Essays in Political Economy

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April 3, 2006

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Introduction

1 – Overview

In these four essays, I consider the effectiveness of alternative democratic systems in selecting policies representing the political center. Democratic systems vary greatly, across counties and across the states within the U.S. Even more variety is possible if one considers various proposals for democratic reform. These differences effect how the preferences of the electorate are translated into political outcomes.

In the United States, policy outcomes are determined in a multi-stage process. Candidates compete for votes, voters choose from among the candidates, members of the Senate and House of Representatives bargain over outcomes, etc. In each of these stages, there is a potential for variation in institutional form. I will consider (i) the decision making rules within legislative bodies, (ii) candidate positioning in single-winner elections, (iii) and voting rules for selecting among multiple alternatives. These are, by no means, the only relevant issues in determining the effectiveness of a Democratic system (or the U.S. system in particular) of finding the political center, but they are three important ones.

In the first essay, I will consider supermajority requirements for passing legislation. I will argue that supermajority requirements can lead to more moderate policy outcomes, if the majority party is able to control the agenda. The second and third essays will focus on candidate positioning in single-winner elections. I will argue that vote-maximizing behavior will incentivize candidates to move to the center, while policy-motivation will provide some incentives for candidates to move away from the center. The fourth essay will consider voting rules for selecting a single winner from among multiple alternatives. Here, I will find that the commonly used voting rules will often fail to select a centrist outcome. Multi-stage voting rules can be devised that guarantee the selection of a centrist outcome.

2 – Decision Making in Legislatures
The first essay, “Checks and Balances in a Two-Party System”, considers the effect of supermajority requirements on policy outcomes in a legislative body. Legislative governance in the United States is organized in an apparently contradictory fashion. On the one hand, the majority party has significant control over the agenda in both the Senate and the House of Representatives, and the President has exclusive proposal power for treaties and nominations. Such power is tempered by an intricate system of checks and balances. These include supermajority requirements (such as the filibuster and the two-thirds requirement for international treaties), bicameralism, and the presidential veto.

Of these checks and balances, the filibuster has received a great deal of scrutiny. Its scope has been gradually reduced throughout the twentieth century, and hundreds of newspaper editorials have called for its elimination. The filibuster is argued to lead excessive gridlock and frustrate popular majorities. I develop a theory of legislative outcomes which incorporates supermajority requirements. This theory can be used to evaluate the effect of supermajority requirements on legislative outcomes. I argue that the supermajority requirements can, in fact, serve an important purpose in balancing concentrated agenda setting power. In these circumstances, supermajority requirements will often lead to more moderate policy outcomes than would occur under bare-majority requirements.

I will measure policy moderation by how close policy outcomes are to the median voter’s preferred policy. If control of the agenda were perfectly distributed within the legislature, then we would expect moderate outcomes under bare-majority requirements, and we would expect that supermajority requirements would prevent moderate policies from being realized. Concentrated agenda control, however, leads to a friction which prevents policy outcomes from following the median voter. For example, the majority party has significant influence over the agenda, the president has the power to veto legislation, and the president has exclusive proposal power for treaties and nominations. In the presence of concentrated agenda control, supermajority requirements can lead policy outcomes to actually follow the median voter more closely. This holds for both positive and negative forms of agenda control.

My results suggest that small changes in the composition of the legislature can leads to large fluctuations in policy under majority rule- a phenomenon I will refer to as ‘over-responsiveness’. Supermajority requirements can be effective in reducing over-responsiveness. Of course, excessive supermajority requirements are undesirable as well, as they lead to under-responsiveness. I find that
supermajority requirements of around 60% are optimal for the Senate, if the aim is to select policies preferred by the median voter. Smaller supermajority requirements are optimal for the House.

3 – Candidate Positioning

The second essay, “Securing the Base: Electoral Competition under Variable Turnout”, considers the incentives for U.S. presidential candidates to position away from the political center. I identify two arguments made by politicians, political strategists, and pundits, as to why candidates should position close to their core supporters (i.e. adopt the Securing the Base strategy). First, the candidates may be able to raise turnout among their core supporters. Second, they may be able to reduce defections of their core supporters to third party candidates.

To see why the Securing the Base strategy may be justified, I consider the theories of abstention due to alienation and abstention due to indifference. The theory of abstention due to alienation states that a voter will be less likely to vote if he is dissatisfied with both major party candidates (or alternatively, all candidates). The theory of abstention due to indifference states that a voter will be more likely to vote if the candidates are not equally far away from him. I will refer to both these cases collectively as ‘variable turnout’, since both imply that voter turnout depends on the positions that the candidates take. The presence of third party candidates may provide a second motivation for candidates to move away from the political center. Some suggested that in 2000, Al Gore could have increased his vote share (at the expense of Ralph Nader’s) by moving further to the left.

My approach to evaluating the effectiveness of the Securing the Base and Swing Voter strategies will be empirical. I identified two effects that may cause the candidates to gain votes by moving away from the center- the turnout effect and the third party effect. I will specify an empirical model that will allow me to determine the size of each of these effects.

I find strong support for abstention due to indifference and moderate support for abstention due to alienation. The marginal effects of candidate positioning on turnout are quite large- a one unit change in a candidate’s position can lead to as much as a 7% increase in turnout. Moreover, these effects are often asymmetric. In 1976, for example, a leftward move by Jimmy Carter would increase turnout among the
most liberal voters by 18%, but would not increase turnout among the most conservative voters. Similarly, I find that candidate positioning has a strong effect on the vote share of third party candidates. This effect is asymmetric as well. I find that in 1992 and 1996, Perot voters would have heavily favored the Republican candidate had Perot not run.

Despite these findings, neither the turnout effect nor the third party effect is strong enough to incentivize candidates to move away from the center. Despite the fact that they may gain votes among their core supporters, the candidates would lose too many votes among the ideological center to make an outward move profitable. I thus find evidence in support of the Swing Voter strategy over the Securing the Base strategy.

The third essay, “The Spatial Model with Non-Policy Factors: Office-Motivated and Policy-Motivated Candidates” considers an extension of the two candidate multi-dimensional probabilistic voting model that includes non-policy factors. Non-policy factors affect the voters’ preferences between the candidates, but the candidates cannot compete over them in the same way as they compete over policy. Non-policy factors are an extension of ‘valence’. Unlike valence, non-policy factors are not perceived uniformly across the population. Because of this, they are general enough to include identification with a political party, incumbency (which may be perceived positively by some voters and negatively by others), retrospective evaluations of the candidates’ performance, and multiple types of valence (charisma, height, good looks, etc.).

My analysis considers both vote-maximizing candidates and policy-motivated candidates. Non-policy factors will lead to a striking difference between the equilibrium outcomes in each case. Under vote-maximizing behavior, the candidates will converge in equilibrium, mimicking the results when no non-policy factors are present. Alternatively, when the candidates are policy-motivated and one candidate has a non-policy advantage over the other, a divergent equilibrium will result. The advantaged candidate will move away from the vote-maximizing equilibrium and towards his ideal point. However, when non-policy factors are perfectly balanced so that neither candidate is advantaged, both vote-maximizing and policy-motivated behavior will lead to the same equilibrium outcomes.

I use my results to study threats by interest groups against the candidates. For example, suppose that a Republican candidate has a non-policy advantage over a Democratic candidate. A conservative
interest group may threaten to encourage their members to stay home on election day if the Republican candidate does not position himself sufficiently to the right. My results suggest that such threats are likely to be self-defeating. If the candidate himself is a conservative ideologue, he already should have placed himself as far right as he could while still winning the election. The threat by the conservative group can have two effects then. Either the conservative candidate will move further to left in order to remain competitive, or the candidate will fail to do so and thus lose the election. Alternatively, my results indicate that a moderate group can effectively threaten a candidate in order to induce him to move towards the political center.

4 - Voting Rules

The fourth essay, “Multi-Stage Voting Rules”, studies procedures for selecting a single winner from among multiple alternatives. Plurality Rule is used for group decision making in a wide variety of circumstances including presidential and congressional elections. A number of alternative voting rules are in current use and many more have been proposed. The most prominent alternatives are Majority Rule, Approval Voting, and Single Transferable Vote. The fact that Plurality Rule is used in so many situations would seem to suggest its superiority, but there is some evidence to suggest the contrary. Majority Rule, Approval Voting, and Single Transferable Vote have been widely suggested as superior alternatives, both by political reformers and academics.

I begin by considering a number of commonly used voting rules—Plurality Rule, Majority Rule, Approval Voting, and Single-Transferable Vote. All of these suffer from multiple Undominated Nash Equilibria, and will often fail to select the Condorcet Winner when one exists. I study multi-stage voting rules as a solution to this problem. I find that three classes of multi-stage voting rules select the Condorcet Winner (under suitable conditions). Multi-Stage Runoff will select the Condorcet Winner so long as the Majority Rule social preference is quasi-transitive (a condition weaker than single-peakedness). Binary Voting Trees will always select an element in the Uncovered Set, and consequently, will select a Condorcet Winner when one exists. The Nominate-Two rule will select the Condorcet Winner under weak conditions as well.
Though single-stage voting rules are deficient in terms of selecting Condorcet outcomes, the lack of practical alternatives means that single-stage voting rules will continue to be used in many circumstances. However, there are a range of circumstances where multi-stage voting rules remain practical. The Nominate-Two Rule is the most practical for large elections since it requires only a single-stage of voting in the large electorate. However, it presumes the existence of a small committee whose preferences do not differ too much from the electorate. There are some cases where this condition is easily met, however. The Nominate-Two Rule can be used to select a prime minister in a parliamentary democracy or can be used to select judges. Multi-Stage Runoff and Binary Voting Trees are more practical for decision making in small committees or organizations. Various forms of Binary Voting Trees are already used within most legislatures.
1 – Introduction

Legislative governance in the United States is organized in an apparently contradictory fashion. On the one hand, agenda setting power is concentrated. The majority party has significant control over the agenda in both the Senate and the House of Representatives, and the President has exclusive proposal power for treaties and nominations. Such power is tempered by an intricate system of checks and balances. These include supermajority requirements (such as the filibuster and the two-thirds requirement for international treaties), bicameralism, and the presidential veto.

Of these checks and balances, the filibuster has received a great deal of scrutiny. Its scope has been gradually reduced throughout the twentieth century, and hundreds of newspaper editorials have called for its elimination. The filibuster is argued to lead excessive gridlock and frustrate popular majorities. We will develop a theory of legislative outcomes which incorporates supermajority requirements. Our theory will then allow us to critically evaluate their effect on legislative outcomes. We will argue that the supermajority requirements can, in fact, serve an important purpose in balancing concentrated agenda setting power. In these circumstances, supermajority requirements will often lead to more moderate policy outcomes than would occur under bare-majority requirements.

The filibuster was effectively created in 1806 when the previous question was removed from the Senate rules. Since the cloture rule was not yet established, any one Senator could prevent a vote simply through delay, although this possibility was not taken advantage of until years later. In the later part of the nineteenth century, the Senate’s workload increased dramatically. Consequently, the filibuster became a more effective tactic and was employed with increased frequency. In 1917, Rule 22 was adopted, which allowed two-thirds of the Senators present and voting to invoke cloture. A number of amendments to Rule 22 have subsequently weakened the filibuster. In 1975, Rule 22 was amended to require only 60 votes to invoke cloture, although two-thirds of the Senate was still required to consider changes in the Senate rules.
In 1979, a limitation of 100 hours was placed on post cloture debate, and was reduced to 30 hours in 1986 (Binder and Smith, 1997).

Throughout the twentieth century, there have been a number of failed attempts to eliminate the filibuster completely by allowing 51 Senators to invoke cloture. In 1995, Republicans unanimously opposed a proposal sponsored by Democrats Tom Harkin and Joseph Lieberman which would have gradually ratcheted down the number of votes required to invoke a cloture to a bare majority. This incident is rather unique in the history of the filibuster in that a move to weaken the filibuster was initiated by the minority party and opposed by the majority party. It is worth noting that the Republicans had been the minority party in the Senate only one year prior.

More recently, the minority party Democrats blocked many of George W. Bush’s appellate court nominees using the filibuster. Republican Majority Leader Bill Frist, in turn, threatened to eliminate the filibuster through a parliamentary tactic know as the ‘Nuclear Option’ (Klotz, 2004). This possibility has been postponed due to an agreement stuck between 14 moderate Democrats and Republicans which would require the signatories to oppose a change to the Senate rules and allow votes for judicial nominees in all but the most extreme circumstances. Nonetheless, there is a real possibility that the agreement could unravel and that we could witness the end of the judicial filibuster. And if the judicial filibuster is eliminated, there is no reason why the legislative filibuster could not be eliminated in the future, using the same parliamentary tactic.

The filibuster retains a number of defenders. Some stress the importance of preserving minority rights in the Senate (e.g., Henderson and Moore, 2005). Others mention the desirability of preserving the differences among the chambers of congress, of which the filibuster is an important component. The filibuster is argued to asymmetrically benefit those who prefer small government (Healy, 2005, Yglesias; 2005). Others have argued that supermajority requirements are useful in preventing majority rule from resulting in chaos (Buchanan and Tullock, 1962; Caplin and Nalebuff, 1988; Hammond and Miller, 1987; Miller and Hammond, 1989; Levmore, 1992).

Binder and Smith (1997) argue that the Senate rules should be changed to allow a bare majority of Senators to invoke cloture. They claim that supermajority requirements and unlimited debate were not features that the framers of the constitution would have wanted, as the conventional wisdom would suggest.
Most of the Senate leaders of the nineteenth century favored making it easier to limit debate, but were often thwarted by minorities. Furthermore, partisan motives are the most important factor in explaining votes on cloture and cloture reform. Finally, they argue that “…there is no necessary theoretical connection between supermajority requirements and policy moderation” (Binder and Smith, 1997, p. 203). It is this point that we will study in detail in this paper.

We will measure policy moderation by how close policy outcomes are to the median voter’s preferred policy (Powell, 2000). If control of the agenda were perfectly distributed within the legislature, then we would expect moderate outcomes under bare majority requirements, and we would expect that supermajority requirements would prevent moderate policies from being realized. Concentrated agenda control, however, leads to a friction which prevents policy outcomes from following the median voter. For example, the majority party has significant influence over the agenda, the president has the power to veto legislation, and the president has exclusive proposal power for treaties and nominations. In the presence of concentrated agenda control, supermajority requirements can lead policy outcomes to actually follow the median voter more closely. This holds for both positive and negative forms of agenda control.

Consequently, Binder and Smith’s claim of a lack of theoretical connection between supermajority requirements and policy moderation hinges on a strong assumption, and one that many political scientists would find unrealistic- that the majority party does not enjoy an advantage in controlling the agenda.

Since our argument in favor of supermajority requirements depends on the assumption of majority party agenda power, this issue warrants further discussion. We note that committee chairmen are always members of the majority party in both chambers of congress. Each committee contains a majority from the majority party, with the exception of the Ethics committee in the House. Kiewiet and McCubbins (1991) document that the leadership in either chamber typically has voting behavior which represents the median member of their party, and not the median member of the whole chamber. Cox and McCubbins (2002) present evidence that suggests that the majority party is less likely to be ‘rolled’ than the minority party.

Crombez, Groseclose, and Krehbiel (forthcoming) argue that “due to its absence of a germaneness rule, the Senate has a de facto semi-automatic weapon with which to discharge its committees”. Any Senator can force the floor to consider a bill by introducing is as an amendment to an unrelated bill. The same tactic cannot be used in the House since the chamber’s rules requirement that amendments be
germane. However, the discharge petition can be used to similar effect, allowing a coalition of 218 House
members to get a bill considered on the floor, under an open rule, eventually. Patty and Penn (2004) present
a different view of the discharge petition, arguing that it limits, but does not eliminate the majority party’s
power to influence the agenda through the scheduling process. Beth (1998) presents evidence that the
discharge petition is used very infrequently, although Burden (2004) argues that public disclosure of
signatories has led to increased use of the discharge petition in recent years. Krehbiel and Meirowitz (2002)
argue that the motion to recommit gives the minority party final proposal power in the House, although
that this motion is only in order in a limited number of circumstances.

Clearly, there are legitimate arguments on both sides of this issue\(^1\). We think that the evidence in
favor of majority party agenda control is strong enough to merit investigation of its effects on legislative
outcomes. We will proceed by developing three models that describe the relationship between
supermajority requirements and policy outcomes when the majority party has control of the agenda. We
will consider both negative and positive forms of agenda control. Under the Gatekeeping Model, the
majority party will be able to block legislation over an issue if it thinks the floor outcome will not be to its
liking. Under the Closed Rule Model, the majority party will be able to present the floor with a take-it-or-
leave-it offer. For comparison purposes, we will also consider the Majoritarian Model, where the median
legislator effectively controls the agenda.

Some readers may think that the Gatekeeping and Closed Rule models are appropriate for the
House while the Majoritarian model is appropriate for the Senate. This is based on the observation that in
the House, legislation is often considered under a restrictive rule. This is not true of the Senate. We think
the conclusion is wrong, however. The models of agenda control are meant to capture not only the rules in
place, but also the legislators’ behavior on agenda issues. In particular, they capture the fact that the
majority party is more cohesive on procedural matters than on final passage. Legislators are more polarized
than their constituents. If their constituents can more easily monitor voting behavior on final passage than
on procedural issues, we would expect greater majority party cohesiveness over the agenda. We thus think

\(^1\) For more on this debate, see Rohde (1991), Krehbiel (1993), Aldrich (1995), Krehbiel (1999), Groseclose
it is wrong to conclude that the Gatekeeping and Closed Rule models are only appropriate for the House. In fact, Kiewiet and McCubbins (1989) finding that committee chairs tend to represent the median of their party is no less true for the Senate than for the House. The same can be said of Cox and McCubbins (2002) finding that the majority party is less likely to be rolled.

The models we consider are similar to some that have been considered in the literature. For example, the Majoritarian Model follows work by Krehbiel (1998). The Gatekeeping Model follows work by Denzau and MacKay (1983) and Cox and McCubbins (1993). The Closed Rule Model follows work by Romer and Rosenthal (1978). In contrast to the papers mentioned above, we will allow for flexibility regarding supermajority requirements, as this feature will be central to our analysis.

The theories we develop will take as inputs the preferences of legislators and the status quo. In order to derive implications of these models for real-world institutions, we must be able to obtain these inputs. In order to represent the preferences of the legislators, we will use the DW-Nominate scores (Poole and Rosenthal, 1991, 1997). We will assume that today’s policy outcome becomes tomorrow’s status quo, subject to some random fluctuations (Cox and McCubbins, 2002).

Finally, we will use the inputs we have obtained to evaluate the effects of supermajority requirements on legislative outcomes. Our results suggest that small changes in the composition of the legislature can lead to large fluctuations in policy under majority rule - a phenomenon we will refer to as ‘over-responsiveness’. Supermajority requirements can be effective in reducing over-responsiveness. Of course, excessive supermajority requirements are also undesirable, as they lead to under-responsiveness. We find that supermajority requirements of around 60% are optimal for the Senate, if the aim is to select policies preferred by the median voter. Smaller supermajority requirements are optimal for the House.

2 – A Model of Legislative Outcomes

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2 Anecdotal evidence for majority party cohesiveness over agenda issues is supplied by the Bipartisan Campaign Finance Reform Act of 2002. A version of the bill was originally introduced in 1995, but the Republican Senate leadership refused to allow it to come to a vote. It eventually came to a vote only after Jim Jeffords began caucusing with the Democrats, thus handing the Senate leadership to them. The bill eventually passed the Senate with 59 votes, indicating that a number of Republicans were willing to impede popular legislation through procedural votes, but not willing to vote against this legislation on final passage.
Throughout, we will model policy outcomes using the one-dimensional spatial model. The legislature will consist of \( N \) members, where \( N \) is odd. We assume that the legislators never abstain from voting. Member \( n \) has a utility function given by,

\[
u_n(x, q_n) = -|x - q_n|\]

Here, \( x \in \mathbb{R} \) is the policy outcome and \( q_n \in \mathbb{R} \) is the ideal point of legislator \( n \). Without loss of generality, we will assume that the legislator ideal points are ordered such that,

\[ q_1 \leq q_2 \leq \ldots \leq q_N \]

In order for a bill to defeat the status quo, it must receive at least \( M \) votes where \( M > \frac{1}{2}N \). In order for a bill to be amended from \( b \) to \( b' \), \( b' \) must receive at least \( \frac{N+1}{2} \) votes. In other words, a supermajority of \( M \) is needed to pass legislation while only a bare-majority of votes is needed to amend legislation prior to passage. For notational convenience, we will let \( I = N + 1 - M \), \( m = \frac{N+1}{2} \), and \( u = M \). Thus, \( q_I, q_m, \) and \( q_u \) will denote the ideal points of the lower-pivotal, median, and upper-pivotal legislators.

The legislature’s activity will be described by a multi-stage game, although the exact form of the game will depend on the allocation of agenda setting power. We will consider three different models- the Majoritarian Model, the Gatekeeping Model, and the Closed Rule Model.

**The Majoritarian Model**

Under the Majoritarian model, there are two stages to consider. In the first stage, a bill \( b \) is selected. In the second stage, the legislature chooses between the bill \( b \) and the status quo \( s \). We will analyze this game using backwards induction, starting from the second stage.

Since the second stage is the final stage in the game, and the legislators are choosing between two alternatives, they have weakly-dominant strategies of voting sincerely. To avoid open-set problems in the previous stages, we will assume that legislators who are indifferent between the bill and the status quo vote

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3 Our approach is similar to Aghion, Alesina, and Trebbi (2004), who search for optimal supermajority requirements in the context of providing a check on executive power.
for the bill. We will refer to the set of all bills that defeat the status quo as the Winset. The Winset is given by,

\[ W(s) = \{ b : \# \{ n : u_n(b; q_n) \geq u_n(s; q_n) \} \geq M \} \]

In words, the Winset is the set of bills such that the requisite number of voters prefer the bill to the status quo. The following proposition will characterize the Winset.

**Proposition 1:** The Winset is given by,

\[
W(s) = \begin{cases} 
  [s, 2q_i - s], & s \leq q_i \\
  s, & q_i < s < q_u \\
  [2q_u - s, s], & s \geq q_u 
\end{cases}
\]

Proposition 1 fully characterizes the outcome of the final stage- the bill \( b \) will pass if and only if \( b \in W(s) \). Notice that when \( q_i < s < q_u \), \( W(s) \) is empty, indicating that no bill can defeat the status quo.

We now proceed to analyze the first stage. Let \( v_n(b) \) be the utility legislator \( n \) receives from the bill \( b \) being chosen to be pitted against the status quo. Using backwards induction, we can determine that,

\[
v_n(b) = \begin{cases} 
  -|b - q_u|, & b \in W(s) \\
  -|s - q_u|, & b \notin W(s) 
\end{cases}
\]

Thus, the utility of voter receives from bill \( b \) being chosen is \(-|b - q_u|\) if the bill would defeat the status quo and \(-|s - q_u|\) otherwise.

We assume that legislators keep amending the bill until there are no further amendments that will be approved by the legislature.

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4 See Duggan (forthcoming) for a discussion of open set problems. As Krehbiel (1998) points out, such an assumption is not even technically necessary since the case where indifferent voters vote against the bill cannot comprise a subgame perfect Nash Equilibrium.

5 Here, \( \# A \) refers to the cardinality of set \( A \).

6 Here, \( \emptyset \) refers to the empty set, indicating that no bill can defeat the status quo when \( q_i < s < q_u \).

7 This assumption differs somewhat for the U.S. Senate, where amendments are subject to an effective supermajority requirement as well. If amendments are subject to the filibuster, the initial proposer retains some power. If we assume that the initial agenda setter is the median member of the majority party and the same supermajority requirement applies for amendments and final passage, this actually yields a model which is equivalent to the gatekeeping model we study.
**Definition:** The bill $b$ is *Amendment-Proof* if there does not exist another bill $b'$ such that,

$$\# \{ n : v_n(b') > v_n(b) \} \geq \frac{n+1}{2}$$

Thus, a bill is Amendment-Proof if there does not exist an alternative bill that is preferred to the current bill by a majority of the legislature. The following proposition shows that a bill is Amendment-Proof if and only if it is preferred by the median voter to all bills in the Winset.

**Proposition 2:** The bill $b \in W(s)$ is Amendment-Proof if and only if $|b' - q_m| \geq |b - q_m|$ for all $b' \in W(s)$.

Let $B(s)$ represent the set of Amendment-Proof bills. From Proposition 2, we know that this set is characterized by,

$$B(s) = \{ b \in W(s) : |b' - q_m| \geq |b - q_m| \text{ for all } b' \in W(s) \}$$

Proposition 3 will more fully characterize this set.

**Proposition 3:** The set of Amendment-Proof bills is given by,

$$B(s) = \begin{cases} q_m, & s \leq 2q_i - q_m \\ 2q_i - s, & 2q_i - q_m \leq s \leq q_i \\ \mathbb{R}, & q_i \leq s \leq q_u \\ 2q_u - s, & q_u \leq s \leq 2q_u - q_m \\ q_m, & s \geq 2q_u - q_m \end{cases}$$

Notice that when $q_i \leq s \leq q_u$, every bill $b \in \mathbb{R}$ is Amendment-Proof. This occurs because there does not exist a bill that defeats the status quo, and thus no one in the legislature cares which bill is selected in this case.

Using Proposition 3, we can immediately see that the policy outcomes will be characterized by,

$$x = \begin{cases} q_m, & s < 2q_i - q_m \\ 2q_i - s, & 2q_i - q_m \leq s \leq q_i \\ s, & q_i < s < q_u \\ 2q_u - s, & q_u \leq s \leq 2q_u - q_m \\ q_m, & s > 2q_u - q_m \end{cases}$$
This is depicted graphically in Figure 1. We refer to the interval $q_t < s < q_u$ as the gridlock interval, since the policy outcome is equal to the status quo. In this range, there does not exist a bill that would garner $M$ votes when pitted against the status quo. If the status quo is relatively extreme, $s < 2q_t - q_u$ or $s > 2q_u - q_t$, then the policy outcome will move all the way to the median voter’s ideal point. Here, the status quo is so extreme that the lower-pivotal and upper-pivotal voters have no incentive to block the legislation from passing. If $2q_t - q_m \leq s \leq q_t$ or $q_u \leq s \leq 2q_u - q_m$, the policy outcome will move only part of the way from the status quo towards the median voter’s position.

In the special case where only a bare majority is required for passing legislation, we have $q_t = q_m = q_u$. The policy outcome becomes $x = q_u$, as we would expect.

**The Gatekeeping Model**

The Gatekeeping Model is described by a three-stage game. In the first stage, the agenda setter chooses whether to block legislation. If the agenda setter blocks, the status quo becomes the policy outcome. Otherwise, the game proceeds to the second stage. In the second stage, the floor chooses a bill $b$ to pit against the status quo $s$. In the third stage, the legislature chooses between the bill and the status quo.

Let $a$ represent the ideal point of the agenda setter. We will assume further that the agenda setter is the median member of the majority party. This approach is supported by the work of Kiewiet and McCubbins (1991), who show that members of the majority party leadership in the House and Senate are typically moderates within their parties. Our approach also follows Cox and McCubbins (1993).

Notice that the last two stages of the Gatekeeping Model resemble the Majoritarian Model, so that if the agenda setter chooses not to gatekeep, the outcome will be identical to the Majoritarian Model. Let $f$ represent the outcome if the bill is allowed to go to the floor. The analysis we conducted for the Majoritarian Model shows that,
The agenda setter will choose to gatekeep if $|s - a| < |f - a|$, i.e. if the agenda setter prefers the status quo to the potential floor outcome. There are three cases to consider. The first case is where $q_l < a \leq q_u$. Here, the agenda setter always prefers the floor’s outcome to the status quo, so we get,

$$f = \begin{cases} 
q_m, & s < 2q_l - q_m \\
2q_l - s, & 2q_l - q_m \leq s \leq q_l \\
s, & q_l < s < q_u \\
2q_u - s, & q_u \leq s \leq 2q_u - q_m \\
q_m, & s > 2q_u - q_m 
\end{cases}$$

The second case occurs when $a < q_l$. In this case, we have,

$$x = \begin{cases} 
q_m, & s \leq 2a - q_m \\
s, & 2a - q_m \leq s \leq q_u \\
2q_u - s, & q_u \leq s \leq 2q_u - q_m \\
q_m, & s \geq 2q_u - q_m 
\end{cases}$$

The third case occurs when $a > q_u$, in which case we have,

$$x = \begin{cases} 
q_m, & s \leq 2q_l - q_m \\
2q_l - s, & 2q_l - q_m \leq s \leq q_l \\
s, & q_l \leq s \leq 2a - q_m \\
q_m, & s \geq 2a - q_m 
\end{cases}$$

Like the Majoritarian model, the Gatekeeping Model will contain a gridlock interval. The gridlock interval will, in general, be larger and will depend on whether the agenda setter is located relative to the lower and upper pivotal voters. In all three relevant cases, when the status quo is extreme enough, the policy outcome will move all the way towards the median voter’s ideal point.

**The Closed Rule Model**

The Closed Rule model involves a two-stage game. In the first stage, the agenda setter selects the bill $b$. In the second stage, the legislature chooses between the bill and the status quo. Since the bill $b$ will
pass if and only if the bill is in the Winset $W(s)$, we can characterize the agenda setter’s utility from choosing the bill $b$ as,

$$v_a(b) = \begin{cases} -|b-a|, & b \in W(s) \\ -|s-a|, & b \notin W(s) \end{cases}$$

Assuming that the agenda setter chooses $b$ to maximize $v_a(b)$, we can show the following policy outcome will result,

$$x = \begin{cases} a, & s < 2m_{la} - a \\ 2m_{la} - s, & 2m_{la} - a \leq s \leq m_{la} \\ s, & m_{la} < s < M_{wa} \\ 2M_{wa} - s, & M_{wa} \leq s \leq 2M_{wa} - a \\ a, & s > 2M_{wa} - a \end{cases}$$

where $m_{la} = \min\{q_l, a\}$ and $M_{wa} = \max\{q_u, a\}$.

This situation is depicted graphically in Figure 2. Here, the gridlock interval is $m_{la} < s < M_{wa}$.

When the status quo is very extreme ($s < 2m_{la} - a$ or $s > 2M_{wa} - a$), the agenda setter will be able to move the policy outcome all the way to his ideal point. When the status quo is in a moderate range ($2m_{la} - a \leq s \leq m_{la}$ or $M_{wa} \leq s \leq 2M_{wa} - a$), the agenda setter will be able to move the policy outcome part way towards his ideal point. Figure 2 (the Closed Rule model) differs from the Figure 1 (the Majoritarian model) in two ways. First the gridlock interval is bigger in Figure II since the agenda setter can block legislation, in addition to the lower-pivotal and upper-pivotal voters. Second, when the status quo is extreme, the agenda setter now gets to select the policy outcome rather than the median voter.

3 – Evaluating Check and Balances

In the previous section, we described a model which predicts likely policy outcomes based on the preferences of legislators and the status quo. The analysis is conditional on the allocation of agenda setting power as well as the supermajority required for the passage of legislation. We would like to use this model to gauge the effects of supermajority requirements on political outcomes.
We will evaluate a legislative institution by how close the policy outcome is to the ideal point of the median legislator in the chamber. In this sense, we are assuming that the median legislator is a good proxy for the median voter in the electorate, and thus reflects ‘moderate’ policy outcomes. While we think that the median voter better captures policy moderation than the mean voter, we note that this choice does not matter in our application. We will use Poole and Rosenthal’s DW-Nominate scores as our measure of ideology and these measures indicate that the median and mean legislators are nearly identical. Let $x_t$ represent the policy outcome in period $t$ and let $q_m^t$ denote the ideal point of the median legislator in period $t$. We will measure the closeness of policy outcomes to the median voter using a measure we will refer to as Average Squared Error

$$ASE = \frac{1}{T} \sum_{t=1}^{T} (x_t - q_m^t)^2$$

This measure captures the average dispersion of the policy outcome from the median voters position.

Consider first the Majoritarian model. When a bare-majority requirement is used, it is clear that $x_t = q_m^t$, which implies $ASE = 0$. When a supermajority is required, we will not always have $x_t = q_m^t$, which implies that $ASE > 0$. Thus, a bare-majority requirement is optimal under the Majoritarian Model. This is, of course, what we expected.

Consider, alternatively, the Closed Rule model and assume that the status quo is to the far right of the median legislator. Suppose a temporary change in the preferences of the voters leads the Democratic party to gain a small majority in the legislature. Under a bare-majority requirement, the Democratic agenda setter will be able to propose a fairly substantial move to the left. This bill will pass because the median legislator, though a very moderate liberal, still prefers a policy outcome far to the left to the status quo which is far to the right.

The above example shows that under a bare-majority requirement, a small change in the composition of the legislature can lead to a very substantial change in the policy outcome, if such a change

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8 Chapter 7 of Powell (2000) offers a defense of using the distance between policy outcomes and the median voter, as a way evaluating the desirability of different political institutions.

9 Another equally valid measure would be the Average Deviation Error given by $ADE = \frac{1}{T} \sum_{t=1}^{T} |x_t - q_m^t|$. We found that this measure led to comparable results, so we will only report results based on Average Squared Error.
involves a switch in majority party status. We refer to this tendency as ‘over-responsiveness’. If we were to impose a supermajority requirement, the Democratic agenda setter would now have to propose a more moderate policy in order to win the support of a larger coalition. This would result in a less extreme policy outcome. Nonetheless, under the Closed Rule model, policy outcomes fail to perfectly track the median voter as they do under the Majoritarian model with a bare majority requirement\textsuperscript{10}.

Though it is not immediately obvious, supermajority requirements can lead to increased policy moderation under the Gatekeeping Model. Suppose that temporary increase in the Republican’s electoral fortune leads the policy outcome to move to the far right. Suppose that circumstances then returned back to a more normal state in which the Republicans held a small majority in the legislature. If legislation were allowed to go to the floor, the policy outcome would be moved back towards the center since the median legislator is a very moderate Republican. However, the Republican agenda setter would have a strong incentive to exercise his gate-keeping power and prevent this from happening. Employing supermajority requirements cannot prevent him from exercising his gate-keeping power, but it can greatly decrease the likelihood that the status quo moves to the far right (or left) in the first place.

To elaborate on these points, we will compute Average Squared Error for various supermajority requirements. First, we need to specify certain inputs to the model- the preferences of legislators and the evolution of the status quo. For the preferences of the legislature, we will use the DW-Nominate scores (Poole and Rosenthal, 1991, 1997), which provide ideal point estimates for both the Senate and the House of Representatives. We will use data for the 60th to the 108th Congress\textsuperscript{11}.

We assume that today’s status quo depends of yesterday’s policy outcome as follows,

\[ s_t = x_{t-1} + \delta \varepsilon_t \]

Here, \( \varepsilon_t \) represents a policy shock (which we assume is normally distributed with mean zero and variance one) and \( \delta \) represents the magnitude of the shock. Since there is no obvious procedure for selecting a single most appropriate value for \( \delta \), we will consider a number of different values- \( \delta \in \{0.0, 0.05, 0.2\} \).

\textsuperscript{10} The results of the Closed Rule model differ from Baron (1996). He assumes that policy in each period is selected under a closed rule, and that the agenda setter is randomly selected. Under these assumptions, the policy outcome eventually converges to the median voter. In our case, policy outcomes fail to converge in the same way because we allow the position of the median voter to change over time.

\textsuperscript{11} Prior to this congress, the number of Senators was rapidly increasing due to the admission of new states to the union.
Low values of $\delta$ mean that the status quo is a good proxy for the median voter’s ideal point where as high value of $\delta$ indicate that the status quo is essentially random. Computing the ASE will involve integrating over various realizations of $\varepsilon$, and we will compute these integrals using simulation methods. In all cases, we use $K = 1000$ replications of the time series. We can then calculate the ASE for various supermajority requirements. The level of supermajority requirements that produces the lowest value of ASE will be referred to as the optimal supermajority requirement.

Results for the optimal supermajority requirements in the Senate are given in Table 1. As we expect, the optimal supermajority requirement under the Majoritarian Model is 51. For the Gatekeeping Model, the optimal supermajority requirement ranges between 53 and 55, and for the Closed Rule model, the optimal supermajority requirement is 60. We report similar results for the House of Representatives in Table 2. Once again, a bare majority requirement is optimal under the Majoritarian model. The optimal supermajority requirement ranges between 224 (52%) and 234 (54%) for the Gatekeeping Model and between 234 (54%) and 238 (55%) for the Closed Rule model.

The results indicate that when control over the agenda is concentrated, substantial supermajority requirements may be preferable to bare majority requirements. The case is stronger when the agenda setter has positive agenda control (the Closed Rule model) than when the agenda setter has negative control (the Gatekeeping model). One may find it interesting that the optimal supermajority requirement for the Senate under the Closed Rule is exactly 60, mimicking the number of voters now needed to invoke cloture and end a filibuster. While this coincidence is reassuring to some degree, we will avoid the stronger interpretation that this value was chosen because it was optimal.

To get a better sense for the gains or losses in employing supermajority requirements, Figure 3 plots Average Squared Error for the Senate when $\delta = 0.05$. For the Majoritarian model, the ASE increases as the supermajority requirement increases. For the Gatekeeping model, the ASE is essentially flat in the 51 to 61 range, although it does contain a small dip around 55. In this sense, under the Gatekeeping model, moderate supermajority requirements are neither helpful nor harmful. For the Closed Rule model, the ASE contains a substantial dip around 60. If fact, a bare majority requirement is as bad as a 77 vote
supermajority requirement\textsuperscript{12}. These results indicate that there may in fact be substantial gains from employing supermajority requirements, although once again, the case is stronger when the agenda setter has positive power.

To demonstrate the properties of the model more thoroughly, Figure 4 and Figure 5 plot a single realization of the time series of policy outcomes. Figure 4 covers the Majoritarian Model for supermajority requirements of 51, 56, 61, and 66. We know that bare-majority requirements are optimal for the Majoritarian model, and as we would expect, the policy outcome follows the median voter’s position exactly with a bare majority requirement. The policy outcome tracks the median voter’s position less closely when the requisite supermajority is increased.

Contrast this with the Closed Rule model, depicted in Figure 5. With a bare majority requirement, we can clearly observe over-responsiveness. The policy outcome seems to swing back and forth between extremes, with a small change in the median voter’s position leading to drastic changes in the policy outcome from period to period. As the supermajority requirement is increased to 56 and 61, this effect diminishes greatly. Essentially, this occurs because the agenda setter is forced to moderate his proposal in order to ensure that his proposal will receive the requisite supermajority.

4 – Conclusions

In recent years, the filibuster has been used with increasing frequency. Changes to Rule 22 made over the past century have gradually made it easier to invoke cloture and limit post cloture debate. Many have argued for eliminating the 60 vote supermajority requirement inherent in the filibuster, allowing cloture to be invoked by a bare-majority of Senators. Binder and Smith (1997) present a number of arguments for eliminating this supermajority requirement. Among their claims is that “…there is no necessary theoretical connection between supermajority requirements and policy moderation”.

We have argued that supermajority requirements lead to increased policy moderation if control over the agenda is concentrated. This can occur if the majority party has either positive or negative control

\textsuperscript{12} The ASE from the three models converges as the supermajority requirement gets large because legislation almost never passes.
of the agenda. More specifically, concentrated control over the agenda leads to over-responsiveness, where small changes in the composition of the legislature can lead to large changes in policy outcomes. We have shown that supermajority requirements can successfully mitigate over-responsiveness, leading policy outcomes to track the preferences of the electorate more closely. For the Senate, we have found that supermajority requirements around 60% are optimal for passing legislation. Smaller supermajority requirements are optimal for the House.

While we think that our theory is appropriate for studying legislation involving policy, the theory is poorly suited to study legislation of distributive content. The one-dimensional spatial model is clearly not appropriate in this case, and we can no longer use the distance of policy outcomes to the median legislator as a metric for evaluating supermajority requirements. Furthermore, the assumptions we make about the evolution of the status quo are not defensible in this context. The effect of supermajority requirements on distributive legislation could potentially be studied using the divide-the-dollar models of Baron and Ferejohn (1989) and Banks and Duggan (2001), but we have not covered this in this paper.

Similarly, our framework is inappropriate for studying judicial nominations. While the one-dimensional spatial model may still be appropriate, our assumption about the evolution of the status quo is not. During a Supreme Court vacancy, for example, the court is reduced to eight voting members. Even though the court is reduced in size, overturning a lower court decision still requires the support of five justices (if all eight justices are present and voting). Thus, the effective ‘status quo’ makes overturning lower court decisions more difficult.

While we have not dealt with this explicitly, we think our theories are appropriate for studying nominations (other than judicial) and treaties. For nominations, the President clearly has exclusive proposal power, while the President has a large degree of agenda control over international treaties.

References


Appendix I – Proof of Propositions

Proposition 1: The Winset is given by,

\[ W(s) = \begin{cases} [s,2q_l-s], & s \leq q_l \\ \emptyset, & q_l < s < q_u \\ [2q_u-s,s], & s \geq q_u \end{cases} \]

Proof: We can characterize the Winset by,

\[ W(s) = \{ b : |(n : |b - q_u|)\leq s - q_u|\geq M \} \]

We will first show that \( b \in W(s) \) if and only if \( |b - q_l| \leq s - q_l \) and \( |b - q_u| \leq s - q_u \). Suppose that \( |b - q_l| > s - q_l \). Then all voters with \( n \leq l \) must also have \( |b - q_u| > s - q_u \). Since there are \( N + 1 - M \) such voters, there cannot be \( M \) voters with \( |b - q_u| \leq s - q_u \), so \( b \notin W(s) \). A similar argument holds if \( |b - q_u| > s - q_u \), so in this case, we also have \( b \notin W(s) \).

Now suppose that \( |b - q_l| \leq s - q_l \) and \( |b - q_u| \leq s - q_u \). If \( b \leq s \), then \( |b - q_u| \leq s - q_u \) implies that all legislators with \( n \leq M \) have \( |b - q_u| \leq s - q_u \), so at least \( M \) legislators have \( |b - q_u| \leq s - q_u \) and \( b \in W(s) \). If \( b \geq s \), then \( |b - q_l| \leq s - q_l \) implies that all legislators with \( n \geq N + 1 - M \) have \( |b - q_u| \leq s - q_u \), so at least \( M \) legislators have \( |b - q_u| \leq s - q_u \) and \( b \in W(s) \). Combining these gives,

\[ W(s) = \{ b : |b - q_l| \leq s - q_l, |b - q_u| \leq s - q_u \} \]

Now we can consider 3 cases. If \( s \leq q_l \leq q_u \), we get \( W(s) = [s,2q_l-s] \), if \( q_l \leq q_u \leq s \), we get \( W(s) = [2q_u-s,s] \), and if \( q_l < s < q_u \), then we get \( W(s) = \emptyset \).

Proposition 2: The bill \( b \in W(s) \) is Amendment-Proof if and only if \( |b' - q_u| \geq |b - q_u| \) for all \( b' \in W(s) \).

Proof: To show necessity, suppose that \( |b' - q_u| \geq |b - q_u| \) for all \( b' \in W(s) \). For all \( b' \leq b \), it must be the case that \( |b' - q_u| \leq |b - q_u| \) for all \( n \geq m \), so that a majority weakly-prefer to select \( b \) over \( b' \). For all \( b' \geq b \), it must be the case that \( |b' - q_u| \leq |b - q_u| \) for all \( n \leq m \), so again a majority weakly-prefer to select
b over b'. Thus, b is Amendment-Proof. We show sufficiency using the contra-positive. Suppose that there exists b' ∈ W(s) such that |b' − q_m| > |b − q_m| . If b' ≤ b, then |b' − q_m| > |b − q_m| must hold for all n ≤ m . Similarly, if b' ≥ b, then |b' − q_m| > |b − q_m| must hold for all n ≥ m . In both cases, a majority of legislators strictly-prefer b' to b, so that b cannot be Amendment-Proof.

Proposition 3: The set of Amendment-Proof bills is given by,

\[
B(s) = \begin{cases} 
q_m, & s \leq 2q_i - q_m \\
2q_i - s, & 2q_i - q_m \leq s \leq q_i \\
\mathbb{R}, & q_i \leq s \leq q_u \\
2q_u - s, & q_u \leq s \leq 2q_u - q_m \\
q_m, & s \geq 2q_u - q_m 
\end{cases}
\]

Proof: Consider first the case where s ≤ 2q_i − q_m . In this case, s ≤ q_m ≤ 2q_i − s so that q_m ∈ W(s) . Since |b − q_m| = 0 when b = q_m , it follows that q_m ∈ B(s) . Furthermore, we cannot have b ∈ B(s) for b ≠ q_m since b' = q_m would imply |b' − q_m| > |b − q_m| . Thus, B(s) = q_m when s ≤ 2q_i − q_m . Similarly, we can show that when s ≥ 2q_u − q_m , then B(s) = q_m . Now consider the case where 2q_i − q_m ≤ s ≤ q_i . Clearly b = 2q_i − s implies that b ∈ W(s) . Notice that |b' − q_m| > |b − q_m| implies that b' > b = 2q_i − s , but this implies that b' ∉ W(s) , so 2q_i − s ∈ B(s) . Notice that b > 2q_i − s implies that b ∉ W(s) , so that b ∉ B(s) . If b < 2q_i − s , then b' = 2q_i − s implies that b' ∈ W(s) and |b' − q_m| > |b − q_m| . Thus, B(s) = 2q_i − s when 2q_i − q_m ≤ s ≤ q_i . Similar logic shows that B(s) = 2q_u − s when q_u ≤ s ≤ 2q_u − q_m . Finally, when q_i ≤ s ≤ q_u , W(s) = \emptyset , so clearly B(s) = \mathbb{R} .
Appendix II – Tables and Figures

Table 1 – Optimal Supermajority Requirements (Senate)\textsuperscript{13}

<table>
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<th>Delta</th>
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<th>Gatekeeping Model</th>
<th>Closed Rule Model</th>
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<td>52.48%</td>
<td>59.41%</td>
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<tr>
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<td>53</td>
<td>60</td>
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<td>52.48%</td>
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<td>55</td>
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<tr>
<td></td>
<td>50.50%</td>
<td>54.46%</td>
<td>59.41%</td>
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Table 2 – Optimal Supermajority Requirements (House)\textsuperscript{14}

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<td>51.49%</td>
<td>54.71%</td>
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</table>

\textsuperscript{13} Table 1 reports the optimal supermajority requirements for the Senate, out of a total of 101 voting members. Results are reported for the Majoritarian, Gatekeeping, and Closed Rule models.

\textsuperscript{14} Table 2 reports the optimal supermajority requirements for the House, out of a total of 435 voting members. Results are reported for the Majoritarian, Gatekeeping, and Closed Rule models.
Figure 1 – Majoritarian Model

Figure 1 graphs the policy outcome $x$ as a function of the status quo $s$ for the Majoritarian model. Here, $q_l$ refers to the ideal point of the lower-pivotal voter, $q_m$ to the ideal point of the median voter, and $q_u$ to the ideal point of the upper-pivotal voter.
Figure 2 – Closed Rule Model

Figure 2 graphs the policy outcome $x$ as a function of the status quo $s$ for the Closed Rule model. Here, $q_l$ refers to the ideal point of lower-pivotal voter, $q_m$ to the ideal point of the median voter, $q_u$ to the ideal point of the upper-pivotal voter, and $a$ to the ideal point of the agenda setter. $m_u = \text{Min}\{q_u, a\}$ and $M_u = \text{Max}\{q_u, a\}$.
Figure 3 – Average Squared Error\textsuperscript{17}

\textsuperscript{17} Figure 3 plots the Average Squared Error as a function of the supermajority requirement in the Senate, for the Majoritarian, Open Rule, and Closed Rule models. The supermajority requirement is varied between $M = 51$ and $M = 101$, out of a total of $N = 101$ voting members.
Figure 4 - Simulated Policy Outcomes (Majoritarian Model)\(^{18}\)

Figure 4 plots one realization of the simulated policy outcomes along with the median legislator for the Majoritarian model, for various supermajority requirements. In this case, the optimal supermajority requirement is 51. The graphs indicate that as the supermajority requirement increases from 51 to 66, the policy outcome follows the median legislator less closely.

\(^{18}\)Figure 4 plots one realization of the simulated policy outcomes along with the median legislator for the Majoritarian model, for various supermajority requirements. In this case, the optimal supermajority requirement is 51. The graphs indicate that as the supermajority requirement increases from 51 to 66, the policy outcome follows the median legislator less closely.
Figure 5 – Simulated Policy Outcomes (Closed Rule Model)\textsuperscript{19}

\textsuperscript{19} Figure 5 plots one realization of the simulated policy outcomes along with the median legislator for the Closed Rule model, for various supermajority requirements. In this case, the optimal supermajority requirement is 60. The graphs indicate that as the supermajority requirement increases from 51 to 61, the policy outcomes begin to track the median legislator more closely. When the supermajority requirement increases to 66, the trend reverses.
Essay 2 - Securing the Base: Electoral Competition under Variable Turnout

1 – Introduction

Many theories of political competition in single-winner elections imply that candidates should adopt centrist positions in order to maximize their chances of being elected. In the Downsian model (Downs, 1957; Black, 1958), both candidates converge to the median voter’s position in equilibrium. The probabilistic voting model (Hinich, 1977; Coughlin and Nitzan, 1981) implies a similar result, with both candidates converging to the mean voter’s position when the voters have quadratic utility functions. For a more general class of utility functions, both candidates converge to the position that maximizes the Nash social welfare function.

In reality, we observe candidates taking divergent positions. One can rationalize this by questioning the assumptions inherent in the Downsian and Probabilistic Voting models. Both models assume that candidates can credibly commit to change their positions. The candidates’ ability to change their positions may be severely limited, meaning that losing candidates would like to move to centrist positions, but cannot. The candidates may be forced to take off-center positions in order to succeed in their party’s primary. Some argue that while taking extreme positions may be against a party’s short term interests, it may have the effect of swaying voters in the long term.

All of the above explanations suggest that candidates may diverge despite that fact that it costs them votes in the general election. Many partisans, however, do not believe that taking extreme positions hurts their candidate’s electoral prospects. According to this view, in order to win elections, candidates must appease their core supporters so that they turn-out in large numbers and do not defect to extremist third party candidates. A number of popular press articles have documented that this stance has received increasing acceptance in recent years, and is generally accepted on both sides of the political spectrum (Nagourney, 2003; Milbank and Allen, 2004; Brownstein, 2004; Miniter, 2005).
Matthew Dowd, a senior advisor to George W. Bush’s re-election campaign, said “there's a realization, having looked at the past few elections, that the party that motivates their base- that makes their base emotional and turn-out- has a much higher likelihood of success on Election Day” (quoted in Nagourney, 2003). Stanley Greenberg, the Democratic pollster who advised Bill Clinton in the 1992 election agreed, saying that “things have changed over the decade since 1992. The partisans are much more polarized. And turnout has actually gone up because the partisans have turned-out in much greater numbers and greater unity” (quoted in Nagourney, 2003). Green party activist Charlene Spretnak argued that “Gore could have demonstrated that he had read the Green Party platform… and had identified several issues on which he could publicly promise action that the Democratic Party was otherwise likely to take. In that way, he surely would have gained badly needed votes.” (Spretnak, 2000).

Many politicians, party activists, and political consultants have come to believe in the ‘Securing the Base’ strategy- that by moving away from the center, a candidate may actually gain more votes (due to increased turnout among his supporters and decreased defections of his supporters to third party candidates) than he loses (due to increased defections of swing voters to his opponent). The Securing the Base strategy has become a widely accepted alternative to the Swing Voter strategy, which suggests that a candidate should position in the political center in order to pick up as many moderate voters as possible 20.

The theories of abstention due to alienation and abstention due to indifference provide a possible justification for the Securing the Base strategy. Abstention due to alienation (Converse, 1966) posits that a voter will be less likely to vote if he is dissatisfied with both major party candidates (or alternatively, all candidates). Abstention due to indifference (Downs, 1957) posits that a voter will be more likely to vote if the candidates are not equally far away from him 21. We will refer to both these cases collectively as ‘variable turnout’, since both imply that voter turnout depends on the positions that the candidates take. If the Downsian or Probabilistic Voting models are altered to allow for abstention due to alienation or

---

20 Notice that we are defining the Securing the Base and Swing Voter strategies in terms of candidate positioning alone. A separate issue is the targeting of get-out-the-vote efforts and persuasive advertising. The conventional wisdom here suggests that get-out-the-vote should be targeted towards ones core supporters while persuasive advertising should be targeted towards swing voters. Rosenstone and Hansen (1993) suggest that partisans are more likely to recall seeing political advertisements and to recall being contacted by party representatives.

21 Here, we refer to the policy formulation of these theories. Both abstention due to alienation and abstention due to indifference have utility formulations, e.g. a voter is more likely to abstain if neither candidate provides him sufficient utility, or if both candidates provide him with the similar utility.
indifference, divergent equilibria may result (Hinich and Ordeshook, 1969; Adams and Merrill, 2003; Glaeser, Ponzetto, and Shapiro, 2004; Adams, Merrill, and Grofman, 2005).

Brody and Page (1973) test both theories of variable turnout using thermometer data from the 1968 U.S. Presidential Election. They find support for abstention due to indifference, but not for abstention due to alienation. Zipp (1985) tests these theories using issue placement data for the 1968-1980 U.S. Presidential elections. He finds support for both forms of variable turnout, although he finds that indifference has a larger effect. Thurner and Eymann (2000) use the German part of the International Comparative Nation Election Project for the 1990 all-German general election. Since Germany is a multi-party system for which the one-dimensional spatial model is likely not appropriate, they test for issue-by-issue abstention due to alienation and indifference. They find support for abstention due to indifference in immigration policy, and no support for abstention due to alienation. Plane and Gershtenson (2004) find support for both abstention due to alienation and abstention due to indifference in U.S. Senate elections.

The presence of third party candidates may provide a second motivation for candidates to move away from the political center. Some suggested that in 2000, Al Gore could have increased his vote share (at the expense of Ralph Nader’s) by moving further to the left. Such a move would probably have increased the number of strong liberals voting for him, but would also have decreased the number of moderate voters who voted for him. In order to determine the electoral consequences of such a move, we need to quantitatively evaluate which effect is stronger.

It is worth noting that the Swing Voter strategy used to be more widely accepted than the Securing the Base strategy. Many believed that Bill Clinton’s electoral successes in 1992 and 1996 were due to his ability to appeal to swing voters. The primary goal of this paper will be to determine which view of political competition is the correct one. We will make the case that the Swing Voter strategy is more effective. We find that candidate positioning has an appreciable effect on voter turnout and third party voting. Moreover, these effects are often asymmetric, benefiting one candidate and not the other. However, these effects are not strong enough to cause the candidates to gain votes from moving away from the center.

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While moving away from the center will often increase turnout among a candidates base and reduce third party voting, this increase is not large enough to offset the loss of votes from the center of the political spectrum.

2 – Theoretical Background

Here, we outline a model of voter behavior. Voters choose whether to vote, and for which candidate to vote. We will assume that, in making their voting decisions, voters consider both the policy positions of the candidates as well as non-policy factors (chapter 3). While non-policy factors affect the voters’ decisions, the candidates cannot compete over them in the same way as they compete over policy. Non-policy factors are an extension of ‘valence’ (Stokes, 1963). Unlike valence, non-policy factors are not perceived uniformly across the population.

Voters are characterized by their ideal point \( v \) and by their non-policy factors \( z = (z_D, z_R) \). Let \( f(v, z) \) signify the distribution of the voter’s characteristics. Let \( y_D \) denote the position of the Democratic candidate and let \( y_R \) denote the position of the Republican candidate. The utility a voter characterized by \( (v, z) \) receives from voting for candidate \( k \) is given by \( u(y_k - v, z_k) = z_k - \rho |y_k - v| \), where \( \rho > 0 \). Here, we can see that a voter’s utility depends on the closeness of the policy outcome to their ideal point, as well as the voter’s non-policy evaluation of the candidate.

Voters may abstain, and moreover, the probability that a voter abstains may depend on the positions that the candidates take. The probability that a voter turns out will be represented by \( p(v, z_D, z_R, y_D, y_R) \). This feature of the model allows us to incorporate abstention due alienation and indifference. For example, we could consider,

\[
p(v, z_D, z_R, y_D, y_R) = \Phi(\alpha + \beta \text{Min}(|y_D - v|, |y_R - v|) + \gamma |y_D - v| - |y_R - v|)^{23}
\]

The term \( \text{Min}(|y_D - v|, |y_R - v|) \) represents abstention due to alienation since the probability that a voter votes decreases with the voter’s distance from the nearest candidate. The term \( |y_D - v| - |y_R - v| \)

\[^{23}\text{Here, } \Phi \text{ represents the standard normal cumulative distribution function.}\]
similarly represents abstention due to indifference. This version of the theory, sometimes referred to as the policy formulation of abstention due to alienation and indifference, is used in the empirical work of Zipp (1985), Thurner and Eymann (2000), and Plane and Gershtenson (2004). An alternative speciation is,

\[ p(v, z_D, z_R, y_D, y_R) = \Phi(\alpha + \beta \text{Max}\{u(y_D - v, z_D), u(y_R - v, z_R)\} + \gamma[u(y_D - v, z_D) - u(y_R - v, z_R)]) \]

This specification is referred to as the utility formulation of abstention due to alienation and indifference, and is used in the empirical work of Adams, Dow, and Merrill (2005). Our approach will rely on the policy formulation.

Define,

\[ A_D(y_D, y_R) = \{(v, z) : u(|y_D - v|, z_D) \geq u(|y_R - v|, z_R)\} \]

\[ A_R(y_D, y_R) = \{(v, z) : u(|y_R - v|, z_R) \geq u(|y_D - v|, z_D)\} \]

as the set of voters that prefer the Democratic and Republican candidates, respectively. The candidate’s vote shares will be characterized by,

\[ s_D(y_D, y_R) = \int_{z_D} p(v, z_D, z_R, y_D, y_R) f(v, z) dv dz, \quad s_R(y_D, y_R) = \int_{z_R} p(v, z_D, z_R, y_D, y_R) f(v, z) dv dz \]

The Democratic candidate will win the election if \( s_D(y_D, y_R) > s_R(y_D, y_R) \) and the Republican candidate will win the election \( s_D(y_D, y_R) < s_R(y_D, y_R) \).

We say that a candidate has a non-policy advantage if that candidate would win the election if both candidates were to locate in the same positions. Define,

\[ \tilde{z} = \int [1(z_D > z_R) - 1(z_R > z_D)] p(v, z_D, z_R, y_D, y_R) f(v, z) dv dz \]

Here, \( 1\{\cdot\} \) represents the indicator function, so that \( \tilde{z} \) equals the share of voters that prefer the Democratic candidate based on non-policy factors minus the share of voters that prefer the Republican candidate based on non-policy factors. If \( \tilde{z} > 0 \), then the Democratic candidate has a non-policy advantage and if \( \tilde{z} < 0 \), the Republican candidate has a non-policy advantage. Otherwise, we say that neither candidate has a non-policy advantage. If we did not include non-policy factors in the model, then the winning candidate would always be the candidate who is located closer to the median voter. This is not always the case (as we will see later), so non-policy factors are necessary to explain this discrepancy. A candidate who is located further from the median voter may still win the election if he has a non-policy advantage.
3 – A Model of Voting and Turnout

We will describe estimation of a statistical model consistent with the theoretical model we described in the previous section. This model compares with previous work but has some important differences. Much of this work does not model the turnout decision. Lacy and Burden (1999, 2001) include only abstention due to alienation. Adams and Merrill (2003) and Adams, Dow and Merrill (2005) include both abstention due to alienation and indifference. These specifications are not ideal for our purposes since they assume that abstention due to alienation and indifference are the only sources of voter abstention. We would like to use a framework that allows us to test for both types of variable abstention and allow turnout to differ from other sources, such as demographic characteristics. The specification we will use will be consistent with, but will not necessarily imply, both types of abstention. Thus, it will allow us to test whether each type of abstention occurs, as well as determine the consequences of variable turnout.

Two Candidate Competition

Let $y_{n,k}$ denote respondent $n$’s placement of candidate $k$, and let $v_n$ denote the respondents ideal point. The variable $\text{MinDist}_n = \text{Min} \{|y_{n,D} - v_n|, |y_{n,A} - v_n|\}$ will represent abstention due to alienation, and the variable $\text{DiffDist}_n = |y_{n,D} - v_n| - |y_{n,A} - v_n|$ will represent abstention due to indifference. If abstention due to alienation occurs, we would expect the coefficient on $\text{MinDist}_n$ to be negative. If abstention due to indifference occurs, we would expect the coefficient on $\text{DiffDist}_n$ to be positive. We will specify the turnout equation as follows,

$$t_n = \delta X_n + \beta_{\text{MinDist}} \text{MinDist}_n + \gamma \text{DiffDist}_n + \epsilon_n$$

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25 Lacy and Burden (1999, 2001), Adams and Merrill (2003), and Adams, Dow, and Merrill (2005) allow demographics to affect turnout only by altering the alienation and indifference thresholds.
Here, $t_n$ is a latent variable representing the propensity to vote, $X_n$ is a vector of control variables, and $\varepsilon^\prime_n$ is a disturbance term. We assume that the voter votes if $t_n \geq 0$ and does not vote otherwise.

Let $Dist_{n,k} = |y_{n,k} - v_n|$ be the distance between the voter and the candidate. We will model the utility a voter receives from voting for the candidates using the equations,

$$u_{n,D} = \alpha_D X_n - \rho Dist_{n,D} + \varepsilon^D_n$$
$$u_{n,R} = \alpha_R X_n - \rho Dist_{n,R} + \varepsilon^R_n$$

Here, $z_n = (\alpha_D - \alpha_R) X_n + \varepsilon^D_n - \varepsilon^R_n$ represents the respondent’s non-policy factor. For identification purposes, we will normalize $\alpha_R = 0$. In our estimation, we control for gender, race, the South, party identification, age, education, and income.

We assume that $\varepsilon_n = (\varepsilon_n^\prime, \varepsilon_n^D, \varepsilon_n^R)$ is normally distributed with mean zero and variance matrix $\Omega$. This specification thus yields a bivariate binomial probit model. The identification issues that arise in this model are similar to those that arise in the multinomial probit model. If we estimated the turnout and candidate choice equations separately, each equation would yield a binomial probit model. By estimating a joint model of turnout and candidate choice, we allow for a correlation between the error terms in the turnout and candidate choice equations. Lacy and Burden (1999) have shown that inclusion of such a term affects the inferences one makes. They update the work of Alvarez and Nagler (1995) to include such a correlation. While Alvarez and Nagler (1995) find that Bill Clinton’s margin of victory over George H. W. Bush would have decreased if Ross H. Perot had not run for president, Lacy and Burden (1999) argue that Bill Clinton’s margin of victory actually would have increased.

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26 We generate an exclusion restriction by constraining $\rho$ to have the same value in each of the candidate choice equations. Keane (1992) argues that such exclusions restrictions are necessary to properly identify multinomial probit models. Though our model is not a multinomial probit, a similar argument applies. Alvarez and Nagler (1995) use a similar exclusion restriction in their work while Lacy and Burden (1999, 2001) generate an exclusion restriction using party identification.

27 For identification purposes, we normalize $\sigma^2_v = \sigma_D^2 = 1$ and $\sigma_{VR} = \sigma_{DR} = \sigma_R^2 = 0$, where,

$$\Omega = \begin{pmatrix} \sigma^2_v & \sigma_{VD} & \sigma_{VR} \\ \sigma_{DV} & \sigma_D^2 & \sigma_{DR} \\ \sigma_{VR} & \sigma_{DR} & \sigma_R^2 \end{pmatrix}$$

28 More accurately, it is a bivariate binomial probit model where one equation is censored by the other, i.e., the outcome of the candidate choice equation is observed only if the voter votes.
Three Candidate Competition

For 1980, 1992, and 1996, we will include three candidates in our analysis. The turnout equation now becomes,

\[ t_n = \delta X_n + \beta_2 \text{MinDist}_2 + \beta_3 \text{MinDist}_3 + \gamma \text{DiffDist}_n + \epsilon_n^r \]

where \( \text{MinDist}_3 = \text{Min}( \left\{ |y_{n,D} - v_n|, |y_{n,R} - v_n|, |y_{n,T} - v_n| \right\} ) \). Since the theory of abstention due to alienation is ambiguous as to whether the voter should consider only the major party candidates or all three candidates, we include both possibilities in our analysis.

We also add an equation representing the respondents’ utility for the third party candidate,

\[ u_{n,T} = \alpha_T X_n - \rho \text{Dist}_{n,T} + \epsilon_n^T \]

We assume that \( \epsilon_n = (\epsilon_n^r, \epsilon_n^D, \epsilon_n^R, \epsilon_n^T) \) is normally distributed. This specification yields a bivariate multinomial probit model.

Maximum Likelihood Estimation

Consider first the two candidate case. Let \( Y_n = 0 \) if the voter does not vote, \( Y_n = 1 \) if the voter votes for the Democratic candidate, and \( Y_n = 2 \) if the voter votes for the Republican candidate. Let \( \theta = (\delta, \alpha_D, \alpha_R, \rho, \beta_2, \gamma, \Omega) \) denote the vector of parameters.

The probability that an individual does not vote is given by,

\[ P(Y_n = 0 | X_n, \theta) = P(t_n < 0 | X_n, \theta) = P(\epsilon_n^r < -\delta X_n - \beta_2 \text{MinDist}_2 - \gamma \text{DiffDist}_n) \]

\[ = \Phi(-\delta X_n - \beta_2 \text{MinDist}_2 - \gamma \text{DiffDist}_n) \]

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29 See Keane (1992) and Alvarez and Nagler (1995) for discussion of identification issues in the multinomial probit model.
30 While we would have liked to include Ralph Nader in our analysis for 2000, we were unable to do so. The ANES did not ask respondents to place Nader on the liberal-conservative scale, and the ANES sample contains a very small number of Nader voters.
31 For identification purposes, we normalize \( \alpha_T = 0 \), \( \sigma_r^2 = \sigma_D^2 = 1 \), and \( \sigma_{RT} = \sigma_{DR} = \sigma_{TR} = \sigma_r^2 = 0 \).
where $\Phi$ denote the normal cumulative distribution function. The probabilities of voting for the Democratic and Republican candidates are given by,

$$
P(Y_n = 1 \mid X_n, \theta) = P(t_n \geq 0, u_{n,D} \geq u_{n,R} \mid X_n, \theta) = \int_{t \geq 0, u_{n,D} \geq u_{n,R}} \phi(c_{n}^{D}, c_{n}^{R}, e_{n}^{R}; 0, \Omega) d c_{n}^{D} d c_{n}^{R}
$$

$$
P(Y_n = 2 \mid X_n, \theta) = P(t_n \geq 0, u_{n,D} \geq u_{n,R} \mid X_n, \theta) = \int_{t \geq 0, u_{n,D} \geq u_{n,R}} \phi(c_{n}^{D}, c_{n}^{R}, e_{n}^{R}; 0, \Omega) d c_{n}^{D} d c_{n}^{R}
$$

where $\phi$ denotes the normal probability density function.

The above integrals involve computing rectangles of the normal distribution and have no closed-form solution. We compute these integrals using the GHK method, which computes these integrals using simulation methods. The GHK method has been successfully applied to estimate multinomial probit, multivariate probit, and panel probit models (Geweke, Keane, and Runkle, 1994, 1997; Greene, 2002) and is used in popular statistics packages such as Stata and Limdep.

We form the likelihood function in the usual way,

$$
l(Y_n \mid X_n, \theta) = \sum_{n=1}^{N} \{1[Y_n = 0] \log P(Y_n = 0 \mid X_n, \theta) + 1[Y_n = 1] \log P(Y_n = 1 \mid X_n, \theta) + 1[Y_n = 2] \log P(Y_n = 2 \mid X_n, \theta)\}
$$

We follow a similar procedure for forming the log-likelihood function for the three candidate case.

4 – Data

In order to estimate the parameters of the model, we will rely on data from the American National Election Studies (ANES). This is a bi-yearly survey, corresponding to presidential and midterm elections. We will analyze the presidential elections between 1976 and 2004\textsuperscript{32}. The ANES interviews respondents both before and after the election. The pre-election survey provides us with the respondents’ characteristics (gender, income, age, etc.), party identification, self-placement on a liberal-conservative scale, and placement of the candidates on the same scale. The post-election survey provides us with whether the respondents voted and for whom they voted.

\textsuperscript{32} This includes all the studies for presidential elections since the ANES began asking respondents to place the candidates on a liberal-conservative scale.
Perhaps the most significant problem with the ANES data is that turnout is self-reported. Turnout among ANES respondents who participated in the post-election survey and answered the turnout question ranged between 74% to 81%. There are a number of possible reasons for the discrepancy between turnout in the sample and actual turnout—(a) the ANES post-election survey may be over-sampling voters relative to non-voters, (b) the respondents may lie or incorrectly recall voting, and (c) the act of participating in a political survey may motivate the respondent to vote (the ‘instrumentation’ problem).

The first cause is the least problematic since it can be corrected for using simple weighting. The second cause is potentially more problematic. Katz (2001) reports that between 8% and 13% of respondents who claim to have voted did not in fact vote. Fortunately, the ANES provides validated vote turnout for the 1980, 1984, and 1988 surveys. Our own calculations indicate that over-sampling voters is the most important cause of the discrepancy between sample turnout and actual turnout. Because of this, we will use simple weights when performing inferences, and we will use the validated turnout data for the years in which it is available. The presence of instrumentation may potentially bias our inferences about the population, but since the magnitude of this effect is unknown, it cannot be corrected for. Nonetheless, Burden (2001) argues that the instrumentation effect is small.

5 – Estimation Results

We report our full estimation results in Tables I through IV. Though we report the turnout and candidate choice equations separately, we note that we estimated the parameters of these equations jointly. In general, we find that partisans, the old, and the rich are more likely to vote, and that southerners are less likely to vote. Our results for voter turnout correspond closely with Wolfinger and Rosenstone’s study of

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33 In particular, a higher proportion of non-voters drop out of the post-election survey and a vast majority of respondents coded as missing values for self-reported turnout did not vote.

34 As a robustness check, we replicated our results using the self-reported turnout data for 1980, 1984, and 1988, and found no differences in our substantial conclusions. Thus, we expect that using self-reported turnout will not be a problem in the other years we study.

35 In order to determine the size of this effect, we would require a design that randomly assigned people into a post-election survey group and a pre and post-election survey group. The ANES has never included such a design- the presidential election survey always includes both a pre and post-election survey and the midterm survey always includes only a post-election component.
turnout in the 1972 election (Wolfinger and Rosenstone, 1980), as well the more recent work on voter turnout (Lacy and Burden, 1999, 2001).

Party identification has the expected effect on voting behavior with self-identified Democrats more likely to vote for the Democratic candidate and self-identified Republicans more likely to vote for the Republican candidate. Ideological distance has a negative and highly significant effect in all the elections we study. Blacks are much less likely to vote for third party candidates such as John Anderson and Ross Perot. The rest of the demographic variables do not have an effect that is stable across elections.

One way to measure the fit of the model is to determine the fraction of outcomes that are correctly predicted by the model. These numbers are given in Tables III and IV. We correctly predicted the voter’s turnout decision between 68.6% and 80.6% of the time. The voter’s choice of candidate is predicted between 81.9% and 91.0% of the time in the two candidate elections we consider and between 71.2% and 83.2% of the time in the three candidate elections we consider. This indicates that our model fits the data quite well and is in line with other work in this area36.

6 – Candidate Characteristics

We want to ensure that voting behavior in the sample accurately reflects voting behavior in the population. We mentioned earlier that the ANES over-samples voters relative to non-voters. There is the potential that the ANES also over-samples Democratic voters over Republicans voters, or visa versa. To correct for both potential problems, we will use simple weighting37. For each observation in the sample, we will construct a weight $w_s$ such that,

(i) The fraction of voters voting for any of the two (or three) major candidates in the sample equals the corresponding fraction in the population, estimated by dividing the number of votes for the two (or three) major candidates by the size of the voting-age population.

36 Wolfinger and Rosenstone (1980) report correct predictions rates of 72.9% and 71.4% for two probit regressions they run for the 1972 election. Lacy and Burden (1999) correctly predict both the turnout and candidate choice 50.6% of the time for the 1992 presidential election.
(ii) The fraction of voters voting for each of the two (or three) major candidates equals the fraction of voters voting for those candidates in the population.

All quantities of interest are computed using these weights (e.g. the median voter’s position we report is the weighted-median in the sample).

The weighting scheme that we use has a substantial effect on voter turnout since, as we argued earlier, the ANES over-samples voters by a significant amount. Contrarily, the ANES produces an accurate sample of the candidate vote shares. In most years, the candidates’ vote shares in the sample where within one percentage point of their actual vote shares, and thus weighting has little effect here. The exceptions were in 1976, where the ANES over-sampled Ford voters by a few percentage points and in 1980 where Anderson voters were over-sampled and Carter voters were under-sampled. We note that even when the ANES sample is only off by a fraction of a percentage point, weighting the observations makes interpreting the results far easier.

Table V reports the positions of the median voter and the candidates in each election\textsuperscript{38}. In each year, we highlight the candidate that is closer to the median voter. If voters voted completely based on the policy positions of the candidates, we would expect the candidate that is closer to the median voter to win in every election. This does not occur—Gerald Ford, Jimmy Carter (in 1980), Bob Dole, and George W. Bush (in 2000) are all examples of popular vote losers who were closer to the median voter. This discrepancy can be explained either by non-policy voting, or by the presence of third party competition.

In Table VI, we report the non-policy factors of the candidates. These factors represent the share of votes the candidates would receive if all the candidates we located at the same position (e.g., in 1976, if both Carter and Ford were located at the same ideological position, Carter would receive 51.3% of the vote and Ford would receive 48.7% of the vote). These measures capture the fact that some voters who are closer to candidate A than candidate B, may still vote for candidate B for some other reasons (e.g. charisma, party identification, retrospective economic evaluations, etc.). If a Republican candidate has a strong non-policy advantage over the Democratic candidate, this means that the Republican can position

\textsuperscript{37} Note that we use weighting only in making inferences about the population, and not in the estimation procedure.

\textsuperscript{38} We measure the candidate’s position using the median placement of the candidate in the ANES survey.
himself far to the right, and still win the election. Indeed, the candidates we identified that lost the popular vote despite being closer to the median voter were disadvantaged based on non-policy factors.

7 – The Effect of Variable Turnout

The effect of variable turnout is summarized in the highlighted portions of Tables I and II. Here, we can see very strong support for abstention due to indifference and moderate support for abstention due to alienation. The coefficients on abstention due to alienation are statistically significant in two out of the eight elections we consider. The coefficient on abstention due to indifference is statistically significant in all but one election. Our findings here are similar to Zipp (1985) and Thurner and Eymann (2000). This provides a contrast with the work of Plane and Gershtenson (2004) who find support for both types of variable abstention in U.S. Senate elections between 1988 and 1992.

While our main results use the policy formulation of abstention due to alienation and indifference, we also considered the utility-based theory as a robustness check. This amounts to re-specifying the turnout equation as follows,

\[ t_n = \delta X_n + \beta_2 \max \{u_{n,D}, u_{n,R} \} + \gamma (u_{n,D} - u_{n,R}) + \sigma_n^\varepsilon \]

Computing the choice probabilities, once again, involves integrating rectangles of the normal distribution. This means we can again apply the GHK method to compute the likelihood function. The results are similar to what we found for the policy-based theory. Indifference is statistically significant in 1984, 1988, 2000, and 2004. In 1976, alienation is borderline insignificant (recall that it was borderline significant under the policy-based specification). In addition, we used Vuong’s likelihood ratio test for non-nested models (Vuong, 1989) to see if we could reject one model over the other. In all the years we considered, we could not reject the policy formulation in favor of the utility formulation (nor could we reject the utility formulation in favor of the policy formulation).

8 – Candidate Positioning in Two Candidate Elections
The effect of candidate positioning on the candidate’s votes shares will depend on both the sensitivity of voter turnout to candidate positioning and the sensitivity of candidate choice to candidate positioning. In order for the candidates to be incentivized to move away from the center, three conditions must be met. First, candidate positioning must have a substantial effect on voter turnout (Condition A). Second, this effect must be asymmetric in a way that benefits the mover more than his opponent (Condition B). Third, this effect must be large enough to compensate for the loss of swing voters (Condition C). We consider each of these conditions in turn.

Table VII reports the marginal effect of candidate positioning on aggregate voter turnout. We consider the effect of a one unit change in a candidate’s position on turnout. Turnout in the baseline column matches actual turnout exactly, due to the weighting scheme we used. We then compute turnout under four alternative scenarios- the Democratic candidate moves one unit to the left, the Democratic candidate moves one unit to the right, the Republican candidate moves one unit to the left, and the Republican candidate moves one unit to the right. The alternative scenarios often lead to large changes in turnout (as much as 7% in 1996). Moving the candidates further apart always leads to an increase in turnout over the baseline. This move has the effect of making most voters less alienated and indifferent, thus increasing the likelihood that they vote. Thus, we can see that Condition A is met.

Figure I plots turnout as a function of ideology. To save space, we only report two representative elections- 1976 and 2004. Consider first 1976. In the baseline scenario, moderate voters turn out at low rates. Mainstream liberals and mainstream conservatives turnout at the highest rates while extreme liberals and extreme conservative turnout at lower rates. If Jimmy Carter moved one unit to the left, turnout would increase dramatically among the most liberal voters- from 46% to 59%. This move leads to a much smaller increase in turnout among the most conservative voters- from 47% to 49%. Similarly, a rightward move by Gerald Ford would increase turnout among the most conservative voters from 47% to 59%, but would only increase turnout among the most liberal voter from 46% to 47%. This indicates that in 1976 at

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39 We preformed this check only for the two-candidate elections.
40 The magnitude of effect we find is similar to Adams, Dow, and Merrill (2005).
41 This pattern, while unusual, is not an artifact of the model, and is present in the ANES data for 1976. This is the pattern of turnout predicted by the alienation and indifference theories if the candidates both located relatively close to the center, which was the case in 1976.
least, candidate positioning has a very asymmetric effect on turnout (and the asymmetry is in the direction that would incentivize the candidates to leave the center).

This asymmetry was not observed in all the years we considered. In 2004, the baseline scenario has turnout highest among the most extreme voters and lowest among moderate voters. If John Kerry were to move one unit to the left, our model predicts a moderate increase in turnout among conservative voters and a slight increase in turnout among liberal voters. If George W. Bush were to move one unit to the right, this would lead to a moderate increase in liberal turnout and a slight increase in conservative turnout. In 2004, candidate positioning has an asymmetric effect on turnout, but the asymmetry is in the opposite direction.

In general, we found that candidate positioning most often had an asymmetric effect on turnout. The direction of the asymmetry varied from year to year, sometimes benefiting the candidate moving away from the center and sometimes benefiting the other candidate. Overall, we find that Condition B is met in about half the elections we consider.

Figure II graphs the candidates’ vote margins over their opponents, as a function of their position. A candidate will win the election when their vote margin is positive, or when the corresponding line in Figure II is above zero. Table X reports each candidate’s vote-maximizing position, fixing the other candidate at their actual position. Proponents of the Securing the Base strategy claim that a candidate may actually gain votes by moving away from the center. The methodology used in this paper has the potential to either confirm or refute this conclusion since variable turnout can potentially cause the vote margin functions to peak away from the center of the ideology distribution. Our results indicate that candidates would gain votes from moving towards the center and lose votes from moving away from the center. If one of the candidates were to move away from the center, this would increase turnout substantially, often in an asymmetric way. However, candidates lose too many swing voters by moving away from the center and the net effect this negative.

Another way of stating the main result of the paper is as follows. While turnout is sensitive to candidate positioning, this effect is not asymmetric enough to incentivize the candidates to move away from the center. In order to demonstrate that our framework provides a meaningful test, we considered the following experiment. For 2004, we replicated the graph in Figure II, changing only one thing. We
multiplied the coefficient representing alienation by 25. In this case, the graph no longer peaked in the center, indicating that the candidates would gain votes by moving away from the center. Similarly, we replicated this figure while multiplying the indifference coefficient by 25. We once again observed that the vote margin function peaked away from the center, and that the candidates would gain votes by moving away from the center. This demonstrates that lack of support for the Securing the Base strategy is not ‘hard-wired’ into our framework, but occurs because the data do not support the degree of asymmetry necessary to justify the Securing the Base strategy.

9 – The Impact of Candidate Positioning in Three-Candidate Elections

In the three-candidate elections, there is an additional mechanism which could incentivize candidates to move away from the center. By moving away from the center, candidates may be able to reduce defections of their supporters to third party candidates. In order for this to be the case, three conditions must be met. First, candidate positioning must have a substantial effect on third party voting (Condition D). Second, third party voters must defect in an asymmetric way (Condition E). Third, this effect must be large enough to compensate for the loss of swing voters (Condition F).

Table VIII summarizes the effect of candidate positioning on the third party candidate’s vote share. We find a particularly large effect on Anderson’s vote share in 1980. For example, if Reagan were to move one unit to the right, Anderson’s vote share would increase from 6.7% to 11.9%. The effect on Perot’s vote share in 1992 and 1996 is more moderate, ranging from 0.2% to 1.9%. This indicates that Condition D is met.

To evaluate the effect of the presence of a third party candidate, we will use an approach similar to Alvarez and Nagler (1995) and Lacy and Burden (1999, 2001). Alvarez and Nagler (1995) used a multinomial probit model to argue that the presence of Ross Perot increased Bill Clinton’s margin of victory in 1992, although not by enough to affect the outcome of the election. Contrarily, Lacy and Burden (1999) argue that the presence of Ross Perot decreased Bill Clinton’s margin of victory. Lacy and Burden (2001) argue that third party candidates almost always benefit the incumbent over the major party challenger.
We proceed by eliminating the third party candidate from the voters’ choice set and computing the resulting vote shares for the major party candidates. We present our results in Table IX. Consider first 1980, when John Anderson ran as an independent and garnered 6.6% of the vote. Jimmy Carter received 44.7% of the two party vote while Ronald Reagan received 55.3%. Anderson was located only slightly to the right of Jimmy Carter. This suggests that many of Anderson’s voters would have voted for Carter had Anderson not run. However, when we take non-policy factors into account, we find that Carter would have received 44.3% of the two party vote had Anderson not run. This indicates that Jimmy Carter’s policy advantage among the Anderson voters was almost exactly canceled out by Reagan’s non-policy advantage among these voters. We also find that Anderson’s presences had a negligible effect on voter turnout. Consequently, we can say that Anderson’s presence had almost no effect on the election.

Now consider 1992, when Ross Perot ran as an independent and garnered 18.9% of the vote. Bill Clinton received 53.5% of the two party vote and George H.W. Bush received 46.5%. Had Perot not run, Clinton would have received 50.2% of the two party vote while Bush would have received 49.8%, indicating an extremely close election. In 1996, Ross Perot ran as a reform party candidate and garnered 8.4% of the vote. Bill Clinton received 54.7% of the two party vote and Robert Dole received 45.3%. If Perot would not have run, Clinton would have received 52.8% of the two party vote and Dole would have received 47.2%.

Our results thus indicate that reductions in third party voting can often affect the major party candidates asymmetrically (and consequently Condition E is sometimes met). This also indicates that third party candidates can have a significant effect on electoral outcomes. This effect was particularly strong in 1992. Interestingly, our results differ significantly from Lacy and Burden (1999, 2001). These papers predict that Bush would have had a much smaller share of the two-party vote. Lacy and Burden (2001) also predict that the 1996 election would have been a landslide for Clinton without Perot. These differences could be due to a number of factors- they do not weight the sample vote shares to equal the actual vote shares as we do and they to not use a measure of policy distance in their specification. More significantly, their framework imposes that all abstention is due to alienation, and they thus find that a large fraction of Perot votes would have abstained had Perot not run while we find that most of these voters would have
voted for Dole. Our results also indicate that Perot’s presence had a limited effect on turnout. In both years, the effect was less than 0.5%.

Let us consider Figure II once again, which displays the effect of candidate positioning on the candidate vote margins. Our results indicate that candidates would gain votes from moving towards the center and lose votes from moving away from the center. This is never violated in any of the elections we consider, although in 1980, Carter’s margin-maximizing position involves moving only slightly to the right. The vote share function for both candidates peak near the median voter’s position. The largest discrepancy occurs in 1980, where Carter had an optimal position of 3.54. If one of the candidates were to move away from the center, this would affect both turnout and third party voting. However, the net effect would be that the moving candidate would decrease his vote margin. Since an outward move necessitates losing some support among swing voters, the net effect of an outward move is for the candidate to decrease their vote margin.

In 1976, 1988, 1992, 2000, and 2004, our results indicate that the candidate that lost the popular vote could have won by positioning himself closer to the center. In 1980, 1984, and 1996, the winning candidate’s non-policy advantage was so strong that the losing candidate could not have changed the election outcome by positioning himself differently. Thus, our results suggest that the swing voter strategy is a more effective approach to increasing a candidate’s vote margin.

An implicit assumption that we have made in this analysis is that the choice of a third party candidate to run in exogenous. Of course, there are always third party candidates running, so the real question is whether the choice of a nationally prestigious candidate to run is independent of the positioning of the major party candidates. At first, this may seem like a serious problem. But we think that there is little need for concern here. A third party candidate is at a decided disadvantage against the major party candidates. If he is to be successful, he must commit to run quite early in the race. The major party candidates have plenty of time to position after the candidate has already committed to run.

Accumulated evidence suggests that other factors are more important in explaining third party entry. Rosenstone, Behr, and Lazarus (1984) present evidence that a small number of variables correctly

\[42\] This is not surprising given what we found in section 6. If Carter were to move to the left, many moderate voters would defect to Anderson. If Carter were to move to the right, he would loose liberal
predicted whether a prestigious third party candidate ran in 34 out of the 36 U.S. Presidential elections they considered (the exception were in 1848 and 1852). They find that the age of the party system, major party factionalizing, whether an incumbent president is running, the closeness of the previous election, and whether the incumbent president was denied the nomination, are all important predictors. This suggests that there is not much left for candidate positioning to explain.\(^{43}\)

10 – Conclusions

We have attempted to evaluate the Securing the Base strategy as an alternative to the Swing Voter strategy. Proponents of the Securing the Base strategy most often argue that even though moving away from the center will cause swing voters to defect to a candidate’s opponent, that candidate’s core supporters will turnout out at higher rates. A somewhat less prominent argument is that moving away from the center will reduce defections of a candidate’s supporters to third party candidates. This argument was particularly common in relation to the 2000 U.S. Presidential election, when some suggested that Al Gore should move to the left in order to reduce the defection of liberal voters to Ralph Nader. Our results refute both of these arguments.

In our estimation results, we found strong support for abstention due to indifference and moderate support for abstention due to alienation. Moreover, we found that candidate positioning has a substantial effect on aggregate turnout and third party voting. This leaves open the possibility that the candidates may gain votes from moving away from the center. We find, however, that this is not the case. In all the elections we considered, both major party candidates would gain votes by positioning themselves closer to the center. In five of the eight elections we considered, the losing candidate could have won by positioning himself closer to the center. In 1980, 1984, and 1996, the winning candidate had a non-policy advantage so large that he was immune to policy competition.

\(^{43}\) We note that this is not inconsistent with the other claims that Rosenstone et. al. (1984) make- that the attractiveness of the third party candidates affects the third party vote share.
While the variable turnout and third party effects are the most prominent arguments in favor of the Securing the Base strategy, we can think of a few others. If the candidates can effectively use campaign resources to persuade voters, and if the candidates’ ability to raise money depends on the positions they take, then this may lead the candidates to gain voters by moving away from the center. In fact, Moon (2004) presents a model that generates such an effect.

Another plausible justification relates to get out the vote operations. Gerber and Green (2004) study a number of different ways to increase turnout including door to door canvassing, leaflets, direct mail, phone calls, and email. They find that all the methods besides email increase voter turnout, though these methods differ in their cost effectiveness. Much like we hypothesized that turnout depends directly on the positions that the candidates, the effectiveness of get-out-the-vote and voter registration operations may depend directly on the positions the candidates take. For example, the likelihood that door to door canvassing will bring new voters to the polls may itself be subject to alienation and indifference effects. This too may potentially provide a justification for the Securing the Base strategy.

Finally, we note that the claims we have made about the alternative electoral strategies must be tempered by several strong assumptions we have made. We have assumed that candidates can only alter their electoral prospects through movements along a single policy dimension. This assumption may fail in a number of ways. First, there may be more that one relevant policy dimension that the candidates can compete over. We assumed that candidates cannot compete over the ‘non-policy factors’, but some of what we measured as ‘non-policy factors’ may in fact be mutable by the candidates. Contrarily, ideological positions may not be easily altered by the candidates. Ideological positions and non-policy factors may not be completely separable. For example, it may be that a candidate must take extreme positions in order to be perceived as a ‘leader’, by eliminating, creating, or drastically changing government programs. The electorate may be biased towards viewing a moderate as indecisive or opportunistic. This is, in fact part of the argument that extremists on both sides make for abandoning the quest for swing voters.

While this paper has focused exclusively on Presidential elections, it would be interesting to extend this study to midterm elections. It is well known that the party holding the presidency typically loses seats in the forthcoming midterm elections- the so called ‘midterm effect’. Many have argued that the midterm effect is mediated by turnout (Campbell, 1960; Campbell, 1991; Patty, forthcoming). Since the
base rate of turnout is smaller in midterm elections, there is more room for variable turnout to have a substantial effect. In fact, Plane and Gershtenson (2004) find more convincing evidence for abstention due to alienation in midterm Senate elections. This offers the possibility of an area when the positions taken by the candidates have a more substantial effect on turnout, and where candidates would be incentivized to move away from the center.

References


### Appendix – Tables and Figures

#### Table I – The Turnout Equation

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44 Table I reports the estimation results for the turnout equation for the two candidate elections. MinDist2 represents abstention due to alienation and DistDiff represents abstention due to indifference. One asterisk represents significance at the 5% level and two asterisks represent significance at the 1% level.
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Table II reports the estimation results for the turnout equation for the three candidate elections. MinDist2 and Mindist3 represent abstention due to alienation and DistDiff represents abstention due to indifference. One asterisk represents significance at the 5% level and two asterisks represent significance at the 1% level. In 1980, the coefficients of MinDist2 and MinDist3 were jointly statistically significant at the 5% level.
Table III – The Candidate Choice Equation (2 Candidates)\textsuperscript{47}

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\textsuperscript{47} Table III reports the estimation results from the candidate choice equation for the two candidate elections.
Table IV – The Candidate Choice Equation (3 Candidates)\(^{48}\)

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<th></th>
<th></th>
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</tr>
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<td>-</td>
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<td>-</td>
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<td>(0.169)</td>
</tr>
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<tr>
<td>Income: Quartile 1</td>
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<td>(0.168)</td>
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<td>(0.203)</td>
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<td>0.294</td>
<td>0.096</td>
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<td>(0.120)</td>
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<td>(0.188)</td>
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\(^{48}\) Table IV reports the estimation results from the candidate choice equation for the three candidate elections.
### Table V – Candidate Positions

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<th>Year</th>
<th>Median Voter</th>
<th>Democratic Position</th>
<th>Republican Position</th>
<th>Third Party Position</th>
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<td>4.256</td>
<td>2.977</td>
<td>5.198</td>
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<td>4.450</td>
<td>3.516</td>
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<td>1984</td>
<td>4.971</td>
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<td>1988</td>
<td>4.550</td>
<td>2.684</td>
<td>5.658</td>
<td>-</td>
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<td>4.214</td>
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<td>4.837</td>
<td>2.657</td>
<td>5.661</td>
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<td>4.792</td>
<td>3.528</td>
<td>5.738</td>
<td>-</td>
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<td>2004</td>
<td>4.355</td>
<td>2.471</td>
<td>5.982</td>
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### Table VI – Non-Policy Factors

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<th>Third Party Candidate</th>
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<td>-</td>
</tr>
<tr>
<td>1980</td>
<td>36.3%</td>
<td>57.0%</td>
<td>6.7%</td>
</tr>
<tr>
<td>1984</td>
<td>40.1%</td>
<td>59.9%</td>
<td>-</td>
</tr>
<tr>
<td>1988</td>
<td>48.9%</td>
<td>51.1%</td>
<td>-</td>
</tr>
<tr>
<td>1992</td>
<td>46.6%</td>
<td>33.9%</td>
<td>19.5%</td>
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<tr>
<td>1996</td>
<td>62.7%</td>
<td>28.5%</td>
<td>8.8%</td>
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<tr>
<td>2000</td>
<td>51.9%</td>
<td>48.1%</td>
<td>-</td>
</tr>
<tr>
<td>2004</td>
<td>47.2%</td>
<td>52.8%</td>
<td>-</td>
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</table>

49 Table V reports the position of the median voter as well as the positions of the candidates. The candidates’ positions are measured by the median respondent’s placement. Bold indicates the candidate that is closer to the median voter.

50 Table VI reports the non-policy factors of the candidates, which indicates the vote share the candidates would receive if all the candidates located in the same position.
Table VII – Marginal Effects of Candidate Positioning of Voter Turnout

<table>
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<tr>
<th>Year</th>
<th>Baseline</th>
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<th>Democrat Moves Right</th>
<th>Republican Moves Left</th>
<th>Republican Moves Right</th>
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<tr>
<td>1976</td>
<td>52.5%</td>
<td>53.8% (0.9%)</td>
<td>53.8% (0.6%)*</td>
<td>53.7% (0.7%)</td>
<td>54.2% (0.9%)</td>
</tr>
<tr>
<td>1980</td>
<td>51.7%</td>
<td>55.7% (1.3%)**</td>
<td>51.5% (1.2%)</td>
<td>51.2% (1.1%)</td>
<td>54.3% (1.2%)*</td>
</tr>
<tr>
<td>1984</td>
<td>52.9%</td>
<td>55.6% (0.7%)**</td>
<td>51.5% (0.6%)*</td>
<td>52.6% (0.5%)</td>
<td>54.1% (0.7%)</td>
</tr>
<tr>
<td>1988</td>
<td>49.8%</td>
<td>52.5% (0.8%)**</td>
<td>48.8% (0.6%)</td>
<td>49.0% (0.4%)*</td>
<td>51.7% (0.8%)*</td>
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<tr>
<td>1992</td>
<td>54.7%</td>
<td>59.7% (1.0%)**</td>
<td>55.6% (0.9%)</td>
<td>55.0% (0.9%)</td>
<td>60.4% (1.1%)**</td>
</tr>
<tr>
<td>1996</td>
<td>48.2%</td>
<td>55.0% (1.2%)**</td>
<td>46.3% (1.1%)</td>
<td>47.5% (1.1%)</td>
<td>51.7% (1.2%)**</td>
</tr>
<tr>
<td>2000</td>
<td>49.3%</td>
<td>54.9% (1.0%)**</td>
<td>48.3% (0.7%)</td>
<td>49.1% (0.6%)</td>
<td>53.1% (0.8%)**</td>
</tr>
<tr>
<td>2004</td>
<td>54.7%</td>
<td>57.6% (1.0%)**</td>
<td>55.2% (0.7%)</td>
<td>54.5% (0.7%)</td>
<td>57.9% (1.0%)**</td>
</tr>
</tbody>
</table>

Table VIII – Marginal Effects of Candidate Positioning of Third Party Vote Share

<table>
<thead>
<tr>
<th>Year</th>
<th>Baseline</th>
<th>Democrat Moves Left</th>
<th>Democrat Moves Right</th>
<th>Republican Moves Left</th>
<th>Republican Moves Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>6.7%</td>
<td>8.5% (0.5%)**</td>
<td>9.4% (0.9%)*</td>
<td>4.9% (0.8%)*</td>
<td>11.9% (1.1%)**</td>
</tr>
<tr>
<td>1992</td>
<td>19.0%</td>
<td>20.5% (0.5%)**</td>
<td>19.5% (0.3%)</td>
<td>19.9% (0.5%)</td>
<td>21.9% (0.6%)*</td>
</tr>
<tr>
<td>1996</td>
<td>8.5%</td>
<td>9.1% (0.5%)</td>
<td>8.7% (0.4%)</td>
<td>9.0% (0.3%)</td>
<td>9.0% (0.4%)</td>
</tr>
</tbody>
</table>

51 Table VII reports the marginal effects of candidate positioning on voter turnout. Since the observations are weighted, the baseline levels of turnout match the actual fraction of the voting-age population that voted. Standard errors for the difference between the baseline and alternative scenarios are in parentheses. One asterisk indicates significance at the 5% level and two asterisks indicate significance at the 1% level.

52 Table VIII reports the marginal effects of candidate positioning on the vote share of the third party candidate. Since the observations are weighted, the baseline third party vote share matches the actual vote share in the population. Standard errors for the difference between the baseline and alternative scenarios are in parentheses. One asterisk indicates significance at the 5% level and two asterisks indicate significance at the 1% level.
Table IX - The Effect of Third Party Candidates

<table>
<thead>
<tr>
<th>Year</th>
<th>With Third Party Competition</th>
<th>Without Third Party Competition</th>
<th>Difference * Dem. Margins</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Democrat</td>
<td>Republican</td>
<td>Democrat</td>
</tr>
<tr>
<td>1980</td>
<td>44.7%</td>
<td>55.3%</td>
<td>44.3%</td>
</tr>
<tr>
<td>1992</td>
<td>53.5%</td>
<td>46.5%</td>
<td>50.2%</td>
</tr>
<tr>
<td>1996</td>
<td>54.7%</td>
<td>45.3%</td>
<td>52.8%</td>
</tr>
</tbody>
</table>

Table X – Candidate Best Responses

<table>
<thead>
<tr>
<th>Year</th>
<th>Median Voter</th>
<th>Best Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Democrat</td>
<td>Republican</td>
</tr>
<tr>
<td>1976</td>
<td>4.26</td>
<td>4.00</td>
</tr>
<tr>
<td>1980</td>
<td>4.45</td>
<td>3.54</td>
</tr>
<tr>
<td>1984</td>
<td>4.97</td>
<td>4.83</td>
</tr>
<tr>
<td>1988</td>
<td>4.55</td>
<td>4.52</td>
</tr>
<tr>
<td>1992</td>
<td>4.21</td>
<td>4.30</td>
</tr>
<tr>
<td>1996</td>
<td>4.84</td>
<td>4.71</td>
</tr>
<tr>
<td>2000</td>
<td>4.79</td>
<td>5.03</td>
</tr>
<tr>
<td>2004</td>
<td>4.36</td>
<td>4.20</td>
</tr>
</tbody>
</table>

53 Table IX reports the major party candidates’ share of the two-party vote under the baseline scenario and an alternative scenario where the third party candidate does not run. Since the observations are weighted, the candidates’ vote shares in the baseline scenario matches the actual vote shares exactly. Standard errors for the difference between the margins with and without third party competition are in parentheses. One asterisk indicates significance at the 5% level and two asterisks indicate significance at the 1% level.

54 Table X reports the margin-maximizing positions of the Democratic and Republican candidates. These values correspond to the peaks of the curves plotted in Figure II.
Figure I represents predicted voter turnout under two alternative scenarios. The baseline scenario, depicted with a dotted line, has the candidates positioned in their current position. The alternative scenario, depicted with a solid line, depicts either the Democratic candidate moving one unit to the left, or the Republican candidate moving one unit to the right.
Figure II represents the vote-margin of each candidate over his opponent, as a function of the candidate’s position, holding the other candidate’s position constant at his actual position. Solid lines represent the Democratic candidate’s vote margin and dashed lines represent the Republican candidate’s vote margin. A candidate will win the election when the line is above zero, indicating a positive vote margin over his opponent.
Essay 3 – The Spatial Model with Non-Policy Factors

1 - Introduction

In this paper, I consider an extension of the two candidate multi-dimensional probabilistic voting model that includes non-policy factors. Non-policy factors affect the voters’ preferences between the candidates, but the candidates cannot compete over them in the same way as they compete over policy. Non-policy factors are an extension of ‘valence’ (Stokes, 1963). Unlike valence, non-policy factors are not perceived uniformly across the population. Because of this, they are general enough to include identification with a political party, incumbency (which may be perceived positively by some voters and negatively by others), retrospective evaluations of the candidates’ performance, and multiple types of valence (charisma, height, good looks, etc.).

The model we will study is an extension of the two-candidate probabilistic voting model, introduced by Hinich (1977). Coughlin and Nitzan (1981) extend this model and show that an equilibrium exists with both candidates locating at the policy position which maximizes social welfare. Banks and Duggan (2003) extend the work of Coughlin and Nitzan (1981) and derive further existence and uniqueness results. Lin et al. (1999) study the multi-candidate probabilistic voting model, and find that there exists an equilibrium at the social welfare optimum if the utility shocks have a large enough variance. McKelvey and Patty (2003) generalize the multi-candidate probabilistic voting model and allow for strategic voting and abstention. They prove that an equilibrium exists at the social welfare optimum if the number of voters is ‘large enough’. Schofield (2003, 2004) studies the multi-candidate probabilistic voting model with valence and argues that convergent equilibria will almost never result.

Enelow and Hinich (1982) were perhaps the first to consider a valence issue in a model of spatial competition. They find that when each candidate has a valence advantage among an equal share of the population, the resulting equilibrium will involve both candidates locating at the mean voter’s position. Groseclose (1999) shows that the median voter result of Black (1958) extends to the case where valence characteristics are present and voters have single-peaked preferences over policy. Whitman (2001) argues
the presence of valence characteristics can lead majority rule to be transitive over multi-dimensional policy
spaces\textsuperscript{57}. Ansolabehere and Snyder (2000) show that when valence characteristics are present, the multi-
dimensional Downsian model may admit pure-strategy equilibria\textsuperscript{58}. Groseclose (2001) extends the
Downsian model to include uncertainty and valence characteristics. He finds that when the candidates are
policy-motivated, divergent equilibria will result. Aragones and Palfrey (2003) study a model where
candidates are policy-motivated, one candidate has a valence advantage, and the voters are uncertain about
the candidates’ ideal points.

A key motivation to understating political equilibrium when non-policy factors are present (and
not simply valence) is that these are the specifications that are most often used in empirical work. Examples
include Erikson and Romero (1990), Alvarez and Nagler (1995), Schofield et al. (1998), Martin et al.
(1999), Lacy and Burden (1999, 2001), Adams and Merrill (2003), and Peress (2005). Typically, one
estimates a discrete choice model such as Probit to describe the choice between the candidates. The
specification includes ideological distance as well as other regressors. One can then perform counterfactual
experiments by altering the candidates’ policy positions and observing the affect on the candidates’ vote
shares. The additional regressors as well as the stochastic error term naturally become the ‘non-policy
factors’ since they cannot be altered by the candidates.

Our approach leads to an interesting interpretation of the probabilistic voting model. Under
probabilistic voting, voters vote for the candidate they prefer on policy grounds with some probability less
than one. This probability decreases when the distance between the voter and the candidate increases. The
fact that voters do not vote for their preferred candidate with probability one is sometimes justified by
arguing that voters make mistakes. Our approach naturally leads to a different interpretation- that the voters
care both about the policy positions of the candidates as well as some other characteristics which are not
mutable by the candidates.

The framework we describe here nests the standard probabilistic voting model (Hinich, 1977;
Coughlin and Nitzan, 1981) when the non-policy factors also have mean zero. Our framework nests the
‘probabilistic voting model with valence’ of Schofield (2003, 2004) when the voter’s ideal points and the

\textsuperscript{57} This is in contrast to McKelvey (1976) and Schofield (1978) who show that majority rule is generically
intransitive for multi-dimensional policy spaces.
non-policy factors are independent. Our results will differ from all the results in the literature in that vote-maximizing candidates will not always converge to the social welfare optimum. Instead, they will converge to the point which maximizes the social welfare of the voters with non-policy factors equal to zero. Thus, the candidates focus their attention on those voters who are the most sensitive to position-taking by the candidates.

The effect of candidate motivations has been studied for both the Downsian and Probabilistic Voting models. Whitman (1983) finds that in the Downsian model, both office-motivated and policy-motivated candidates will locate at the median voter’s ideal point in equilibrium. Calvert (1985) finds that when the candidates are uncertain about the location of the median voter, then policy-motivated candidates will diverge from the median voter’s position (while office-motivated candidates will not). Roemer (2001) finds that a similar result holds in the probabilistic voting model. Duggan and Fey (forthcoming) show that when candidates are policy-motivated, equilibria do not generically fail to exist in the multidimensional Downsian model (contrary to the case with purely-office motivated candidates)\(^{59}\).

Our analysis will consider both vote-maximizing candidates and policy-motivated candidates. Non-policy factors will lead to a striking difference between the equilibrium outcomes in each case. Under vote-maximizing behavior, the candidates will converge in equilibrium, mimicking the results when no non-policy factors are present. Alternatively, when the candidates are policy-motivated and one candidate has a non-policy advantage over the other, a divergent equilibrium will result. The advantaged candidate will move away from the vote-maximizing equilibrium and towards his ideal point. However, when non-policy factors are perfectly balanced so that neither candidate is advantaged, both vote-maximizing and policy-motivated behavior will lead to the same equilibrium outcomes.

We will use our results to study threats by interest groups against the candidates. For example, suppose that a Republican candidate has a non-policy advantage over a Democratic candidate. A conservative interest group may threaten to encourage their members to stay home on election day if the Republican candidate does not position himself sufficiently to the right. Our results suggest that such

\(^{58}\) This is in contrast to Plott (1967), who finds that very restrictive conditions are necessary for an equilibrium to exist when valence characteristics are not present.

\(^{59}\) A related branch of the literature asks whether maximizing the probability of victory is equivalent to maximizing expected votes, when candidates are uncertain about the voters’ ideal points. Patty (2002,
threats are likely to be self-defeating. If the candidate himself is a conservative ideologue, he already should have placed himself as far right as he could while still winning the election. The threat by the conservative group can have two effects then. Either the conservative candidate will move further to left in order to remain competitive, or the candidate will fail to do so and thus lose the election. Alternatively, our results indicate that a moderate group can effectively threaten a candidate in order to induce him to move towards the political center.

2 - The Model

We assume there are two candidates competing for office- candidate D and candidate R (who for expositional purposes we will think of as the Democratic and Republican candidates). Policy is characterized using the \( J \)-dimensional spatial model. We denote the positions of candidate D and candidate R by \( y_D \in Y \) and \( y_R \in Y \) where \( Y \subseteq \mathbb{R}^J \) is the set of positions that the candidates are allowed to take. Voters are completely characterized by their ideal points \( v \in \mathbb{R}^J \) and by their non-policy factors \( (z_D, z_R) \in \mathbb{R}^2 \). There is a continuum of voters represented by the density function \( f(v, z_D, z_R) \). The utility a voter characterized by \( (v, z_D, z_R) \) receives from voting for candidate \( k \) is represented by

\[
u_k(v, z_D, z_R) = g(z_k) + h(v, z_k - v)\]

where \( g : \mathbb{R} \to \mathbb{R} \) and \( h : \mathbb{R}^J \to \mathbb{R} \). Furthermore, we assume that the following properties hold,

(A1) \( h \) is second-order differentiable at all points except zero and is continuous at zero.

(A2) \( \frac{\partial^2 h}{\partial x_j^2} h(x) < 0 \) when \( x_j > 0 \) and \( \frac{\partial^2 h}{\partial x_j^2} h(x) > 0 \) when \( x_j < 0 \).

(A3) \( \frac{\partial^2}{\partial x_1 \partial x_2} h(x) \) is negative definite for all \( x \neq 0 \).

2005) and Duggan (2000) derive sufficient conditions for the two types of motivation to lead to the same equilibrium outcomes. Patty (forthcoming) shows that these conditions will generically fail to hold.

60 We assume that \( f(v, z_D, z_R) \) represents the subpopulation of voters rather than the entire population. The restriction of full turnout is without loss of generality so long as the voters’ turnout decisions do not depend on the positions that the candidates take.
Here, Assumption (A2) indicates that the voters care about the closeness of the policy outcome to their ideal points.

For expositional convenience, we will write $z = g(z_D) - g(z_R)$ and replace the distribution $f(v, z_D, z_R)$ with the distribution $f(v, z)$ since only the difference $g(z_D) - g(z_R)$ matters in terms of partitioning the set of voters among the candidates. The set $u(y_D - v, z_D) > u(y_R - v, z_R)$ can be characterized as follows,

$$\{(v, z) : z > h(y_R - v) - h(y_D - v)\}$$

We can write the vote shares of the candidates at positions $y_D$ and $y_R$ as,

$$s_p(y_D, y_R) = \int_{v \in \mathbb{R}} \int_{z = h(y_R - v) - h(y_D - v)}^{\infty} f(v, z) dz dv$$

$$s_R(y_D, y_R) = \int_{v \in \mathbb{R}} \int_{z = -\infty}^{h(y_D - v) - h(y_R - v)} f(v, z) dz dv$$

Two particularly interesting choices for the utility function are,

$$u(y - v, z) = z - \frac{1}{2}(y - v)A^T(y - v)$$

where $A$ is positive semi-definite and,

$$u(y - v, z) = z - \sum_{j=1}^{J} w_j |y_j - v_j|$$

where $w_j > 0$ for all $j$. The assumption that $v$ and $z$ are independent and $z$ is mean zero yields the standard probabilistic voting model (Coughlin and Nitzan, 1981; Banks and Duggan, 2003; McKelvey and Patty, 2003). In this case, the unique equilibrium will correspond with the social welfare optimum. When the utility function is quadratic, a ‘mean-voter theorem’ will hold. When the absolute value utility function is considered, a multi-dimensional median voter theorem will hold. As we will show later, these conclusions hold regardless of whether the candidates are vote-maximizing or policy-motivated. For a more general choice of $f$, this equivalence no longer holds.

3 – Office-Motivated Candidates
We define an equilibrium for vote-maximizing candidates as a point \((y_D^*, y_R^*)\) such that \(y_D^*\) maximizes \(s_D(y_D, y_R^*)\) over \(y_D \in Y\) and \(y_R^*\) maximizes \(s_R(y_D^*, y_R)\) over \(y_R \in Y\). Such an equilibrium will not always exist, but when an (interior) equilibrium does exist, it will be characterized by,

\[
\frac{\partial}{\partial y_D} s_D(y_D^*, y_R^*) = 0, \quad \frac{\partial^2}{\partial y_D \partial y_R} s_D(y_D^*, y_R^*) \text{ is negative semi-definite}
\]

\[
\frac{\partial}{\partial y_R} s_R(y_D^*, y_R^*) = 0, \quad \frac{\partial^2}{\partial y_R \partial y_D} s_R(y_D^*, y_R^*) \text{ is negative semi-definite}
\]

The following proposition shows that the candidates must converge in equilibrium.

**Proposition 1:** Suppose that assumptions (A1)-(A3) hold. Then any vote-maximizing equilibrium \((y_D^*, y_R^*)\) with \(y_D^* \in \text{int}(Y)\) and \(y_R^* \in \text{int}(Y)\) must satisfy,

(i) \(y_D^* = y_R^* = y^*\)

(ii) \(\int_{v \in \mathbb{R}} \frac{\partial}{\partial y} (y^* - v) f_{y_D}(v | 0) dv = 0\)

**Proof:** Taking first-order conditions leads to the following necessary conditions for equilibrium\(^{63}\),

\[
\int_{v \in \mathbb{R}} \frac{\partial}{\partial y} (y_D - v) f(v, h(y_R - v) - h(y_D - v)) dv = 0 \quad \text{for } j = 1, \ldots, J
\]

\[
\int_{v \in \mathbb{R}} \frac{\partial}{\partial y} (y_R - v) f(v, h(y_R - v) - h(y_D - v)) dv = 0 \quad \text{for } j = 1, \ldots, J
\]

We first show that there does not exist an equilibrium with \(y_D^* \neq y_R^*\). The two necessary conditions above imply that,

\[
\int_{v \in \mathbb{R}} [\frac{\partial}{\partial y} (y_R - v) - \frac{\partial}{\partial y} (y_D - v)] f(v, h(y_R - v) - h(y_D - v)) dv = 0 \quad \text{for } j = 1, \ldots, J
\]

This implies that,

\[
x^T \int_{v \in \mathbb{R}} [\frac{\partial}{\partial y} (y_R - v) - \frac{\partial}{\partial y} (y_D - v)] f(v, h(y_R - v) - h(y_D - v)) dv = 0
\]

\(^{61}\) Here, \(\text{int}(Y)\) denotes the interior of the set \(Y\).

\(^{62}\) Here, \(f_{y_D}(v | z)\) denotes the conditional distribution of \(v\) given \(z\).

\(^{63}\) These derivatives are computed using Leibniz’s rule. Since \(h\) may potentially be non-differentiable at 0, these derivatives must be computed piecewise over the intervals \((\infty, y_D, k)\), \((y_D, y_R, k)\), and \((y_R, \infty)\) (if \(y_D < y_R\)).
for all \( x \in \mathbb{R}^J \). Taking a first-order Taylor expansion of \( \frac{\partial}{\partial a} (y_a - v) \) around \( y_a - v \) yields

\[
x^T \left[ \frac{\partial}{\partial a} (y_a - v) - \frac{\partial}{\partial a} (y_a - v) \right] = x^T H(y_a - y_d) \text{ where } H \text{ is equal to } \frac{\partial^2 h}{\partial a^2} h(a) \text{ and } a \text{ are the mean values from the Taylor expansion. Setting } x = y_a - y_d, \text{ Assumption (A3) implies that}
\]

\[
x^T \left[ \frac{\partial}{\partial a} (y_a - v) - \frac{\partial}{\partial a} (y_a - v) \right] = x^T Hz < 0 \text{ for all } v. \text{ Since } \int f(y) \text{ has full support, } x^T \left[ \frac{\partial}{\partial a} (y_a - v) - \frac{\partial}{\partial a} (y_a - v) \right] < 0 \text{ for all } v \text{ implies that,}
\]

\[
x^T \int \left[ \frac{\partial}{\partial a} (y_a - v) - \frac{\partial}{\partial a} (y_a - v) \right] f(v, h(y_a - v) - h(y_a - v)) dv = 0
\]

cannot hold. Thus, (i) is a necessary condition for equilibrium.

Plugging in \( y_d^* = y_k^* = y^* \) to the first-order conditions gives,

\[
\int v \in \mathcal{V} \left[ \frac{\partial^2 h}{\partial a^2} (y^* - v) \right] f(v, 0) dv = 0 \text{ for } j = 1, \ldots, J
\]

which immediately shows that (ii) is a necessary condition.

In the standard Probabilistic voting models (Banks and Duggan, 2003; McKelvey and Patty, 2003), the unique equilibrium \((y^*, y^*)\) maximizes the utilitarian social welfare function. Consider choosing \( y^* \in Y \) and \( k \in \{D, R\} \) to maximize

\[
\int _{(v, z_k)} \left[ g(z_k) + h(y^* - v) \right] f_k(v, z_k) dv dz_k
\]

where \( f_k \) represents the marginal density of \((v, z_k)\). This leads to the first-order conditions,

\[
\int _{(v, z_k)} \left[ \frac{\partial}{\partial a} (y^* - v) \right] f_k(v, z_k) dv dz_k = 0
\]

This condition is, in general, not equivalent to condition (ii) of Proposition 1. When \( v \) and \( z \) are independent however, equivalence does hold. In both cases, we get the condition,

\[
\int \left[ \frac{\partial^2 h}{\partial a^2} (y^* - v) \right] f_k(v) dv = 0
\]

Further necessary conditions for equilibrium can be derived using the candidates’ second-order conditions. Since these conditions do not lead to a general proof that a local equilibrium exists, we omit this step. Banks and Duggan (2003) and McKelvey and Patty (2003) show that it is possible to develop general conditions for existence of an equilibrium when non-policy factors are absent. When non-policy factors are present, then it is no longer possible to develop general conditions for existence. In the next two sub-
sections, we will show that for the quadratic and absolute value utility functions, a vote-maximizing (pure strategy) equilibrium will fail to exist if the non-policy factors strongly favor one of the candidates.

Quadratic Utility Function

Consider the case where \( h(x) = -\frac{1}{2} x^T Ax \) and \( A \) is symmetric and positive semi-definite. In this case, the results of the previous section imply that the equilibrium has the form \( y_p = y_e = y^* \) where,

\[
y^* = \int_{x \in \mathbb{R}^T} y f_e(v | 0) dv
\]

In other words, for the quadratic utility specification, a mean-voter theorem holds.

The second-order conditions from this problem require that the following matrices are negative semi-definite,

\[
M_D = \int_{x \in \mathbb{R}^T} \frac{\partial h}{\partial y} (y^* - v) f(v, 0) dv - \int_{x \in \mathbb{R}^T} \frac{\partial h}{\partial v} (y^* - v) \frac{\partial}{\partial y} (y^* - v)^T f_z(v, 0) dv
\]

\[
M_e = \int_{x \in \mathbb{R}^T} \frac{\partial h}{\partial z} (y^* - v) f(v, 0) dv + \int_{x \in \mathbb{R}^T} \frac{\partial h}{\partial v} (y^* - v) \frac{\partial}{\partial z} (y^* - v)^T f_z(v, 0) dv
\]

If we assume that \( J = 1 \), \( A = [a] \), and \( f(v, z) = f_z(v) f_e(z) \), we get,

\[
\left| f'_z(0) \right| < \frac{1}{\sigma^2} f_z(0)
\]

where \( \sigma^2 = \int_{x \in \mathbb{R}^T} (y^* - v)^2 f_z(v) dv \). If \( z \) is normally distributed with mean \( \lambda \) and variance 1, then

\[
f_z(0) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad \text{and} \quad f'_z(0) = \lambda \frac{1}{\sqrt{2\pi}} e^{-z^2/2},
\]

so that \( |\lambda| < \frac{1}{\sigma^2} \). This indicates that an equilibrium will fail to exist if the valence term is relatively large.

Absolute Value Utility Function

A second interesting specification is \( h(x) = -\sum_{j=1}^{I} w_j \, |x_j| \) with \( w_j > 0 \) for all \( j \). In this case,

\[
h_j(x) = \begin{cases} w_j & x_j \leq 0 \\ -w_j & x_j > 0 \end{cases}
\]

The equilibrium will have the form \( y_p = y_e = y^* \) where,
\[ \int_{v_1 \in \mathbb{R} \ldots v_J \in (-\infty, y^\ast) \ldots v_J \in \mathbb{R}} f_{v_j}(v,0)dv = \frac{1}{2} \text{ for } j = 1, \ldots, J \]

so that a median voter theorem hold in this case (where the median is taken dimension by dimension).

Assume further that \( J = 1 \) and \( w = [a] \). In this case, the second-order conditions yield\(^6^4\),

\[-2af(y^\ast,0) - a^2 \int_{v=-\infty}^{\infty} f_2(v,0)dv < 0, \quad -2af(y^\ast,0) + a^2 \int_{v=-\infty}^{\infty} f_2(v,0)dv < 0\]

A necessary and sufficient condition is,

\[
\left| \int_{v=-\infty}^{\infty} f_2(v,0)dv \right| < \frac{2}{a} f(y^\ast,0)
\]

For example, when \( v \) and \( z \) are independent and \( z \) is normally distributed with mean \( \lambda \) and variance 1, we get \( |\lambda| < \frac{2}{a} f_1(y^\ast) \). Again, we have that an equilibrium fails to exist if the valence term is relatively large.

4 – Policy-Motivated Candidates

The results considered in the previous section can be viewed as a problem for models of spatial competition. When vote-maximizing candidates are considered, convergence will result. Furthermore, the point to which the candidates converge will not depend on the degree of advantage one candidate has.

Realistically, we would expect a candidate with a non-policy advantage will strive to move away from the vote-maximizing equilibrium and towards his own ideal point. The more substantial his advantage, the closer he should be able to move to his ideal point. We can show that such behavior will result if policy-motivation is assumed instead of vote-maximizing behavior.

We assume that candidate \( k \) has a utility function of the form, \( U_k(x - q_k, w_k) \), where \( x \) denotes the policy outcome (determined by the candidate that is elected to office), \( w_k \) indicates whether candidate \( k \) wins the election, and \( q_k \) is the ideal point of candidate \( k \). We further assume that,

(B1) \( U_k(x, w_k) \) is continuous in \( x \).
Here, Assumption (B2) indicates that the candidates care about the closeness of the policy outcome to their ideal point. Assumption (B3) indicates that the candidates also care about holding office directly.

Before we can proceed, we must deal with a technical problem. Suppose that we were to assume that in case of a tie, each candidate is chosen as the winner with equal probability. This leads to the ‘open-set problem’ described in Groseclose (2001)\textsuperscript{65}. Essentially, since the candidates’ utility functions depend on the identity of the winner and the identity of the winner is not a continuous function of the positions of the candidates, this can lead to a situation where an equilibrium fails to exists. Groseclose (2001) proposes to solve the ‘open-set’ problem by introducing aggregate uncertainty. We will take a different approach. Following Duggan (forthcoming), we will consider deterministic tie-breaking rules. This will allow us to replicate the limiting equilibria that result from considering small amounts of aggregate uncertainty, without introducing uncertainty explicitly.

Let $\tau \in [0,\frac{1}{2},1]$ represent the probability that candidate D is selected as the winner in case $s_D(y_D^*, y_R^*) = \frac{1}{2}$ (i.e., in case of a tie). The candidates’ utility functions over policy positions are given by,

$$V_t(y_D, y_R, \tau) = \begin{cases} U_A(y_D - q_D, 1) & s_D(y_D, y_R) > \frac{1}{2} \\ tU_A(y_D - q_D, 1) + (1-t)U_A(y_R - q_R, 0) & s_D(y_D, y_R) = \frac{1}{2} \\ U_A(y_R - q_R, 0) & s_D(y_D, y_R) < \frac{1}{2} \end{cases}$$

We define an equilibrium as pair of points in the policy space $(y_D^*, y_R^*)$ and a tie-breaking rule $\tau^*$ such that $y_D^*$ maximizes $V_D(y_D, y_R^*, \tau^*)$ over $y_D \in Y$ and $y_R^*$ maximizes $V_R(y_D^*, y_R, \tau^*)$ over $y_R \in Y$.

The tie-breaking rule will be one of three varieties- select candidate D in case of a tie, select candidate R in case of a tie, or select either candidates with equal probability in case of a tie.

Define,

\textsuperscript{64} Here, the derivatives must be taken piecewise over $(-\infty, y_{D,\delta}), (y_{D,\delta}, y_{R,\delta})$, and $(y_{R,\delta}, \infty)$ (if $y_{D,\delta} < y_{R,\delta}$) since $h$ is not differentiable at 0.
\[ z = \int_{v \in R', z > 0} f(v, z) dv dz - \int_{v \in R', z < 0} f(v, z) dv dz \]

If \( z > 0 \), we say that candidate D has a non-policy advantage. If \( z < 0 \), we say that candidate R has a non-policy advantage. Otherwise, we say that neither candidate has a non-policy advantage. If one candidate has a non-policy advantage, then that candidate would win if both candidates were to locate at the same position. To see that this is the case, suppose that \( y_D = y_R = y \). Then,

\[
s_D(y) - s_R(y) = \int_{s(v) = y} f(v, z) dv dz - \int_{s(v) = y} f(v, z) dv dz = \tilde{z} = \int_{v \in R', z > 0} f(v, z) dv dz - \int_{v \in R', z < 0} f(v, z) dv dz = \tilde{z} \]

Thus, \( s_D(y) > s_R(y) \) if \( \tilde{z} > 0 \) and \( s_D(y) < s_R(y) \) if \( \tilde{z} < 0 \).

We will assume that neither candidate has an ideal point that coincides with the vote-maximizing equilibrium, i.e.,

\[ (C1) \quad q_D \neq y^* \quad \text{and} \quad q_R \neq y^* \quad \text{where} \quad y^* \in \text{int}(Y) \quad \text{satisfies} \quad \int_{v \in R'} \frac{\partial}{\partial v} (y^* - v) f(v|v) dv = 0. \]

We show below that any equilibrium must either have both candidates receiving equal vote shares, or must have the winning candidate located at his policy ideal point.

**Proposition 2:** Suppose that assumptions (A1)-(A3), (B1)-(B4), and (C1) hold.

(i) If \( \tilde{z} > 0 \), then any equilibrium \((y_D^*, y_R^*, t^*)\) must satisfy \( s_D(y_D^*, y_R^*) = s_R(y_D^*, y_R^*) \) or \( y_D^* = q_D \).

(ii) If \( \tilde{z} = 0 \), then any equilibrium \((y_D^*, y_R^*, t^*)\) must satisfy \( s_D(y_D^*, y_R^*) = s_R(y_D^*, y_R^*) \).

(iii) If \( \tilde{z} < 0 \), then any equilibrium \((y_D^*, y_R^*, t^*)\) must satisfy \( s_D(y_D^*, y_R^*) = s_R(y_D^*, y_R^*) \) or \( y_R^* = q_R \).

**Proof:** (i) Suppose that \((y_D^*, y_R^*, t^*)\) satisfies \( s_D(y_D^*, y_R^*) < s_R(y_D^*, y_R^*) \). Then candidate D can clearly improve his utility by moving to \( y_R^* \) since this will lead to the same policy outcome, but will increase his

---

65 Duggan (forthcoming) discusses the open-set problem in the context of legislative agendas.
probability of holding office. Alternatively, suppose that \( s_D(y_D^*, y_R^*) > s_R(y_D^*, y_R^*) \) and \( y_D^* \neq q_D \). Since \( s_D(y_D, y_R) \) is continuous in \( y_D \), there exists an \( \varepsilon > 0 \) small enough such that \( s_D(y_D + \varepsilon p, y_R) > \frac{1}{2} \) for any direction \( p \in \mathbb{R}^J \) with \( \|p\| = 1 \). Furthermore, since \( y_D^* \neq q_D \), there exist an \( p^* \in \mathbb{R}^J \) with \( \|p^*\| = 1 \) such that \( U_D(y_D^* + \varepsilon p^*, 1) > U_D(y_D^*, 1) \) for \( \varepsilon > 0 \) small enough. Picking \( \varepsilon = \min\{\varepsilon, \varepsilon_2\} \), we can see that candidate D can improve his utility by moving to \( y_D^* + \varepsilon p^* \).

(ii) If \( t^* = \frac{1}{2} \) and \( s_D(y_D^*, y_R^*) \neq s_R(y_D^*, y_R^*) \), then the losing candidate can improve his utility by moving to the position of the winning candidate. Now consider the case where \( t^* = 1 \). If \( s_D(y_D^*, y_R^*) < s_R(y_D^*, y_R^*) \), then candidate D can improve his utility by moving to \( y_R^* \). Now suppose that \( s_D(y_D^*, y_R^*) < s_R(y_D^*, y_R^*) \). The argument in part (i) implies that we must have \( y_D^* = q_D \). We know that \( s_D(q_D, q_D) = \frac{1}{2} \). Suppose that there exist an \( p \in \mathbb{R}^J \) with \( \|p\| = 1 \) and \( \varepsilon > 0 \) such that

\[
\frac{1}{2}(q_D, q_D + \varepsilon p) < \frac{1}{2}.
\]

The candidate R can improve his utility by moving to \( q_D + \varepsilon p \). But for this condition to fail, \( (q_D, q_D) \) must be a local equilibrium which violates the assumption that \( q_D \neq y^* \). Thus, when \( t^* = 1 \), we must have \( s_D(y_D^*, y_R^*) = s_R(y_D^*, y_R^*) \). The case where \( t^* = 0 \) can be proved similarly.

(iii) This follows from (i) by symmetry.

We can further characterize the equilibria using the following proposition.

**Proposition 3:** Suppose that assumptions (A1)-(A3), (B1)-(B4), and (C1) hold.

(i) If \( \tilde{z} > 0 \), then any equilibrium \( (y_D^*, y_R^*, t^*) \) satisfies either (a) \( s_R(y_D^*, y_R^*) = \frac{1}{2} \), \( \frac{\partial}{\partial y_D} s_R(y_D^*, y_R^*) = 0 \), and \( \frac{\partial^2}{\partial y_D^2} s_R(y_D^*, y_R^*) \) is negative semi-definite, or (b) \( s_D(y_D^*, y_R^*) \geq \frac{1}{2} \) and \( y_D^* = q_D \).

(ii) If \( \tilde{z} = 0 \), then any equilibrium \( (y_D^*, y_R^*, t^*) \) satisfies \( s_D(y_D^*, y_R^*) = \frac{1}{2} \), \( \frac{\partial}{\partial y_D} s_D(y_D^*, y_R^*) = 0 \), and \( \frac{\partial^2}{\partial y_D^2} s_D(y_D^*, y_R^*) \) is negative semi-definite, \( \frac{\partial}{\partial y_R} s_D(y_D^*, y_R^*) = 0 \), and \( \frac{\partial^2}{\partial y_R^2} s_D(y_D^*, y_R^*) \) is negative semi-definite.
(iii) If $\bar{z} < 0$ then any equilibrium $(y_D^*, y_R^*, t^*)$ satisfies either (a) $s_D(y_D^*, y_R^*) = \frac{1}{2}$, \( \frac{\partial}{\partial y_D} s_D(y_D^*, y_R^*) = 0 \), and $\frac{\partial^2}{\partial y_D \partial y_R} s_D(y_D^*, y_R^*)$ is negative semi-definite, or (b) $s_R(y_D^*, y_R^*) \geq \frac{1}{2}$ and $y_R^* = q_R$.

Proof: (i) By Proposition 1, part (i), we know that either $s_D(y_D^*, y_R^*) = \frac{1}{2}$ or $y_D^* = q_D$. Suppose that the first case is true. If $\frac{\partial}{\partial y_D} s_D(y_D^*, y_R^*) = 0$ did not hold, this implies that their exist an $\varepsilon > 0$ small enough such that $s_R(y_D^*, y_R^* + \varepsilon e) > \frac{1}{2}$ for any direction $p$ with $\|p\| = 1$. Since $U_y(x, w)$ is continuous in $x$, there must be an $\varepsilon > 0$ small enough such that $U_R(y_D^*, y_R^* + \varepsilon e, 1) > U_R(y_D, 0)$. Similarly, if $\frac{\partial^2}{\partial y_D \partial y_R} s_D(y_D^*, y_R^*)$ failed to be negative semi-definite, candidate $R$ would be able to improve his utility by moving $\varepsilon$ in some direction $p$. Alternatively, if $y_D^* = q_D$, then $s_D(y_D^*, y_R^*) \geq \frac{1}{2}$ since if $s_D(y_D^*, y_R^*) < \frac{1}{2}$, candidate $D$ could improve his utility by moving to $y_R^*$.

(ii) $s_D(y_D^*, y_R^*) = \frac{1}{2}$ is shown in Proposition 2, part (ii). The remaining conditions are implied by an identical argument to the one used in the proof of part (i).

(iii) This follows from part (i) by symmetry.

If a candidate has a non-policy advantage, then that candidate must be able to move the policy outcome away from the vote-maximizing equilibrium and towards his ideal point. If neither candidate has a non-policy advantage and both a vote-maximizing equilibrium and a policy-motivated equilibrium exist, they must be equivalent. Both of these results are shown below.

**Proposition 4:** Suppose that assumptions (A1)-(A3), (B1)-(B4), and (C1) hold. Let $(y^*, y^*)$ be the vote-maximizing local equilibrium.

(i) If $\bar{z} > 0$ and $y^* \neq q_D$, then any policy-motivated equilibrium $(y_D^*, y_R^*, t^*)$ must have $U_D(y_D^*, 1) > U_D(y^*, 1)$.

(ii) If $\bar{z} = 0$, then any policy-motivated equilibrium must satisfy, $y_D^* = y^*$ and $y_R^* = y^*$. 

81
(iii) If \( \bar{z} < 0 \) and \( y^* \neq q_R \), then any policy-motivated equilibrium \((y^*_D, y^*_R, t^*)\) must have

\[
U_R(y^*_R, 1) > U_R(y^*, 1).
\]

Proof: (i) We know that \( s_D(y^*, y^*) > \frac{1}{2} \). We must also have that \( s_D(y^*, y^*) < s_D(y^*, y^*_R) \) for \( y^*_R \geq y^* \), by the definition of \( y^* \). Let \( p \) be a direction such that \( \|p\| = 1 \). Consider a move by candidate D to \( y^* + \varepsilon p \).

Since \( s_D(y^*, y^*_R) > \frac{1}{2} \), continuity of \( s_D \) implies that there must exist an \( \varepsilon > 0 \) small enough such that

\[
s_D(y^* + \varepsilon p, y^*_R) > \frac{1}{2}.
\]

Assumption (B2) and (B3) imply that there must exist an \( p \) such that

\[
U_D(y^* + \varepsilon p, 1) > U_D(y^*, 1).
\]

Since candidate D would win the election with probability by positioning at \( y^* + \varepsilon p \), no matter where candidate R locates, and equilibrium \((y^*_D, y^*_R)\) must have

\[
U_D(y^*_D, 1) > U_D(y^*, 1).
\]

(ii) This follows directly from the fact that the first order conditions are equivalent.

(iii) This follows form (i) by symmetry.

We show below that a candidate with a non-policy advantage must win the election with probability one, and that all equilibria involve tie-breaking rules that break in favor of the advantaged candidate (except when the advantaged candidate is located at his ideal point).

**Proposition 5:** Suppose that assumptions (A1)-(A3), (B1)-(B4), and (C1) hold. Let \((y^*, y^*)\) be the vote-maximizing equilibrium.

(i) If \( \bar{z} > 0 \), then any equilibrium \((y^*_D, y^*_R, t^*)\) must have candidate D winning the election with probability one. Furthermore, if \( y^*_D \neq q_D \), then the equilibrium must have \( t^* = 1 \).

(ii) If \( \bar{z} < 0 \), then any equilibrium \((y^*_D, y^*_R, t^*)\) must have candidate R winning the election with probability one. Furthermore, if \( y^*_R \neq q_R \), then the equilibrium must have \( t^* = 0 \).
Proof: (i) We know that \( \frac{\partial}{\partial y_D} s_D(y_D^*, y_R^*) \neq 0 \) since if this were that case, it would imply that

\[ y_D^* = y_R^* = y^* \]

thus violating Proposition 3. Thus, there exist an \( p \) with \( \|p\| = 1 \) and \( \varepsilon > 0 \) small enough such that \( s_D(y_D^* + \varepsilon p, y_R^*) > \frac{1}{2} \). If candidate D is not winning the election with probability one, then he can move to \( y_D^* + \varepsilon p \) an improve his utility for small enough \( \varepsilon \). If we further assume that \( y_D^* \neq q_D \), then Proposition 2 implies that \( s_D(y_D^*, y_R^*) = \frac{1}{2} \) must hold in equilibrium. From this, it is clear that we must have \( t^* = 1 \).

(ii) This follows from part (i) by symmetry.

Finally, below we will show that if a candidate has a non-policy advantage, that candidate will position himself as close to his ideal point as he can while still winning the election.

**Proposition 6:** Suppose that assumptions (A1)-(A3) and (B1)-(B4) hold.

(i) Define \( W_D = \{y_D \in Y : \min_{y \in Y} s_D(y_D, y_R) \geq \frac{1}{2} \} \). If \( \tilde{z} > 0 \), then \( y_D^* = \arg \max_{y \in W_D} U_D(y_D, 1) \) must hold for any equilibrium \((y_D^*, y_R^*, t^*)\).

(ii) Define \( W_R = \{y_R \in Y : \min_{y \in Y} s_D(y_D, y_R) \geq \frac{1}{2} \} \). If \( \tilde{z} < 0 \), then \( y_R^* = \arg \max_{y \in W_R} U_R(y_R, 1) \) must hold for any equilibrium \((y_D^*, y_R^*, t^*)\).

**Proof:** (i) Proposition 5 implies that \( y_D^* \in W_D \). Now suppose that there exist a point \( y_D \in W_D \) such that

\[ s_D(y_D^*, y_R) > \frac{1}{2} \]

for all \( y_R \), and \( U_D(y_D^*, 1) > U_D(y_D^*, 1) \). Then \((y_D^*, y_R^*, t^*)\) cannot be an equilibrium since candidate D can improve his utility by moving to \( y_D \).

(ii) This follows from part (i) by symmetry.

We can summarize our results as follows. If one candidate has a non-policy advantage, then that candidate will win the election in equilibrium. Either the candidate will be able to achieve his ideal point, or all equilibria will have tie-breaking rules favoring the advantaged candidate and both candidates
receiving equal vote shares in equilibrium. Furthermore, the advantaged candidate will move as close to his ideal point as he can while still winning the election. If neither candidate has non-policy advantage, then both candidates must locate at a local vote-maximizing point in equilibrium (although an equilibrium may fail to exist). The type of tie-breaking rule used determines who wins the elections in equilibrium, but does not affect the policy outcome.

5 – Threats by Interest Groups

An interesting application of our results is in analyzing threats by ideological groups. For example, a labor union may threaten to reduce their get-out-the-vote effort if the Democratic candidate does not position himself far enough to the left. Suppose for the moment that these threats are credible. We will analyze whether these threats can have the desired effect.

Assume that the Democratic candidate has a non-policy advantage. If the candidates are indeed vote-maximizing, then the Democratic candidate will receive more than a 50% vote share in equilibrium. In this case, there is a real possibility that such a threat would induce the Democratic candidate to move to the left. However, if the candidates are themselves policy-motivated, this will no longer be the case. In equilibrium, the Democratic candidate will be receiving exactly a 50% vote share (except when he is able to move all the way towards his ideal point). Since this candidate is only receiving 50% of the vote, he has already moved as far left as he could while still winning the election. If the labor union were to withdraw its’ support, then the Democratic candidate would be forced to move to the left in order to recoup the support that he lost, which is clearly not the effect desired by the group.

On the other hand, it is possible for a moderate interest group to carry out an effective threat. Suppose again that the Democratic candidate has a non-policy advantage and that the interest group consists of voters that are left-of-center. If the candidates are policy-motivated, then by threatening to stay at home, these voters could induce the Democratic candidate to move closer to the center then he otherwise would have located.

6 – Conclusions and Discussion
The results of this paper show that when non-policy factors are present, the location and properties of the equilibrium depends crucially on the motivation of the candidates. While vote-maximizing candidates converge to a central position of the ideology distribution, policy-motivated candidates will diverge. When candidates are policy-motivated, the candidate with a non-policy advantage is able to move away from the vote-maximizing equilibrium and towards his own ideal point.

The distinction is important because candidates seen to behave like policy-motivated candidates in reality. For example, sitting Senators and Congressmen in the U.S. typically enjoy an incumbency advantage. This means that some voters who ideologically agree with their opponents will still vote for these incumbents. Incumbents would like to be re-elected, but they also have policy preferences different from the median voter. Thus, they attempt to move as far as they can in the direction of their preferred policy platform while still begin re-elected.

This paper offers an interesting contrast to the results of Schofield et al. (1998) and Schofield (2003, 2004). These papers show that divergence in multi-party systems can be explained by the presence of valence asymmetries. Their model predicts that vote-maximizing parties with high valence will tend to locate near the center while low valence parties will tend to locate at the extremes. Their predicted equilibria fit well with observed party positions in many multi-party democracies such as Germany, Israel, and the Netherlands. We illustrate a contrasting point- divergence in two-party democracies cannot be explained by valence asymmetries alone, although divergence can result from a combination of non-policy factors and policy-motivated candidates.

Finally, we note that when analyzing the behavior of interest groups, the motivation of the candidates matter. When the candidates are vote-maximizers, the equilibrium will involve the advantaged candidate receiving more than 50% of the votes. Thus, an extreme interest group can effectively incentivize the winning candidate to move in their direction by threatening to withdraw their support. Such a treat will no longer be effective if the candidates are policy-motivated. If the threat is carried out, the advantaged candidate will be forced to move away from the interest group in order to recoup lost electoral support.

References


Essay 4 - Multi-Stage Voting Rules

1 – Introduction

Plurality Rule is used for group decision making in a wide variety of circumstances including presidential and congressional elections. A number of alternative voting rules are in current use and many more have been proposed. The most prominent alternatives are Majority Rule, Approval Voting, and Single Transferable Vote. The fact that Plurality Rule is used in so many situations would seem to suggest its superiority, but there is some evidence to suggest the contrary. Majority Rule, Approval Voting, and Single Transferable Vote have been widely suggested as superior alternatives, both by political reformers and academics.

One potential failing of Plurality Rule is that it can select a candidate who receives a relatively small percentage of the total votes. Majority Rule and Single-Transferable Vote both prevent this by requiring that the winning candidate receive at least a majority of the votes. Approval Voting makes it very unlikely that the winning candidate will receive less than a majority of the votes. In addition, all three rules are argued to reduce the possibility of a ‘spoiler’ candidate affecting the outcome of the election.

Among the three popular alternatives to Plurality Rule, Single Transferable Vote is a clear favorite among political reformers. Those advocating Single Transferable Vote for elections in the United States include U.S. Senator John McCain, Democratic presidential candidates Howard Dean and Dennis Kucinich, the National Green Party, the American Reform Party, the Midwest Democracy Center, the Center for Voting and Democracy, and U.S.A. Today. Recently, San Francisco passed a voter initiative establishing Instant Runoff elections and 49 towns in Vermont have passed voter initiatives recommending their use to the state legislature66. Advocates of this voting rule often stress two apparent benefits- that it eliminates the ‘wasted vote’ phenomenon that occurs under Plurality Rule and that it eliminates the need for strategic behavior on the part of the voter.

66 Instant Runoff refers to Single-Transferable Vote and a slight variant of Single-Transferable Vote where all but the top two vote-getting candidates are eliminated from consideration after the first round.
Brams and Fishburn (1978, 1983) have investigated strategic behavior under Approval Voting. They argue in favor of Approval Voting by considering an election with three candidates. There is one left-wing candidate and two right-wing candidates, and the right-wing candidates are preferred by a majority of voters. If the right-wing voters split their votes between the two candidates, the left-wing candidate may win. The left-wing candidate would however lose in a head-to-head race against either right-wing candidate (that is, the left-wing candidate is the Condorcet Loser). Approval Voting would allow the right-wing voters to approve both candidates. They show that in the case of dichotomous preferences, Approval Voting is the only voting rule that is strategy-proof and selects the Condorcet Winner\(^{67}\). Alternatively, Niemi (1984) shows that Approval Voting can select the Condorcet Loser if preferences are not dichotomous, and argues that since Approval Voting admits multiple sincere strategies, the incentives for strategic voting may be even higher than under Plurality Rule.

Palfrey (1989), Myerson and Weber (1993), Cox (1994), and Fey (1997) study voting under incomplete information. These models differ from Brams and Fishburn (1982) and Niemi (1984) in that voters almost always have a positive probability of being pivotal. Rather than fully characterizing the set of equilibria of various voting rules, these papers focus on a mathematical proof of Duverger’s Law\(^{68}\). The results suggest that the outcome of the election under Plurality Rule will depend strongly on the beliefs that the voters have.

Sanver and Sertel (2003) study voting rules in a cooperative context. They show that for a large class of voting rules, the Strong Equilibria are the ‘generalized’ Condorcet Winners. Brams and Sanver (2003) also study voting rules under strong equilibria, paying particular attention to Approval Voting. They show that Approval Voting is unique in allowing voters to achieve strong equilibrium using sincere strategies. Barberá and Coelho (2004) study the Rule of k-Names using an approach similar to Sanver and Sertel (2003).

A large literature on Implementation Theory considers designing new voting rules rather than studying the properties of existing voting rules. Gibbard (1973) and Satterthwaite (1975) independently

\(^{67}\) Under dichotomous preferences, every voter can divide all the candidates into two groups such that he is indifferent between all the candidates within these groups. Under this assumption, a weakly-dominant strategy is to approve all candidates in the preferred group and not approve the other candidates.
showed that no non-dictatorial social choice rule can be truthfully implemented in dominant strategies under an unrestricted preference domain. This result leaves open the possibility of dominant strategy implementation in restricted preference domains and implementation under alternative equilibrium concepts. Moulin (1980) takes up the first point by characterizing implementable social choice functions under Single-Peaked preferences. Maskin (1977) takes up the second point by showing that Monotonic social choice rules can be implemented in Nash Equilibrium.

In order to continue our analysis, we must decide how we can evaluate a voting rule. Much of the work in social choice theory evaluates a voting rule based on whether it satisfies certain axioms (Monotonicity, the Weak Defensive Strategy Criterion, the Sumability Criterion, etc.). The drawback of this approach is that it places restrictions on the mechanism rather than the outcomes. Implementation Theory avoids this criticism by finding mechanisms that select the outcome prescribed by some social choice rule. This approach only allows us to construct a mechanism to implement an arbitrary social choice rule. It does not allow us to evaluate a particular voting rule, or to compare two voting rules. We will instead select a particular social choice rule. That is, we will look for voting rules that select the Condorcet Winner, when one exists. More generally, we will look for voting rules that select elements of the Uncovered Set. In addition, we will consider imposing restrictions relating to the practicality of a mechanism, i.e., the number of rounds of play, the information and coordination requirements placed on the voters, etc.

We will begin by considering a number of commonly used voting rules- Plurality Rule, Majority Rule, Approval Voting, and Single-Transferable Vote. All of these suffer from multiple Undominated Nash Equilibria, and will often fail to select the Condorcet Winner when one exists. In this paper, we will primarily consider multi-stage voting rules as alternatives to the commonly used voting rules. Three classes of multi-stage voting rules select the Condorcet Winner under suitable conditions. Multi-Stage Runoff will select the Condorcet Winner so long as the Majority Rule social preference is quasi-transitive (a condition

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68 Duverger’s Law states that under Plurality elections, only two candidates will be considered serious contenders and the rest of the candidates will eventually lose their support. See Grofman and Lijphart (1988) and Cox (1997) for further discussion.

69 Of course, this does not apply to work in social choice that includes Strategy-Proofness as an axiom, as is done in Moulin (1980).

70 The Uncovered Set defines a social choice Rule that satisfies all of the axioms in Arrow (1951) except the Weak Axiom of Revealed Preference.
weaker than single-peakedness). Binary Voting Trees will always select an element in the Uncovered Set, and consequently, will select a Condorcet Winner when one exists. The Nominate-Two rule will select the Condorcet Winner under weak conditions as well.

2 – Single Stage Voting Rules

Voters and Candidates

The strategic voting game consists of a finite set of voters \( (N) \) who have strict preferences over a set of candidates or policy alternatives \( (K) \). Using a slight abuse of notation, we will let \( N \) and \( K \) denote the cardinality of these sets as well. We assume that voters have strict preferences and denote the strict preferences of voter \( n \) by \( \succ_n \). The Majority Rule social preference will be given by \( \succ \) where,

\[
i \succ j \text{ if and only if } |\{n : i \succ_n j\}| > \frac{1}{2}N
\]

Majority indifference will be denoted by \( \sim \) where,

\[
i \sim j \text{ if and only if } |\{n : i \succ_n j\}| = \frac{1}{2}N
\]

The preference relation will also be defined over sets of candidates. The outcome \( C \subset K \) will denote a tie between a number of candidates. This tie is broken by selecting each candidate with equal probability. We will require that the preferences of the voters satisfy \( C_1 \cup C_2 \succ_n C_3 \) if \( C_1 \succ_n C_3 \) and \( C_2 \succ_n C_3 \) where \( C_1 \), \( C_2 \), and \( C_3 \) are sets of candidates. In particular, this property will hold if voters have utility functions that admit the expected utility form.

There are three restricted preference domains that we will encounter later,

(1) A Condorcet Winner exists - \( \succ \) has a unique maximal element.

(2) The relation \( \succ \) is quasi-transitive - For all \( i, j, k \), \( i \succ j \) and \( j \succ k \) implies \( i \succ k \).

(3) Single-Peaked preferences - There exists candidate positions \( (q_1, q_2, \ldots, q_K) \in \mathbb{R}^K \) and voter ideal points \( (z_1, z_2, \ldots, z_N) \in \mathbb{R}^N \) where if \( q_i < q_j < z_n \) or \( q_i > q_j > z_n \), we have \( i \succ_n j \).
Define the covering relation \( \triangleright \) by,

\[
i \triangleright j \text{ if and only if } i \triangleright j \text{ and } k \triangleright i \Rightarrow k \triangleright j
\]

The Uncovered Set is the set of maximal elements of the covering relation, which we will denote by \( M(K, \triangleright) \). In general, we will look for voting rules such that the only equilibrium outcomes of the strategic voting game are contained in the Uncovered Set. This may not be possible in the most general preference domains, so we will also consider voting rules that select the Condorcet Winner in one of the restricted preference domains mentioned above.

**Voting Rules**

Let \( A \subset \mathbb{R}^K \) be the set of admissible strategies for each voter. We define a voting rule to be a function \( \phi \) that assigns a subset of \( K \) to points in \( A^N \). That is, it assigns a subset of the candidates to each admissible configuration of the voter strategies. All of the commonly used voting rules (with the exception of Majority Rule) fall into our definition of voting rules. Majority Rule will be discussed in Section 3 when we discuss multi-stage voting rules.

The simplest equilibrium concept we can use to solve the strategic voting game is Nash Equilibrium. This concept is not a very useful one for voting rules because voters are hardly ever pivotal. We can restrict attention to Nash equilibria in which voters play undominated strategies.

**Undominated Set:** The Undominated Set of voter \( n \), \( U_n \), is given by the set of strategies \( a_n \) such that there does not exist a strategy \( a'_n \in A \) where \( \phi(a_1, \ldots, a_{n-1}, a'_n, a_{n+1}, \ldots a_N) \) is weakly-preferred to \( \phi(a_1, \ldots, a_{n-1}, a_n, a_{n+1}, \ldots a_N) \) for all \( (a_1, \ldots, a_{n-1}, a_{n+1}, \ldots, a_N) \in A^{N-1} \) and strict preference holds for some \( (a_1, \ldots, a_{n-1}, a_{n+1}, \ldots, a_N) \in A^{N-1} \).
We can then define an Undominated Nash Equilibrium as a Nash Equilibrium that involves voters playing strategies in their Undominated Set.

**Undominated Nash Equilibrium:** $a \in (U_1, U_2, ..., U_N)$ is an Undominated Nash Equilibrium if there does not exist an $n$ and $a_n' \in U_n$ such that $\phi(a_1, ..., a_{n-1}, a_n', a_{n+1}, ..., a_N)$ is strictly preferred to $\phi(a_1, ..., a_{n-1}, a_n, a_{n+1}, ..., a_N)$.

**Plurality Rule**

Under Plurality Rule, each voter is allowed to vote for a single candidate and the candidate with the most votes wins the election (the winner is selected randomly if there is a tie). In our notation, $A$ consists of all vectors $a_n$ such that exactly one component is 1 and the others are 0 and $\phi(a) = \arg\max_{i \in K} \sum_n a_{ni}$. It is well known that under Plurality Rule, the only weakly dominated strategy is to vote for your last choice (Brams and Fishburn, 1983). It is therefore not surprising that even if we eliminate these strategies, Plurality Rule will still admit multiple Nash Equilibrium outcomes.

**Proposition 1:** Under Plurality Rule, (i) voting for your last choice is a weakly-dominated strategy. (ii) No other strategies are weakly-dominated. (iii) If $N \geq 4$ and candidate $i$ is not a Condorcet Loser, there exists an Undominated Nash Equilibrium where candidate $i$ wins the election.

**Approval Voting**

Under Approval Voting, voters may vote for (approve) as many candidates as they like, and the candidate with the highest number of votes is declared the winner. Here, $A = \{0,1\}^K$ and

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71 Different definitions of the Uncovered Set have been considered, which may not be equivalent when the space of alternatives if infinite (Penk, 2002). These alternate definitions are equivalent for finite sets of alternatives.
\[ \phi(a) = \arg\max_{i \in K} \sum_{n} a_{ni} . \] Under Approval Voting, not approving of your first choice is weakly-dominated since this can only have the effect of swinging the election from your first choice to another candidate. Similarly, approving your last choice is weakly-dominated since this can only have the effect of swinging the election from another candidate to your last choice. Since there are more weakly-dominated strategies under Approval Voting than Plurality Rule, the set of equilibrium outcomes will often be smaller under Approval Voting.

**Proposition 2:** Under Approval Voting, (i) If \( i \) is the first choice of voter \( n \), then any strategy in which the voter does not approve \( i \) is weakly dominated. (ii) If \( j \) is the last choice of voter \( n \), then any strategy where the voter approves \( j \) is weakly dominated. (iii) No other strategies are weakly dominated. (iv) If the number of voters for whom \( i \) is not the last choice is at least two greater than the number of voters who prefer \( j \) first for all \( j \neq i \), then there exists an Undominated Nash Equilibrium where \( i \) wins the election. (v) If the number of voters for whom \( i \) is not the last choice is not less than the number of voters who prefer \( j \) first for some \( j \neq i \), then \( i \) cannot win in Undominated Nash Equilibrium\(^{72}\).

The above proposition comes close to an if and only if result, with the exception of the case where the number of voters who prefer \( i \) not last and the number of voters who prefer \( j \) first differ by less than one. Unlike Plurality Rule, Approval Voting may sometimes have a unique Undominated Nash Equilibrium even when there are more than two candidates competing, as we show in the following example.

**Example 3:**

Suppose that there are three candidates running for office, \( i \), \( j \), and \( k \). If 25 voters have preferences \( i \succ \) \( j \succ k \), 30 voters have preferences \( j \succ i \succ k \), 30 voters have preferences \( j \succ k \succ i \), and 15 voters have preferences \( k \succ j \succ i \), then under Approval Voting, there exists a unique

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\(^{72}\) Parts (i), (ii), and (iii) were first proved in Brams and Fishburn (1978).
Undominated Nash Equilibrium outcome where $j$ wins the election. Under Plurality Rule, both $i$ and $j$ can win in equilibrium.

**Single Transferable Vote**

Under Single Transferable Vote, each voter ranks all the candidates. In the first round, candidates receive votes based on the number of times they were ranked first by a voter. If no candidate receives a majority of the votes, the candidate with the least number of first preferences is eliminated and this candidate’s votes are transferred to the other candidates based on the preference rankings. This process is continued until one candidate has a majority of the votes. In this case, $A$ will be the set of permutations of $\{1, 2, \ldots, K\}$. Let $\pi^{-1}(i) = \{j : \pi(j) = i\}$. We can then define $\phi_z(a)$ recursively as follows,

$$
\phi_1(a) = K
$$

$$
\phi_{z+1}(a) = \phi_z(a) - \{i : \pi^{-1}(i) < \pi^{-1}(j) \forall j \in \phi_z(a) - i\}
$$

and we will set $\phi(a) = \phi_z(a)$.

Advocates on Single Transferable Vote often stress two benefits of this voting rule. They claim that it eliminates the ‘wasted vote’ phenomenon that occurs under Plurality rule, and that it eliminates the need for strategic behavior. Contrary to what one might expect, this voting rule need not select the Condorcet Winner. It is difficult to fully characterize the set of weakly-dominated strategies. Fortunately, we do not have to do this in order to show that this voting rule suffers from multiple equilibria. First, we will give an example to show that voting sincerely is not a weakly-dominant strategy.

**Example 4:**

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73 Single Transferable Vote goes by a number of different names including Hare’s Rule, the Alternative Vote, Preferential Voting, and Instant Runoff.

74 Note that the above definition excluded the possibility of a tie for last. In case of a tie, each of the last place candidates is eliminated with equal probability. If we were more precise with the notation, we would define $\phi_z(.)$ to take values of sets of sets rather than sets. But this notation would introduce needless complication. Instead, we will simply keep the special case in mind in our later proofs.
Suppose there are 3 candidates, \( i \), \( j \), and \( k \). If 40 voters have preferences \( i \succ_j k \), 30 voters have preferences \( j \succ_k i \), and 30 have preferences \( k \succ_i j \), then a voter who prefers \( i \succ_j k \) will want to rank \( j \) first. If this voter were to rank \( i \) first, then outcome of the election would be \( j \) with probability one half and \( k \) with probability one half, while if this voter were to ranks \( j \) first, than \( j \) would win with probability one. Since for voter \( n \), \( j \succ_n \{j, k\} \), this voter will not want to vote sincerely in this case.

Using a similar construction to the one we use in the example, we can show that for all \( j \) such that candidate \( j \) is not a voter’s last choice, there will be a strategy that ranks \( j \) first, which is not weakly-dominated. We can then use this result to construct multiple equilibria.

**Proposition 3:** Under Single Transferable Vote, (i) If \( j \) is the candidate least favored by a voter, then under Single Transferable Vote, there is some strategy that ranks \( i \) first that is not weakly-dominated for all \( i \neq j \). (ii) If \( N \geq 4 \) and \( i \) is not a Condorcet Loser, then candidate \( i \) can win the election in Undominated Nash Equilibrium.

The above result shows that Single Transferable Vote will have multiple equilibria. Like Plurality Rule, any candidate that is not a Condorcet Loser can win the election. As opposed to Plurality Rule, Single Transferable Vote cannot select the Condorcet Loser if the voters vote sincerely.

### 3 – Multi-Stage Voting Rules

The single-stage voting rules considered in the previous section failed to do what we wanted them do- that is, select the Condorcet Winner. This section considers one possible solution to this problem- multi-stage voting rules. Multi-stage voting rules differ from single-stage voting rules because the voters make multiple decisions and receive new information throughout the stages of the game.
We mentioned earlier that Majority Rule did not fall into the definition of a voting rule that we described in Section 2. This is because Majority Rule is a multi-stage voting rule (voters know the outcome of the first stage before they have to commit to a strategy for the runoff stage). Under Majority Rule, a first round is held where each voter votes for a single candidate. The votes are tallied up and if one candidates receives a majority of the votes (more than half), then this candidate is declared the winner. Otherwise, a runoff election is held between the two candidates that received the most votes in the first round. In this section, we will define multi-stage voting rules, and present an appropriate equilibrium concept. We will then characterize the equilibrium for Multi-Stage Runoff, Majority Rule, Binary Voting Trees, and the Nomination-Two rule.

**Multi-Stage Voting Rules**

Let $A_t(a_{t-1}, a_{t-2}, \ldots, a_t)$ be the set of admissible strategies at state $t$ of the election, which will depend on the strategies played by the voters in the previous stages. Then a multi-stage voting rule will be a function $\phi(a, a_{-1}, \ldots, a_t) : A_{t+1} \times \cdots \times A_{T} \rightarrow \{0, 1\}^K$. For multi-stage voting rules, we will define an equilibrium concept based on Subgame Perfection.

**Subgame Undominated Set:** The Subgame Undominated Set of voter $n$ in stage $t$, $U_{t,n}$, is given by the set of strategies $a_{t,n}$ such that there does not exists a strategy $a_{t,n}' \in A_t$ where

$$\phi(a_T, \ldots, (a_{i,1}, \ldots, a_{i,n-1}, a_{i,n}', a_{i,n+1}, \ldots, a_{i,N}), \ldots, a_t)$$

is weakly-preferred to,

$$\phi(a_T, \ldots, (a_{i,1}, \ldots, a_{i,n-1}, a_{i,n}, a_{i,n+1}, \ldots, a_{i,N}), \ldots, a_t)$$

for all $a_{t,n} \in A_t, \ldots, a_{i,n} \in A_i, a_{i+1,n} \in U_{i+1,n}, \ldots, a_{T,n} \in U_{T,n}$, and where strict preference holds for some $a_{t,n} \in A_t, \ldots, a_{i,n} \in A_i, a_{i+1,n} \in U_{i+1,n}, \ldots, a_{T,n} \in U_{T,n}$.
Subgame Perfect Equilibrium: $a \in (U_1, U_2, \ldots, U_n)$ is a Subgame Perfect Equilibrium if there does not exist an $n$ and $t$ such that $a_{i,n} \not\in U_{i,n}$ and,

$$\phi(a_1, \ldots, (a_{i,n-2}, a_{i,n-1}, a_{i,n}^{'}, a_{i,n+1}, \ldots, a_{i,n}), \ldots, a_i)$$

is strictly preferred to,

$$\phi(a_1, \ldots, (a_{i,n-2}, a_{i,n-1}, a_{i,n}, a_{i,n+1}, \ldots, a_{i,n}), \ldots, a_i).$$

To illustrate the definition, we will show $\phi$ for Majority Rule,

$A_i$ is the set of vectors with one component equal to 1 and the other components equal to 0.

$$A_2(a_i) = \{i, j\} \text{ where for all } k \not\in \{i, j\}, \sum_{a} a_{i,n} > \sum_{a} a_{i,k} \text{ and } \sum_{a} a_{k,n} > \sum_{a} a_{k,k}$$

$$\phi(a_i, a_j) = \arg \max_{i \in \{1, 2, \ldots, K\}} \sum_{a} a_{2,n}$$

The definition is somewhat imprecise since it does not say what happens if there is a tie for second place.

We will assume that if there is a tie for second place, each of the tied candidates will enter the second round with equal probability.

**Multi-Stage Runoff**

Here, we will discuss Multi-Stage Runoff. Unlike the other voting rules we have considered so far, Multi-Stage Runoff will select the Condorcet Winner under Subgame Perfect Equilibrium as long as $\succ$ is quasi-transitive. Two-Stage Runoff is a slight variation of Majority Rule. Under Majority Rule, a runoff between the top two vote-getters in the first round is only held if no candidate has received a majority of the votes. Under Two-Stage Runoff, a runoff is always held. If there are three candidates competing, the Condorcet Winner will win the election (provided one exists). A generalization of Two-Stage Runoff is Multi-Stage Runoff, where the election is held in $K-1$ stages if there are $K$ candidates. In each stage, the candidate with the least number of votes is eliminated.
Proposition 4: If $\succ$ is quasi-transitive, then Multi-Stage Runoff has a unique Subgame Perfect Equilibrium outcome corresponding to the Condorcet Winner.

Note that the result of Proposition 4 does not hold for the Exhaustive Ballot, a voting rule which is seemingly identical to Multi-Stage Runoff. The difference is that under Multi-Stage Runoff, K-1 stages are always held if there are K candidate competing, while under the Exhaustive Ballot, voting stops if one candidate receives a majority. This seemingly unimportant difference affects Undominated Set, and hence changes the set of equilibrium outcomes.

Majority Rule

Unlike Multi-Stage Runoff, Majority Rule need not select the Condorcet Winner. This holds even if there are only three candidates and we restrict preferences to be Single-Peaked. This is surprising because Majority Rule seems to resemble Multi-Stage Runoff very closely when there are three candidates competing. The difference between these two rules is that under Two-Stage Runoff, a runoff is always held, while under Majority Rule, no runoff is held if some candidate receives a majority in the first round.

Proposition 5: Under Majority Rule with $K = 3$ candidates, (i) it is weakly-dominant to vote sincerely in the second stage, (ii) a Condorcet Loser cannot win the election. (iii) If at least a third of the voters prefer the Condorcet Winner first, the unique equilibrium outcome corresponding to the Condorcet Winner. (iv) Otherwise, any candidate that is not a Condorcet Loser can win the election in Subgame Perfect Equilibrium.

When there are more than three candidates, the results are significantly more complicated. In general, increasing $K$ will make the set of equilibria larger.

Consider the situation whether the voter’s preferences are generated from a one-dimensional spatial model and the center candidate is the Condorcet Winner. The Condorcet Winner will be the unique

\[75\] While quasi-transitivity is quite restrictive when the policy space is infinite, it is a more reasonable
equilibrium if the positions that the left and right candidates are extreme enough. The condition is fairly restrictive, but is significantly less restrictive than the condition necessary for a unique equilibrium under Approval Voting.

**Binary Voting Trees**

Multi-Stage Runoff is not the only multi-stage voting rule that will select the Condorcet Winner. In fact, we can find a voting rule that will always select an outcome from the Uncovered Set, even if \( \succ \) is not quasi-transitive. Binary Voting Trees form a sequence of binary elections defined by a tree structure. Consider the example in Figure I.

In this tree, voters first decide whether to choose candidate \( i \) over candidate \( j \) (node A). Which ever candidate is selected will advance to the next round and face candidate \( k \) in an election (nodes B and C).

If there is an even number of voters, then it is possible to have ties. There are two possibilities for selecting the winner in case of a tie. A first approach is to resolve ties using a coin flip. Alternatively, we may designate a tie-breaking rule. For example, in the Binary Voting Tree pictured in Figure I, we may impose that \( i \) wins at node B in the event of a tie, \( j \) wins at node C in the event of a tie, and B wins at node A in the event of a tie. This is the approach we will use.

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76 An earlier version of this paper considered random tie-breaking rules as well. The results in propositions 6 and 7 are not changed except that each now requires the assumption that the Majority Rule social preference is never indifferent.
First, we will put Binary Voting Trees into our framework. A Binary Voting Tree will consist of a finite collection of nodes, $V$, such that,

(i) Each node points to an outcome $i \in K$, or contains two branches pointing to other nodes in $V$ (one branch is labeled Left and the other branch is labeled Right).

(ii) There does not exist a cycle among the nodes in $V$.

Each time the voters reach a node, they choose whether to proceed along the Left branch or the Right branch. Under Binary Voting Trees, Left is chosen if Left receives at least half the votes and otherwise, Right is chosen. Thus, ties are broken by selecting the Left branch.

For a given set of candidates, $K$, there are an infinite number of Binary Voting Trees. We can show, however, that so long some member of the Uncovered Set appears a terminal node, then a Subgame Perfect Equilibrium must be contained in the Uncovered Set.

**Proposition 6:** For all Binary Voting Trees such that some member of the Uncovered Set appears in at least one terminal node, a Subgame Perfect Equilibrium outcome must be contained in the Uncovered Set.

Proposition 6 shows that electing a Condorcet Winner can be achieved under weaker conditions than are required for Multi-Stage Runoff. It is not necessary for $\succ$ to be quasi-transitive. It is only necessary that the Condorcet Winner be in at least one terminal node.

The result we get is somewhat surprising since it does not depend on which Binary Voting Tree is used, indicating that if voters vote strategically, the agenda setter (the person who decides how the tree is to be arranged) has minimal influence on the outcome. The result contrasts with McKelvey (1976), which shows that under the assumption that votes vote sincerely over a sequence of binary outcomes, the agenda setter can achieve any outcome in the space so long as $\succ$ is not rational. If voters vote strategically, then the agenda setter cannot prevent a Condorcet Winner from being elected.

Of course, this is not the first paper to make this point. Miller (1977, 1980) studies strategic voting under amendment procedures and successive procedures. In both cases, he finds that the sophisticated voting outcomes must lie in the Uncovered Set. Deterministic Binary Voting Trees nest both amendment procedures and successive procedures, so Proposition 6 offers a generalization of Miller’s result. This
generalization is interesting in the sense that amendment procedures and successive procedures require that $K - 1$ successive votes be taken to decide among $K$ alternatives. Alternatively, we can design a Binary Voting Tree such that at most $\lceil \log_2 K + 1 \rceil - 1$ successive votes are needed\(^{77}\).

Shepsle and Weingast (1984) show that under an amendment procedure, the alternative chosen under strategic voting is the earliest alternative considered that is strictly majority-preferred to all the latter alternatives. Banks (1985) uses this fact to show that the set of possible outcomes under amendment procedures is in fact restricted to lie in a smaller set that the Uncovered Set. This set was later termed the Banks Set. For Deterministic Binary Voting Trees, no such restriction is possible, as we formalize in the following proposition. We define a Complete Binary Voting Tree to be one such that all elements of the set of candidates $K$ appears in some terminal node.

**Proposition 7**: Suppose that $i$ is an element in the Uncovered Set. Then there exists a Complete Binary Voting Tree that selects $i$ in equilibrium.

**Proposition 7** is proved by constructing a Complete Deterministic Binary Voting Tree that selects $i$ in equilibrium. The tree constructed in the proof does not represent a valid amendment agenda, and thus does not contradict Bank’s (1985) result. In addition, Proposition 6 and Proposition 7 give a full characterization of the possible equilibria and the degree of control the agenda setter can exercise under a more general class of voting procedures.

**Nomination Procedures**

There is one more interesting class of multi-stage voting rules that will select the Condorcet Winner under appropriate conditions. Barberá and Coelho (2004) consider a class of voting rules which they refer to as the Rule of $k$-Names. Under the Rule of $k$-Names, one electorate proposes $k$ alternatives to a second electorate and the second electorate chooses one of the nominated alternatives. Their paper gives a

\[^{77}\text{Here, } \lfloor j \rfloor \text{ represents the greatest integer less than or equal to } j.\]
survey of the various real-world instances in which this rule is used, and characterizes their properties using cooperative equilibrium concepts, following the approach of Sanver and Sertel (forthcoming).

We will focus on the special case where \( k = 2 \) and both electorates use Plurality Rule to decide the winner. We will refer to this voting rule and the Nominate-Two Rule. Thus, there is a first stage where a set of voters \( N_1 \) each vote for a candidate in \( K \). The two candidates with the highest vote totals advance to the next stage. Then, a second set of voters, \( N_2 \), select a candidate from the nominated candidates.

We will think of the first electorate as a small legislature and the second electorate as a large group of voters. This will suggest solving the first stage using cooperative equilibrium concepts and then solving the second stage using non-cooperative concepts. We will refer to such an equilibrium as a Subgame Perfect (First-Stage Strong) Equilibrium. The set of strategies at each stage is,

\[
A_i = \{ a_n \in \{0,1\}^k : \sum_k a_{nk} = 1 \}
\]

\[
A_2(a_i) = \{i,j\} \text{ where } \sum_n a_{ni} > \sum_n a_{nj} > \sum_n a_{nk} \text{ for all } k
\]

Let \( \succ^1_n \) represent the preferences of the first electorate, let \( \succ^2_n \) represent the preferences of the second electorate, and let \( \succ^1 \) and \( \succ^2 \) represent the majority preferences of each group. All ties are broken by flipping a coin. That is, if \( \sum_n a_{2nj} = \sum_n a_{2ni} \), then we will assume that \( \phi(a_i,a_j) = \{i,j\} \) and if \( a_{nj} = 1 \) for all \( n \), \( A_2(a_i) = \{i,K-i\} \) (i.e. each candidate except \( i \) advances to the next stage with equal probability).

**Proposition 8:** Suppose that \( i \succ^2 j \) for all \( j \in K \). Then \( i \) is the unique Subgame Perfect (First Stage Strong) Equilibrium outcome if \( | \{ n : i \succ^1_n j \} | > \frac{1}{2} | N_1 | \).

The conditions of Proposition 8 are rather easy to meet, indicating that this voting rule is effective in selecting the Condorcet Winner so long as the preferences of the nominating committee do not differ too much from the preferences of the electorate.

### 4 – Implications of the Theory
In this paper, we have argued that the voting rules commonly used for single-district elections suffer from multiple equilibria. Multiple equilibria are pervasive under Plurality Rule and Single Transferable Vote. The result for Single Transferable Vote is somewhat surprising given the enthusiasm that political reformers have for this voting rule. Contrary to what is popularly believed, the incentives for strategic voting under Single Transferable Vote are great. Approval Voting and Majority Rule perform somewhat better—there is sometimes a unique equilibrium in which the Condorcet Winner is selected.

The solution we consider in this paper is multi-stage voting rules. We found three classes of multi-stage voting rules that will guarantee that Condorcet Winners are elected: Multi-Stage Runoff, Binary Voting Trees, and the Nominate-Two Rule. Multi-stage voting rules have the obvious disadvantage of requiring multiple stages for implementation, which is particularly cumbersome when the number of candidates or alternatives is large. If there are $K$ alternatives, Multi-Stage Runoff requires $K-1$ stages, Binary Voting Trees require at least $\lceil \log_2 K + 1 \rceil - 1$ stages, and the Nominate-Two Rule requires 2 stages.

Though single-stage voting rules are deficient in terms of selecting Condorcet outcomes, the lack of practical alternatives means that single-stage voting rules will continue to be used in many circumstances. However, there are a range of circumstances where multi-stage voting rules remain practical. The Nominate-Two Rule is the most practical for large elections since it requires only a single-stage of voting in the large electorate. However, it presumes that existence of a small committee whose preferences do not differ too much from the electorate. Multi-Stage Runoff and Binary Voting Trees are more practical for decision making in small committees or organizations.

The Nominate-Two Rule requires that a small committee exists, but we can think of a number of important situations where this condition is met. When electing a prime minister in a parliamentary democracy, the parliament can serve as the small committee. When appointing judges, the national and state legislatures can act as the committee. In addition, the Nominate-Two Rule offers an alternative form for direct democracy.

In most parliamentary democracies, the prime minister is selected either by the parliament or through direct election. The first alternative has the drawback that voters have no direct control over who is selected. The second alternative has the drawback that it does not ensure that a Condorcet Winner will be
selected. The Nominate-Two Rule offers a third option. The parliament selects two candidates for prime minister and the electorate than directly chooses between these alternatives.

Under the current U.S. system for selecting Supreme Court and Appellate Court judges, the president recommends a candidate to the Senate, and the Senate may either confirm or reject the candidate. This has the drawback that nomination power is concentrated in a single individual and that voters have not direct control over the selection process. The Nomination-Two Rule provides an alternative way to select judges. For example, the U.S. could be divided into nine districts, each one selecting a single member of the Supreme Court. The Senate would nominate two candidates for the position, and the voters in the district would select the candidate they prefer. This system would ensure that judges are selected in a democratic way and that the makeup of the court reflects the preferences of the voters.

Nomination procedures also allow for an alternative form of direct democracy. The most common forms of direct democracy in the U.S. are the referendum and the initiative. These differ in who is allowed to propose action—under a referendum, the legislative or executive branch proposes action while under the initiative, a group of citizens propose action. Both have the drawback that agenda setting power is concentrated in either the majority party (in case of the referendum) or a well-funded interest group (in case of the initiative). The Nominate-Two Rule offers an alternative here as well. The legislature would make two proposals to the voters, neither of which would be required to be the status quo. While the other forms of direct democracy are not guaranteed to select the Condorcet Winner, the Nominate-Two Rule would lead to competition in the proposal process and would lead to the selection of a Condorcet Winning alternative.

For decision making within committee or small organizations, both Multi-Stage Runoff and Binary Voting Trees are viable alternatives. The advantage of Multi-Stage Runoff is that the rule is simpler to describe than Binary Voting Trees. Furthermore, there is only one form of Multi-Stage Runoff while Binary Voting Trees take many forms. Binary Voting Trees have the advantage that they select the Condorcet Winner even when the Majority Rule social preference fails to be quasi-transitive, and that they select elements from within the Uncovered Set even when a Condorcet Winner does not exist. Furthermore, they require fewer stages of voting. Binary Voting Trees are already being used heavily in the legislatress throughout the world. Amendment procedures and successive procedures are most often used. Our results
suggest that these are not the only options that will produce reasonable results. The equilibrium outcome will not depend on the form of the Binary Voting Tree, so long as a Condorcet Winner exists among the terminal nodes of the tree.

References


Appendix - Proofs of the Propositions

Proposition 1: Under Plurality Rule, (i) voting for your last choice is a weakly-dominated strategy. (ii) No other strategies are weakly-dominated. (iii) If $N \geq 4$ and candidate $i$ is not a Condorcet Loser, there exists an Undominated Nash Equilibrium where candidate $i$ wins the election.

Proof: Parts (i) and (ii) are proved in Brams and Fishburn (1983), so we prove only party (iii). Suppose that candidate $i$ is not a Condorcet Loser. Then there is a candidate $j$ such that $i \succ j$. We can construct an equilibrium where all voters vote for $i$ if $i \succ j$ and $j$ otherwise. Switching ones vote to $k \in \{i, j\}$ can only increase a voter's utility if $k$ wins the election with some probability. This cannot be the case since $|N|-1 \geq 3$ implies that $i$ must be receiving at least 2 votes. Similarly, switching ones vote to the least preferred candidate cannot increase one's utility since this is weakly dominated strategy. Thus, this consists of an equilibrium where $i$ to wins the election with probability 1. Since this equilibrium does not involve voters playing weakly-dominated strategies, we have an Undominated Nash Equilibrium.

Proposition 2: Under Approval Voting, (i) If $i$ is the first choice of voter $n$, then any strategy in which the voter does not approve $i$ is weakly dominated. (ii) If $j$ is the last choice of voter $n$, then any strategy where the voter approves $j$ is weakly dominated. (iii) No other strategies are weakly dominated. (iv) If the number of voters for whom $i$ is not the last choice is at least two greater than the number of votes who prefer $j$ first for all $j \neq i$, then there exists an Undominated Nash Equilibrium where $i$ wins the election. (v) If the number of voters for whom $i$ is not the last choice is at less than the number of voters who prefer $j$ first for some $j \neq i$, then $i$ cannot win in Undominated Nash Equilibrium.

Proof: Parts (i), (ii), and (iii) are proved in Brams and Fishburn (1983), so we prove only parts (iv) and (v). (iv) Suppose that the number of voters who don’t rank $i$ last is at least two more than the number of candidates that prefer $j$ first for all $j$. We will construct an equilibrium where $i$ wins the election. Suppose that all voters who don’t prefer $i$ last approve only $i$ and their first choice, and that all other
voters only prefer their first choice. Part (iii) implies that no voters are playing weakly dominated strategies. Since \( i \) has at least two more votes than all the other candidates, this is an Undominated Nash Equilibrium, since no single voter can change the outcome of the election by deviating.

(v) Notice that if voters play undominated strategies, the maximum number of votes that \( i \) can get (which is equal to number of voters that do not prefer \( j \) last) is less than the minimum number of \( k \) can get (which is equal to the number of voters that prefer \( j \) first). Since \( i \) will always have less votes than \( j \), \( i \) cannot win in equilibrium.

**Proposition 3:** Under Single Transferable Vote, (i) If \( j \) is the candidate least favored by a voter, then under Single Transferable Vote, there is some strategy that ranks \( i \) first that is not weakly-dominated for all \( i \neq j \). (ii) If \( N \geq 4 \) and \( i \) is not a Condorcet Loser, then candidate \( i \) can win the election in Undominated Nash Equilibrium.

**Proof:** (i) Suppose that voter \( n \) prefers \( j \) last and that \( i \) is preferred to \( j \). Suppose \( c \) voters rank \( j \) first and \( i \) second, \( c \) voters rank \( i \) first and \( j \) second, and \( c_1 \) voters rank \( k \) first, \( i \) second, and \( j \) third for all other candidates \( k \) where \( c_1 < c < 2c \). Then the voter will want to rank \( i \) first since if \( i \) is eliminated \( j \) will win the election and \( i \) will win otherwise. Thus, there is some strategy ranking \( i \) first that is not weakly-dominated.

(ii) Now let \( i \) be a candidate that is no a Condorcet Loser and let \( i \succ j \). Suppose all voters who prefer \( i \) to \( j \) rank \( i \) first and all other voters rank \( j \) first. Switching to a strategy where \( k \not\in \{i, j\} \) is ranked first cannot improve ones utility since candidate \( k \) is still guaranteed to be eliminated in the first round. Similarly, switching to a strategy that ranks your least preferred candidate first cannot improve one’s utility. Any other switch cannot effect the election outcome either since \( i \) and \( j \) are guaranteed to advance to the last stage of the election. Result (i) of this proposition shows us that this equilibrium does not involve weakly-dominated strategies, so we have found an Undominated Nash Equilibrium where \( i \) wins.
Proposition 4: If $>$ is quasi-transitive, then Multi-Stage Runoff has a unique Subgame Perfect Equilibrium outcome corresponding to the Condorcet Winner.

Proof: We will show that this holds by induction. The base case of $T = 1$ stages and $K = 2$ candidates holds trivially. Now suppose the theorem holds for $T = t$ stages. We will show that it must also hold for $T = t + 1$ stages. Let $i$ be the Condorcet Winner and let $j$ be the Condorcet Winner among $\{1, 2, \ldots, K\} - i$.

We will show that for all voters who prefer $i >_n j$, voting for $i$ is a weakly-dominant strategy. Suppose that the voter is pivotal between $i$ and some other candidate $k$. If this voter votes for $i$ in the first round, then $i$ will advance to the next stage. By the induction hypothesis, $i$ will win the election. If $i$ does not advance to the next stage, then by the induction hypothesis, $j$ will win the election. So for all voters with $i >_n j$, voting for $i$ is a weakly-dominant strategy. Since $i > j$, more than half of the votes will vote for $i$, and $i$ will win the election.

Proposition 5: Under Majority Rule with $K = 3$ candidates, (i) it is weakly-dominant to vote sincerely in the second stage, (ii) a Condorcet Loser cannot win the election. (iii) If at least a third of the voters prefer the Condorcet Winner first, the unique equilibrium outcome corresponding to the Condorcet Winner. (iv) Otherwise, any candidate that is not a Condorcet Loser can win the election in Subgame Perfect Equilibrium.

Proof: (i) Since the voter can only be pivotal between $i$ or $j$ at his point, it is weakly-dominant to vote for the preferred candidate among $i$ and $j$. (ii) Since voters vote sincerely in the second stage, the candidate who wins at this point must be preferred by the majority to some other candidate, and thus cannot be a Condorcet Loser. (iii) Voters who prefer the Condorcet Winner first will have a weakly-dominant strategy in the first stage of voting for this candidate. This is because the Condorcet Loser cannot win the election, so the voter can only be pivotal between the Condorcet Winner and the other candidate. But since a third of the voters vote for the Condorcet Winner in the first stage, the Condorcet Winner advances to the second stage and is selected. (iv) If a candidate does not have a third of the first preferences, it is possible for the
Proposition 6: For all Binary Voting Trees such that some member of the Uncovered Set appears in at least one terminal node, the Subgame Perfect Equilibrium outcome must be contained in the Uncovered Set.

Proof: We prove this statement by induction. Suppose that $T = 1$ and $A_r = \{i, j\}$ where $i \in M(K, \triangleright)$. If $j \in M(K, \triangleright)$, the result holds trivially. If $j \not\in M(K, \triangleright)$, then $i \triangleright j$ and $i$ will win the election, since all voters are voting sincerely. Now suppose the proposition holds for $T = t$. We will show that it also holds for $T = t + 1$. Suppose that $A_t = \{i, j\}$ with $i$ being chosen in the event of a tie. If $\sum a_{t-1, k, j} \geq \frac{1}{2}$, then the game will reduce to a contest among the candidates in the set $C_1$. If $\sum a_{t-1, k, j} > \frac{1}{2}$, then the game will reduce to a contest among the candidate the candidate in the set $C_2$. We know that either $C_1 \cap M(K, \triangleright) \neq \emptyset$ or $C_2 \cap M(K, \triangleright) \neq \emptyset$. If $C_1 \cap M(K, \triangleright) = \emptyset$, then $k \not\in M(K, \triangleright)$ will win the election if $i$ is chosen. By the induction hypothesis, some candidate $l \in M(K, \triangleright)$ will win the election if $j$ is chosen. Thus, since $l \triangleright k$, $j$ will be chosen and $l \in M(K, \triangleright)$ will win the election. A symmetric argument holds when $C_2 \cap M(K, \triangleright) = \emptyset$. Finally, if $C_1 \cap M(K, \triangleright) \neq \emptyset$ and $C_2 \cap M(K, \triangleright) \neq \emptyset$, then by the induction hypothesis a candidate in $M(K, \triangleright)$ will win regardless of whether $i$ or $j$ is chosen.

Proposition 7: Suppose that $i$ is an element in the Uncovered Set. Then there exists a Complete Deterministic Binary Voting Tree that selects $i$ in equilibrium.

Proof: We will prove the result by constructing such a Complete Deterministic Binary Voting Tree where $i$ is selected in equilibrium. Let $Y = \{y_1, y_2, \ldots, y_{k-1}\}$ denote the set of all alternatives excluding $i$. For each alternative $y_k$, there must exist an alternative $z_k$ such that $z_k \triangleright y_k$, but not $z_k \triangleright i$. Suppose that each node $A_k$ has two terminal branches- one leading to $z_k$ and the other to $y_k$. Each node $B_k$ leads to $B_{k+1}$ on
one side and \( A_k \) on the other side for \( k < K - 1 \) (with \( B_{k+1} \) being chosen in the event of a tie). \( B_{k+1} \) leads to \( i \) on one side and \( A_{K-1} \) on the other (with \( i \) being chosen in the event of a tie). We can then use backwards induction to solve this game. At each node, \( A_s, z_s \) is chosen since voters vote sincerely and \( z_s > y_s \). Then, moving from the end of the tree forward, voter will choose \( x \) over \( A_{K-1} \) and \( B_{k+1} \) over \( A_k \) since \( z_s > i \) does not hold. Thus, \( i \) will be the unique equilibrium outcome.

**Proposition 8:** Suppose that \( i >^2 j \) for all \( j \in K \). Then \( i \) is the unique Subgame Perfect (First Stage Strong) Equilibrium outcome if \( \{ n : i >^1_j \} > \frac{1}{2} |N_1| \).

**Proof:** Define \( \tilde{\phi}(a_l) = \phi(a_l, \tilde{a}_l(a_l)) \) where \( \tilde{a}_{x} = 1 \) if \( i >^2 j \), \( \tilde{a}_{\bar{x}} = 1 \) if \( j >^2 i \), and \( A_x(a_l) = \{i,j\} \). Then \( (a_l,\tilde{a}_l) \) is an equilibrium if and only if there does not exist coalition of voters \( L \subset N_1 \) and first stage strategies \( a_l \) such that \( a_{tx} = a_{tx} \) for all \( n \not\in L \) and \( \tilde{\phi}(a_l,') >^1_n \tilde{\phi}(a_l) \) for all \( n \in L \). To construct an such an equilibrium, suppose that \( a_{tx} = 1 \) for all \( n \in N_1 \). If the coalition \( L \) has \( \|L\| < \frac{1}{2} |N_1| \), then it must still be the case that \( i \in A_x(a_l) \) which implies \( \tilde{\phi}(a_l) = \phi(a_l) = i \). Alternatively, \( \|L\| \geq \frac{1}{2} |N_1| \), then there must be an \( n \in N_1 \) such that \( i >^1_j \) so that \( \tilde{\phi}(a_l,') >^1_n \tilde{\phi}(a_l) \) for all \( n \in L \) cannot hold. Thus, \( (a_l,\tilde{a}_l(a_l)) \) must be an equilibrium.

We also need to show that \( j \neq i \) cannot be an equilibrium. Since \( i >^2 j \), \( j \) could only be selected if \( i \not\in A_x(a_l) \). In this case, there is a coalition \( L \subset N_1 \) or voters such that \( i >^1_j \) for all \( n \in L \) and \( \|L\| > \frac{1}{2} |N_1| \) by assumption. If these voters all nominate \( i \), then \( i \) will win the election. Thus, \( j \neq i \) cannot be an equilibrium.