s_t), \tag{2.13}
\]

where \(\lambda_0\) is an \(m \times 1\) vector, \(\lambda_1\) is an \(m \times k\) matrix and \(\Lambda_{k,m}\) is a \(k \times m\) matrix that becomes \(\mathbb{I}\) when \(k = m\). The quadratic term \(\frac{1}{2} \lambda(s_t)^\top \Sigma \lambda(s_t)\) is a correction that preserves the linearity of interest rates in the log-normal framework. This characterization conforms to the essentially affine framework in Duffee (2002), Dai and Singleton (2002) or Ang and Piazzesi (2003).

From this specification for the stochastic discount factor and the bond pricing equation (2.9)

we can write interest rates as
$$i_t^{(n)} = \frac{1}{n} \left[ A_n + B_n^\top s_t \right],$$
where the parameters defining bond yields, $A_n$ and the $k \times 1$ vector $B_n$, are found recursively to be
$$A_n = \Gamma_0 + A_{n-1} + B_{n-1}^\top \left[ (I - \Phi) \theta - \Sigma \psi A_{k,m} \lambda_0 \right] - \frac{1}{2} B_{n-1}^\top \Sigma \psi B_{n-1},$$
$$B_n^\top = \Gamma_1^\top + B_{n-1}^\top \left[ \Phi - \Sigma \psi A_{k,m} \lambda_1 \right],$$
where $\Sigma \psi = \Psi^\top \Sigma \Psi$ with initial conditions $A_0 = 0$ and $B_0 = 0$. The recursive equation for the factor loading parameters $B_n$ allows us to observe the evolution across maturity of these loadings. In particular, if the market prices of risk are constant ($\lambda_1 = 0$), the adjustment in the factor loadings is given by the autocorrelation matrix of the state variables. When the market prices of risk are time-varying the adjustment depends on the modified autocorrelation matrix $\Phi - \Sigma \psi A_{k,m} \lambda_1$ which can be seen as the autocorrelation matrix for the state variables under the risk-neutral measure.

The equilibrium term structure for the economy can be represented as an affine term structure model of the class described above by imposing significant restrictions on the parameters. Proposition 10 presents a complete characterization for the term structure model that is consistent with equilibrium. It shows that three state variables are required to describe the dynamics of the term structure. The representation in the proposition is given for simplicity in terms of three macroeconomic variables: natural consumption growth and current and lagged inflation.

**Proposition 10.** The equilibrium characteristics of the economy and its associated nominal pricing kernel are represented by equations (2.11), (2.12) and (2.13) when
$$s_t \equiv (\Delta c_t^{nat}, \pi_t, \pi_{t-1})^\top$$
and
$$\Phi = \begin{bmatrix} \phi_c & 0 & 0 \\ 0 & \phi_\epsilon & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \theta = \begin{pmatrix} \theta_c \\ \delta_\epsilon \\ -\delta_\epsilon \end{pmatrix}, \quad \Psi^\top = \begin{bmatrix} 1 & 0 \\ 0 & \delta_\epsilon \end{bmatrix}$$
$$\Sigma^{1/2} = \text{diag} \{ \sigma_c, \sigma_\epsilon \}, \quad \varepsilon = (\varepsilon_c, \varepsilon_\epsilon)^\top,$$
$$\Gamma_0 = -\log \beta + \theta_\pi + \gamma (1 - \phi_c) \theta_c + (1 - \gamma \rho) (1 - \phi_\epsilon) \delta_\epsilon - \frac{1}{2} \gamma^2 \sigma_c^2 - \frac{1}{2} (1 - \gamma \rho)^2 \delta_\epsilon^2 \sigma_\epsilon^2,$$
$$\Gamma_1 = \begin{bmatrix} \Gamma_{1,1} & \Gamma_{1,2} & \Gamma_{1,3} \end{bmatrix}^\top = \begin{bmatrix} \gamma (\phi_c - \eta \sigma_c^2) + \eta \mu & \phi_c + (1 - \phi_\epsilon + \eta \sigma_\epsilon^2) \gamma \rho - \eta \rho \mu & -\eta \rho (\gamma \sigma_c^2 - \mu) \end{bmatrix}^\top,$$
\[ \lambda_0 = \begin{pmatrix} \gamma \\ 1 - \gamma \rho \end{pmatrix}, \quad \lambda_1 = \eta \begin{bmatrix} 1 & -\rho & \rho \\ -\rho & \rho^2 & -\rho^2 \end{bmatrix}, \quad A_{3,2} = \begin{bmatrix} I_{2 \times 2} \\ 0_{1 \times 2} \end{bmatrix}. \]

and

\[ \mu = \rho (1 - \gamma \rho) \delta_\tau^2 \sigma_\tau^2. \]

The policy-dependent parameter \( \rho \) plays a significant role in the determination of the term structure given its impact on consumption growth and inflation. The relative weight of inflation in the policy objective function affects not only the sensitivity \( \Gamma_1 \) of \( M_{t,t+1} \) to macroeconomic variables, but also it affects the market prices of natural consumption growth risk and cost-push shocks. We can notice that the constant component of the market price of supply shocks in \( \lambda_0 \) can be positive or negative depending on whether the effect of inflation on the (nominal) marginal utility of consumption is greater than the effect of the output gap. In particular, the constant component of the market price of supply shocks is positive if \( \gamma^{-1} > \rho \), that is, if the relative weight of inflation in the policy objective function is low enough, a unit of supply shocks has greater effects on inflation than on consumption growth, in marginal utility terms.

Proposition 10 also shows us that the time-varying components of the market price of risk in \( \lambda_1 \) depend on the external habit parameter \( \eta \) and the monetary policy parameter \( \rho \). The particular case \( \eta = 0 \), or constant relative risk aversion, implies a constant price of risk. For \( \eta \neq 0 \), the price of the two sources of uncertainty depend on the state of the economy. The time-varying component of the price of risk is

\[ \lambda_1 s_t = \eta \begin{bmatrix} 1 \\ -\rho \end{bmatrix} \Delta c_t^{\text{nat}} + \eta \rho \begin{bmatrix} -1 \\ \rho \end{bmatrix} \Delta \pi_t. \]

Then, the negative dependence of consumption growth on inflation, generated by the optimal monetary policy, makes the price of risk to depend on inflation changes. A shock that moves the price of natural consumption growth in one direction moves the price of supply shocks in the opposite direction.

**Sensitivity of the Market Prices of Risk to the Relative Weight of Inflation in the Policy Objective**

Changes in the relative weight of inflation in the objective function of monetary policy, \( \rho \), affect the price of natural consumption growth risk and supply shocks. An increase in the weight \( \rho \) increases the sensitivity of inflation and the output gap to supply shocks and affects both prices of risk. Consider first the effect on the price of natural consumption growth. The effect is entirely driven by the existence of preference shocks (\( \eta \neq 0 \)) that are affected by lagged consumption growth. A positive (lagged) supply shock generates a greater reduction in lagged output gap (less lagged inflation), and thus a greater negative effect on aggregate consumption growth. If \( \eta > 0 \),
the marginal utility of consumption increases and the price of natural consumption risk declines. Effectively, it can be seen as a reduction in risk aversion for each unit of positive supply shocks that affect consumption growth. The effect is the opposite if $\eta$ is negative.

With respect to the price of supply shocks, it is affected even in the absence of preference shocks. The constant component $(1 - \gamma \rho) \delta_t$ of this price of risk decreases when the relative weight of inflation in the policy objective function increases. The reason is less inflation for each unit of supply shocks, which reduces the nominal part of the pricing kernel, and more effects on the output gap, which reduces the related real component of the pricing kernel due to its negative effects on consumption growth. In addition, the existence of preference shocks makes the price of supply shocks to depend on lagged changes in supply shocks (lagged changes in inflation). The net effect of a higher $\rho$ depends on the combination of two effects. A higher relative weight on inflation increases the sensitivity of this price of risk to lagged consumption growth and, simultaneously, has negative effects on the output gap. It translates into higher sensitivities of the price of risk to lagged natural consumption growth and lagged changes in supply shocks. If $\eta > 0$, it implies a lower price of risk for positive lagged natural consumption growth and a higher price of risk for positive lagged changes in supply shocks.

2.3 Analysis

Our main interest is to understand how the term structure of interest rates is useful for the analysis of optimal monetary policy. We consider three possible applications of the yield curve for policy making and analyze them using the equilibrium framework presented above. First, the term structure of interest rates can be used to extract expectations about future short-term interest rates. This expectations can be seen as expectations about future monetary policy if the monetary authority implements the policy using an interest-rate rule. Second, the term structure is useful to extract macroeconomic information. Interest rates contain information about the current value and the expected evolution of macroeconomic variables such as consumption growth and inflation. Third, monetary policy might be implemented with interest rate rules that are based on term structure information such as interest rate levels or term spreads. The three type of analysis are described below and are complemented with a calibration exercise and a policy experiment in section sec:calibI. The purpose of the policy experiment is to understand how changes in monetary policy affect the three applications of the term structure to policy analysis. The change in monetary policy that is considered is a change in the relative weight that the monetary policy assigns to inflation in the policy objective function. This change can be linked to economic welfare as shown in the exercise.

A Useful Change of State Variables
It is convenient at this point to describe a tool that is used in the analysis below. We want to describe interest rates not only in terms of macroeconomic variables as in Proposition 10 but also in terms of financial variables such as term spreads. At the same time it is helpful to describe macroeconomic variables in terms of interest-rate information. These two tasks can be accomplished with appropriate changes of the set of state variables. Consider a new set of \( l \leq k \) state variables represented by the \( l \times 1 \) vector \( \hat{s}_t \). These variables can be written as linear combinations of the original state variables in the form

\[
\hat{s}_t = C + Ds_t,
\]

where \( C \) is an \( l \times 1 \) vector and \( D \) an \( l \times k \) matrix. If \( l = k \) and \( D \) is invertible we can obtain again the original set of state variables as

\[
s_t = D^{-1}(\hat{s}_t - C).
\]

2.3.1 Time-Varying Term Premia and the Expectations Hypothesis of Interest Rates

Long-term interest rates can be seen as risk-adjusted expectations of future short-term interest rates. Under the theory of the expectations hypothesis of interest rates, the risk adjustment of the expectations is constant or zero (pure expectations hypothesis). This theory is convenient to interpret the information of the term structure: changes in long-term rates simply reflect changes in expected future short-term rates. Unfortunately, empirical studies such as Campbell and Shiller (1991) or Fama and Bliss (1987) provide evidence of deviations from the expectations hypothesis in the data. This rejection, that can be interpreted as evidence of time-varying term premia, makes it difficult the interpretation of the information of the yield curve.

Time variation in term premia presents a non-trivial challenge for policy makers that want to use the term structure for forecasting purposes. If the term premia are significantly large and volatile, using the expectations hypothesis of interest rates to forecast short-term rates can be misleading. As an alternative, term premia can be extracted from an equilibrium model. However, the latter approach is exposed to the validity of the model. This subsection presents the implications of the equilibrium term structure model studied above for time variation in term premia.

Consider the one-period term premium on an \( n \)-period bond defined by

\[
\xi_t^{(n)} \equiv i_t^{(n)} - \frac{1}{n} \left[ i_t + (n - 1)E_t i_{t+1}^{(n-1)} \right].
\]
The no-arbitrage recursive equations (2.14) imply that
\[ \xi_t^{(n)} = \frac{1}{n} \left[ \xi_{A,n} + \xi_{B,n}^\top s_t \right] \] (2.17)
with coefficients given by
\[ \xi_{A,n} = -B_{n-1}^\top \Sigma \psi \left( \beta_{k,m} \lambda_0 + \frac{1}{2} B_{n-1} \right) \quad \text{and} \quad \xi_{B,n}^\top = -B_{n-1}^\top \Sigma \psi \beta_{k,m} \lambda_1. \]

It can be inferred from these equations that a necessary condition for time-variation in term premia is a time-varying market price of risk ($\lambda_1 \neq 0$). As a result, the habit in the economic model and the relative weight assigned to inflation in the policy objective are fundamental to characterize deviations from the expectations hypothesis. To see this, consider the Campbell and Shiller (1991) coefficients, $\beta^{(n)}$, associated to the regression
\[ i_t^{(n-1)} - i_t^{(n)} = \alpha^{(n)} + \beta_{CS}^{(n)} \left( i_t^{(n-1)} - i_t \right) + \varepsilon_{CS,t}^{(n)}. \] (2.18)

Under the expectations hypothesis the $\beta_{CS}^{(n)}$ coefficients are equal to 1. Using equation (2.16) these coefficients can be written as
\[ \beta_{CS}^{(n)} = 1 - n \frac{\text{cov}(i_t^{(n)} - i_t, \xi_t^{(n)})}{\text{var}(i_t^{(n)} - i_t)}, \]
implying that deviations from the expectations hypothesis are potentially captured by the correlation between term spreads and one-period term premia.

To simplify the analysis, consider the relation between expected excess returns and term premia $\xi_t^{(n)}$. Let $x_{t,t+1}^{(n)}$ be the one-period excess return for holding an $n$-period bond with respect to a one-period risk-free bond. The expected excess return is
\[ \mathbb{E}_t \left[ x_{t,t+1}^{(n)} \right] = -(n + 1) \mathbb{E}_t i_{t+1}^{(n-1)} + n i_t^{(n)} - i_t = n \xi_t^{(n)}. \]

From this representation for expected excess returns and using equation (2.16) recursively, the one-period interest rate is
\[ i_t^{(n)} = \frac{1}{n} \mathbb{E}_t \left[ \sum_{j=0}^{n-1} i_{t+j} \right] + \frac{1}{n} \mathbb{E}_t \left[ \sum_{j=0}^{n-2} x_{t,t+j}^{(n-j)} \right]. \] (2.19)

The first component of the short-term rate reflects the pure expectations hypothesis. The second component contains the sum of the one-period expected excess returns embedded in the
an n-period bond until maturity. Deviations from the expectations hypothesis can be captured if this component is not constant.

### 2.3.2 Expectations About Future Monetary Policy

If the expectations hypothesis holds in an economy where the monetary authority implements monetary policy using the short-term rate, one-period forward rates for different horizons are good indicators of expected future monetary policy. However, time variation in term premia complicates the analysis. The forward rate in this environment can be a misleading indicator of future monetary policy. In this subsection we provide a characterization of the differences between forward rates and expected future short rates and show that this difference may depend on the type of policy that is conducted.

Denote the one-period forward rate at time $t$ between $t + n$ and $t + n + 1$ by $f_{t,n}^{(1)} = f_{t,n}$. By no-arbitrage this rate has to be equal to

$$f_{t,n} = (n + 1)\mu_{t,n}^{(n+1)} - n\mu_{t}^{(n)}.$$ 

Differencing (2.19) we obtain that the difference between the expected value of the one-period interest rate $n$ periods in the future and the forward rate is

$$E_t[i_{t+n}] - f_{t,n} = -E_t[xr_{t,t+1}^{(n+1)}] - E \left[ \sum_{j=2}^{n-2} xr_{t+1,t+2}^{(n-j)} - xr_{t+1,t+2}^{(n-j)} \right]$$

$$= -E_t[xr_{t,t+1}^{(n+1)}] + \sum_{j=2}^{n-2} \xi_{t,j}^T (\Phi - \Phi)(\theta - E_t[s_{t+j}])$$

Thus, the expected future short-rate differs from the forward rate in the presence of term premia. The first component of the difference is the expected one-period excess return from $n$ to $n + 1$. This term survives even if term premia are constant. The second term results from time variation in term premia. It captures expected changes in one-period excess returns during the life of the bond with maturity at $t + n$. It can be seen that this term contributes to the difference between the expected short rate and the forward rate as long as the conditional expectation of the state variables during the life of the bond differ from the unconditional mean. Therefore, its effects are not observed on the long-run difference but can be analyzed using impulse responses to shocks that make the current economic conditions deviate from their long-run mean. This decomposition allows us to describe the usefulness of the forward rate as an indicator of expected monetary policy for policies with different distaste for inflation and different economic conditions.

One way to obtain expected future monetary policy given the limitations of looking directly
at the appropriate forward rate is to use additional information from the term structure. As an example consider the change of state variables represented in equation (2.15) where the new set of state variables is

\[ \tilde{s}_t \equiv (i_t, f_{t,n}, \pi_{t-1}) \]

That is, besides the forward rate, we can use the short term rate and lagged inflation to obtain the expected short rate for any horizon. If the expectations hypothesis holds we should obtain a loading coefficient of 1 for the forward rate and zero coefficients for the other two variables in the representation

\[ \mathbb{E}_t[i_{t+n}] = A_1 + \Gamma_1^T (I - \Phi^n) \theta - \Gamma_1^T \Phi^n D^{-1} C + \Gamma_1^T \Phi^n D^{-1} \tilde{s}_t, \]

where

\[ C = \begin{pmatrix} A_1 \\ A_{n-1} - A_n \\ 0 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} B_1^T \\ B_{n-1}^T - B_n^T \\ e_3^T \end{pmatrix}. \]

The calibration and policy experiment below provide a comparison of expected short-term rates for different maturities and different weights for inflation in the policy objective function.

2.3.3 The Macroeconomic Content of the Term Structure

An important indicator of the effectiveness of monetary policy and, thus, a guideline for policy making is the expected future inflation. This indicator provides an idea of the credibility of monetary policy about stabilizing prices in the future and might help take appropriate measures when it signals a decline in credibility. The term structure can be used to extract information about expectations of future inflation for different horizons as well as for other macroeconomic indicators. This task is, however, far from simple even under the expectations hypothesis.

Consider, for instance, the expected inflation one-period in the future implied by the term structure. Let \( r_t \) be the one-period continuously-compounded real rate. From the no-arbitrage price of a one-period real bond we can obtain the interest rate as

\[ e^{-r_t} = \mathbb{E}_t \left[ M_{t+1}^{real} \right] = \mathbb{E}_t \left[ \exp(\log \beta - \gamma \Delta c_{t+1} + \Delta q_{t+1}) \right]. \]

Using this equation and equation (2.10) for the nominal one-period rate, we can express the difference between the nominal rate and the real rate plus expected inflation as

\[ i_t - (r_t + \theta_{\pi} + \mathbb{E}_t \pi_{t+1}) = -(\gamma + \eta \Delta c_t) \text{cov}_t(\Delta c_{t+1}, \pi_{t+1}) - \frac{1}{2} \text{var}_t(\pi_{t+1}) \]

\[ = \left[ (\gamma + \eta \Delta c_t) \rho - \frac{1}{2} \right] \text{var}_t(\pi_{t+1}) = \left[ e_1^T \Lambda(s_t) - \frac{1}{2} \right] e_2^T \Sigma \phi e_2, (2.21) \]

63
where \( e_j \) denotes the \( k \times 1 \) vector with a one in the \( j \)-th position and zeros everywhere else. This equation is helpful to understand that the difference between the nominal interest rate and the real rate is not only expected inflation when inflation and consumption growth are correlated. This correlation is determined by the optimal policy followed by the monetary authority. In addition, the variability of inflation adds a precautionary savings effect to the difference. The last equality shows that the difference depends on the market price of risk. When it is time-varying, the expectations hypothesis does not hold and the difference is not constant. Then, the existence of time-varying risk premia makes it difficult to extract expected inflation from the yield curve. In this subsection we show how the term structure can be used to overcome these difficulties.

Let \( mv_t \) be the set of macroeconomic variables to be analyzed. Here we restrict the analysis to \( mv_t = \{ \Delta c_t, \pi_t \} \). The current values for these variables and current expectations about future values can be described in terms of yield curve information such as rate levels or term spreads. Our main interest is to understand the explanatory power of term structure information about these variables. This task can be accomplished running regressions of the form

\[
mv_{t+h} = \alpha_{mv,h} + \beta_{mv,h}^\top \hat{s}_t + \varepsilon_{mv,h},
\]

for different horizons \( h \geq 0 \), where \( \beta \) is an \( l \times 1 \) vector (for \( l \leq k \)) that contains the loadings on the \( l \) explanatory variables in the set of state variables \( \hat{s}_t \). The explanatory variables can be described in terms of the original state variables \( s_t \) using the change of variables characterized in equation (2.15). In addition, the macroeconomic variables can be written in terms of the original variables as \( mv_{t+h} = e_{mv}^\top s_{t+h} \) for the appropriate \( k \times 1 \) vector \( e_{mv} \). Therefore, the regression loadings are given by

\[
\beta_{mv,h} = \left( D \text{var}(s_t) D^\top \right)^{-1} D \text{var}(s_t) (\Phi^h)^\top e_{mv},
\]

where the unconditional variance of the state variables, \( \text{var}(s_t) \), satisfies the Lyapunov equation \( \text{var}(s_t) = \Phi \text{var}(s_t) \Phi^\top + \Sigma_q \). This derivation is obtained using the fact, from equation (2.11), that the process for the state variables at time \( t+h \) is

\[
s_{t+h} = (I - \Phi^h) \theta + \Phi^h s_t + \sum_{j=1}^h \Phi^{h-j} \Psi^\top \Sigma^{1/2} \varepsilon_{t+j}.
\]

It is also useful to understand the fraction of the variability of current and future macroeconomic conditions that is explained by the regression explanatory variables. It can be obtained by
computing the model-implied $R^2$’s associated with each regression, given by

$$R^2_{mv,h} = \frac{\beta_{mv,h}^\top D \text{var}(s_t) D^\top \beta_{mv,h}}{e_{mv}^\top \Phi^h \text{var}(s_t)(\Phi^h)^\top + \sum_{j=1}^h \Phi^{h-j} \Sigma \Psi (\Phi^{h-j})^\top e_{mv}}.$$  

The calibration and policy experiment below present different regressions that are useful to understand the macroeconomic content of the term structure depending on the relative weight of inflation in the policy objective function. In particular, regressions with one, two and three regressors are presented. Univariate regressions where the regressor is the term spread or its expectations hypothesis component are analyzed. It provides insights about the importance of deviations from the expectations hypothesis (time-varying term premia) explaining the current and future macroeconomic conditions. In the bivariate regressions, the regressors are the one-period interest rate and the term spread for some maturity. These regressions are useful to ask whether the predictability of different macro variables is explained by the short-term rate or the spread. The analysis is complemented with regressions where $l = k = 3$. The idea of this regressions is to add to the list of regressors one lagged macro variable and observe the maximum explanatory power that the system allows. Moreover, this type of regressions is useful to obtain an accurate value for a macroeconomic variable $mv_t$ from the term structure. This can be a useful tool when the current macroeconomic value is unobservable at time $t$ or reported with noise. This is possible under the reasonable assumption that the current term structure and one-period lagged macro variables are known.

### 2.3.4 Term-Structure-Based Policy Rules

The optimal monetary policy described in section 2.2 can be implemented using an interest-rate policy rule. The level of the one-period nominal interest rate is then determined by a reaction function that responds to a set of variables known at the moment of implementation. A popular example of this implementation procedure is the Taylor (1993) rule, where the interest rate is a deterministic function of the output gap (de-trended GDP) and inflation (deviations from a trend). However, the operational feasibility of such a rule can be questioned given the informational requirements that it implies. For instance, the output gap is an unobservable variable and its measurement implies noise. Moreover, information about output growth and inflation takes time to collect and is subject to sampling error and revisions. In summary, the monetary authority is limited by the availability and quality of economic data.

Subsection 2.3.3 shows that current economic conditions can be extracted from long-term interest rates using an equilibrium (no-arbitrage) model of the term structure. Since developed bond markets provide accurate interest rates for different maturities on real-time, it suggests that
non-operational macroeconomic-based policy rules can be re-expressed as operational rules where the short-term interest rate is set responding to term-structure information. An analysis of this type of rules is presented here.

The equilibrium one-period interest rate in equation (2.10) can be written as

\[ i_t = \Gamma_0 + \Gamma_{1,1} \Delta c^{nat}_t + \Gamma_{1,2} \pi_t + \Gamma_{1,3} \pi_{t-1} \]

from Proposition 10. Assuming that this equilibrium representation satisfies some conditions,\(^8\) this representation provides a rule to set the interest rate. This rule can be expressed in terms of term-structure information using an appropriate change of variables. In particular, we have interest in rules that react to current and lagged term structure information such as interest rate levels and term spreads. These rules would allow us to see which sectors of the yield curve are important for policy purposes and how the rule should be modified to implement changes in the relative weight of inflation in the policy objective function (2.2).

The first group of rules to analyze are those where the short-term rate reacts to the current term spread for a specific maturity, the current level of a macroeconomic variable \( mv_t = \{ \Delta c_t, \pi_t \} \) and the one-period lagged inflation. These rules can be obtained from a change of state variables as in equation (2.15) where \( \hat{s}_t = (i^{(n)}_t - i_t, mv_t, \pi_{t-1})^\top \) and

\[
C = \begin{pmatrix}
\frac{1}{n} A_n - A_1 \\
0 \\
0
\end{pmatrix}
\quad \text{and} \quad
D = \begin{pmatrix}
\frac{1}{n} B_n^\top - B_1^\top \\
e_{mv}^\top \\
e_3^\top
\end{pmatrix}.
\]

The rule for the one-period interest rate becomes

\[ i_t = A_1 - B_1^\top D^{-1} C + B_1^\top D^{-1} \hat{s}_t. \]

The calibration and policy experiment in subsection 2.3.5 provide numerical examples of these rules for different maturities and different weights for inflation in the objective function (2.2). In particular, this exercise allows to see what sectors of the yield curve should be used in order to minimize the weight of the contemporaneous macro variable in the rule.

The second group of rules are those rules that include a lagged value for the short-term interest rate. These rules capture the idea of interest-rate smoothing by the monetary authority. The appendix shows an extension of the procedure in Gallmeyer, Hollifield and Zin (2005) to

---

\(^8\)These conditions, known as the Taylor principle, deal with the indeterminacy of equilibrium that might arise from using an interest-rate rule. See Woodford (2003) and Cochrane (2006) for determinacy and uniqueness of equilibrium in this setting.
characterize these rules. Consider the affine vector-representation
\[
\begin{pmatrix}
  i_t \\
  i_t^{(n)} - i_t
\end{pmatrix}
= \mathcal{F} + \mathcal{G}
\begin{pmatrix}
  i_{t-1} \\
  i_{t-1}^{(n)} - i_{t-1}
\end{pmatrix}
+ \mathcal{H}\varepsilon_t + \mathcal{I}\varepsilon_{t-1},
\]
where \(\mathcal{G}, \mathcal{H}\) and \(\mathcal{I}\) are 2\(\times\)2 matrices. Manipulating this system as shown in the appendix, provides a representation for the short-term interest rate given by
\[
i_t = \mathcal{F}_1 - \mathcal{G}_{12/22}\mathcal{F}_2 + (\mathcal{G}_{11} - \mathcal{G}_{12/22}\mathcal{G}_{21})i_{t-1} + \frac{\mathcal{G}_{12}}{\mathcal{G}_{22}}(i_t^{(n)} - i_t)
+ \left[\mathcal{H}_1 - \mathcal{G}_{12/22}\mathcal{H}_2\right]\varepsilon_t + \left[\mathcal{I}_1 - \mathcal{G}_{12/22}\mathcal{I}_2\right]\varepsilon_{t-1},
\]
where \(\mathcal{G}_{12/22} = \frac{\mathcal{G}_{12}}{\mathcal{G}_{22}}\) and the matrix and vector sub-indices denote partitions that are described in the appendix.

This particular type of representation suggests a policy rule where the interest rate is set responding systematically to the lagged value of the interest rate and the current value of an interest rate spread. The additional uncertainty terms can be seen as monetary policy shocks. These rules are analyzed in subsection 2.3.5 using spreads of different maturities to understand the dependence of an interest-rate smoothing rule on the term structure.

### 2.3.5 Calibration and Policy Experiment

This section contains a numerical exercise where the model is calibrated to data and the analysis described in section 2.3 is performed. In addition, a policy experiment that consists in reducing the weight of inflation in the policy objective function is conducted. This experiment is justified by the possibility of implementing welfare-improving policies by changing the weight that the monetary authority assigns to inflation in the objective function. Therefore, comparisons in several dimensions between policies with high and low distastes for inflation are provided.

#### Data and Parameter Values

We use quarterly U.S. data from 1976:1 to 2005:4 for interest rates, consumption and consumer prices. The zero-coupon yields for 1 to 10 year bonds are obtained using the Svensson (1994) methodology applied to off-the-run Treasury coupon securities at the Federal Reserve Board.\(^{10}\) The short-term nominal interest rate is the 3-month T-bill rate from the Fama risk-free rates database. The consumption growth series was constructed using quarterly data on real per capita

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\(^9\)The extension consists in finding the appropriate representation for the interest rate when there are current and lagged values of the same variable in the set of state variables \(s_t\), that is, when the matrix \(\Phi\) in equation (2.11) is not full rank.

consumption of nondurables and services from the Bureau of Economic Analysis. The inflation series was constructed to capture inflation related only to non-durables and services consumption, following the methodology in Piazzesi and Schneider (2006).

The model parameters were chosen to match selected macroeconomic and term structure statistics. Appendix 2.5.2 provides analytical computations of the model-implied statistics and some details of the calibration. The procedure allows us to perfectly match the means, standard deviations and the autocorrelations of consumption growth and inflation. We obtain the weight of inflation in the policy objective function (2.2) by matching the correlation between consumption growth and inflation. This weight is obtained from

\[ \hat{\rho} = -\frac{1}{1 - \hat{\phi}_c} \frac{\text{cov}(\Delta c_t, \pi_t)}{\text{var}(\pi_t)}, \]

where the ”\&” symbol denotes the selected parameter value and ”\~” is the sample statistic. It is clear from the equation above that the model associates stronger negative correlations between consumption growth and inflation with a stronger distate for inflation.

Table 2.1: Parameter Values

<table>
<thead>
<tr>
<th>(\beta)</th>
<th>(\phi_c)</th>
<th>(\theta_\pi)</th>
<th>(\rho)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9999</td>
<td>0.9874</td>
<td>1.0228 \times 10^{-2}</td>
<td>0.8437</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(\sigma_c)</td>
<td>(x^*)</td>
<td>(\rho)</td>
</tr>
<tr>
<td>0.975</td>
<td>4.1503 \times 10^{-4}</td>
<td>0</td>
<td>0.8298</td>
</tr>
<tr>
<td>(\theta_c)</td>
<td>(\eta)</td>
<td>(\kappa)</td>
<td>(\sigma_\epsilon)</td>
</tr>
<tr>
<td>4.9312 \times 10^{-3}</td>
<td>22445</td>
<td>0.024</td>
<td>6.4259 \times 10^{-4}</td>
</tr>
</tbody>
</table>

Table 2.1 shows the parameter values used in the calibration. It is assumed for simplicity that \(\hat{x}^* = 0\). That is, the natural rate of consumption in the model economy is efficient and, as a consequence, there is no inflation bias (\(\hat{\delta}_\pi = 0\)). The coefficient \(\kappa\) that captures the relation between inflation and the output gap in the aggregate supply equation was obtained from the monetary policy literature. The implied weight of inflation in the policy function is high, given by \(\frac{\hat{\rho}}{\hat{\rho} + \hat{\kappa}} = 97.23\%\), as well as the autocorrelation of the natural consumption growth, \(\phi_c\). The latter is required to match the variability of consumption growth. Given that the autocorrelation of the supply shock, \(\phi_c\), is set to match the autocorrelation of inflation during the sample period, the high level of \(\phi_c\) plays a significant role trying to capture the volatility of long-term rates. The preference shock parameter \(\eta\) is positive, implying that positive lagged consumption growth decreases the level of utility today. Since consumption growth is negatively related to changes in inflation, a positive \(\eta\) also implies that the level of utility increases with positive lagged changes in inflation.
A Welfare-Improving Policy and a Policy Experiment

The policy analyzed in section 2.2 is the optimal policy under discretion. Kydland and Prescott (1977) show that, given the forward-looking nature of economic agents, a monetary authority can obtain better results in terms of economic welfare if it is able to conduct a policy under commitment. This is possible given the ability of commitment affecting agents’ expectations. A welfare-improving policy has direct effects on the equilibrium macroeconomic dynamics and, therefore, it is reasonable to think that the dynamics of the term structure, its predictive power and economic content will be different under such a policy than under discretion. We analyze here the effects of conducting a “restricted optimal” policy under commitment. It is restricted because it is optimal within a family of policies where the level of inflation and the output gap depend linearly on supply shocks.

Woodford (2003) suggests decomposing the policy objective function (2.2) as

$$
\mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t \left[ \rho \pi_t^2 + \kappa (x_t - x^*)^2 \right] \right\} = \sum_{t=0}^{\infty} \beta^t \left( \rho \mathbb{E}[\pi_t]^2 + \kappa \mathbb{E}[x_t - x^*]^2 \right) + \sum_{t=0}^{\infty} \beta^t \left( \rho \text{var}(\pi_t) + \kappa \text{var}(x_t) \right). \quad (2.25)
$$

He refers to the second term as the stabilization component and proposes to rank policies (for welfare purposes) according to the size of this component. This criterion is used here to find the optimal policy within a restricted family of policies.

Consider the family of policies where the output gap and inflation depend linearly on the supply shock. The appendix shows the derivation of the policy under commitment within this family that minimizes the stabilization component above. This policy implies the targeting rule

$$
x_t - x^* = -\frac{\rho}{1 - \beta \phi_\epsilon} \pi_t = -\rho_c \pi_t.
$$

If supply shocks are autocorrelated, the “restricted optimal” policy involves a stronger reaction of the output gap to inflation than in the discretionary case in equation (2.5). Moreover, this policy can be implemented under discretion by increasing the relative weight of inflation in the policy objective function to $\rho_c$. As a result, the analysis in section (2.2) remains valid for this type of policy rules.

The considerations above constitute the basis for the policy experiment conducted here, allowing us to provide welfare interpretations. We assume that during the sample period the Federal Reserve implemented the “restricted optimal” policy under commitment by raising the weight of
inflation in the policy objective function. It implies that \( \hat{\rho} = \rho_{rc} \), and therefore the “true” relative inflation weight was

\[
\rho_{low} = \hat{\rho}(1 - \hat{\beta}\hat{\phi}_\epsilon) = 0.1437
\]

and \( \frac{\rho_{low}}{\rho_{low} + \kappa} = 85.69\% \). The policy experiment consists then on reducing the relative weight of inflation to \( \rho_{low} \) to observe its macroeconomic and term structure implications. We refer to the calibrated model and the experiment model as the \( \rho \)-economy and \( \rho_{low} \)-economy, respectively.

**Consumption Growth, Inflation and the Yield Curve**

This section analyzes the macroeconomic and interest-rate differences between the two economies described above. Table 2.2 shows selected statistics for the data and the two model economies.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>( \rho )</th>
<th>( \rho_{low} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E[\Delta c_t] \times 4 )</td>
<td>1.97%</td>
<td>1.97%</td>
<td>1.97%</td>
</tr>
<tr>
<td>( E[\pi_t] \times 4 )</td>
<td>4.09%</td>
<td>4.09%</td>
<td>4.09%</td>
</tr>
<tr>
<td>( \sigma(\Delta c_t) \times 4 )</td>
<td>1.59%</td>
<td>1.59%</td>
<td>1.07%</td>
</tr>
<tr>
<td>( \sigma(\pi_t) \times 4 )</td>
<td>2.42%</td>
<td>2.42%</td>
<td>2.65%</td>
</tr>
<tr>
<td>( \text{corr}(\Delta c_t, \Delta c_{t-1}) )</td>
<td>0.38</td>
<td>0.38</td>
<td>0.94</td>
</tr>
<tr>
<td>( \text{corr}(\pi_t, \pi_{t-1}) )</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>( \text{corr}(\Delta c_t, \pi_t) )</td>
<td>-0.22</td>
<td>-0.22</td>
<td>-0.06</td>
</tr>
<tr>
<td>( E[i_t] \times 4 )</td>
<td>6.12%</td>
<td>6.12%</td>
<td>6.12%</td>
</tr>
<tr>
<td>( E[i_t^{(20)}] \times 4 )</td>
<td>7.38%</td>
<td>7.59%</td>
<td>6.30%</td>
</tr>
<tr>
<td>( E[i_t^{(40)}] \times 4 )</td>
<td>7.80%</td>
<td>7.80%</td>
<td>6.26%</td>
</tr>
<tr>
<td>( \sigma(i_t) \times 4 )</td>
<td>3.21%</td>
<td>2.56%</td>
<td>2.49%</td>
</tr>
<tr>
<td>( \sigma(i_t^{(20)}) \times 4 )</td>
<td>2.78%</td>
<td>1.83%</td>
<td>1.23%</td>
</tr>
<tr>
<td>( \sigma(i_t^{(40)}) \times 4 )</td>
<td>2.52%</td>
<td>1.64%</td>
<td>0.98%</td>
</tr>
<tr>
<td>( \text{corr}(i_t, i_{t-1}) )</td>
<td>0.93</td>
<td>0.86</td>
<td>0.86</td>
</tr>
<tr>
<td>( \text{corr}(i_t, i_t^{(20)}) )</td>
<td>0.87</td>
<td>0.75</td>
<td>0.84</td>
</tr>
<tr>
<td>( \text{corr}(i_t, i_t^{(40)}) )</td>
<td>0.84</td>
<td>0.61</td>
<td>0.70</td>
</tr>
<tr>
<td>( \text{corr}(i_t, \Delta c_t) )</td>
<td>-0.08</td>
<td>0.08</td>
<td>0.36</td>
</tr>
<tr>
<td>( \text{corr}(i_t, \pi_t) )</td>
<td>0.68</td>
<td>0.91</td>
<td>0.91</td>
</tr>
</tbody>
</table>

**Macroeconomic Variables**

The \( \rho \)-economy matches the macroeconomic statistics of the data by construction. It is observed that the policy that is less conservative about inflation does not affect the long-run averages for consumption growth and inflation. In the absence of an inflation bias, the monetary authority is able on average to reach the inflation target. Also, prices are flexible on the long-run and
consumption growth follows the trend of natural consumption growth. The differences between
the two economies are observed in terms of the variability of the macroeconomic variables. The
optimal targeting equation (2.5) shows us that a less conservative policy about inflation allows less
deviations in the output gap from its target. It translates into a higher volatility of inflation and
a lower volatility of consumption growth. Simultaneously, a policy under discretion is not able
to break the direct link between the persistence of the supply shocks and the persistence of inflation.
Therefore, the autocorrelation of inflation is the same in the two economies. However, the greater
weight of the output gap in the policy objective function makes consumption growth to increase
the correlation with the natural rate of consumption growth. Given the high autocorrelation of
natural consumption growth in the calibration, it implies an increase in the autocorrelation of
consumption growth in the $\rho_{low}$-economy. As mentioned before, a lower inflation weight reduces
the negative correlation between consumption growth and inflation.

The lowest panel of Table 2.2 shows the correlation of the one-period rate with consumption
growth and inflation. While the correlation with inflation is practically unaffected by $\rho$, the
correlation with consumption growth increases, reflecting the reduction of the negative impact of
inflation on consumption. The $\rho$-economy implies a slightly positive correlation of the short-term
rate with consumption growth which differs from the slightly negative correlation observed in
the data. With respect to the correlation with inflation, it is positive and higher than the one
observed during the sample period.

**The Term Structure of Interest Rates**

Some important properties of interest rates implied by the data and the model economies are
presented in Table 2.2 and Figure 2.1. The model parameters $\beta$, $\gamma$ and $\eta$ for the $\rho$-economy were
used to match the level of the 3-month and 10-year interest rates and minimize the difference
between the short-term rate volatility implied by the model and that of the data. Although
the slope of the yield curve is captured, Panel A of the figure shows that the level of interest
rates for intermediate maturities are not closely matched. Panel B allows us to see the inability
of the model to capture simultaneously the high volatility of interest rates and the volatility of
consumption growth and inflation. Interest rates are not volatile enough when the model captures
macroeconomic volatility. However, the model is successful generating significant volatility of
long-term rates relative to the volatility of short-term rates. The ratios 10-year rate volatility to 3-
month rate volatility implied by the data and the $\rho$-economy are 61% and 41%, respectively. This
achievement is due to the high persistence of the natural consumption growth process. However,
the ratio decreases to 15% in the $\rho_{low}$-economy. This is the result of a reduced time variation in
the market prices of risk when the monetary authority is less concerned about inflation.

Panel C of Figure 2.1 also shows an important decrease in the size of one-period term premia
(and their volatility). It explains the significant decline in the slope of the yield curve. Thus,
the model implies that if the monetary authority assigns more weight to the output gap in the objective function, we should observe lower term premia. Equivalently, a high relative weight on inflation creates significant distortions in the real side of the economy, and bondholders demand high compensations for holding long-term bonds despite the benefits of a reduced inflation risk. However, the reduction in the inflation risk is significant enough to offset the higher systematic risk of holding long-term bonds and it allows for an increase in economic welfare.

Expected Future Monetary Policy

The analysis in section 2.3.2 evidences the difficulties in extracting expectations about future short-term rates from forward rates when the yield curve exhibits time-varying expected returns. It also shows how these difficulties might be overcome using, in addition to the forward rate, macroeconomic and other term structure information to extract expectations about future monetary policy. This section intends to quantify the differences between the forward rate and the expected short-rate in the ρ-economy and the ρlow-economy, on average and when some shocks impact the economy. In addition, it presents the sensitivity of expected future one-period rates for different horizons to the current short-term rate, the appropriate forward rate and lagged
It can be observed from equation (2.20) that the difference between the forward rate and the expected future short-term rate has two components. The first component is the one-period expected excess return and the second component captures changes in expected excess returns during the life of the bond linked to the forward rate. Unconditionally, the second component vanishes and the long-run differences between forward rates and expected short rates are reflected only in the first component. Figure 2.2 presents these differences for the two model economies. When the weight of inflation is high in the policy objective function, the differences between the forward rate and the average short-rate tend to increase with maturity (this is consequence of the upward sloping yield curve). When the weight of inflation decreases, the yield curve tends to be flatter, the differences for intermediate maturities are high (but much lower than in the $\rho$-economy and decline for longer maturities.

Figure 2.2 also shows variations in the difference between forward rates and expected short rates as a result of shocks to natural consumption growth and supply shocks. While supply shocks do not considerably affect this difference, shocks to natural consumption growth do. The impact is quantitatively more significant in the $\rho$-economy than in the $\rho_{low}$-economy. A positive shock to natural consumption increases the forward rate more than the associated expected short-rate, implying also changes in expected excess returns for different maturities. It indicates that a shock to natural consumption growth does not require the one-to-one reaction in future monetary policy (interest rate) that seems to require a supply shock. In summary, the different responses

Figure 2.2: Difference between forward rates and expected one-period rate for different horizons. The “Initial fwd. curve” line denotes the unconditional difference $\mathbb{E}[f_{t+n} - i_t + n]$. The other two lines show the differences conditional on a natural consumption growth shock and a supply shock, respectively, of size one.
of forward rates and expected short rates to different shocks make it difficult to use the forward rate as a reliable indicator of future monetary policy.

Given the difficulties discussed above, an alternative way to obtain indicators of future monetary policy is using term structure information, in addition to the forward rate, and macro-economic information that may be available to policy makers. Here we consider the case of representing expected future one-period rates in terms of the current level of the rate, the forward rate and lagged inflation. Figure 2.3 contains the loading coefficients of functions of the form

$$\mathbb{E}_t[i_{t+n}] = \alpha_n + \beta_{i,n}i_t + \beta_{f,n}f_{t,n} + \beta_{\pi,n}\pi_{t-1} + \epsilon_{f,t+n}.$$ 

Under the expectations hypothesis, the loading coefficient on the forward rate ($\beta_{f,n}$) is 1 and zero for the other two explanatory variables. The existence of time-varying term premia makes these coefficients to depend on the weight of inflation on the policy function. Panel B of the figure shows that the loading coefficients on the forward rates are different from 1. In the $\rho$-economy, term premia are more volatile than under the $\rho_{low}$-economy and for that reason the correlation between the forward rate and expected future short-rates is lower. Therefore, for short-term horizons the current level of the short-term rate and lagged inflation have important explanatory power about future monetary policy when the weight of inflation is high. However, this explanatory power decreases with maturity and tend to be very similar to that under the low inflation weight for long-term maturities. As a result, a policy maker that is conservative with respect to inflation should not rely only on the forward rate to predict changes in expectations of future monetary policy for short horizons but also should pay attention to other indicators as, for instance, the current level of the short-term rate. For longer horizons the forward rate is a useful indicator for

![Figure 2.3: Loading coefficients for the regression $i_{t+n} = \alpha_n + \beta_{i,n}i_t + \beta_{f,n}f_{t,n} + \beta_{\pi,n}\pi_{t-1} + \epsilon_{f,t+n}$ for one-period forward rates of different horizons.](image-url)
future monetary policy independently of how conservative is the monetary authority.

The Economic Content of the Term Structure

Section 2.3.3 shows that the term structure of interest is a rich source of macroeconomic information that, however, is not easy to extract. In this section we examine the model implications of considering expected inflation simply as the difference between the nominal and the real interest rate. We also analyze how the economic content of different maturity sectors of the yield curve and their predictive power are affected by the weight that the monetary authority gives to inflation in the policy function.

The Term Structure of Real Interest Rates

As shown in equation (2.21), time-varying term premia make it difficult to extract expected future inflation from the yield curve. It is useful to compute the model-implied term structure of real interest rates and decompose the difference between nominal and interest rates into their expected inflation and term premium components. This decomposition allows us to observe the possible mistakes involved in using the difference between nominal and real rates as indicator of future inflation.

The real term structure is an affine term structure of the class presented in section 2.2.3. The characterization of the associated real stochastic discount factor is presented in Proposition 11.

Proposition 11. The real pricing kernel $M_{t,t+1}^{real}$ can be represented by equations (2.12) and (2.13) and the parameterization for the state variables in Proposition 11 when

$$s_t \equiv (\Delta c_t^{nat}, \pi_t, \pi_{t-1})^\top$$

and

$$\Gamma_0^{real} = -\log \beta + \gamma(1-\phi_c)\theta_c - \gamma \rho (1-\phi_c) \delta_\pi - \frac{1}{2} \gamma^2 \sigma_\pi^2,$$

$$\Gamma_1^{real} = \left[ \gamma(\phi_c - \eta \sigma_\mu^2) \quad (1-\phi_c + \eta \sigma_\mu^2)\gamma \rho \quad - \eta \rho \gamma \sigma_\mu^2 \right]^\top,$$

$$\lambda_0^{real} = \gamma \begin{pmatrix} 1 \\ -\rho \end{pmatrix}, \quad \lambda_1^{real} = \lambda_1, \quad \text{and} \quad \sigma_\mu^2 = \sigma_\epsilon^2 + \rho^2 \delta_\epsilon^2 \sigma_\epsilon^2.$$

A comparison of the coefficients in propositions 10 and 11 is helpful to understand the differences between the real and nominal pricing kernels. The absence of inflation in the real pricing kernel eliminates not only a constant precautionary savings motive for inflation but also a term capturing the correlation between inflation and consumption growth. In addition, the constant component of the price of risk $\lambda_0^{real} \neq \lambda_0$. However, this component is still sensitive to inflation.
given the fact that consumption growth is negatively correlated with inflation. The sensitivity depends then on $\rho$. It is important to note, however, that the time varying component of the market price of risk is the same as for the nominal pricing kernel. It is the case, since the variability of the price of risk is entirely driven by the preference shock.

![Figure 2.4: Real Interest Rates and Impulse Responses of the difference $i_t - r_t - \bar{\pi}_t[\theta + \pi_{t+1}]$.](image)

Now we can analyze the differences between the nominal and real term structures. Panel A of Figure 2.4 shows the real term structures for the two model economies. While the real yield curve is upward sloping when the weight of inflation is high, it becomes downward sloping when the output gap gains relative importance in the policy function. It implies that a policy that pays significant attention to the stabilization of the output gap reduces the sources of real risk and agents perceive long-term real bonds as low-risk instruments. However, if the monetary authority becomes more conservative about inflation, consumption growth is less correlated to natural consumption growth and this deviation represents a real risk that translates into positive expected excess returns in long-term real bonds. Additionally, Figure 2.11 in the appendix shows the difference between the average nominal rates and the sum of the average real rates and unconditional expected inflation. For the one-period interest rates, the difference is high (more than 40 bps.) when the inflation weight in the policy function is high and it becomes small if the monetary authority reduces the weight on inflation. Therefore, the significant amount of term premia in nominal and real rates when the monetary authority is more conservative about inflation makes the difference between the nominal and real rate a poor indicator of one-period ahead inflation.

It is also useful to observe how economic shocks affect the difference between the nominal rate and the real rate plus expected inflation. Panels B and C of Figure 2.4 contain impulses responses to a shock to natural consumption growth and a supply shock, respectively. The difference is
more sensitive to the shocks when monetary policy is more conservative about inflation. A shock
to the natural rate of consumption has effects that die out slowly but, at the same time, the
magnitude of the effect is small even for a high inflation weight. This is no longer true for a
supply shock. A positive supply shock initially reduces the difference between the nominal rate
and the real rate plus expected inflation. The initial reaction is followed by an increase in the
difference that disappears over time. The effect is highly amplified by a high weight of inflation
in the policy function and can produce a significant initial reduction in the difference (20 bps.
in the graph). The reason for the reduction is the big impact of the supply shock on the output
gap as a result of the targeting policy. The subsequent amplification in the difference is the
result of the dependence of term premia on changes in the output gap rather than the level of
the output gap. Then, the initial reduction produced by the supply shock is offset by the lagged
effect one period after. To summarize, when monetary policy pays significant attention to price
stabilization, expected inflation becomes more difficult to extract from the yield curve given the
increased variability in term premia.

Predictability of Macro Variables and the Expectations Hypothesis

Forward-looking economic agents use long-term bonds to implement consumption and invest-
ment contingent plans over time. As a result, long-term interest rates can be useful to predict
future economic activity. As shown in equation (2.19), long-term rates can be seen as risk-adjusted
expectations of future short-term rates. Then, it is helpful to ask whether any predictability of
interest rates comes from their expectations hypothesis or term premium components, and how
this predictability is affected by the inflation weight in the policy objective function.

Consider univariate regressions of the type described by equation (2.22), where the regressor
is the term spread or its expectations hypothesis component. The comparison of these two
regressions provides an idea of the role of term premia in the predictability of future economic
conditions. Figure 2.5 presents the slope coefficients for this type of regressions for consumption
growth and inflation one period ahead. The regressions are run for spreads of different maturities.
Panels A and B show that the predictive power of spreads about future consumption growth
depends on the type of monetary policy that is conducted. A policy paying significant attention
to price stability involves greater predictive ability of the term structure than a policy which
assigns more weight to output gap deviations. The reason is the existence of persistent output
gaps when the inflation weight is high. It can also be seen that the predictive power declines
with maturity. The differences in the magnitude of the coefficients between the term spread and
its expectations hypothesis component show that the time-varying term premia in interest rates
reduce the predictive power for consumption growth. The $R^2$'s for the regressions in Figure 2.12
in the appendix show the low ability of spreads to explain the variability of consumption growth.

With respect to the predictability of inflation, Panels C and D of Figure 2.5 show small
Figure 2.5: Slope coefficients for univariate regressions $mv_{t+1} = \alpha_{mv,1} + \beta_{mv,1}(i_t^{(u)} - i_t) + \varepsilon_{mv,1}$ for $mv = \{\Delta c, \pi\}$. The regressor is the interest rate spread or its expectations hypothesis component for different maturities.

differences between policies with high or low weight on inflation. However, the directions predicted by the spread and its expectations hypothesis are different. While a positive spread predicts deflation (relative to the trend), the related expectations hypothesis component predicts positive inflation. It suggests a risk premium component in interest rates that entirely shifts the predictive ability of the yield curve with respect to inflation. That is, positive expected excess return signal low inflation in the future. Figure 2.12 in the appendix show that term-spreads explain a significant fraction of the variability of inflation. This fraction is affected by the weight of inflation in the policy function for short-term spreads. A high weight reduces the explanatory power of spreads.

**Predictive Power of the Yield Curve. Short-Term Rate vs. Term Spreads**

It is also natural to ask whether the one-period interest rate has more predictive power of future economic conditions than term spreads. This question can be approached running bivariate regressions where the regressors are the one-period rate and a term spread. Figure 2.6 presents the slope coefficients for this type of regressions for consumption growth and inflation using yields of different maturities. The one-period nominal rate has important explanatory power for future consumption growth and inflation as can be inferred from Panels C and F in the figure. This
power depends on the type of monetary policy that is conducted. In particular, the $R^2$’s for the consumption growth regressions are significantly lower in the $\rho$-economy than in the $\rho_{low}$-economy. Also, the loadings on the short rate for the consumption growth regressions are smaller than in the $\rho_{low}$-economy and tend to increase with the maturity of the term spread. The loadings on the short rate for the inflation-related regressions are higher when the inflation weight in the policy function is high. In summary, the current level of the one-period interest rate contains considerable amount of information about consumption growth one period ahead, which is not contained in spreads. It does not add much information about future inflation that is not already provided by the spread.

Extracting Current Consumption Growth and Inflation from Interest Rates

The economic model implies that the current values for consumption growth and inflation can be obtained from term structure information without error. This possibility can be seen as a useful tool for policy makers that have to make decisions in an environment where macroeconomic data are collected and processed with a lag and may involve some noise. Figure 2.7 shows the
coefficients of representations of consumption growth and inflation in terms of the short-term rate, term spreads and lagged inflation. Lagged inflation was chosen for simplicity given that it is part of the original set of state variables. It can be replaced with another lagged value of a macro variable or with an additional interest rate spread or a linear combination of spreads. Qualitatively, the implications are similar to those for the predictability of macro variables one period ahead. In particular, the presence of a third state variable (lagged inflation in this case) is helpful to obtain the value of consumption growth when the weight of inflation in the policy function is high ($\rho$-economy). It is due to the lack of enough explanatory power of the short-term rate and one spread under these circumstances.

**Policy Rules**

Section 2.3.4 describes how the equilibrium one-period interest rate can be written in terms of different macroeconomic and term structure information. It also suggests considering the equilibrium rate as a policy rule to implement the optimal monetary policy associated with the
objective function (2.2). We analyze here different policy rules that react to macroeconomic and
term structure information for the two model economies (high and low inflation weight in the
objective function). We characterize first a rule that reacts only to macroeconomic variables for
an initial comparison of the policies in the two model economies. Then we evaluate four different
types of rules: rules that only respond to term spreads, rules that react to both macro variables
and interest rates, rules that respond to a lagged short-term rate, current spreads and lagged
inflation and, finally, rules with interest rate smoothing.

A Macroeconomic Rule

Table 2.3 contains the coefficients of a rule that sets the one-period rate responding to con-
sumption growth and the current and lagged levels of inflation. The rules for the two economies
assign positive coefficients to consumption growth and current inflation and a negative coefficient
to lagged inflation. However, the coefficients for inflation are higher when the monetary authority
is more conservative about inflation. This is consistent with the empirical evidence presented in
Clarida, Galí and Gertler (2000) describing monetary policy during the period when Paul Volcker
and Alan Greenspan were chairmen of the Federal Reserve System.

Table 2.3: Coefficients for the Macroeconomic Rules - High an Low Inflation Weight Economies

<table>
<thead>
<tr>
<th>Economy</th>
<th>Constant</th>
<th>Δc_t</th>
<th>π_t</th>
<th>π_{t-1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>0.0416</td>
<td>0.9971</td>
<td>1.7821</td>
<td>-0.8122</td>
</tr>
<tr>
<td>ρ_{low}</td>
<td>0.0416</td>
<td>0.9969</td>
<td>0.9920</td>
<td>-0.1383</td>
</tr>
</tbody>
</table>

A Term Structure Rule

The macroeconomic rule presented above can be written in terms of interest rate spreads. Table 2.4 shows the coefficients of a rule that reacts to three different sectors of the term struc-
ture: the two-year, five-year and ten-year spreads. The rules for the ρ and ρ_{low} economies are
qualitatively the same. There are negative reactions to the short and long ends of the curve (2
and 10 year spreads) and a positive reaction to the 5-year spread. Quantitatively, the rule when
the policy is more conservative about inflation implies stronger reactions to the three sectors of
the curve.

Table 2.4: Coefficients for the Macroeconomic Rules - High an Low Inflation Weight Economies

<table>
<thead>
<tr>
<th>Economy</th>
<th>Constant</th>
<th>i_t^{(8)} - i_t</th>
<th>i_t^{(20)} - i_t</th>
<th>i_t^{(40)} - i_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>0.0709</td>
<td>-5.0748</td>
<td>14.4206</td>
<td>-10.4906</td>
</tr>
<tr>
<td>ρ_{low}</td>
<td>0.0578</td>
<td>-4.2440</td>
<td>13.0671</td>
<td>-9.9480</td>
</tr>
</tbody>
</table>
A Macroeconomic and Term Structure Rule

We can also analyze a representation for the short-term rate that combines macroeconomic and term structure information. This type of representations are useful to understand what type of information can be captured by the term structure that is not already contained in the macroeconomic variables that are part of the policy rule. Figure 2.8 shows the coefficients for policy rules that react to consumption growth, an interest rate spread and lagged inflation, using different maturities for the spread. This rules can be compared against the macroeconomic rule presented above, to see how replacing current inflation with a term spread affects a policy rule. The figure shows that the implied rules under the \( \rho \)-economy are much more sensitive to the variables than rules for the \( \rho_{low} \)-economy. For the \( \rho \)-economy the coefficient for consumption growth increases significantly when a spread with intermediate maturity is used, but for longer maturities the coefficient decreases. When the weight of inflation is low, the coefficients do not vary significantly with the maturity of the spread. The coefficients of the spreads are negative and increase (less negative) with maturity. While the spread coefficients get closer to zero quickly with maturity when the weight of inflation is low in the policy function, they are important describing the short-term rate when the weight of inflation is high. This difference in the sensitivity of the policy rule to the spreads in the two economies is related to the fact that long-term rates are more volatile in the \( \rho \)-economy.

A Policy Rule with Interest Rate Smoothing

Equation (2.24) shows us that it is possible to represent the equilibrium short-term rate depending on its one-lag value, an interest-rate spread and some error terms. This suggests a policy rule that smooths interest rates over time. Figure 2.9 shows the coefficients for the lagged interest rate and the spread for this type of rules using different maturities for the spread.

Figure 2.8: Loading coefficients for a policy rule that reacts to consumption growth, an interest-rate spread and lagged inflation, for different spread maturities.
The loadings on the lagged value for the interest rate and the spread are positive. While the loading on the interest rate increases with the maturity of the spread the loading on the spread decreases. Therefore, the information that spreads contain about the short-term rate declines with the maturity of the spread in this representation. A reduction in the weight of inflation in the policy objective function increases the lagged interest rate loading to levels that can be greater than one for some spreads. It can also be seen that using short-term spreads the loading on the spread in the $\rho$-economy is significantly higher than in the low inflation weight economy. However, the differences die out fast as longer maturities are used in the rule.

### 2.4 Conclusion

This chapter studies the economic content of interest rates using an equilibrium theory of the term structure with time-varying term premia and optimal monetary policy. The study of the economic information contained in interest rates is of great importance for policy makers who require tools to understand current economic conditions and make appropriate decisions. It is shown that the economic content of the yield curve considerably depends on the actual monetary policy regime. The power of interest rates to predict future monetary policy, real economic activity and inflation is highly sensitive to the weight that the monetary authority assigns to inflation on the policy objective. A high weight for inflation allows greater instability in consumption that increases the compensation for real risk in nominal interest rates. It is also shown that an optimal monetary policy can be implemented through interest-rate rules that react to term structure information. These rules offer informational advantages over macroeconomic rules and then may imply welfare benefits. The specific characteristics of these rules, such as the responses to different maturity
sectors in the term structure, are highly sensitive to the policy objective.
2.5 Appendix

2.5.1 Term-Structure-Based Policy Rule with Interest Rate Smoothing

The original set of state variables $s_t \equiv (\Delta c_t^{(n)}, \pi_t, \pi_{t-1})$ contains two contemporaneous values and one lagged valued. Consider the reduced set of contemporaneous state variables $s_r \equiv (\Delta c_t^{(n)}, \pi_t)$, with process

$$s_{r+1}^r = (I_2 \otimes 2 - \Phi_r)\theta_r + \Phi_r s_r^r + \Psi_r^\top \Sigma^{1/2} \varepsilon_{t+1}, \quad (2.26)$$

where $\Phi_r = \text{diag}(\phi_c, \phi_\epsilon)$, $\theta_r = (\theta_c, \delta_\pi)^\top$ and $\Psi_r = \text{diag}(1, \delta_\epsilon)$. The one-period interest rate and a term spread for some maturity can be represented by $s_r^r$ in the system

$$\begin{align*}
\begin{pmatrix}
i_t \\ i_t^{(n)} - i_t
\end{pmatrix} &= \begin{pmatrix} A_1 \\ \frac{1}{n}A_n - A_1 \end{pmatrix} + \begin{pmatrix} B_1^\top e_1 \\ (\frac{1}{n}B_n - B_1) e_1 \\ (\frac{1}{n}B_n - B_1) ^\top e_2 \end{pmatrix} s_r^r \\
&\quad + \begin{pmatrix} 0 \\ B_1^\top e_3 \\ 0 \end{pmatrix}(\frac{1}{n}B_n - B_1) ^\top e_3 s_{r-1}^r = \mathbb{K} + Ls_r^r + Ms_{r-1}^r \\
&= \mathbb{K} + L(\mathbb{I}_{2 \times 2} - \Phi_r)\theta_r + (L\Phi_r + M)s_{r-1}^r + L\Psi_r^\top \Sigma^{1/2} \varepsilon_t \\
&= \tilde{C} + \tilde{D}s_{r-1}^r + \mathcal{H}\varepsilon_t 
\end{align*}
$$

Therefore, if $\tilde{D}$ is an invertible matrix, the reduced set of state variables can be written in terms of the short-term rate and the term spread as

$$s_{r-1}^r = \tilde{D}^{-1} \begin{pmatrix}
i_t \\ i_t^{(n)} - i_t
\end{pmatrix} - \tilde{C} - \mathcal{H}\varepsilon_t.$$

Replacing it into equation (2.26) and then into equation (2.27) again, we obtain the partitioned representation

$$\begin{pmatrix}
i_t \\ i_t^{(n)} - i_t
\end{pmatrix} = \begin{pmatrix} \mathcal{F}_1 \\ \mathcal{F}_2 \end{pmatrix} + \begin{pmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} \\ \mathcal{G}_{21} & \mathcal{G}_{22} \end{pmatrix} \begin{pmatrix}
i_{t-1} \\ i_t^{(n)} - i_{t-1}
\end{pmatrix} + \begin{pmatrix} \mathcal{H}_{1(1\times2)} \\ \mathcal{H}_{2(1\times2)} \end{pmatrix} \varepsilon_t + \begin{pmatrix} \mathcal{I}_{1(1\times2)} \\ \mathcal{I}_{2(1\times2)} \end{pmatrix} \varepsilon_{t-1},$$

where

$$\begin{pmatrix} \mathcal{F}_1 \\ \mathcal{F}_2 \end{pmatrix} = \tilde{C} + \tilde{D}(\mathbb{I}_{2 \times 2} - \Phi_r)\theta_r - \tilde{D}\Phi_r\tilde{D}^{-1} \tilde{C}, \quad \begin{pmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} \\ \mathcal{G}_{21} & \mathcal{G}_{22} \end{pmatrix} = \tilde{D}\Phi_r \tilde{D}^{-1}$$

and

$$\begin{pmatrix} \mathcal{I}_1 \\ \mathcal{I}_2 \end{pmatrix} = \tilde{D}(\mathbb{I}_{2 \times 2} - \Phi_r\tilde{D}^{-1}\mathbb{L})\Psi_r^\top \Sigma^{1/2}.$$

If $\mathcal{G}_{22} \neq 0$, the inferior partition allows us to write the lagged spread in terms of the current spread, the lagged short-term rate and some errors. Replacing that representation for the lagged
spread into the superior partition, we obtain the interest-rate smoothing representation (2.24) for
the one-period rate.

2.5.2 Model-Implied Moments and Calibration Details

Let $\pi_t^{total} = \theta_{\pi} + \pi_t$ be the level of inflation. The moments that were considered for the calibration
are presented in Table 2.5.

Table 2.5: Analytical Characterization of Some Model-Implied Statistics

<table>
<thead>
<tr>
<th>$\mathbb{E}[\pi_t^{total}] = \theta_{\pi} + \delta_{\pi}$</th>
<th>$\mathbb{E}[\Delta c_t] = \mathbb{E}[\Delta c_t^n] = \theta_{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{var}(\pi_t^{total}) = \text{var}(\pi_t) = \delta_{\pi}^2 \text{var}(\epsilon_t)$</td>
<td>$\text{var}(\Delta c_t) = \text{var}(\Delta c_t^n) + 2\rho^2(1 - \phi_{\pi})\text{var}(\pi_t)$</td>
</tr>
<tr>
<td>$\text{cov}(\pi_t^{total}, \pi_{t-1}^{total}) = \text{cov}(\pi_t, \pi_{t-1}) = \phi_{\pi}\text{var}(\pi_t)$</td>
<td>$\text{cov}(\Delta c_t, \Delta c_{t-1}) = \phi_{c}\text{var}(\Delta c_t) - \rho^2(1 - \phi_{\pi})^2\text{var}(\pi_t)$</td>
</tr>
<tr>
<td>$\text{cov}(\Delta c_t, \pi_t^{total}) = -\rho(1 - \phi_{\pi})\text{var}(\pi_t)$</td>
<td>$\text{var}(\Delta c_t^n) = \frac{\sigma_{\epsilon,n}^2}{1 - \phi_{\epsilon}^2}$</td>
</tr>
<tr>
<td>$\text{var}(\epsilon_t) = \frac{\sigma_{\epsilon}^2}{1 - \phi_{\epsilon}^2}$</td>
<td>$\text{var}(\pi_t^{total})$</td>
</tr>
</tbody>
</table>

There are 12 parameters involved in the calibration: $\beta, \gamma, \eta, \phi_{c}, \theta_{c}, \sigma_{c}, \phi_{\epsilon}, \sigma_{\epsilon}, \kappa, \rho, x_{\text{star}}, \theta_{\pi}$. The value for the parameters ($x$) in the calibration are denoted by $\hat{x}$ and the sample values for the statistics ($y$) are denoted by $\tilde{y}$.

Let $\hat{x}^* = 0$, then $\hat{\delta}_{\pi} = 0$ and $\hat{\theta}_{\pi} = \hat{\mathbb{E}}[\pi_t^{total}]$. The other parameters were selected to match the moments presented above. Then,

$$\hat{\theta}_c = \hat{\mathbb{E}}[\Delta c_t], \quad \hat{\phi}_c = \frac{\text{cov}(\pi_t^{total}, \pi_{t-1}^{total})}{\text{var}(\pi_t^{total})},$$

$$\hat{\rho} = -\frac{1}{1 - \hat{\phi}_c} \frac{\text{cov}(\Delta c_t, \pi_t^{total})}{\text{var}(\pi_t^{total})},$$

$$\hat{\phi}_c = \frac{1}{\text{var}(\Delta c_t^n)} \left[ \text{cov}(\Delta c_t, \Delta c_{t-1}) + \rho^2(1 - \hat{\phi}_{\pi})^2\text{var}(\pi_t^{total}) \right],$$

$$\hat{\sigma}_{\epsilon}^2 = (1 - \hat{\phi}_{\epsilon}^2)(1 - \hat{\beta}\hat{\phi}_{\epsilon} + \hat{\rho}\kappa)^2\text{var}(\pi_t^{total})$$
where $\hat{\beta}$ is found to match the one-period interest rate as follows:

$$
\log \hat{\beta} = -\tilde{E}[i_t] + \hat{\theta}_c + \hat{\gamma}\hat{\theta}_c - \hat{\gamma}\hat{\eta}\hat{\sigma}_c^2\hat{\theta}_c + (1 - \hat{\gamma}\hat{\rho})\hat{\eta}(1 - \hat{\phi}_c^2)\text{var}(\pi_{t}^{\text{total}}) \hat{\theta}_c
$$

$$
- \frac{1}{2}\hat{\gamma}^2\hat{\sigma}_c^2 - \frac{1}{2}(1 - \hat{\gamma}\hat{\rho})^2(1 - \hat{\phi}_c^2)\text{var}(\pi_{t}^{\text{total}})
$$

The parameter value $\hat{\gamma}$ was set to 0.975 and $\hat{\eta}$ was set to match the slope of the curve. The value $\hat{\gamma}$ minimizes the difference between the volatility of the short-term rate in the data and the model. The parameter value for $\kappa$ was obtained from the monetary policy literature.

### 2.5.3 Welfare-Improving Policies

Consider the problem of finding $\pi_t$ and $x_t$ to minimize the stabilization component of the policy objective function in equation (2.25) subject to the aggregate supply equation (2.4) and restricting attention to policies were the equilibrium inflation and output gaps depend linearly on supply shocks.

Guess solutions of the form $\pi_t = \bar{\pi} + \pi_\epsilon\epsilon_t$ and $x_t = \bar{x} + x_\epsilon\epsilon_t$. In order to satisfy (2.4), it is necessary that

$$
\bar{\pi} = \frac{\kappa x^*}{1 - \beta} \quad \text{and} \quad \pi_\epsilon = \frac{\kappa x_\epsilon + 1}{1 - \beta\phi_\epsilon}.
$$

Given this restrictions, the problem for the monetary authority can be written as

$$
\min_{\pi_\epsilon} \frac{1}{2} \left[ \pi_\epsilon^2 + \frac{1}{\kappa \rho} ((1 - \beta\phi_\epsilon)\pi_\epsilon - 1)^2 \right] \sum_{t=0}^\infty \beta^t \text{var}(\epsilon_t).
$$

Solving this optimization problem it follows that

$$
\pi_\epsilon = \frac{1}{1 - \beta\phi_\epsilon + \frac{\kappa \rho}{(1 - \beta\phi_\epsilon)}} = \frac{1}{1 - \beta\phi_\epsilon + \kappa \rho rc}.
$$

From the restrictions above, the equilibrium output gap is found and the targeting rule follows.
2.5.4 Additional Figures

Figure 2.10: Model-implied Campbell-Shiller coefficients.

Figure 2.11: Real Interest Rates and Difference $E[i_t] - E[r_t + \theta \pi + \pi_{t+1}]$. 

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Figure 2.12: $R^2$'s for univariate regressions $mv_{t+1} = \alpha_{mv,1} + \beta_{mv,1}(i_{t}^{(n)} - i_t) + \varepsilon_{mv,1}$, for $mv = \{\Delta c, \pi\}$. The regressor is the interest rate spread or its expectations hypothesis component for different maturities.
Chapter 3

Bond Pricing, Habits and a Simple Policy Rule

3.1 Introduction

This chapter explores the bond-pricing implications of a model economy where (i) inflation is endogenously determined using a simple interest-rate monetary policy rule and (ii) non-trivial term-premium dynamics are generated by stochastic habit formation in preferences. The model is compared to a similar one with exogenous inflation. The two models capture the average level and shape of the yield curve, the volatility of the short-term interest rate and selected descriptive statistics of consumption growth and inflation. However, the exogenous-inflation model is not able to generate the observed volatility of long-term rates. The endogenous-inflation model with a countercyclical price of consumption growth risk, highly persistent policy shocks and negative correlation between consumption growth and inflation is able to capture long-term rate volatility. Its success relies on an equilibrium inflation process that depends on highly autocorrelated policy shocks. The endogenous-inflation model is used to analyze the effects on the yield curve of policy rules with different responses to economic conditions. It is found that a policy rule with a stronger reaction to inflation explains recent developments in the dynamics of interest rates and inflation.

3.2 Model

This section describes a general class of affine term structure models and presents an economic model that delivers an equilibrium term structure which fits in this class. The economic model can be seen as providing theoretical restrictions for the parameters of the term structure model.
3.2.1  Affine Term-Structure Models with Stochastic Price of Risk

To explore the relationship between monetary-policy-induced changes in short interest rates and the entire term structure, we study an equilibrium model that imply the Duffie and Kan (1996) affine term-structure model. Here we briefly review this class of models.

The Duffie and Kan (1996) class of arbitrage-free term-structure models are models where the prices of multi-period default-free pure discount bonds are affine functions of the model’s state variables \( s_t \). This class is based on a \( k \)-dimensional vector of state variables \( s_t \) that follows a first-order vector autoregression

\[
s_{t+1} = (I - \Phi) \theta + \Phi s_t + \Sigma(s_t)^{1/2} \epsilon_{t+1},
\]

where \( \{\epsilon_t\} \sim \text{IID} N(0, I) \), \( \Phi \) is a \( k \times k \) matrix of autoregressive parameters assumed to be stable with positive diagonal elements, and \( \theta \) is a \( k \times 1 \) vector of drift parameters. The conditional covariance matrix, \( \Sigma(s_t) \), can depend on the state vector in specific functional forms considered below.

Prices for real and nominal default-free bonds are given by the “fundamental equation” of asset pricing

\[
b_t^{(n)} = \mathbb{E}_t[M_{t+1} b_{t+1}^{(n-1)}],
\]

where \( b_t^{(n)} \) is the price at date \( t \) of a default-free pure-discount bond that pays 1 at date \( t + n \) where \( b_t^{(0)} = 1 \). The positive random variable \( M_{t+1} \) is the “asset-pricing kernel” or the “stochastic discount factor”. In our structural model below, \( M_{t+1} \) will be interpreted as the equilibrium marginal utility of the representative household.

By specifying a particular functional form for the pricing kernel and the variance-covariance matrix \( \Sigma(s_t) \), bond prices of all maturities are log-linear functions of the state,

\[
-\log b_t^{(n)} = \mathcal{A}^{(n)} + \mathcal{B}^{(n)\top} s_t,
\]

where \( \mathcal{A}^{(n)} \) is a scalar, and \( \mathcal{B}^{(n)} \) is a \( k \times 1 \) vector. Equivalently, continuously compounded yields, \( i_t^{(n)} \), defined by \( b_t^{(n)} = \exp(-ni_t^{(n)}) \), are also affine functions of the state variables,

\[
i_t^{(n)} = \frac{1}{n} \left[ \mathcal{A}^{(n)} + \mathcal{B}^{(n)\top} s_t \right].
\]

Empirical work by Duffee (2002) and Dai and Singleton (2002, 2003) has shown that an affine model in which the state dependence of the risk premium is driven by the price of risk rather

than volatility provides a much better empirical model. Duffee (2002) and Ang and Piazzesi (2003) show in continuous and discrete-time settings, respectively, that such a model, known as an “essentially” affine model, is equally tractable. Here we use the discrete-time version of the model and assume that the variance-covariance matrix of the state vector $s_t$ is a constant $\Sigma(s_t) \equiv \Sigma$.

In order to capture the state dependence of the market price of risk in an essentially affine framework, the pricing kernel takes the form

$$-\log M_{t+1} = \Gamma_0 + \Gamma_1^T s_t + \frac{1}{2} \lambda(s_t)^T \Sigma \lambda(s_t) + \lambda(s_t)^T \Sigma^{1/2} \varepsilon_{t+1}. \quad (3.3)$$

The $k \times 1$ vector $\Gamma_1$ represents the “factor loadings” for the pricing kernel and the $k \times 1$ vector $\lambda(s_t)$ is the state-dependent price of risk which is also assumed to be affine in the state vector

$$\lambda(s_t) = \lambda_0 + \lambda_1 s_t, \quad (3.4)$$

where $\lambda_0$ is a $k \times 1$ vector of constants and $\lambda_1$ is a $k \times k$ matrix of constants. Therefore, the quadratic term $\frac{1}{2} \lambda(s_t)^T \Sigma \lambda(s_t)$ in (3.3) can be seen as a correction term that preserves the linearity of interest rates in the log-normal framework.

From this assumption on the structure of the pricing kernel, the parameters defining the bond yields, $A_n$ and $B_n$, can again be found recursively by substituting into the bond pricing equation (3.2) yielding

$$A_n = \Gamma_0 + A_{n-1} + B_{n-1}^T [(I - \Phi) \theta - \Sigma \lambda_0] - \frac{1}{2} B_{n-1}^T \Sigma B_{n-1},$$

$$B_n^T = \Gamma_1^T + B_{n-1}^T [\Phi - \Sigma \lambda_1]. \quad (3.5)$$

Since $b^{(0)} = 1$, the initial conditions for the recursions are $A_0 = 0$ and $B_0 = 0$.

### 3.2.2 Some Properties of the Affine Term-Structure Model

A successful term-structure model must be able to capture salient properties of interest rates such as time-varying expected returns on bonds, upward sloping yield curves on average and volatile long-term interest rates. This section presents the implications of the affine model presented above on these dimensions.

---

2 Other applications of the “essentially” affine model in discrete-time include Brandt and Chapman (2003) and Dai and Philippon (2004).
Expected Excess Returns

The fundamental pricing equation (3.2) tells us that long-term bonds can be seen as one-period instruments with uncertain payoff \( i_{t+1}^{(n-1)} \). It implies that, from a one-period holding perspective, long-term bonds might involve compensations for risk that must be reflected in expected excess returns over the one-period risk-free rate \( i_t \). Define then the one-period term premium involved in an \( n \)-period bond as

\[
\xi_t^{(n)} \equiv i_t^{(n)} - \frac{1}{n} \left[ i_t + (n - 1)E_t i_{t+1}^{(n-1)} \right].
\]  

(3.6)

Using the recursive equations (3.5), the term premium of an \( n \)-period bond can be written in the affine form

\[
\xi_t^{(n)} = \frac{1}{n} \left[ \xi_{A,n} + \xi_{B,n}^\top s_t \right],
\]  

(3.7)

with coefficients given by

\[
\xi_{A,n} = -B_{n-1}^\top \Sigma \left( \lambda_0 + \frac{1}{2}B_{n-1} \right)
\]  

and

\[
\xi_{B,n}^\top = -B_{n-1}^\top \Sigma \lambda_1.
\]  

From these equations we can infer that term premia in the affine framework are time-varying as long as the market price of risk is not constant (\( \lambda_1 \neq 0 \)). This characteristic is essential to capture deviations from the expectations hypothesis. To see this, consider the Campbell and Shiller (1991) coefficients, \( \beta^{(n)} \), associated to the regression

\[
i_t^{(n-1)} - i_t^{(n)} = \alpha^{(n)} + \frac{\beta^{(n)}}{n} (i_t^{(n)} - i_t) + \varepsilon_t^{(n)}.
\]  

(3.8)

Under the expectations hypothesis the \( \beta^{(n)} \) coefficients are equal to 1. Using equation (3.6) it can be shown that these coefficients can be written as

\[
\beta^{(n)} = 1 - n \frac{\text{cov}(i_t^{(n)} - i_t, \xi_t)}{\text{var}(i_t^{(n)} - i_t)}.
\]

It becomes clear that deviations from the expectations hypothesis can be explained by time-varying term premia whose variation is correlated with the variability of interest-rate spreads. This variability in the model is entirely driven by the existence of time variation in the market price of risk.
The term premia defined above correspond to one-period holding expected excess returns scaled down by maturity. To see this, denote by $x_{t,t+1}^{(n)}$ the one-period excess return at time $t+1$ of a bond with maturity at time $t+n$. This return is given by

$$x_{t,t+1}^{(n)} = \log \left( \frac{i_{t+1}^{(n-1)}}{b_t^{(n)}} \right) - i_t = -(n-1)i_{t+1}^{(n-1)} + n\alpha_t^{(n)} - i_t$$

and from equation (3.6) it follows that $\mathbb{E}_t \left[ x_{t,t+1}^{(n)} \right] = n\alpha_t^{(n)}$.

**Average Yield Curve**

A robust empirical observation is that, on average, long-term rates tend to be higher than short-term rates and then we observe on average positive interest rates spreads. Therefore, the ability to capture upward sloping yield curves is a desired feature for a term-structure model.

It can be shown from the affine representation for interest rates that the average spread between an $n$-period bond yield and a one-period interest rate is

$$\mathbb{E}[i_t^{(n)} - i_t] = \frac{n-1}{n} \mathbb{E}[i_t^{(n-1)} - i_t] + \frac{1}{n} \mathbb{E}[x_{t,t+1}^{(n)}] = \frac{1}{n} \mathbb{E} \left[ \sum_{j=2}^{n} x_{j,t+1}^{(j)} \right].$$

The recursive representation for the average spread above shows us that the unconditional spread associated to a specific maturity can be expressed as the weighted average of the unconditional spread linked to a bond with shorter maturity and a maturity-specific holding-period expected excess return. Then, when interest rates can be represented by stationary state variables, expected excess returns must be positive enough in order to obtain on average upward sloping yield curves. This imposes important restrictions on the parameters of the market price of risk.

**Term-Structure Volatility**

In order to obtain the volatility of long-term interest rates implied by the affine-term structure model, consider the non-recursive solution for the vector of factor sensitivities in equation (3.5),

$$\mathcal{B}_n = (I - \Phi_\lambda)^{-1} (I - \Phi_\lambda^\top) \mathcal{B}_1,$$

where

$$\Phi_\lambda = [\Phi - \Sigma \lambda_1]^\top.$$

The matrix $\Phi_\lambda$ can be seen as the autoregressive matrix for the state variables under the risk-neutral measure. This matrix is different from the autocorrelation matrix under the actual mea-
sure as long as the market price of risk is time varying. From this representation, we observe that the unconditional variance of interest rates is

$$\text{var}(i_t^{(n)}) = \frac{1}{n} (\mathbb{I} - \Phi_{\lambda})^{-1} (\mathbb{I} - \Phi_{\lambda}^n) \text{var}(i_t) (\mathbb{I} - \Phi_{\lambda}^n)^\top [(\mathbb{I} - \Phi_{\lambda})^{-1}]^\top.$$ 

For the particular one state variable case \((k = 1)\), the volatility of interest rates simplifies to

$$\sigma(i_t^{(n)}) = \frac{1}{n} \frac{1 - \Phi_{\lambda}^n}{1 - \Phi_{\lambda}} \sigma(i_t).$$

Figure 3.1 presents the volatility of long-term interest rates implied by the formula above for different coefficients \(\Phi_{\lambda}\) as a proportion of the volatility of the one-period interest rate. This curve implies that volatility dies out quickly unless \(\Phi_{\lambda}\) is very close to one. For models with constant market price of risk \(\Phi_{\lambda} = \Phi\), the volatility of interest rates depends entirely on the autocorrelation of the state variables. Thus, in order to capture a slow declining volatility across maturities, stationary state variables need to be very persistent processes. This is consistent with the finding by Backus and Zin (1994) that under stationary state factors the volatility of interest rates converges to zero. From this point of view, the existence of state-dependent market prices of risk such that \(\lambda_1 < 0\), potentially overcomes the lack of persistence in the state variables and helps increasing the volatility of interest rates.

### 3.2.3 Economic Model

To better understand the features of bond prices that can be captured in the context of a pure-exchange economy with a time-varying market price of risk, we derive both real and nominal term structures in a pure exchange economy where the representative household’s preferences contains
a stochastic preference shock. The stochastic preference shock takes the form of stochastic risk aversion and can be interpreted as a form of external habits. The model adapted here is a variation of the one considered in Gallmeyer, Hollifield and Zin (2005) without inflation modeled through firms’ staggered price setting.

In this discrete-time economy, households derive lifetime consumption from a single good. The supply of this good is exogenously specified. To isolate the asset pricing role of the representative household with a preference shock, real and nominal bonds are in zero net supply. We abstract away from models with price rigidities that would lead to inflation influencing the real side of the economy; instead, nominal prices are constructed in our economy either by exogenously specifying an inflation process or imposing a monetary policy rule that links the inflation process to the nominal short-rate in the economy.

Consumption and Households

Consumption is exogenous in our pure-exchange setting. The process for consumption growth is given by

\[
\Delta c_{t+1} = (1 - \phi_c)\theta_c + \phi_c \Delta c_t + \sigma_c \varepsilon_{c,t+1}
\]

with \(\varepsilon_{c,t+1} \sim N(0,1)\).

The representative household is infinitely lived and derives utility from consumption \(C_t\). The representative household solves the intertemporal optimization problem

\[
\max_{\{C_t\}_{t=0}} \mathbb{E} \left[ \sum_{t=0}^{\infty} e^{-\delta t} \frac{C_t^{1-\gamma}}{1-\gamma} Q_t \right]
\]

subject to the intertemporal budget constraint

\[
\mathbb{E} \left[ \sum_{t=0}^{\infty} M_t C_t \right] \leq w_0.
\]

Here \(\delta\) denotes the time preference parameter, \(\gamma\) is the local curvature of the utility function, \(Q_t\) is the time \(t\) preference shock and \(w_0\) is the initial household’s wealth. The preference shock is taken as exogenous by the representative household. The stochastic preference shock, expressed as the change in the logarithm of the shock \(\Delta q_{t+1} \equiv \log Q_{t+1} - \log Q_t\), is linearly related to shocks in consumption growth \(\Delta c_{t+1} \equiv \log C_{t+1} - \log C_t\), with a coefficient that varies linearly with the current level of consumption growth and an exogenous variable \(\nu_t\):

\[
-\Delta q_{t+1} = \frac{1}{2} (\eta_c \Delta c_t + \eta_c \nu_t)^2 \text{var}_t \Delta c_{t+1} + (\eta_c \Delta c_t + \eta_c \nu_t) (\Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1}).
\]
The preference shock allows for exogenously varying stochastic risk aversion through external habit formation. The representative household’s overall sensitivity to consumption growth is \( \gamma + (\eta_c \Delta c_t + \eta_v \nu_t) \). The term \( (\eta_c \Delta c_t + \eta_v \nu_t) \) can be interpreted as the stochastic part of the representative household’s risk aversion. To complete the specification of the preference shock, \( \nu_t \) has autoregressive dynamics given by

\[
\nu_{t+1} = \phi \nu_t + \sigma \varepsilon_{t+1}^\nu 
\]  

with \( \varepsilon_{\nu,t+1} \sim \text{IID} N(0,1) \). The shock \( \varepsilon_{\nu,t+1} \) is independent of the consumption growth shock \( \varepsilon_{c,t+1} \).

Note that the term \(-\frac{1}{2} (\eta_c \Delta c_t + \eta_v \nu_t)^2 \text{var}_t \Delta c_{t+1} \) in the stochastic preference shocks implies that the conditional mean of the growth of the preference shock is

\[
E_t \left[ \frac{Q_{t+1}}{Q_t} \right] = 1, 
\]

implying that the process for the preference shock is a martingale. The coefficient \( \eta_c \) measures the sensitivity of the representative household’s level of risk-aversion to the current growth rate of aggregate consumption and is a form of sensitivity to habits as in Campbell and Cochrane (1999), Dai (2000), and Wachter (2005). The coefficient \( \eta_v \) measures the sensitivity of the representative household’s level of risk aversion to the process \( \nu_t \) which is independent of consumption growth.

The intertemporal budget constraint (3.10) contains the stochastic discount factor or real pricing kernel \( M_t \). From the household’s first-order conditions we obtain a pricing kernel given by the intertemporal marginal rate of substitution

\[
M_{t+1} = e^{-\delta} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{Q_{t+1}}{Q_t} \right). 
\]

Therefore, the logarithmic real pricing kernel \( m_{t+1} \equiv \log M_{t+1} \) is given by

\[
-m_{t+1} = \delta + \gamma \Delta c_{t+1} - \Delta q_{t+1} 
= \delta + \gamma (1 - \phi_c) \theta_c + \gamma \phi_c \Delta c_t + \frac{1}{2} (\eta_c \Delta c_t + \eta_v \nu_t)^2 \sigma_c^2 
+ (\gamma + \eta_c \Delta c_t + \eta_v \nu_t) \sigma_c \varepsilon_{c,t+1}.
\]

As a result, the real pricing kernel in the habit model can be seen as a 2-factor stochastic price of

\[\text{Note:} \quad \frac{Q_{t+1}}{Q_t} \text{ can be seen as a Radon-Nikodym derivative that represents a change of measure from the pricing kernel of a CRRA economy. This representation for the pricing kernel is isomorphic to the Epstein-Zin pricing kernel presented in Gallmeyer et al. (2007) or the model-uncertainty adjusted pricing kernel in Hansen and Sargent (2006). Although the economic underpinnings in these models are different, they share the purpose of increasing the marginal utility of consumption.}\]
risk affine model with state variables \( s_t = (\Delta c_t, \nu_t)^\top \). Proposition 12 describes the link between the equilibrium for this economy and the affine framework presented above.

**Proposition 12.** The equilibrium characteristics of the economy and its associated real pricing kernel are represented by equations (3.1), (3.3) and (3.4) when

\[
s_t = (\Delta c_t, \nu_t)^\top
\]

and

\[
\Phi = \text{diag}\{\phi_c, \phi_\nu\}, \quad \theta = (\theta_c, 0)^\top, \quad \Sigma^{1/2} = \text{diag}\{\sigma_c, \sigma_\nu\}, \quad \varepsilon = (\varepsilon_c, \varepsilon_\nu)^\top,
\]

\[
\Gamma_0 = \delta + \gamma (1 - \phi_c) \theta_c - \frac{1}{2} \gamma^2 \sigma_c^2, \quad \Gamma_1 = \begin{bmatrix} \gamma (\phi_c - \eta_c \sigma_c^2) \\ -\gamma \eta_\nu \sigma_c^2 \end{bmatrix}^\top, \quad \lambda_0 = [\gamma \quad 0]^\top, \quad \lambda_1 = \begin{bmatrix} \eta_c & \eta_\nu \\ 0 & 0 \end{bmatrix}.
\]

**Proof.** It follows from characterizing the state vector process (3.1) using equations (3.9) and (3.11) and expressing (3.13) in matrix form.

This representation allows us to price real discount bonds using equation (3.5). The equilibrium continuously compounded \( n \)-period real interest rate, \( r_t^{(n)} \), must satisfy the household’s first-order condition for \( n \)-period real bond holdings

\[
e^{-n r_t^{(n)}} = \mathbb{E}_t [M_{t+n}] = \mathbb{E}_t [M_{t+1} e^{-(n-1) r_{t+1}^{(n-1)}}].
\]

Therefore, real interest rates can be expressed as linear combinations of consumption growth and the exogenous variable \( \nu_t \), with loadings given by functions of deep economic parameters.

Relative to a general essentially-affine model, the model’s structural parameters significantly reduce the dimensionality of the parameter space. From the structure of the price of risk \( \lambda(s_t) \), innovations in the pricing kernel are solely driven by shocks to consumption growth \( \varepsilon_{c,t+1} \). The preference shock \( \nu_t \) does however contribute to time variation in the price of risk as long as \( \eta_\nu \neq 0 \). The preference parameters \( \eta_c \) and \( \eta_\nu \) affect the sensitivity of interest rates to the state variables. In particular, a negative value for \( \eta_c \) increases the response of real interest rates to consumption growth and implies a countercyclical price of consumption growth risk.

### 3.2.4 Nominal Bond Pricing

We are interested in pricing nominal bonds that pay in units of money. To take into account the different numeraire, we need to transform the real pricing kernel described by equation (3.12) into a nominal pricing kernel that takes into account the changes over time of the relative price.
of money with respect to the price of goods. By defining $P_t$ as the nominal price level at time $t$, the nominal pricing kernel is given by

$$M_{t+1}^S = e^{-\delta} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{Q_{t+1}}{Q_t} \right) \left( \frac{P_{t+1}}{P_t} \right)^{-1}. \quad (3.15)$$

Therefore, denoting by $i^{(n)}_t$ the continuously compounded $n$-period nominal interest rate, the household’s first-order condition for $n$-period nominal bond holdings is

$$e^{-i^{(n)}_t} = E_t \left[ M_{t+n}^S \right].$$

The logarithm of the nominal pricing kernel is then $m_{t+1}^S = -m_t^S + \pi_{t+1}$, where $\pi_{t+1} = \log P_{t+1} - \log P_t$ is the log of the money-price of goods at $t + 1$ relative to $t$, or inflation.

In order to close the nominal side of the model, we need to derive a process for the evolution of prices in the economy. For simplicity, we consider two approaches for pinning down inflation. Our first approach is to directly specify its dynamics. Our second approach is to exogenously specify a Taylor rule describing Fed policy which links the nominal short-rate and inflation.

**Exogenous Inflation Nominal Pricing Kernel**

By expanding the state space to include an exogenous inflation process $\pi_t$, the state vector becomes $s_t^S = (\Delta c_t, \nu_t, \pi_t)^\top$. Further, we assume that the stochastic process for inflation is given by

$$\pi_{t+1} = (1 - \phi_\pi) \theta_\pi + \phi_\pi \pi_t + \sigma_\pi \varepsilon_{\pi,t+1}, \quad (3.16)$$

where $\varepsilon_{\pi,t+1} \sim \text{IID} N(0, 1)$ and is independent of all other shocks in the model. Note in particular that the conditional variance $\text{var}_t(\pi_{t+1}) = \sigma_\pi^2$ is constant and thus the nominal state vector still conforms to the “essentially” affine setting described above.

Based on the equilibrium real and nominal pricing kernels given by equations (3.13) and (3.15), the equilibrium nominal term structure from our habit-based pure exchange economy can be expressed as the 3-factor stochastic price of risk affine model characterized in Proposition 13.

**Proposition 13.** The equilibrium characteristics of the economy under the exogenous inflation process and its associated nominal pricing kernel are represented by equations (3.1), (3.3) and (3.4) when

$$s_t^S = (\Delta c_t, \nu_t, \pi_t)^\top$$

and

$$\Phi^S = \text{diag}(\phi_c, \phi_\nu, \phi_\pi), \quad \Theta^S = (\theta^\top, \theta_\pi)^\top, \quad \Sigma^{1/2,S} = \text{diag}(\sigma_c, \sigma_\nu, \sigma_\pi), \quad \varepsilon^S = (\varepsilon^\top, \varepsilon_\pi)^\top,$$
\[
\Gamma_0^s = \Gamma_0 + (1 - \phi_x)\theta_\pi - \frac{1}{2}\sigma_\pi^2, \quad \Gamma_1^s = [\Gamma_1 \quad \phi_\pi]^\top,
\]
\[
\lambda_0^s = [\lambda_0^\top \quad 1]^\top, \quad \lambda_1^s = \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix}.
\]

**Proof.** It follows from characterizing the state vector process (3.1) using equations (3.9), (3.11) and (3.16), replacing them into the nominal pricing kernel (3.15) and expressing it in matrix form.

From Proposition 13 it is clear that the market prices of risk related to consumption growth and the exogenous variable \(\nu_t\) are the same as in Proposition 12. Equivalently, the compensations for the risks inherent to the consumption growth and the exogenous variable processes are the same for assets with real and nominal payoffs. The last term of \(\lambda(s_t)\) contains the price of inflation risk, which in this context is constant.

**A Monetary-Policy Consistent Nominal Pricing Kernel**

Alternatively, we can derive the nominal pricing kernel by imposing a monetary policy rule linking inflation to the nominal short rate. Assume that monetary policy follows a nominal interest rate rule of the form

\[
i_t = \bar{i} + i_c \Delta c_t + i_\nu \nu_t + u_t
\]

where \(u_t\) is a policy shock capturing the non-systematic component of monetary policy. It is assumed to follow an autoregressive process with dynamics given by

\[
u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u, t+1},
\]

where \(\varepsilon_{u, t+1} \sim \text{IIDN}(0, 1)\) and is independent of all other shocks in the model. The policy rule (3.17) is similar to the one proposed in Taylor (1993). The evident difference between the two rules is that, while under the original Taylor (1993) specification the short-term interest rate rule depends on the output gap level, the rule here reacts to consumption growth. The absence of a production sector with frictions in this endowment economy does not admit an interpretation for an output gap. Therefore, with slight abuse of terminology, we refer to the policy rule as the Taylor rule for the model.

Given that the Taylor rule (3.17) must be consistent with the nominal pricing kernel (3.15), we can use the two equations to solve for an internally consistent process for inflation. This process is given by

\[
\pi_t = \bar{\pi} + \pi_c \Delta c_t + \pi_\nu \nu_t + \pi_u u_t.
\]
The equilibrium constraint imposed by the price of the one-period nominal bond (3.16) implies
loading coefficients for the equilibrium inflation that satisfy
\[
\bar{\pi} = \frac{1}{1 - \bar{\pi}} \left[ \bar{\pi} - \frac{1}{\bar{\pi} - \pi_c} \left( \pi_c - \pi_c \bar{\pi} \eta_c \right) + \frac{1}{2} (\gamma + \pi_c)^2 \sigma_c^2 \right],
\]
\[
\pi_c = \frac{\gamma (\phi_c - \bar{\sigma}_c^2 \eta_c)}{\pi_c}, \quad \pi_\nu = \frac{(\gamma + \pi_c) \sigma_\nu^2 \eta_\nu}{\pi_\nu}, \quad \pi_u = -\frac{1}{\pi_u}.\]

By substituting the monetary-policy consistent inflation process into the nominal pricing kernel (3.15), we obtain a 3-factor “essentially” affine term structure model again. The state vector is given by \( s^t = (\Delta c_t, \nu_t, u_t)^T \) and the dynamics for the state variables and the nominal pricing kernel are characterized in Proposition 14.

**Proposition 14.** The equilibrium characteristics of the economy under the exogenous inflation process and its associated nominal pricing kernel are represented by equations (3.1), (3.3) and (3.4) when
\[
s^t = (\Delta c_t, \nu_t, u_t)^T
\]
and
\[
\Phi^c = \text{diag}\{\phi_c, \phi_\nu, \phi_u\}, \quad \theta^c = (\theta^T, 0)^T, \quad \Sigma^1/2^c = \text{diag}\{\sigma_c, \sigma_\nu, \sigma_u\}, \quad \varepsilon^c = (\varepsilon^T, \varepsilon_\nu)^T,
\]
\[
\Gamma^c_0 = \delta + \bar{\pi} + (\gamma + \pi_c) (1 - \phi_c) \theta_c - 2 (2 (\gamma + \pi_c)^2 \sigma_c^2 - \frac{1}{2} \sigma_\nu^2 \sigma_c^2 - \frac{1}{2} \sigma_u^2 \sigma_c^2),
\]
\[
\Gamma^c_1 = \left[ (\gamma + \pi_c) \left( \phi_c - \eta_c \sigma_c^2 \right) \quad \pi_\nu \phi_\nu - (\gamma + \pi_c) \eta_\nu \sigma_c^2 \quad \pi_u \phi_\nu \right]^T,
\]
\[
\lambda^c_0 = [\gamma + \pi_c \quad \pi_\nu \quad \pi_u]^T, \quad \lambda^c_1 = \left[ \begin{array}{ccc} \lambda_1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right].
\]

**Proof.** It follows from characterizing the state vector process (3.1) using equations (3.9), (3.11) and (3.18), replacing them into the nominal pricing kernel (3.15) and expressing it in matrix form.

The vector \( \lambda^c_0 \) shows that the constant component of the market prices of risk related to consumption growth and \( \nu_t \) are affected by the inflation process. The fact that in equilibrium the inflation process is determined by consumption growth and \( \nu_t \) makes the compensations for risk for assets with nominal payoffs to depend on the response of inflation to these two processes.

### 3.3 Analysis

The purpose of the analysis is to compare the term structure implications of describing the economy using the exogenous or the endogenous processes for inflation presented above. This
analysis is helpful to understand to what extent we can learn about the dynamics of interest rates incorporating monetary policy. In addition, given that modeling monetary policy through a policy rule allows us to evaluate the effects of changes in the rule, we conduct a policy experiment and analyze its implications for the term structure and macroeconomic variables. A description of the data used in the calibration is presented, followed by the calibration exercise for the two models and the policy experiment.

### 3.3.1 Data

We use quarterly U.S. data from 1971:3 to 2005:4 for interest rates, consumption and consumer prices. The zero-coupon yields for yearly maturities from 1 to 10 years are obtained using the Svensson (1994) methodology applied to off-the-run Treasury coupon securities at the Federal Reserve Board. The short-term nominal interest rate is the 3-month T-bill from the Fama risk-free rates database. The consumption growth series was constructed using quarterly data on real per capita consumption of nondurables and services from the Bureau of Economic Analysis. The inflation series was obtained following the methodology in Piazzesi and Schneider (2006). These data capture inflation related only to nondurables and services consumption and therefore, it is consistent with the consumption data. The construction of the inflation data is presented in the Appendix as well as a comparison with the inflation rate measured by changes in the consumer price index.

### 3.3.2 Calibration: Exogenous-\(\pi\) vs. Endogenous-\(\pi\)

In order to understand the main differences for the dynamics of the term structure between a model with exogenous inflation and a model where a monetary policy rule determines inflation endogenously, we calibrate the two models to selected statistics of the U.S. data above. We refer to the models with exogenous and endogenous inflation as exogenous-\(\pi\) and endogenous-\(\pi\), respectively.

For comparison purposes, the two models were calibrated to match the average level and the volatility of the short-term nominal interest rate and the average, volatility and first order autocorrelation of consumption growth and inflation. Analytical representations of macroeconomic and term structure model-implied statistics are provided in the Appendix.

Table 3.1 contains the values for the parameters that are the same in both calibrations. The parameters \(\theta_c\), \(\phi_c\) and \(\sigma_c\) were chosen to match the mean, standard deviation and first-order autocorrelation of consumption growth. We picked the habit parameters \(\eta_c\) and \(\eta_v\) calibrating the exogenous-\(\pi\) model. They were chosen to match the shape of the yield curve and the volatility of

\[4\text{The data series are available at http://www.federalreserve.gov/pubs/feds/2006/200628/feds200628.xls.}\]
Table 3.1: Common parameter values in the two models.

<table>
<thead>
<tr>
<th>θ_c</th>
<th>φ_c</th>
<th>σ_c</th>
<th>η_c</th>
<th>φ_ν</th>
<th>σ_ν</th>
<th>η_ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.938 × 10^{-3}</td>
<td>0.4146</td>
<td>3.962 × 10^{-3}</td>
<td>-28044</td>
<td>0</td>
<td>0.05</td>
<td>-14826</td>
</tr>
</tbody>
</table>

the short term rate. The sensitivity of the habit to consumption growth, η_c < 0, is fundamental to obtain an upward sloping yield curve. Its negative value increases the marginal utility of consumption when consumption growth is low. It can be seen as countercyclical risk aversion. The autoregressive parameter φ_ν is set to zero for simplicity and σ_ν was fixed at 0.05. The magnitude of σ_ν was determined such that η_ν has the same order of magnitude as η_c. The sensitivity of the habit to the exogenous variable, η_ν, allows us to capture the volatility of the short-term rate. When η_ν = 0, the exogenous-π model is able to reproduce the shape of the yield curve but implies a lower volatility for the short-term rate than the observed in the data.

Table 3.2 contains the model-specific parameters. The time preference δ and the local risk aversion γ are different in the two models. The parameter δ is selected to match the average level of the short-term rate (3 month rate). Given the parameters specified in 3.1, δ has to be negative implying a subjective discount factor that is greater than one. This failure is solved by choosing different habit parameters as will be shown for the policy experiment. In the exogenous-π model, the parameters θ_π, φ_π and σ_π were chosen to match the mean, standard deviation and first order autocorrelation of inflation. Also, the local risk aversion γ does not affect the exogenous processes for the macroeconomic variables. It allows us to use γ to match term-structure related statistics. This is not the case in the endogenous-π model where γ plays a role in the determination of inflation in equilibrium. It is used jointly with ̄i, i_c, i_π, and φ_ν to match the mean, standard deviation and first-order autocorrelation of inflation, the shape of the nominal yield curve and the volatility of the 10-year bond.

Table 3.2: Model-specific parameter values.

<table>
<thead>
<tr>
<th>Exogenous-π</th>
<th>Endogenous-π</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ 1.1467 × 10^{-3}</td>
<td>δ -1.5842 × 10^{-3}</td>
</tr>
<tr>
<td>γ 0.42</td>
<td>γ 0.98</td>
</tr>
<tr>
<td>θ_π 1.115 × 10^{-2}</td>
<td>i̅ -0.0223</td>
</tr>
<tr>
<td>φ_π 0.84</td>
<td>i_c 1.8295</td>
</tr>
<tr>
<td>σ_π 3.593 × 10^{-3}</td>
<td>i_π 2.5655</td>
</tr>
<tr>
<td></td>
<td>φ_ν 0.9985</td>
</tr>
<tr>
<td></td>
<td>σ_u 0.0005</td>
</tr>
</tbody>
</table>

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The policy-rule parameters imply positive responses of the monetary authority to consumption growth and the level of inflation. The autoregressive coefficient of the policy shock, $\phi_u$, is very close to one. The inflation process in the endogenous-\(\pi\) model is given by

$$\pi_t = 0.014 - 0.58\Delta c_t + 0.036\nu_t + u_t,$$

where the negative loading on consumption growth induces the negative correlation between consumption growth and inflation that is observed in the data. This correlation is zero by construction under the exogenous-\(\pi\) model.

The first section of Table 3.3 shows us that both models are able to capture important properties of the dynamics of consumption growth and inflation. The endogenous-\(\pi\) model has the advantage over the exogenous-\(\pi\) model that it also captures the negative correlation between consumption growth and inflation.

Table 3.3: Data and model-implied descriptive statistics.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Exogenous-(\pi)</th>
<th>Endogenous-(\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\Delta c_t] \times 4$</td>
<td>1.98%</td>
<td>1.98%</td>
<td>1.98%</td>
</tr>
<tr>
<td>$E[\pi_t] \times 4$</td>
<td>4.46%</td>
<td>4.46%</td>
<td>4.46%</td>
</tr>
<tr>
<td>$\sigma(\Delta c_t) \times 4$</td>
<td>1.74%</td>
<td>1.74%</td>
<td>1.74%</td>
</tr>
<tr>
<td>$\sigma(\pi_t) \times 4$</td>
<td>2.66%</td>
<td>2.66%</td>
<td>2.67%</td>
</tr>
<tr>
<td>corr($\Delta c_t, \Delta c_{t-1}$)</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>corr($\pi_t, \pi_{t-1}$)</td>
<td>0.84</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>corr($\Delta c_t, \pi_t$)</td>
<td>-0.33</td>
<td>0</td>
<td>-0.38</td>
</tr>
<tr>
<td>$E[i_t] \times 4$</td>
<td>6.11%</td>
<td>6.11%</td>
<td>6.11%</td>
</tr>
<tr>
<td>$E[i_{t}^{(20)}] \times 4$</td>
<td>7.31%</td>
<td>7.36%</td>
<td>7.34%</td>
</tr>
<tr>
<td>$E[i_{t}^{(40)}] \times 4$</td>
<td>7.68%</td>
<td>7.62%</td>
<td>7.60%</td>
</tr>
<tr>
<td>$\sigma(i_t) \times 4$</td>
<td>3.04%</td>
<td>3.04%</td>
<td>3.07%</td>
</tr>
<tr>
<td>$\sigma(i_{t}^{(20)}) \times 4$</td>
<td>2.61%</td>
<td>1.16%</td>
<td>2.42%</td>
</tr>
<tr>
<td>$\sigma(i_{t}^{(40)}) \times 4$</td>
<td>2.38%</td>
<td>0.50%</td>
<td>2.32%</td>
</tr>
<tr>
<td>corr($i_t, i_{t-1}$)</td>
<td>0.92</td>
<td>0.48</td>
<td>0.61</td>
</tr>
<tr>
<td>corr($i_t, i_{t}^{(20)}$)</td>
<td>0.86</td>
<td>0.99</td>
<td>0.91</td>
</tr>
<tr>
<td>corr($i_t, i_{t}^{(40)}$)</td>
<td>0.82</td>
<td>0.99</td>
<td>0.85</td>
</tr>
<tr>
<td>corr($i_t, \Delta c_t$)</td>
<td>-0.10</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>corr($i_t, \pi_t$)</td>
<td>0.60</td>
<td>0.73</td>
<td>0.77</td>
</tr>
</tbody>
</table>

The second section of Table 3.3 and Figure 3.2 present selected properties of nominal interest rates. The average level of the yield curves implied by the two models match the empirical counterpart. This achievement is driven by the existence of the habit parameter $\eta_c$. With
respect to the volatility of long-term interest rates, it is very small under the exogenous-π model in comparison to the data. The ratio volatility of the 10-year rate to volatility of the short-term rate is 61.2% in the data and 2.63% in this model. This ratio improves to 57.2% in the endogenous-π model. Based on the analysis in Section 3.2.2, the failure of the exogenous-π model can be explained by the absence of enough persistence in the consumption growth and inflation processes. The existence of time-varying prices of risk ($\lambda_1 \neq 0$) driven by the habit parameters $\eta_c$ and $\eta_\nu$ is not strong enough to increase the volatility of long rates. The success of the endogenous-π model relies on the fact that the policy rule allows us to describe inflation,

![Figure 3.2: Nominal Interest Rate Properties - Exogenous and Endogenous Inflation](image)

and thus, interest rates, in terms of a very persistent process, the policy shock. It allows us to reproduce the empirical properties of inflation while increasing the volatility of long-term nominal interest rates. Therefore, under this calibration the volatility of interest rates does not die fast with bond-maturity because the non-systematic component of the Taylor rule exhibits significant persistence.

It is important to note at this point that in this calibration exercise we are not using an additional degree of freedom in the exogenous-π model. Allowing for positive values for the autoregressive parameter $\phi_\nu$ increases the volatility of long-term rates. However, the value required to match the volatility of the 10-year rate also implies a non-monotonically decreasing shape for volatility across maturities: the volatility of interest rates for some intermediate maturities is significantly higher than the volatility of the short term rate and long-term rates. In the policy experiment below we show that this feature of the model is helpful to capture the very slow decline in volatility observed at short maturities.

Table 3.3 also shows us other differences between the two models. The first-order autocorrelation of the short-term interest rate is too low in the model with exogenous inflation and it increases in the model with endogenous inflation. However, it is still low in comparison to the au-
tocorrelation implied by the sample period. The correlation of the short-term rate and long-term rates implied by the exogenous-\( \pi \) model are too high. The endogenous-\( \pi \) model reduces these correlations to levels that are close to those observed in the data. The last section of the table shows that the correlations between macro variables and the short-term rate implied by the two models are different from those in the data. Specifically, the correlation between the short rate and consumption growth is positive while in the data it is negative and the correlation between the short rate and inflation is higher than in the data. Introducing a monetary policy rule does not seem to reduce it.

Panel C of Figure 3.2 presents the nominal one-period average term premia. For this calibration the average term premia in the two models are basically the same for all maturities. They imply expected excess returns that increase monotonically with maturity that vary from 30 bps. in the 6-month rate to 1.80% in the 10-year bond yield.

The most significant differences between the two models can be observed in the dynamics of the implied real interest rates. When the two models are calibrated to match the level of the average nominal term structure, the level and the slope of the average real term structure, the volatility of real interest rates and the average term premia implied by the endogenous-\( \pi \) model are higher than those under exogenous inflation. Besides the fact that \( \delta \) and \( \gamma \) are different in the two models, the differences in the dynamics of real rates can be understood comparing the prices of risk in Propositions 12 to 14. The prices of risk and the loading coefficients associated to consumption growth and the exogenous variables for assets with real payoffs are the same as those for nominal payoffs in the exogenous-\( \pi \) model. It is due to the fact that inflation is modeled as a process that is uncorrelated to these two factors. Therefore, they don’t have effects on the nominal component of the term structure. This is not longer true in the endogenous-\( \pi \) model. Inflation depends on consumption growth and the exogenous variables. Since \( \pi_c < 0 \), it implies
that the price of consumption growth risk for real payoffs is higher than the price for nominal payoffs. Equivalently, a monetary policy rule implies a reduction in the price of consumption risk in the nominal pricing kernel. Investors are willing to hold financial assets whose nominal payoffs increase with consumption growth for lower expected excess returns given the negative correlation between inflation and consumption growth. Additionally, the volatility of real rates decays quickly across maturities in the endogenous-π model because the volatility of nominal rates is mainly explained by the persistence of policy shocks and these shocks do not affect the real term structure.

We can also compare the sensitivity of interest rates and term premia to the two common state factors in the models: consumption growth and the exogenous variable $\nu_t$. Table 3.6 in the Appendix shows that while the nominal loadings are very similar in the two models, the real loadings are significantly different.

### 3.3.3 Policy Experiment

The endogenous-π model is helpful to analyze the effects of changes in monetary policy on the dynamics of interest rates. These changes can be captured by changes in the functional form of the policy rule or changes in the reaction coefficients of the policy rule presented in Section 3.2.4. Here we follow the latter. We analyze the effects on the dynamics of interest rates of changes in the coefficients of reaction to inflation and consumption growth in the policy rule. The motivation for this exercise is provided by the empirical evidence presented in Clarida, Galí and Gertler (2000). They estimate reaction functions for monetary policy in the U.S. for different periods and find that the policy rule that describes the most recent period in the U.S. economy has a higher coefficient of reaction to the level of inflation than in previous periods. Our objective is to analyze the implications of changes in the reaction to macroeconomic variables on the dynamics of interest rates and try to determine whether these changes are consistent with the evolution of interest rates in recent years.

Table 3.4 shows regressions that are consistent with the policy rule (3.17) for three different periods in the U.S. economy, the whole sample (1971-2005) and the two sub-samples: 1971-1987 and 1987-2005. The second sub-sample corresponds to the Alan Greenspan era.

We perform an initial calibration of the endogenous-π model to obtain a good description of the macro-variable and term-structure dynamics of the data for the whole sample. Based on the baseline calibration we conduct two policy experiments. The two experiments consist of modifying the reaction coefficient $t_\pi$ and $t_c$, respectively, to match the average level of the short-term rate for the Greenspan era, while keeping the value of the other parameters as in the baseline calibration. We refer to the first experiment, as $\Delta t_\pi$ and the second one as $\Delta t_c$. 107
Table 3.4: Regressions $i_t = \bar{i} + i_c \Delta c_t + i_\pi \pi_t + u_t$ for different samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\bar{i}$</th>
<th>$i_c$</th>
<th>$i_\pi$</th>
<th>$R^2$</th>
<th>corr($u_t, u_{t-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971−2005</td>
<td>0.01</td>
<td>0.029</td>
<td>0.458</td>
<td>0.30</td>
<td>0.86</td>
</tr>
<tr>
<td>1971−1987</td>
<td>0.02</td>
<td>-0.360</td>
<td>0.206</td>
<td>0.22</td>
<td>0.72</td>
</tr>
<tr>
<td>1987−2005</td>
<td>0.01</td>
<td>0.016</td>
<td>0.245</td>
<td>0.08</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Table 3.5: Initial parameter values for the policy experiment.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\phi_c$</th>
<th>$\phi_\nu$</th>
<th>$\bar{i}$</th>
<th>$\phi_u$</th>
<th>$\sigma_c$</th>
<th>$\sigma_\nu$</th>
<th>$i_c$</th>
<th>$\sigma_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2888 × 10^{-4}</td>
<td>0.4146</td>
<td>0.1807</td>
<td>-0.012</td>
<td>0.9963</td>
<td>3.962 × 10^{-3}</td>
<td>0.05</td>
<td>1.06</td>
<td>0.0005</td>
</tr>
<tr>
<td>0.65</td>
<td>$\eta_c$</td>
<td>$\eta_\nu$</td>
<td>$\pi_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.938 × 10^{-3}</td>
<td>-34110</td>
<td>-23851</td>
<td>1.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The parameter values for the baseline calibration are presented in Table 3.5. The calibration is very similar to the one presented above but this one implies $\delta > 0$. In addition, allowing $\phi_\nu > 0$ captures better the shape of the term structure of the volatility of interest rates. The parameter values for the Taylor rule that represent the reaction to consumption growth and inflation are both positive and greater than one and the policy shocks are highly persistent. The process that describes inflation is

$$\pi_t = 0.013 - 0.43 \Delta c_t + 0.046 \nu_t + u_t.$$  

Table 3.6 shows some descriptive statistics associated to the two experiments. Experiment $\Delta i_\pi$ requires an increase in $i_\pi$ to 2.72 from 1.98 to match the average short term interest rate for the Greenspan era. Experiment $\Delta i_c$ requires an increase in $i_c$ from 1.45 to 1.05 to do the same. However, the implications for the dynamics of interest rates are completely different.

The $\Delta i_\pi$ experiment is successful in reducing the level of inflation, its volatility and autocorrelation, as well as the less negative correlation between inflation and consumption growth. The $\Delta i_c$ experiment does not capture these developments in the inflation process.

With respect to the term structure properties, despite that $i_\pi$ is the only parameter that is changed in the $\Delta i_\pi$ experiment, Figure 3.4 shows that the implied average yield curve resembles the one observed in the Greenspan era. In particular, an increase in the reaction coefficient to inflation increases the slope of the curve. The $\Delta i_c$ experiment delivers a downward sloping yield curve. The difference between the two experiments can be explained observing Panel C of Figure 3.4. The term premia associated to the $\Delta i_\pi$ experiment are positive and those of the $\Delta i_c$ experiment are negative. While an increase in the $i_\pi$ coefficient decreases the negative sensitivity of inflation to consumption growth to -0.25 from -0.43, the increase in the $i_c$ coefficient increases the
Table 3.6: Data and model-implied descriptive statistics for the policy experiments.

<table>
<thead>
<tr>
<th>Data</th>
<th>Policy Experiment</th>
<th>Baseline</th>
<th>(\Delta \pi )</th>
<th>(\Delta c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E[\Delta c_t] \times 4 )</td>
<td>1.98% 1.83%</td>
<td>1.98% 1.98% 1.98%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E[\pi_t] \times 4 )</td>
<td>4.46% 2.95%</td>
<td>4.46% 2.65% 3.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma (\Delta c_t) \times 4 )</td>
<td>1.74% 1.35%</td>
<td>1.74% 1.74% 1.74%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma (\pi_t) \times 4 )</td>
<td>2.66% 1.26%</td>
<td>2.66% 1.86% 2.85%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr ((\Delta c_t, \Delta c_{t-1}))</td>
<td>0.41 0.28</td>
<td>0.41 0.41 0.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr ((\pi_t, \pi_{t-1}))</td>
<td>0.84 0.54</td>
<td>0.85 0.62 0.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr ((\Delta c_t, \pi_t))</td>
<td>-0.33 -0.19</td>
<td>-0.28 -0.23 -0.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E[i_t] \times 4)</td>
<td>6.11% 4.49%</td>
<td>6.11% 4.49% 4.49%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E[i_{t}^{(20)}] \times 4)</td>
<td>7.31% 5.83%</td>
<td>7.33% 6.17% 3.11%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E[i_{t}^{(40)}] \times 4)</td>
<td>7.68% 6.40%</td>
<td>7.74% 6.27% 2.16%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma (i_t) \times 4)</td>
<td>3.04% 2.05%</td>
<td>3.04% 3.60% 2.74%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma (i_{t}^{(20)}) \times 4)</td>
<td>2.61% 1.73%</td>
<td>2.65% 2.76% 2.49%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma (i_{t}^{(40)}) \times 4)</td>
<td>2.38% 1.50%</td>
<td>2.39% 2.08% 2.30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr ((i_t, i_{t-1}))</td>
<td>0.92 0.97</td>
<td>0.68 0.30 0.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr ((i_t, i_{t}^{(20)}))</td>
<td>0.86 0.89</td>
<td>0.990 0.993 0.992</td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr ((i_t, i_{t}^{(40)}))</td>
<td>0.82 0.79</td>
<td>0.960 0.964 0.972</td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr ((i_t, \Delta c_t))</td>
<td>-0.10 0.08</td>
<td>0.12 0.18 -0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr ((i_t, \pi_t))</td>
<td>0.60 0.44</td>
<td>0.88 0.82 0.89</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Negative correlation to -0.81. It translates into reduced (increased) negative correlation between consumption growth and inflation with a stronger reaction to inflation (consumption growth) that increases (decreases) the expected excess returns on bonds. Equivalently, while a stronger reaction of monetary policy to inflation increases the riskiness of bonds, a stronger reaction to consumption growth increases the hedging benefits of them due to its increased negative impact on inflation.

Panel B of Figure 3.4 shows the implications of the experiments on the volatility of interest rates. The \(\Delta \pi\) experiment implies a high volatility for short-term rates and a quick decline in volatility with maturity. The ratio 10-year rate volatility to short-rate volatility decreases to 0.33 from 0.62. This ratio is very low in comparison to the 0.53 ratio observed on average during the Greenspan era. Therefore, policy shocks in the model lose some of the ability to generate long-term rate volatility. The reason is a reduced response in inflation to policy shocks that is also reflected in the reduced persistence in inflation observed during the period. The \(\Delta c\) does not have great impact on the volatility of interest rates.

Other implications of the \(\Delta \pi\) that are consistent with interest rate developments during
the Greenspan era are the increase (decrease) in the correlation between consumption growth (inflation) and the short-term interest rate. However, the autocorrelation of the short-term rate decreases in the policy experiment while it increased for the sub-sample.

### 3.4 Conclusion

This chapter shows that a consumption-based affine term-structure model is able to capture a salient property of long-term interest rates, the fact that they are as volatile as short term rates. This is an important achievement for an affine term structure model with economic foundations. Affine term structure models require highly persistent state factors to avoid a quick decline in volatility across maturities. This fact apparently disqualifies macroeconomic variables such as consumption growth or inflation as explanatory variables in these models. However, when a monetary policy rule makes inflation to be correlated to real economic activity and a monetary policy shock, it is possible to obtain a high volatility of long-term rates allowing for highly autocorrelated policy shocks while capturing the inflation dynamics. However, the model requires a latent variable with no evident economic interpretation. This drawback requires further study to obtain a fully economic model of the term structure.
3.5 Appendix

3.5.1 Macroeconomic Data

We present a comparison of statistical properties of two different data sets for aggregate consumption and inflation. We use quarterly U.S. data from 1971:3 to 2005:4. In the first set, inflation is constructed using quarterly data on the consumer price index from the Center for Research in Security Prices (CRSP) and the consumption growth series was constructed using quarterly data on real per capita consumption of nondurables and services from the Bureau of Economic Analysis. This data set considers inflation related to aggregate output, and therefore includes durable goods. In the second set, inflation is obtained following the methodology in Piazzesi and Schneider (2006). This data set captures inflation related only to non-durables and services consumption. Therefore, it represents the adequate measure of inflation for the representative agent economy considered here. Inflation is computed as the log-difference in the price index, $PI$,

$$PI_t = PI_{t-1} \sqrt{\frac{P_t Q_{t-1}}{P_{t-1} Q_{t-1}}},$$

The details of the construction of $P$ and $Q$ can be found in http://faculty.chicagogsb.edu/monika.piazzesi/research/macroannual/.

The second series for consumption growth was constructed following the Piazzesi and Schneider methodology, but adjusting it to extract the effect of population growth. Consumption growth is the log-difference in the quantity index, $QI$, given by

$$QI_t = QI_{t-1} \frac{N_{t-1}}{N_t} \sqrt{\frac{P_{t-1} Q_t}{P_{t-1} Q_{t-1}} \frac{P_t Q_t}{P_{t-1} Q_{t-1}}}.$$

where $N$ denotes population. The population series is obtained from the Bureau of Economic Analysis.

The comparison of statistics for the two sets of data is presented in Table 3.7. While the properties of consumption growth are very similar across the two sets, the properties of inflation are significantly different. The series that captures inflation related only to non-durables and services is much less volatile and much more persistent than the series for changes in the consumer price index.

3.5.2 Moment Conditions

Inflation independent processes:
Table 3.7: Consumption Growth and Inflation Statistics. 1971 - 2005

<table>
<thead>
<tr>
<th></th>
<th>Set I</th>
<th>P &amp; S (adj.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\Delta c_t] \times 4$</td>
<td>2.03%</td>
<td>1.98%</td>
</tr>
<tr>
<td>$E[\pi_t] \times 4$</td>
<td>4.58%</td>
<td>4.46%</td>
</tr>
<tr>
<td>$\sigma(\Delta c_t) \times 4$</td>
<td>1.70%</td>
<td>1.74%</td>
</tr>
<tr>
<td>$\sigma(\pi_t) \times 4$</td>
<td>3.66%</td>
<td>2.66%</td>
</tr>
<tr>
<td>corr $(\Delta c_t, \Delta c_{t-1})$</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>corr $(\pi_t, \pi_{t-1})$</td>
<td>0.53</td>
<td>0.84</td>
</tr>
<tr>
<td>corr $(\Delta c_t, \pi_t)$</td>
<td>-0.30</td>
<td>-0.34</td>
</tr>
</tbody>
</table>

$E[\Delta c_t] = \theta_c$, $\sigma^2(\Delta c_t) = \frac{\sigma^2_c}{1 - \phi^2_c}$, $corr(\Delta c_{t+1}, c_t) = \phi_c$.

$\sigma^2(\nu_t) = \frac{\sigma^2_{\nu}}{1 - \phi^2_{\nu}}$, $corr(\nu_{t+1}, \nu_t) = \phi_{\nu}$.

Exogenous inflation:

$E[\pi_t] = \theta_\pi$, $\sigma^2(\pi_t) = \frac{\sigma^2_\pi}{1 - \phi^2_\pi}$, $corr(\pi_{t+1}, \pi_t) = \phi_\pi$

$corr(\Delta c_t, \pi_t) = 0$.

$E[i_t] = \delta + \gamma\theta_c(1 - \eta_c\sigma^2_c) + \theta_\pi - \frac{1}{2}\gamma^2\sigma^2_c - \frac{1}{2}\sigma^2_\pi$.

Endogenous inflation:

$E[\pi_t] = \bar{\pi} + \pi_c\theta_c$, $\sigma(\Delta \pi_t) = (\pi_c^2\sigma^2_c(\Delta c_t) + \pi_{\nu}^2\sigma^2(\nu_t) + \pi_u^2\sigma^2(u_t))^{1/2}$,

$corr(\pi_{t+1}, \pi_t) = 1 - (1 - \phi_c)\pi_c^2\frac{\sigma^2(\Delta c_t)}{\sigma^2(\pi_t)} - (1 - \phi_\nu)\pi_{\nu}^2\frac{\sigma^2(\nu_t)}{\sigma^2(\pi_t)} - (1 - \phi_u)\pi_u^2\frac{\sigma^2(u_t)}{\sigma^2(\pi_t)}$,

$corr(\Delta c_t, \pi_t) = \pi_c \frac{\sigma(\Delta c_t)}{\sigma(\pi_t)}$,

$\sigma^2(u_t) = \frac{\sigma^2_u}{1 - \phi^2_u}$.

$E[i_t] = \delta + \bar{\pi} + (\gamma + \pi_c)\theta_c(1 - \eta_c\sigma^2_c) - \frac{1}{2}(\gamma + \pi_c)^2\sigma^2_c - \frac{1}{2}\pi_{\nu}^2\sigma^2_\nu - \frac{1}{2}\pi_u^2\sigma^2_u$. 

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Figure 3.5: Nominal Interest Rates and Term Premia Loadings
Figure 3.6: Real Interest Rates and Term Premia Loadings
Bibliography


