Essays on Monetary Policy and Macroeconomic Fluctuations

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Abstract

This thesis consists of four essays on macroeconomics, of which two are concerned with the design of monetary policy and the other two with the propagation of shocks to business fluctuations. Chapter 2 investigates implications of search frictions in the labor market for inflation targeting interest rate policy in terms of equilibrium stability. In stark contrast to comparable previous studies with frictionless labor markets, indeterminacy of equilibrium is very likely if the interest rate is adjusted in response solely to expected future inflation. The indeterminacy can be overcome once the interest rate is adjusted also in response to output or the unemployment rate, or if the policy contains interest rate smoothing. When expectational (E-)stability is adopted as an equilibrium selection criterion, a unique E-stable fundamental rational expectations equilibrium is generated under active, but not too strong, policy responses to expected future inflation. Chapter 3 studies implications of on-the-job search for the propagation of monetary shocks in the presence of search frictions. Such frictions induce long-term employment relationships, such that firms’ real marginal cost is determined by real wages and the value of a long-term employment relationship. On-the-job search opens up an extra channel of employment growth that dampens the response of these two components. As a result, on-the-job search gives rise to a strong propagation of monetary shocks that increases output persistence. Chapter 4 investigates the propagation of investment-specific technology shocks that have long-run effects. It finds that markup adjustment is an irrelevant propagation mechanism of such shocks. A positive shock raises the trend component of consumption and reduces its cyclical component. The trend increase can prevent a sharp countercyclical response of consumption. The decrease in its cyclical component implies that a markup adjustment is not required to maintain labor market equilibrium in the face of rising hours worked. In Chapter 5 we ask what prescription for forward-looking interest rate policy overcomes the equilibrium indeterminacy that is induced by such policy in the presence of investment activity. The indeterminacy can be overcome once the forward-looking policy responds also to current output or contains sufficiently strong interest rate smoothing. The forward-looking policy can generate a unique non-explosive E-stable fundamental rational expectations equilibrium.
Chapter 1

Introduction

This thesis consists of four essays on macroeconomics, of which two address an issue about the design of monetary policy and the other two examine a question about the propagation of shocks to business fluctuations. A common theme of these essays is that the analysis is conducted within a dynamic general equilibrium model with nominal price rigidity or “sticky prices”. However, each essay departs from the canonical sticky price model’s simple production side, where firms hire labor in a competitive market to be employed as the only input in production. In particular, the first two essays examine implications of labor market search and matching frictions, whereas the last two essays have a central role for capital accumulation.

There has recently been a surge of interest in the role of labor market search and matching frictions in sticky price models. Chapter 2 (joint with Takushi Kurozumi) investigates implications of search frictions in the labor market for inflation targeting interest rate policy in terms of equilibrium stability. When the interest rate is set in response to past or present inflation, determinacy of equilibrium is ensured, similarly to comparable previous studies with frictionless labor markets. In stark contrast to these studies, however, indeterminacy is very likely if the interest rate is adjusted in response solely to expected future inflation. This is due to a vacancy channel of monetary policy that stems from the labor market frictions and that renders inflation expectations self-fulfilling. The indeterminacy can be overcome once the interest rate is adjusted also in response to output or the unemployment rate, or if the policy contains interest rate smoothing. When expectational (E-)stability is adopted as an equilibrium selection criterion, a unique E-stable fundamental rational expectations equilibrium is generated under active, but not too strong, policy responses only to expected future inflation. This suggests that the problem is not critical from the perspective of learnability of the fundamental equilibrium.

There is a widespread view in the monetary business cycle literature that sticky price models need to be augmented with real rigidities in order to replicate the dy-
namic responses observed in the data. Chapter 3 studies implications of on-the-job search for the propagation of monetary shocks in the presence of search frictions. Such frictions induce long-term employment relationships, such that firms’ real marginal cost is determined by real wages and the value of a long-term employment relationship. On-the-job search opens up an extra channel of employment growth that dampens the response of these two components. Because real marginal cost rigidity induces small price adjustments, on-the-job search gives rise to a strong propagation of monetary shocks that increases output persistence. This contrasts with findings of Krause and Lubik (2007), which suggest that labor market search frictions do not resolve the output persistence problem, even if real wage rigidity is imposed.

Recent studies of dynamic stochastic general equilibrium models point to nominal rigidities as an important propagation mechanism of investment-specific technology shocks. But whereas structural vector autoregression literature documents an important role for investment-specific technology shocks that have long-run output effects (Fisher, 2006), the shocks in these sticky price models have only transitory effects. Chapter 4 investigates whether the transmission channels of transitory investment-specific technology shocks that have recently been put forward are also plausible as an explanation for the importance of a random walk technology process. It finds that markup adjustment is an irrelevant propagation mechanism of shocks that have long run output effects. A positive shock raises the trend component of consumption and reduces its cyclical component. The trend increase can prevent a sharp countercyclical response of consumption. The decrease in its cyclical component implies that a markup adjustment is not required to maintain labor market equilibrium in the face of rising hours worked. In contrast, after a transitory shock, markup adjustment can prevent a countercyclical consumption response that would make the shock an implausible source of business fluctuations.

In Chapter 5 (joint with Takushi Kurozumi) we ask what prescription for forward-looking interest rate policy overcomes the problem of equilibrium indeterminacy that is induced by such policy in the presence of investment activity and price stickiness, as first shown by Carlstrom and Fuerst (2005). We find that this indeterminacy is due to a cost channel of monetary policy, whereby inflation expectations become self-fulfilling. The problem can be overcome once the forward-looking policy responds also to current output or contains sufficiently strong interest rate smoothing, because this prevents the self-fulfilling expectations. We also show that when E-stability is adopted as the selection criterion from multiple equilibria, even the forward-looking policy generates a unique non-explosive E-stable fundamental rational expectations equilibrium, as long as the policy response to expected future inflation is sufficiently strong.
Chapter 2

Labor Market Search and Interest Rate Policy

2.1 Introduction

This essay is based on Kurozumi and Van Zandweghe (2007).

There has recently been a surge of interest in the role of labor market search and matching frictions along the lines of Mortensen and Pissarides (1994) in dynamic stochastic general equilibrium models with sticky prices.¹ Employment adjustment takes time and place at the extensive margin, giving rise to equilibrium unemployment, and wages are determined by Nash bargaining between workers and firms. These features are in stark contrast with Walrasian competitive labor markets, which have been used in the monetary policy literature.²

In this chapter we examine implications of the labor market search and matching frictions for inflation targeting interest rate policy in terms of equilibrium stability. We consider three policy specifications, each of which adjusts the interest rate in response solely to either past inflation (backward-looking), present inflation (current-looking) or expected future inflation (forward-looking). We show that the current-looking and the backward-looking policies ensure (local) determinacy of rational expectations equilibrium (REE) under similar conditions to those obtained in comparable previous studies with frictionless labor markets, such as Bullard and Mitra (2002) and Woodford (2003). Determinacy is guaranteed under active policy responses to current inflation or under active, but not too strong, ones to past inflation. In stark contrast to the previous studies, we find that the forward-looking policy is very likely to induce indeterminacy, and thus makes excessively volatile REE possible. This finding is critical because actual central banks, inflation targeting ones in particular, are

²Some recent exceptions are Blanchard and Galí (2008), Faia (2008) and Thomas (2008), who study optimal monetary policy in the presence of frictional labor markets.
concerned about expected future inflation rather than actual past or present inflation.

Why does the forward-looking policy render REE indeterminate? Passive policy does so, due to the weakness of the conventional aggregate demand channel of monetary policy, as in line with the previous studies. Active policy also induces indeterminacy in the presence of a vacancy channel of monetary policy that stems from the labor market search and matching frictions. This is in stark contrast with the previous studies and it occurs because the vacancy channel makes inflation expectations self-fulfilling. The labor market frictions result in firms’ sluggish adjustment in employment and hence output. Specifically, firms’ reduction in vacancy posting, induced by dampened consumption demand following a rise in the real interest rate, decreases the level of employment available for production in current and future periods. Thus, the real interest rate rise lowers future output supply. At the same time, such a rate rise also prompts households to substitute current with future consumption, so that firms expect consumption demand to recover in future periods after its current decline. From this expected rise in future demand and the diminished future supply, firms anticipate a strong expansion of future vacancy posting, which raises expected future real marginal cost and hence expected future inflation. Therefore, the vacancy channel leads a rise in the real interest rate to increase expected future inflation. This renders inflation expectations self-fulfilling under sufficiently strong active policy responses solely to expected future inflation, thereby inducing indeterminacy of REE.

Actual labor markets are characterized by search and matching frictions, and likewise much evidence suggests that monetary policy in major economies has been forward-looking especially since 1979 (e.g. Clarida, Galí and Gertler, 1998). Yet the actual economy has not exhibited excessive volatility in recent decades as the vacancy channel leads to predict. We examine two possible explanations. The first one is interest rate smoothing or interest rate policy adjustment for output or the unemployment rate in addition to expected future inflation. We then find that the policy adjustment for current output can overcome the indeterminacy as long as a long run version of the Taylor principle is satisfied: in the long run the nominal interest rate should be raised by more than the increase in inflation. With a policy adjustment for expected future output, this amelioration of the problem is limited to mild policy responses, since a strong policy response to expected future output causes indeterminacy as in line with previous studies with frictionless labor markets. The intuition for the amelioration is that indeterminacy is induced by the vacancy channel of the forward-looking policy, in which a rise in the real interest rate stemming from

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3 On the contrary, there is ample evidence of a moderation of U.S. economic aggregates since 1984.
4 The upper bound on the policy coefficient on expected future output that guarantees determinacy is induced by the demand channel of monetary policy, as mentioned later.
inflationary expectations increases expected future inflation and hence such expectations become self-fulfilling. But, the policy adjustment for current or expected future output subdues the real interest rate rise because output falls as a consequence of such a rate rise. With an interest rate policy reaction to the unemployment rate we find that the forward-looking policy brings about determinacy when it satisfies an associated long run version of the Taylor principle. This is because in our model the unemployment rate changes proportionally to fluctuations in production, so that the policy reaction to the unemployment rate yields almost the same result as the one to current output. Finally, we consider interest rate smoothing and find that it helps the forward-looking policy generate determinacy. Such smoothing implies policy responses to lagged interest rates and hence makes the forward-looking policy respond also to current and past inflation like the current-looking and the backward-looking policies, thereby ameliorating the indeterminacy problem. These results provide an additional argument in favor of flexible interest rate policy instead of strict inflation targeting, and the policies thus constitute prescriptions for the indeterminacy.

Next, we consider expectational (or E-)stability as an REE selection criterion and examine whether the forward-looking policy generates a unique E-stable fundamental REE even in cases of indeterminacy.\(^5\) As Evans and Honkapohja (2001) show in a broad class of linear stochastic models, if a fundamental REE is E-stable, it is least-squares learnable, i.e. stable under least-squares learning. Therefore, E-stability is an essential condition for any REE to be regarded as plausible, as stressed by McCallum (2003).\(^6\) We find that a unique E-stable fundamental REE is generated under active, but not too strong, policy responses solely to expected future inflation. This is in stark contrast with Bullard and Mitra (2002), who show that the Taylor principle (i.e. active policy) is a necessary and sufficient condition for the unique E-stable REE in the absence of the labor market search and matching frictions. The presence of such frictions makes the Taylor principle no longer a sufficient condition. Since the interval of the policy coefficient on expected future inflation that generates the unique E-stable REE is wide enough to contain all empirically relevant values, our E-stability result suggests that the indeterminacy problem induced by the forward-looking policy is not critical from the perspective of E-stability or least-squares learnability of fundamental

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\(^5\)Throughout the chapter, “fundamental” refers to Evans and Honkapohja’s (2001) minimal state variable (MSV) solutions to linear RE models so as to distinguish them from McCallum’s (1983) original MSV solution. We do not examine E-stability of non-fundamental REE such as sunspot equilibria, which may exist in cases of indeterminacy. For E-stability analysis of these REE, see e.g. Honkapohja and Mitra (2004), Carlstrom and Fuerst (2004) and Evans and McGough (2005), who all use associated models with frictionless labor markets. We leave E-stability analysis of non-fundamental REE in our model for future work.

\(^6\)McCallum (2003) argues that in cases of indeterminacy there may be a unique REE that is E-stable and thus least-squares learnable, whereas a determinate REE that is E-unstable and thus not least-squares learnable is arguably not a plausible candidate for equilibrium that could be observed in the actual economy.
The findings above are based on our benchmark model in which consumption preferences are standard, job destruction is exogenous and hiring is instantaneous. These findings remain unchanged qualitatively even when we introduce habit formation in consumption preferences or when we consider an alternative labor market specification in which jobs are also endogenously destroyed and new hires become productive in the subsequent period, which is a more conventional one used in previous studies such as Trigari (2005), Walsh (2005) and Krause and Lubik (2007).

Among related literature, Burda and Weder (2002), Giammaroli (2003), and Krause and Lubik (2004) analyze equilibrium determinacy in real business cycle models with labor market search and matching frictions, yielding no implication for monetary policy. Zanetti (2006) investigates monetary policy implications using a sticky price model in which wages and employment are determined via simultaneous Nash bargaining, but such a model involves no search and matching frictions. To our knowledge, the present chapter is the first paper to examine monetary policy implications of labor market search and matching frictions in terms of determinacy and E-stability of REE.

The remainder of the chapter proceeds as follows. Section 2.2 presents an optimizing model with sticky prices and labor market search and matching frictions. Section 2.3 analyzes determinacy of REE under three alternative specifications of inflation targeting interest rate policy. Section 2.4 considers prescriptions for the indeterminacy problem induced by the forward-looking policy. Section 2.5 assesses the problem from the perspective of E-stability. Section 2.6 contains some robustness analysis in which habit formation is introduced in consumption preferences or a more conventional labor market specification is used. Finally, Section 2.7 concludes.

2.2 A model with labor market search and matching frictions

Our model is an optimizing model with sticky prices and labor market search and matching frictions. This model is in line with recent business cycle studies, such as Trigari (2005), Walsh (2005), Christoffel, Kuester and Linzert (2006), Krause and Lubik (2007), Krause, Lopez-Salido and Lubik (2008), Ravenna and Walsh (2007), and Chapter 3. But, it is in stark contrast to recent monetary policy studies with competitive labor markets in that employment adjustment is costly and takes place at the extensive margin, which gives rise to equilibrium unemployment, and wages are determined by Nash bargaining.

The model economy consists of four types of agents: households, perfectly com-
petitive wholesale firms, monopolistically competitive retail firms, and a monetary
authority. We describe each in turn.

### 2.2.1 Households

In the economy there is a continuum of households. To avoid distributional issues,
we assume as in Andolfatto (1996) and Merz (1995) that employed and unemployed
households pool consumption. Thus, we can consider the presence of a representative
household. This household purchases $C_t$ consumption goods, supplies one unit of labor
inelastically, and holds $B_t$ nominal one-period bonds, which earn the gross nominal
interest rate $R_t$ in the subsequent period. The household chooses consumption and
bond holdings so as to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} g^t$$

under the budget constraint

$$P_tC_t + B_t \leq P_tD_t + B_{t-1}R_{t-1},$$

where $\beta \in (0, 1)$ is a discount factor, $\sigma > 0$ measures (relative) risk aversion, $g_t$
is a preference shock, $D_t$ is real income that consists of monopoly profits from retail
firms, rents related to labor market frictions from wholesale firms, either a wage
$w_t$ from employment or a benefit $b$ when unemployed, minus a lump-sum transfer
to finance unemployment benefits. The disutility from employment is normalized
to zero. Consumption $C_t = \left[ \int_0^1 C_t(i)^{1/(\epsilon-1)} di \right]^{(\epsilon-1)/\epsilon}$ is a composite of differentiated
goods produced by retail firms, with an elasticity of substitution $\epsilon > 1$. Thus, cost-
minimizing demand for good $i$ is given by $C_t(i) = \left[ P_t(i)/P_t \right]^{-\epsilon} C_t$, where the aggregate
price index satisfies

$$P_t = \left[ \int_0^1 P_t(i)^{1-1} di \right]^{1/(\epsilon-1)}.$$  \hspace{1cm} (2.1)

The optimality conditions for consumption and bond holdings are given by

$$\lambda_t = C_t^{-\sigma} e^{g_t}, \quad \lambda_t = \beta E_t \lambda_{t+1} \frac{R_t}{\pi_{t+1}},$$

where $\lambda_t/P_t$ is the Lagrange multiplier on the budget constraint and $\pi_t = P_t/P_{t-1}$ is
the gross inflation rate. These conditions yield the consumption Euler equation

$$C_t^{-\sigma} e^{g_t} = \beta E_t C_{t+1}^{-\sigma} e^{g_{t+1}} \frac{R_t}{\pi_{t+1}}.$$  \hspace{1cm} (2.2)

As in recent monetary policy studies, we assume that fiscal policy is ‘Ricardian’, i.e. it appropri-
ately accommodates consequences of monetary policy for the government budget constraint. We
thus leave hidden the government budget constraint and fiscal policy. For recent analyses of equi-
librium determinacy under non-Ricardian fiscal policy and interest rate policy, see e.g. Benhabib,
Schmitt-Grohé and Uribe (2001), Benhabib and Eusepi (2005), Linnemann (2006) and Kurozumi
(2005).
2.2.2 Wholesale firms

Wholesale firms use labor as the only input in production and sell homogeneous goods at a price $P^w_t$ to retail firms in a perfectly competitive market. The labor market is characterized by search and matching frictions. The population size is normalized to one. The time line of period $t$ is as follows. At the beginning of the period there are $n_{t-1}$ matches between workers and wholesale firms. Then, a proportion $\rho \in (0,1)$ of the existing matches is exogenously destroyed, thus job destruction equals $\rho n_{t-1}$. These workers join the pool of searching workers, so that the measure of search unemployment is

$$u_t = 1 - (1 - \rho)n_{t-1}. \quad (2.3)$$

Next, $u_t$ searching workers and $v_t$ vacancies participate in the matching market, giving rise to $m_t$ new matches (i.e. job creation), a number increasing in search unemployment and vacancies according to a constant returns to scale technology $m_t = \psi u_t^\xi v_t^{1-\xi}$, where $\psi > 0$ and $\xi \in (0,1)$ is the search elasticity of matches. We assume as in Blanchard and Galí (2008) and Ravenna and Walsh (2007) that new matches become productive instantaneously. Thus, the number of worker-firm matches that produce in period $t$ is given by

$$n_t = (1 - \rho)n_{t-1} + m_t, \quad (2.4)$$

where the change in employment is equal to the difference between job creation and job destruction. Then, the unemployment rate is

$$U_t = 1 - n_t. \quad (2.5)$$

Each worker-firm match produces one unit of wholesale goods in every period, so that aggregate production of the wholesale sector is

$$y_t = n_t. \quad (2.6)$$

The ratio

$$\theta_t = \frac{v_t}{u_t} \quad (2.7)$$

measures the tightness of the labor market. An unmatched wholesale firm’s probability to fill a vacancy (i.e. the firm matching rate) is

$$q_t \equiv \frac{m_t}{v_t} = \psi \theta_t^{-\xi}, \quad (2.8)$$

---

8The exogenous job destruction rate is empirically supported by Hall (2006) and Shimer (2007), who argue that the job separation rate explains only a small fraction of fluctuations in the unemployment rate.

9In Section 2.6.2 we analyze a more conventional timing in which new matches become productive in the subsequent period and contemporaneous employment adjustment takes place only via job destruction.
which rises when the labor market becomes slack. A searching worker’s probability to find a job (i.e. the worker matching rate) is

\[ p_t \equiv \frac{m_t}{u_t} = \psi \theta^{1-\xi}, \tag{2.9} \]

which is increasing in labor market tightness.

Job creation is costly for wholesale firms, which must pay a fixed cost \( \gamma > 0 \) each period they post vacancies. This cost gives rise to a joint surplus from a match, which is split between the matched worker and firm through Nash bargaining. To determine a wage that gives the worker his/her share of the bargain, it is convenient to consider asset values of matched and unmatched workers and firms. The asset value of a matched firm, \( F^m_t \), is the sum of real net revenue that accrues to the firm in the current period and the discounted present value of this firm in the next period. The match is dissolved with probability \( \rho \), so that the value of a matched firm is given by

\[ F^m_t = z_t - w_t + E_t \beta_{t,t+1} [(1 - \rho)F^m_{t+1} + \rho F^u_{t+1}], \]

where \( \beta_{t,t+j} = \beta^j \lambda_{t+j} / \lambda_t \) is the stochastic discount factor, \( z_t = P^w_t / P_t \) is the real price of wholesale goods, and \( F^u_t \) is the asset value of an unmatched firm in period \( t \). An unmatched firm pays the vacancy posting cost and is matched with probability \( q_t \). Since new matches become productive instantaneously, the value of an unmatched firm is given by

\[ F^u_t = -\gamma + q_t F^m_t + (1 - q_t) E_t \beta_{t,t+1} F^u_{t+1}. \]

Free entry drives the asset value of an unmatched firm to zero in equilibrium. Combining these firm asset values yields a job creation condition that makes the expected cost of a match equal its expected value

\[ \frac{\gamma}{q_t} = z_t - w_t + E_t \beta_{t,t+1} (1 - \rho) \frac{\gamma}{q_{t+1}}. \]

The asset value of a matched (unmatched) worker is the wage (unemployment benefit) plus the expected present discounted value of this worker’s employment status in the next period

\[ W^m_t = w_t + E_t \beta_{t,t+1} \left\{ [1 - \rho(1 - p_{t+1})]W^m_{t+1} + \rho(1 - p_{t+1})W^u_{t+1} \right\}, \]

\[ W^u_t = b + E_t \beta_{t,t+1} \left\{ p_{t+1}W^m_{t+1} + (1 - p_{t+1})W^u_{t+1} \right\}. \]

The Nash bargaining outcome \( \eta F^m_t = (1 - \eta)(W^m_t - W^u_t) \), where \( \eta \in (0, 1) \) is the worker’s share of the surplus (i.e. the worker bargaining power), then results in the wage equation

\[ w_t = \eta \left[ z_t + E_t \beta_{t,t+1} (1 - \rho)p_{t+1} \frac{\gamma}{q_{t+1}} \right] + (1 - \eta) b. \]
The worker is compensated for a fraction $\eta$ of firm revenue and the hiring cost that the firm expects to save thanks to the match. In addition, the worker is compensated for a fraction $1 - \eta$ of the forgone unemployment benefit. Substituting for the wage, the job creation condition becomes

$$
\frac{γ}{qt} = (1 - \eta)(z_t - b) + E_tβ_{t,t+1}(1 - \rho)(1 - \eta pt+1) \frac{γ}{qt+1}.
$$

(2.10)

### 2.2.3 Retail firms

There is a continuum of retail firms $i \in [0, 1]$, each of which produces one unit of differentiated good $i$ from one unit of wholesale goods and sells a quantity $Y_t(i)$ of good $i$ to households in a monopolistically competitive market. Cost minimization implies that each retail firm’s real marginal cost is equal to the wholesale goods’ real price $z_t$. Then, facing households’ demand $Y_t(i) = C_t(i) = [P_t(i)/P_t]^{-\epsilon}C_t$, each retail firm chooses its profit-maximizing price subject to Calvo (1983) and Yun (1996) style price stickiness. That is, each period a fraction $\alpha$ of retail firms does not reoptimize price and instead adjusts it for steady state inflation $\pi$, while the remaining fraction $1 - \alpha$ of firms faces the problem

$$
\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \alpha^j β_{t,t+j}\left[ \frac{P_t(i)\pi^j}{P_{t+j}} - z_{t+j} \right] \left[ \frac{P_t(i)\pi^j}{P_{t+j}} \right]^{-\epsilon} C_{t+j}.
$$

The optimality condition for price setting is

$$
P_t(i) = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty}(\alpha\pi^{-\epsilon})^j β_{t,t+j} P_{t+j}^\epsilon C_{t+j} z_{t+j}}{E_t \sum_{j=0}^{\infty}(\alpha\pi^{1-\epsilon})^j β_{t,t+j} P_{t+j}^{\epsilon-1} C_{t+j}}.
$$

(2.11)

If prices are perfectly flexible (i.e. $\alpha = 0$), (2.11) reduces to $P_t(i) = [\epsilon/(\epsilon - 1)]P_t z_t$, which shows that $1/z = \epsilon/(\epsilon - 1)$ is the steady state markup of each retail firm’s price over its marginal cost. In the presence of price stickiness, the firm’s actual markup differs from, but tends toward, the steady state markup.

### 2.2.4 Monetary authority

The monetary authority conducts inflation targeting policy that adjusts the interest rate in response solely to either past, present or expected future inflation

$$
R_t = R \left( \frac{E_t \pi_{t+j}}{\pi} \right)^{\phi_\pi}, \quad j = -1, 0, 1,
$$

(2.12)

where $R$ is the steady state nominal interest rate and $\phi_\pi$ is a non-negative policy coefficient on inflation. These three policy specifications are referred to as, respectively, backward-looking ($j = -1$), current-looking ($j = 0$) and forward-looking ($j = 1$) in what follows.
2.2.5 Equilibrium and calibration

A rational expectations equilibrium (REE) is a set of processes for all the endogenous variables satisfying (2.1)–(2.12), the aggregate resource constraint \( y_t = Y_t + \gamma v_t \), and the market clearing condition \( Y_t(i) = C_t(i) \) for each retail good \( i \in [0, 1] \), which implies \( Y_t = \Delta_t C_t \), where \( \Delta_t \equiv \int_0^t \left[ P_t(i)/P_t \right]^{-\epsilon} \, dt \) measures relative price dispersion across retail firms. Log-linearizing these equilibrium conditions around the steady state and rearranging the resulting equations yields

\[
\hat{\theta}_t = \hat{v}_t + \frac{1-u}{u} \hat{n}_{t-1}, \tag{2.13}
\]

\[
\hat{n}_t = (1-\rho)\hat{n}_{t-1} + \rho \left( \hat{v}_t - \xi \hat{\theta}_t \right), \tag{2.14}
\]

\[
\xi \hat{\theta}_t = \chi \hat{z}_t + \beta (1-\rho) \left( \xi - \eta \right) \tilde{E}_t \hat{\theta}_{t+1} - \beta (1-\rho) (1-\eta \eta) \left( \tilde{R}_t - \tilde{E}_t \tilde{\pi}_{t+1} \right), \tag{2.15}
\]

\[
\hat{\pi}_t = \beta \tilde{E}_t \tilde{\pi}_{t+1} + \kappa \hat{z}_t, \tag{2.18}
\]

\[
\hat{R}_t = \phi \tilde{E}_t \tilde{\pi}_{t+j}, \quad j = -1, 0, 1, \tag{2.19}
\]

where \( \chi \equiv z(1-\eta)/(\gamma/q) > 0 \), \( s_v \equiv \gamma v/y \) is the steady state vacancy creation share of production, \( s_c \equiv 1 - \gamma v/y \) is the steady state consumption share of production, and \( \kappa \equiv (1-\alpha)(1-\alpha \beta)/\alpha > 0 \) is the real marginal cost elasticity of inflation. The preference shock, \( g_t \), is assumed to follow a stationary first order autoregressive process with a parameter \( \rho_g \in (-1, 1) \) and a white noise \( \varepsilon_t \)

\[
g_t = \rho_g g_{t-1} + \varepsilon_t. \tag{2.20}
\]

In the presence of search and matching frictions in the labor market, firms’ adjustment in employment and output is persistent and as a consequence, the transmission mechanism of monetary policy consists of a vacancy channel in addition to an aggregate demand channel which is the only channel in the absence of the labor market frictions. One point here is that these two channels have opposing effects on inflation.

To see this, consider the effect of a rise in the real interest rate. Then, households reduce their current consumption according to the Euler equation (2.17). This decreases retail firms’ current output because of monopolistic competition and hence dampens these firms’ current demand for wholesale goods. In response to this dampened demand, wholesale firms reduce current vacancy posting and hence the current labor market becomes slack via (2.13), which lowers retail firms’ current real marginal cost via the job creation condition (2.15) because the labor market slackness decreases the hiring cost and hence wholesale goods’ price. Consequently, current inflation

\[^{10}\text{The real interest rate rise also reduces the labor market tightness directly by lowering the expected...}\]
is reduced via the Phillips curve (2.18). This is the aggregate demand channel of monetary policy, through which a higher real interest rate lowers current inflation.

The vacancy channel of monetary policy, on the contrary, leads a rise in the real interest rate to increase expected future and current inflation. A real interest rate rise implies that consumption demand is expected to recover in future periods after its current decline, and hence expected future demand for wholesale goods increases above its current level. As explained just above, the rate rise also reduces wholesale firms’ current vacancy posting and hence lowers the level of employment available for production in the subsequent periods. Then, facing the expected recovery in future demand, wholesale firms anticipate a strong expansion of future vacancy posting and hence a tightened future labor market, which in turn raises expected future real marginal cost via the next-period job creation condition. Thus, expected future and current inflation is raised via the Phillips curve. This is the vacancy channel of monetary policy that stems from the labor market frictions. As shown later, this vacancy channel induces a possibility that inflation expectations become self-fulfilling and the REE is indeterminate if active interest rate policy has sufficiently strong responses solely to expected future inflation.

The ensuing analysis uses a realistic calibration of model parameters to illustrate conditions for determinacy and E-stability. Our baseline calibration for the quarterly model is summarized in Table 5.1. The discount factor $\beta = 0.99$, the risk aversion $\sigma = 1$, the substitution elasticity $\epsilon = 10$ yielding a steady state markup of $1/z = 1.11$, and the probability of no price optimization $\alpha = 0.67$ as in line with recent literature such as Woodford (2003). The labor market parameters are the worker bargaining power $\eta = 0.5$ following most of the literature on labor market search and matching, the search elasticity of matches $\xi = 0.4$ based on the empirical estimates of Blanchard and Diamond (1989), the firm matching rate $q = 0.7$ and the job destruction rate $\rho = 0.1$ taken from den Haan, Ramey and Watson (2000), the steady state unemployment rate $U = 1 - n$ of six percent as in Walsh (2005), and the vacancy posting cost $\gamma = 0.06$ consistent with the value implied by the steady state with endogenous job destruction in Walsh (2005). The steady state relationships then imply values for the remaining parameters: $p$, $u$, $s_c$, $s_v$. value of a match in the job creation condition (2.15). But, under realistic calibrations, this effect is weak enough to ensure procyclical real marginal cost.

This transmission can also be explained from the perspective of the wholesale goods market. This market is perfectly competitive, and wholesale goods’ real price is equal to retail firms’ real marginal cost because such goods are the only input in retail firms’ production. Therefore, the dampened current demand for wholesale goods drives current real marginal cost downward.

Again, this can be explained from the perspective of the wholesale goods market. The expected rise in future demand for wholesale goods drives retail firms’ expected future real marginal cost upward.
Table 2.1: Baseline calibration for our quarterly model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>risk aversion</td>
<td>1</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>elasticity of substitution between retail goods</td>
<td>10</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>probability of not reoptimizing price</td>
<td>0.67</td>
</tr>
<tr>
<td>$\eta$</td>
<td>worker bargaining power</td>
<td>0.5</td>
</tr>
<tr>
<td>$\xi$</td>
<td>search elasticity of matches</td>
<td>0.4</td>
</tr>
<tr>
<td>$q$</td>
<td>firm matching rate</td>
<td>0.7</td>
</tr>
<tr>
<td>$\rho$</td>
<td>job destruction rate</td>
<td>0.1</td>
</tr>
<tr>
<td>$U$</td>
<td>steady state unemployment rate</td>
<td>0.06</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>flow cost of a vacancy</td>
<td>0.06</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>autoregressive coefficient for preference shocks</td>
<td>0.35</td>
</tr>
</tbody>
</table>

2.3 Equilibrium determinacy under interest rate policy

In the model presented above, we examine implications of the labor market search and matching frictions for inflation targeting interest rate policy in terms of determinacy of REE.

2.3.1 Forward-looking policy

We begin with the case of the forward-looking policy, i.e. $j = 1$ in (2.12). The system of log-linearized equilibrium conditions (2.13)–(2.20) can be reduced to a system of the form

$$E_t x_{t+1} = Ax_t + B g_t, \quad (2.21)$$

where $x_t = [\hat{\pi}_t \ \hat{C}_t \ \hat{n}_{t-1}]'$ and the coefficient matrix $A$ is given in Appendix A.1.\footnote{The form of the vector $B$ is omitted since it is not needed in what follows.} In this system the two variables $\hat{\pi}_t$ and $\hat{C}_t$ are non-predetermined, while the remaining one, $\hat{n}_{t-1}$, is predetermined. Therefore, determinacy of REE is generated if and only if the coefficient matrix $A$ has exactly one eigenvalue inside the unit circle and the other two outside the unit circle.\footnote{To be precise, this condition is sufficient for determinacy but only generically necessary. Throughout the chapter, consideration of non-generic boundary cases is omitted.} We thus obtain the following proposition using Proposition C.2 of Woodford (2003).

**Proposition 1** The forward-looking policy, i.e. $j = 1$ in (2.12), ensures determinacy of REE if and only if either of the following two cases is satisfied.
Case I: (2.22), (2.23), and (2.24) or (2.25) hold.

\begin{align*}
\phi_n > 1, \\
\phi_n &< 1 + \frac{2\chi a_1 (1 + \beta)}{\kappa\{s_c (2u - \rho)[\xi + \beta(1 - \rho)(\xi - \eta p)] - 2\beta \sigma a_1 (1 - \rho)(1 - \eta p)\}}, \\
b_3(b_3 - b_1) + b_2 &> 1 \quad \text{or} \quad |b_1| > 3, \\
\phi_n &> 1 + \frac{\chi \sigma [\rho(1 - \xi) - s_c]}{\kappa(1 - \rho)[s_c (\xi - \eta p) - \sigma(1 - \eta p)\rho(1 - \xi) - s_c]},
\end{align*}

where \( a_1 = \rho(1 - \xi)[u - s_c (1 - u)] - s_c (2u - \rho) \) and \( b_i, i = 1, 2, 3 \), are given in Appendix A.2.

Case II: the two strict inequalities opposite to (2.22) and (2.23) hold and (2.24) or the strict inequality opposite to (2.25) holds.

**Proof** See Appendix A.2.

We illustrate the conditions for determinacy with the baseline calibration. Determinacy obtains for a very narrow interval of the policy coefficient on expected future inflation, \( 1 < \phi_n < 1.16 \). Therefore, the forward-looking policy is very likely to render the REE indeterminate.\(^{15}\) Note that Case I is the empirically relevant condition for determinacy, since Case II cannot obtain with realistic calibrations of the model parameters including the baseline one. The lower bound of the determinacy interval is of course given by Taylor principle (2.22), while the upper bound is induced by the first inequality of (2.24), which limits the policy coefficient on expected future inflation very severely. This is in stark contrast to Proposition 4 of Bullard and Mitra (2002) and Proposition 4.5 of Woodford (2003), which show that in the absence of the labor market search and matching frictions the forward-looking policy ensures determinacy if and only if it satisfies the Taylor principle but its response to expected future inflation is not too strong, \( 1 < \phi_n < 1 + 2(1 + \beta)/\kappa = 25 \) under the baseline calibration.\(^{16}\)

What causes the forward-looking policy to induce indeterminacy of REE? Indeterminacy is induced by any inflation coefficient less than one, due to the weakness of the demand channel of monetary policy. That is, passive policy makes the REE indeterminate, as in line with previous monetary policy literature. It is also induced by any inflation coefficient greater than 1.16 in the presence of the vacancy channel of monetary policy that stems from the labor market search and matching frictions.

\(^{15}\)An interval of realistic values of the coefficient on expected future inflation is arguably between 1.5 and 3.1. This interval contains the values reported by Clarida, Gali and Gertler (1998, 2000) and Orphanides (2004), obtained from more flexible forward-looking policy functions estimated on post-1979 data.

\(^{16}\)In this policy coefficient interval, \( \kappa \) is the real marginal cost elasticity of inflation, but not the output (gap) elasticity of inflation, which is given by \( \kappa/\sigma \) and appears in the determinacy interval of the policy coefficient given in Proposition 4 of Bullard and Mitra (2002) and Proposition 4.5 of Woodford (2003).
As noted above, this vacancy channel leads a rise in the real interest rate to increase expected future inflation and therefore makes inflation expectations self-fulfilling if the forward-looking policy has sufficiently strong responses solely to expected future inflation. Consequently, the REE fails to be determinate. Only if the policy coefficient lies in the very narrow interval of (1,1.16), the effect of the vacancy channel is negligible and hence a determinate REE is generated.

The indeterminacy result is robust with respect to the model parameters. As sensitivity analysis, we consider how the upper bound on the policy coefficient on expected future inflation changes for alternative values of parameters that determine the effect of the vacancy channel of monetary policy. As noted above, two factors give rise to the vacancy channel: the sluggish adjustment in output due to the labor market frictions and the expected recovery of future consumption after a rise in the real interest rate. Thus, the effect of the vacancy channel depends on the sluggishness of the labor market holding back the adjustment in future output and on the strength of the recovery of expected future consumption in response to real interest rate changes. In the labor market, we can see from the labor market tightness (2.13) and the employment motion law (2.14) that the dynamics of employment and output are determined by the proportion of separations \( \rho \), the steady state unemployment rate \( U \), and the search elasticity of matches \( \xi \). A small \( \rho \) implies via (2.14) that changes in current output persist strongly into the future and that current vacancies have a small effect on employment creation. Therefore, a smaller \( \rho \) makes the labor market more sluggish and hence the indeterminacy problem is worsened. For a small value of \( \rho = 0.07 \) (e.g. Merz, 1995) the interval of the inflation coefficient for which determinacy is ensured becomes \( 1 < \phi_\pi < 1.13 \), while for a large value of \( \rho = 0.15 \) (e.g. Andolfatto, 1996) it widens to \( 1 < \phi_\pi < 1.20 \). A large \( U \) reduces the employment coefficient \((1-u)/u\) in (2.13) and thus, in combination with (2.14), employment has a more persistent effect on expected future employment. As a consequence, the determinacy interval becomes narrower for a larger steady state unemployment rate; e.g. if \( U = 0.12 \) (0.03) this interval is \( 1 < \phi_\pi < 1.10 \) (1.29). A reduction of \( \xi \) dampens the firm matching rate’s response to changes in labor market tightness. This increases the sluggishness of the labor market by making expected future employment more sensitive to current employment via (2.13) and (2.14). But it also raises the proportion

\[ \phi_\pi \]

The other parameter that determines the effect of the vacancy channel is the probability of not optimizing price \( \alpha \), which measures price stickiness. A smaller \( \alpha \) increases the real marginal cost elasticity of inflation \( \kappa = (1 - \alpha)(1 - \alpha \beta)/\alpha \), which strengthens the vacancy channel effect and hence the indeterminacy problem deteriorates; e.g. if \( \alpha = 0.5 \) (0.8), the determinacy interval is \( 1 < \phi_\pi < 1.05 \) (1.50).

The determinacy interval for alternative unemployment rates is obtained by keeping the job finding rate \( p \) at its baseline value implied by \( U = 0.06 \), to isolated the effect of \( U \) on the dynamics. The change in \( p \) implied by a change in \( U \) would have an opposing effect on the length of the determinacy interval.
of newly matched vacancies and dampens the rise of the expected cost of a match in response to a tightening labor market. For a small value of $\xi = 0.235$ (Hall, 2005) the determinacy interval narrows to $1 < \phi_\pi < 1.11$, while for a large value of $\xi = 0.5$ (e.g. Krause, Lopez-Salido and Lubik, 2008) it becomes $1 < \phi_\pi < 1.26$. As for the strength of the recovery of expected future consumption in response to real interest rate changes, it is determined entirely by households’ degree of risk aversion $\sigma$, the inverse of which measures the intertemporal substitution elasticity of consumption. A smaller degree of risk aversion makes consumption movements more sensitive to real interest rate changes, resulting in a strong expected growth of future consumption after a real interest rate rise. Consequently, the determinacy interval becomes narrower for a smaller $\sigma$; e.g. if $\sigma = 0.16$ (Woodford, 2003) and $\sigma = 5$ (McCallum and Nelson, 1999), the REE is determinate for $1 < \phi_\pi < 1.03$ and $1 < \phi_\pi < 1.35$, respectively.

### 2.3.2 Current-looking and backward-looking policies

We turn next to the current-looking and the backward-looking policies, i.e. $j = 0$ and $j = -1$ in (2.12). These policies yield, respectively, third and fourth order characteristic equations for the systems’ coefficient matrices and then determinacy requires that exactly two solutions to these equations be outside the unit circle and the others lie inside the unit circle. To our knowledge, there is no general result about conditions under which fourth order equations have such solutions, and thus we investigate the backward-looking policy numerically. For the current-looking policy we obtain the following proposition.

**Proposition 2** The current-looking policy, i.e. $j = 0$ in (2.12), ensures determinacy of REE if and only if either of the following two cases is satisfied.

Case I: (2.22), (2.26), and (2.27) or (2.28) hold.

\[
\begin{align*}
\phi_\pi &> -1 - \frac{2c_3(c_3 - c_1)}{\kappa s_c(2u - \rho)[\xi + \beta(1 - \rho)(\xi - \eta_p)] - 2\beta\sigma a_1(1 - \rho)(1 - \eta_p)}, \\
c_3(c_3 - c_1) + c_2 &> 1 \text{ or } |c_1| > 3, \\
-1 &> \frac{\chi\sigma(1 - \xi) - s_c}{\kappa(1 - \rho)[s_c(\xi - \eta_p) - \sigma(1 - \eta_p)](1 - \xi) - s_c},
\end{align*}
\]

where $a_1$ is given in Proposition 1 and $c_i$, $i = 1, 2, 3$, are given in Appendix A.3.

Case II: the two strict inequalities opposite to (2.22) and (2.26) hold and (2.27) or the strict inequality opposite to (2.28) holds.

**Proof** See Appendix A.3. ■

Under the baseline calibration, the current-looking policy guarantees determinacy of REE if and only if Taylor principle (2.22) is satisfied, i.e. $\phi_\pi > 1$. In that calibrated
case, (2.26) and the first inequality of (2.27) are satisfied, such that (2.22) generates determinacy. The backward-looking policy ensures determinacy as long as it meets the Taylor principle but its response to past inflation is not too strong, $1 < \phi_\pi < 10.4$ under the baseline calibration; otherwise, it induces indeterminacy for $0 \leq \phi_\pi < 1$ and makes the REE explosive for $\phi_\pi \geq 10.4$. These results are robust with respect to any realistic value of each model parameter and are in line with Bullard and Mitra (2002) and Woodford (2003) who consider the case of a frictionless labor market.\footnote{For unrealistically large values of the risk aversion $\sigma$, active current-looking and backward-looking policies induce indeterminacy; e.g. in the case of $\sigma = 15$, the current-looking policy with $2.16 \leq \phi_\pi \leq 91.66$ and the backward-looking policy with $4.91 \leq \phi_\pi \leq 19.08$ render REE indeterminate.}

## 2.4 Prescriptions for the indeterminacy problem

We have shown that the forward-looking policy is very likely to render the REE indeterminate due to the vacancy channel of monetary policy that stems from the labor market search and matching frictions. In this section we consider three prescriptions for this indeterminacy problem. Specifically, we examine the following generalization of the forward-looking policy.\footnote{This generalization includes, as the special case in which $\phi_x = \hat{\phi}_x (1 - \phi_R)$ for $x = \pi, Y, U$,}

$$R_t = R^{1-\phi_R} (R_{t-1})^{\phi_R} \left( \frac{E_t \bar{\pi}_{t+1}}{\pi} \right)^{\phi_\pi} \left( \frac{E_t Y_{t+k}}{Y} \right)^{\phi_Y} \left( \frac{1-U_t}{1-U} \right)^{\phi_U}, \quad k = 0, 1, \quad (2.29)$$

where $\phi_R$, $\phi_Y$, $\phi_U$ are non-negative policy coefficients on the lagged interest rate, current or expected future output and the unemployment rate. The log-linearization of (2.29) is given by

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + \phi_\pi E_t \bar{\pi}_{t+1} + \phi_Y E_t \hat{Y}_{t+k} - \phi_U \hat{U}_t, \quad k = 0, 1, \quad (2.30)$$

where $\hat{U}_t = U_t - U$. The first prescription is the policy adjustment for current or expected future output in addition to expected future inflation. Second, we consider interest rate smoothing, i.e. the policy response to the lagged interest rate. These two are motivated by empirical studies such as Clarida, Galí and Gertler (1998, 2000) and Orphanides (2004), who use them as a good description of actual monetary policy conducted in industrialized countries.\footnote{Carlstrom and Fuerst (2005) show in the case of a frictionless labor market that in the presence of investment spending, the forward-looking policy induces indeterminacy of equilibrium. Kurozumi and Van Zandwegrhe (2008) find that this indeterminacy problem can be overcome by a policy response to current output or by interest rate smoothing. See Chapter 5.}

The last prescription is the policy reaction to the unemployment rate. Blanchard and Galí (2008) and Faia (2008) find that interest
rate policy with responses to inflation and unemployment rates can approximate well optimal policy responses to shocks in a sticky price model with labor market frictions.

2.4.1 Policy response to output

We first investigate whether the policy response to current or expected future output as well as expected future inflation, i.e. \( \phi_R = \phi_U = 0 \) in (2.29), can resolve the indeterminacy problem induced by the forward-looking policy.

In the case of the policy response to expected future output, \( k = 1 \) in (2.29), the system consisting of (2.13)–(2.18), (2.20) and (2.30) can be reduced to a system of the same form as (2.21) with a different coefficient matrix \( A \) given in Appendix A.1. Analyzing this coefficient matrix yields the following proposition.

**Proposition 3** Suppose \( \phi_Y \neq \sigma \). If the forward-looking policy responds also to expected future output, i.e. \( k = 1, \phi_R = \phi_U = 0 \) in (2.29), it ensures determinacy of REE if and only if either of the following two cases is satisfied.

Case I: (2.31), (2.32), and (2.33) or (2.34) hold.

\[
\begin{align*}
\phi_n + \frac{\chi(1-\beta)[u(1-\xi) - \xi(s_v/s)]}{\kappa[\xi - \beta(1-\rho)(\xi - \eta p)]} \phi_Y & > 1, \\
\phi_n + \frac{\chi \alpha_i(1+\beta)(2\sigma - \phi_Y)}{\kappa (s_v(2u-\rho)[\xi + \beta(1-\rho)(\xi - \eta p)] - 2\beta \sigma a_i (1-\rho)(1-\eta p))} & > 1,
\end{align*}
\]

\[d_i (d_3 - d_1) + d_2 > 1 \text{ or } |d_i| > 3, \]

\[
\phi_n > 1 + \frac{\chi \rho(1-\xi) - s_v(\sigma - \phi_Y)}{\kappa(1-\rho)\{s_v(\xi - \eta p) - \sigma(1-\eta p)[\rho(1-\xi) - s_v]\}},
\]

where \( a_i \) is given in Proposition 1 and \( d_i, i = 1, 2, 3 \), are given in Appendix A.4.

Case II: the two strict inequalities opposite to (2.31) and (2.32) hold and (2.33) or the strict inequality opposite to (2.34) holds.

**Proof** See Appendix A.4. ■

Like Bullard and Mitra (2002), Woodford (2003) and Kurozumi and Van Zandweghe (2008), (2.31) can be given the following economic interpretation. By equilibrium conditions (2.13)–(2.18), each percentage point of permanently higher inflation implies a permanent increase in output of \( \chi(1-\beta)[u(1-\xi) - \xi(s_v/s)]/\{\kappa[\xi - \beta(1-\rho)(\xi - \eta p)]\} \) percentage points. Hence the left-hand side of (2.31) shows the long run rise in the nominal interest rate by policy (2.30) with \( \phi_R = \phi_U = 0 \) for each percentage point permanent increase in the inflation rate. Therefore, (2.31) can be interpreted as the long run version of the Taylor principle: in the long run the nominal interest rate should be raised by more than the increase in inflation.

Figure 2.1 illustrates a region of policy coefficients on expected future inflation and output \( (\phi_\pi, \phi_Y) \) that generate determinacy under the baseline calibration. Case I
is the empirically relevant condition for determinacy, since Case II cannot obtain under realistic calibrations of the model parameters including the baseline one. We can see that moderate policy adjustment for expected future output ameliorates the indeterminacy problem. The lower bound on the inflation coefficient $\phi_\pi$ is given by Taylor principle (2.31). The first inequality of condition (2.33) yields an upper bound on the inflation coefficient and allows a much wider determinacy interval of the inflation coefficient as the output coefficient $\phi_Y$ increases. However, once it increases beyond a certain threshold given by the intersection of (2.32) and (2.33), the determinacy interval becomes narrower due to (2.32), which imposes the upper bound on the inflation and output coefficients. This upper bound is induced by the demand channel of monetary policy because we can see the corresponding one in Figure 3 of Bullard and Mitra (2002) who examine the case of a frictionless labor market, in which monetary policy contains only the demand channel and the upper bound limits the determinacy region more severely. The intuition for this amelioration of the indeterminacy problem is as follows. Indeterminacy is induced by the vacancy channel of the forward-looking policy, in which a rise in the real interest rate stemming from inflationary expectations increases expected future inflation. But, the policy response to expected future output subdues such a rate rise because this output falls as a consequence of the rate rise. Hence, an expected recovery of future consumption following the real interest rate rise is subdued and the expected need for more future vacancies is prevented. Therefore, determinacy is generated.

In the case of the policy response to current output, $k = 0$ in (2.29), a similar analysis of the system’s coefficient matrix $A$ given in Appendix A.1 yields the following proposition.

**Proposition 4** If the forward-looking policy responds also to current output, i.e. $k = 0$, $\phi_R = \phi_U = 0$ in (2.29), it ensures determinacy of REE if and only if either of the following two cases is satisfied.

**Case I**: (2.31), (2.35), and (2.36) or (2.25) hold.

\[
\phi_\pi < 1 + \frac{\chi a_1(1 + \beta)(2\sigma + \phi_\nu)}{\kappa\{s(2u - \rho)[\xi + \beta(1 - \rho)(\xi - \eta p)] - 2\beta\sigma a_1(1 - \rho)(1 - \eta p)\}}, \tag{2.35}
\]

\[
e_3(e_3 - e_1) + e_2 > 1 \quad \text{or} \quad |e_1| > 3, \tag{2.36}
\]

where $a_1$ is given in Proposition 1 and $e_i$, $i = 1, 2, 3$, are given in Appendix A.5.

**Case II**: the two strict inequalities opposite to (2.31) and (2.35) hold and (2.36) or the strict inequality opposite to (2.25) holds.

**Proof** See Appendix A.5. ■

If the forward-looking policy responds also to current output, the long run version of the Taylor principle yields the same inequality as (2.31) in the case of the policy
Figure 2.1: Determinacy region of interest rate policy coefficients on expected future inflation and expected future output

Figure 2.2: Determinacy region of interest rate policy coefficients on expected future inflation and current output
response to expected future output. Figure 2.2 shows a determinacy region of policy coefficients on expected future inflation and current output under the baseline calibration. Note that Case I is the empirically relevant condition for determinacy because of (2.35) and Taylor principle (2.31), the latter of which gives the lower bound on the inflation coefficient. The upper bound on the inflation coefficient is induced by the first inequality in (2.36) for any output coefficient less than a certain threshold given by the intersection of (2.35) and (2.36), while it is induced by (2.35) for any output coefficient greater than this threshold. This is in contrast with the case of the policy response to expected future output, in which the determinacy interval narrows once the output coefficient increases beyond the threshold specified before. Thus, the policy response to current output is a much better prescription for the indeterminacy problem than the one to expected future output. Intuitively, this amelioration of the indeterminacy problem arises because current output falls as a consequence of a rise in the real interest rate stemming from inflationary expectations and hence the policy response to this output subdues such a rate rise. Therefore, the policy response to current output prevents the inflationary expectations from becoming self-fulfilling and REE from being indeterminate.

2.4.2 Policy response to unemployment

We turn next to the second prescription, the forward-looking policy with responses to the unemployment rate, i.e. $\phi_R = \phi_Y = 0$ in (2.29). Analyzing the system’s coefficient matrix $A$ given in Appendix A.1 yields the following necessary and sufficient condition for determinacy.

**Proposition 5** If the forward-looking policy responds also to the unemployment rate, i.e. $\phi_R = \phi_Y = 0$ in (2.29), it ensures determinacy of REE if and only if either of the following two cases is satisfied.

**Case I:** (2.37), (2.38), and (2.39) or (2.25) hold.

\[
\begin{align*}
\phi_{\pi} + \frac{\chi u(1-\beta)(1-\xi)(1-U)}{\kappa[\xi - \beta(1-\rho)(\xi - \eta p)]} \phi_{U} &> 1, \\
\phi_{\pi} &< 1 + \frac{\chi(1+\beta)[2\sigma a_1 + u\rho(1-\xi)(1-U)\phi_{U}]}{\kappa\{s_1(2u - \rho)[\xi + \beta(1-\rho)(\xi - \eta p)] - 2\beta\sigma a_1(1-\rho)(1-\eta p)\}}, \\
f_3(f_3 - f_1) + f_2 &> 1 \quad \text{or} \quad |f_1| > 3,
\end{align*}
\]  

where $a_1$ is given in Proposition 1 and $f_i$, $i = 1, 2, 3$, are given in Appendix A.6.

**Case II:** the two strict inequalities opposite to (2.37) and (2.38) hold and (2.39) or the strict inequality opposite to (2.25) holds.

**Proof** See Appendix A.6. ■
Percent changes in production are reflected to a very large extent in percentage point changes in the unemployment rate because of the relation \( \hat{U}_t = -(1 - U)\hat{n}_t \), where the steady state unemployment rate is a very small number, e.g. \( U = 0.06 \) under the baseline calibration. Thus, the policy response to the unemployment rate yields almost the same determinacy result as the one to current output, since output also largely reflects production. The intuition for determinacy is also the same. Figure 2.3 illustrates the determinacy region of policy coefficients on expected future inflation and current unemployment rates under the baseline calibration. As is the case with the policy response to current output, (2.37) can be interpreted as the long run version of the Taylor principle and provides the lower bound on the inflation coefficient, while (2.38) and the first inequality of (2.39) induce the upper bound. Therefore, the policy response to the unemployment rate, as well as the one to current output, is a better prescription for the indeterminacy problem.

### 2.4.3 Interest rate smoothing

Finally, we consider whether interest rate smoothing can help the forward-looking policy generate determinacy of REE, i.e. \( \phi_Y = \phi_U = 0 \) in (2.29). It seems hard to

\[ \tag{22} \text{This intersection appears at an inflation coefficient greater than five, so that the upper bound induced by (2.35) does not appear in Figure 2.2.} \]
analytically examine determinacy with this policy specification, since interest rate smoothing leads to a fourth order characteristic equation for a system’s coefficient matrix, which has two predetermined variables, and hence determinacy requires that two solutions are outside the unit circle and the remaining two lie inside the unit circle. There seems to be no general result about conditions for that and thus we numerically investigate determinacy.

![Graph showing determinacy region](image)

**Figure 2.4:** Determinacy region of interest rate policy coefficients on expected future inflation and interest rate smoothing

Figure 2.4 shows the determinacy region of policy coefficients of inflation and interest rate smoothing under the baseline calibration. The long run version of the Taylor principle yields $\phi_\pi > 1 - \phi_R$, which provides the lower bound on the inflation coefficient. We can see that a sufficiently high degree of interest rate smoothing of $\phi_R = 0.3$ or more brings about determinacy as long as the long run Taylor principle is met. The intuition for determinacy is that interest rate smoothing implies the policy responses to lagged interest rates and hence makes the forward-looking policy respond also to current and past inflation like the current-looking and the backward-looking policies, which are likely to generate determinacy. Thus, determinacy is guaranteed with interest rate smoothing. In sum, the forward-looking policy with sufficiently strong interest rate smoothing is also a better prescription for the indeterminacy problem.
2.5 E-stability analysis of the indeterminacy problem

In this section we assess the indeterminacy problem induced by the forward-looking policy from the perspective of E-stability. Specifically, we examine whether the forward-looking policy generates a unique E-stable fundamental REE even in cases of indeterminacy. Following the literature, our E-stability analysis is based on the so-called “Euler equation” approach suggested by Honkapohja, Mitra and Evans (2003): the rational expectations operator \( E_t \) is replaced with a possibly non-rational one \( \hat{E}_t \) in the system of (2.13)–(2.20). This system can be reduced to a system of the form

\[
F \hat{x}_t = G \hat{E}_t \hat{x}_{t+1} + H \hat{n}_{t-1} + J g_t, \tag{2.40}
\]

where \( \hat{x}_t = [\hat{\pi}_t \hat{C}_t \hat{n}_t]' \) and the coefficient matrices \( F, G, H \) are given in Appendix A.7. Then, fundamental RE solutions to system (2.40) are given by

\[
\hat{x}_t = \bar{c} + \bar{\Phi} \hat{n}_{t-1} + \bar{\Gamma} g_t, \tag{2.41}
\]

where the coefficient matrices are determined by

\[
\bar{c} = 0_{3 \times 1}, \quad H = (F - G \bar{\Phi}[0 0 1]) \bar{\Phi}, \quad \bar{\Gamma} = \{F - G \bar{\Phi}[0 0 1] - \rho_g G\}^{-1} J.
\]

Note that \( \bar{\Gamma} \) is uniquely determined given a \( \bar{\Phi} \), but \( \bar{\Phi} \) is not generally uniquely determined, which induces multiplicity of fundamental REE.

Following Section 10.2 of Evans and Honkapohja (2001), we analyze E-stability of fundamental REE. Corresponding to fundamental RE solutions (2.41), all agents are assumed to be endowed with a perceived law of motion (PLM) of \( \hat{x}_t \)

\[
\hat{x}_t = c + \Phi \hat{n}_{t-1} + \Gamma g_t. \tag{2.42}
\]

Using a forecast from the PLM and the relation \( \hat{n}_t = [0 0 1] \hat{x}_t \) to substitute \( \hat{E}_t \hat{x}_{t+1} \) out of (2.40) leads to an actual law of motion (ALM) of \( \hat{x}_t \)

\[
\hat{x}_t = F^{-1} G(I + \Phi[0 0 1]) c + F^{-1} (G \Phi[0 0 1] \Phi + H) \hat{n}_{t-1}
+ F^{-1} \{G(\Phi[0 0 1] \bar{\Gamma} + \rho_g \Gamma) + J\} g_t \tag{2.43}
\]

\[23\]Recall that in this chapter we refer to Evans and Honkapohja’s (2001) MSV solutions to linear RE models as fundamental and do not undertake E-stability analysis of non-fundamental REE.

\[24\]The form of the vector \( J \) is omitted, since it is not needed in what follows.

\[25\]System (2.40) contains a predetermined variable \( \hat{n}_{t-1} \), so that we can consider two learning environments, which are studied respectively in Section 10.2 and 10.3 of Evans and Honkapohja (2001). One environment allows agents to use current endogenous variables in expectation formation, whereas another does not. In this chapter we present only E-stability analysis with the latter environment, as in Bullard and Mitra (2002). This is because any inflation coefficient that generates a unique E-stable fundamental REE in the latter environment does so in the former one, as Kurozumi (2006) shows in the absence of the labor market frictions. An intuition for this is that in forming future expectations, agents have more information by the current endogenous variables and hence E-stability is more likely in the former environment than in the latter one. Another reason for our focus on the latter environment is that the former induces a problem with simultaneous determination of the expectations and current endogenous variables, which is critical to equilibrium under non-rational expectations as indicated by Evans and Honkapohja (2001) and Bullard and Mitra (2002).
provided that $F$ is invertible. Here, $I$ denotes a conformable identity matrix. Then, a mapping $T$ from the PLM (2.42) to the ALM (2.43) can be defined by

$$T(c, \Phi, \Gamma) = (F^{-1}G(I + \Phi[0 0 1])c, F^{-1}(G\Phi[0 0 1]\Phi + H), F^{-1}(G(\Phi[0 0 1]\Gamma + \rho g \Gamma) + J)).$$

For a fundamental RE solution $(\bar{c}, \bar{\Phi}, \bar{\Gamma})$ to be E-stable, the matrix differential equation

$$\frac{d}{d\tau}(c, \Phi, \Gamma) = T(c, \Phi, \Gamma) - (c, \Phi, \Gamma)$$

must have local asymptotic stability at the solution, where $\tau$ denotes a notional time. Then, we have

$$DT_c(c, \Phi) = F^{-1}G(I + \Phi[0 0 1]),$$

$$DT_\Phi(\Phi) = F^{-1}G([0 0 1]\Phi + \Phi[0 0 1]),$$

$$DT_\Gamma(\Phi, \Gamma) = F^{-1}G(\rho g I + \Phi[0 0 1]).$$

Therefore, it follows that a fundamental RE solution $(\bar{c}, \bar{\Phi}, \bar{\Gamma})$ is E-stable if and only if all eigenvalues of three matrices, $DT_c(\bar{c}, \bar{\Phi})$, $DT_\Phi(\bar{\Phi})$, $DT_\Gamma(\bar{\Phi}, \bar{\Gamma})$, have real parts less than one. We summarize this result in the following lemma.

**Lemma 1** Suppose that the coefficient matrix $F$ is invertible. A fundamental RE solution to the system of (2.13)–(2.20) with the forward-looking policy, i.e. $j = 1$ in (2.12), is E-stable if and only if all eigenvalues of three matrices, $F^{-1}G(\varphi I + \Phi[0 0 1])$, $\varphi = 1, \rho_g, \bar{\Phi}_3$, have real parts less than one, where $\bar{\Phi}_3$ is the third element of the RE solution vector $\bar{\Phi}$.

With this lemma we investigate E-stability of fundamental REE numerically, since it seems impossible to analytically solve the matrix equation for $\Phi$ in fundamental RE solutions (2.41) and thus to obtain explicit conditions for E-stability. As pointed out by McCallum (1998), distinct fundamental REE are obtained for different orderings of stable generalized eigenvalues of the matrix pencil for system (2.40).

The E-stability analysis shows that in the presence of the labor market search and matching frictions, the forward-looking policy generates a unique E-stable fundamental REE if the policy response to expected future inflation lies in one of two intervals, which both satisfy the Taylor principle: $1 < \phi_s < 7.25$ and $\phi_s > 25.06$. Only the policy response to expected future inflation in these intervals succeeds in guiding temporary equilibria under non-rational expectations toward the unique E-stable REE.

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26In cases of indeterminacy, the baseline calibration shows order one or two indeterminacy and hence two or three distinct fundamental REE.

27The price stickiness changes these intervals quantitatively. For instance, if $\alpha = 0.5$ (0.8), an inflation coefficient in the interval of $1 < \phi_s < 3.05$ (20.93) or $\phi_s > 8.89$ (77.71) generates a unique E-stable fundamental REE.
Because the first interval is wide enough to contain all empirically relevant values, the result suggests that the indeterminacy problem induced by the forward-looking policy is not critical from the perspective of E-stability or least-squares learnability of fundamental REE.

This result is a generalization of Bullard and Mitra (2002), who examine the case of a frictionless labor market to show that the forward-looking policy generates a unique E-stable fundamental REE if and only if it meets the Taylor principle. In the presence of the labor market search and matching frictions, the vacancy channel emerges and reduces the guiding effect of the demand channel. As a consequence, multiple fundamental REE are E-stable if the policy response to expected future inflation lies in the intermediate one between the two intervals of inflation coefficients that generate the unique E-stable REE.

2.6 Robustness analysis

In this section we analyze the robustness of our results obtained with the benchmark model by introducing habit formation in consumption preferences or considering an alternative labor market specification that is a more conventional one in previous studies.

2.6.1 Habit formation in consumption preferences

As noted before, the vacancy channel of monetary policy causes the forward-looking policy to induce indeterminacy of REE because output supply recovers sluggishly relative to expected future consumption demand after a tightening of the policy. This suggests that the indeterminacy problem might be less severe when habit formation in consumption preferences is taken into account, since such preferences imply that consumption demand adjusts sluggishly to real interest rate changes.\(^{28}\) Thus, in this subsection we assume that the period \(t\) utility is given by

\[
\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma}e^{\epsilon t},
\]

where \(h \in [0,1)\) measures (internal) habit persistence in consumption and the benchmark model examined above is contained as the special case of \(h = 0\). Then, the representative household’s optimality condition for consumption becomes

\[
\lambda_t = (C_t - hC_{t-1})^{-\sigma} - \beta h (E_t C_{t+1} - h C_t)^{-\sigma}.
\]

\(^{28}\)In a sticky price model with internal habit formation in consumption preferences, Walsh (2005) shows that the labor market search and matching frictions affect the dynamics of real marginal cost to the effect of augmenting the persistence in output and inflation. This is not the case in the absence of the habit formation as pointed out by Krause and Lubik (2007).
As predicted, indeterminacy becomes less likely with a large $h$. For instance, in the case of $h = 0.8$, the interval of the policy response to expected future inflation for which determinacy is ensured widens to $1 < \phi < 1.25$. Yet, this interval is still very narrow and hence the indeterminacy problem remains. The prescriptions for this problem examined above are still effective for guaranteeing determinacy. Also, when E-stability is adopted as an REE selection criterion, a unique E-stable fundamental REE is generated by active, but not too strong, responses to expected future inflation, suggesting that the indeterminacy problem is not critical from the perspective of E-stability of fundamental REE.

2.6.2 Alternative labor market specification

In the benchmark model we assume that job destruction is exogenous and hiring is instantaneous. Previous studies, such as Trigari (2005), Walsh (2005) and Krause and Lubik (2007), however, use a distinct labor market specification in which jobs are also endogenously destroyed and new hires become productive in the subsequent period. This specification implies that contemporaneous employment adjustment takes place only via job destruction, rather than only via job creation as in the benchmark model. In this subsection we use the alternative labor market specification to examine the robustness of our results obtained with the benchmark model.

Each worker-firm match surviving in period $t$ produces $a_t$ goods, which is a job-specific productivity level that is drawn from a distribution $F$ with a positive support. There is a threshold level of job productivity, denoted by $\tilde{a}_t$, below which matches are discontinued. Specifically, at the beginning of the period, a fraction $\rho_x$ of existing matches is destroyed exogenously, and so is the share of remaining matches that fall below the productivity threshold. Thus, the rate of job destruction is

$$\rho_t = \rho_x + (1 - \rho_x)F(\tilde{a}_t)$$

---

29Introducing habit persistence into the model with frictionless labor market of Bullard and Mitra (2002) and Woodford (2003) shifts up the upper bound of the determinacy interval for the policy coefficient from 25 ($h = 0$) to 1,862 ($h = 0.8$).

30The current-looking and the backward-looking policies guarantee determinacy of REE under similar conditions to those obtained with the benchmark model.

31This interval of policy responses becomes narrower with larger $h$. There is no second interval of very large policy responses to expected future inflation for which a unique E-stable fundamental REE is generated as in the benchmark model.

32This specification of the labor market corresponds to early evidence that job loss rates rise strongly during recessions. More recently, Shimer (2007) and Hall (2006) argue that estimates of job finding rates account for much of the changes in the unemployment rate, a finding that motivates the constant separation rate in many recent search and matching models. But this view is also contested; e.g. Elsby, Michaels and Solon (2007) find that inflows and outflows of unemployment are both important in explaining cyclical unemployment variation. The results in this subsection indicate that this debate is not relevant for the question about indeterminacy of REE with interest rate policy.
and the measure of search unemployment is
\[ u_t = 1 - (1 - \rho_t)n_{t-1}. \]

We assume that newly formed matches become productive only in the subsequent period. Then, the aggregate production of the wholesale sector becomes
\[ y_t = (1 - \rho_t)n_{t-1}H(\tilde{a}_t), \]
where \( H(\tilde{a}_t) = E[a|a > \tilde{a}_t] = \int_{\tilde{a}_t}^{\infty} a \, dF(a)/[1 - F(\tilde{a}_t)] \), and the employment including hired, but non-productive, workers evolves according to
\[ n_t = (1 - \rho_t)n_{t-1} + m_t. \]

As before, the asset values of a matched firm and worker can be used to obtain the job creation condition and the wage equation from Nash bargaining. The asset value of a matched firm is the current real net revenue plus the expected continuation value of the match. An unmatched firm pays the vacancy posting cost and produces in the next period with probability \((1 - \rho_{t+1})q_t\). Combining these firm asset values yields the job creation condition
\[ \frac{\gamma}{q_t} = E_t\beta_{t+1}(1 - \rho_{t+1}) \left[ z_{t+1}H(\tilde{a}_{t+1}) - \overline{w}_{t+1}(\tilde{a}_{t+1}) + \frac{\gamma}{\tilde{q}_{t+1}} \right], \]
where \( \overline{w}(\cdot) \) is the average wage which is defined below. The asset value of a matched (unmatched) worker is the wage (unemployment benefit) plus the expected present discounted value of this worker’s employment status in the next period. The Nash bargaining outcome then results in the wage equation
\[ w_t(a_t) = \eta \left( z_t a_t + p_t \frac{\gamma}{q_t} \right) + (1 - \eta) b. \]

Thus, the average wage is
\[ \overline{w}_t(\tilde{a}_t) = \int_{\tilde{a}_{t+1}}^{\infty} \frac{w_t(a)}{1 - F(\tilde{a}_{t+1})} \, dF(a) = \eta \left[ z_t H(\tilde{a}_t) + p_t \frac{\gamma}{q_t} \right] + (1 - \eta) b. \]

Substituting the wage equation, the job creation condition becomes
\[ \frac{\gamma}{q_t} = E_t\beta_{t+1}(1 - \rho_{t+1}) \left\{ (1 - \eta) [z_{t+1}H(\tilde{a}_{t+1}) - b] + (1 - \eta p_{t+1}) \frac{\gamma}{q_{t+1}} \right\}. \]

Finally, the threshold value \( \tilde{a}_t \) is determined by \( F^m(\tilde{a}_t) = 0 \), or equivalently,
\[ \tilde{a}_t = \frac{1}{z_t} \left( b - \frac{1 - \eta p_t}{1 - \eta} \frac{\gamma}{q_t} \right). \]
Log-linearizing these labor market conditions around the steady state and rearranging the resulting equations yields

\[ \hat{y}_t = \varepsilon_{H, \hat{a}} \hat{a}_t + \hat{n}_{t-1} - \frac{\rho}{1 - \rho} \hat{\rho}_t, \]  
(2.44)

\[ \hat{\rho}_t = \frac{\rho - \rho_x}{\rho} \varepsilon_{F, \hat{a}} \hat{a}_t, \]  
(2.45)

\[ \hat{\theta}_t = \hat{v}_t + \frac{1 - u}{u} \left( \hat{n}_{t-1} - \frac{\rho}{1 - \rho} \hat{\rho}_t \right), \]  
(2.46)

\[ \hat{n}_t = (1 - \rho) \left( \hat{n}_{t-1} - \frac{\rho}{1 - \rho} \hat{\rho}_t \right) + \rho \left( \hat{v}_t - \xi \hat{\theta}_t \right), \]  
(2.47)

\[ \xi \hat{\theta}_t = \tilde{\chi} \left( E_t \hat{\dot{z}}_{t+1} + \varepsilon_{H, \hat{a}} E_t \hat{a}_{t+1} \right) - \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right) \]  
+ \beta (1 - \rho) (\xi - \eta p) E_t \hat{\theta}_{t+1} - \frac{\rho}{1 - \rho} E_t \hat{\rho}_{t+1}, \]  
(2.48)

\[ \hat{a}_t = -\frac{(\gamma / q) (\xi - \eta p)}{(1 - \eta) b - (\gamma / q) (1 - \eta p)} \hat{\theta}_t - \hat{z}_t, \]  
(2.49)

\[ \hat{y}_t = s_c \hat{C}_t + s_v \hat{v}_t, \]  
(2.50)

where \( \tilde{\chi} \equiv \beta (1 - \rho) H(\hat{a}) \chi > 0 \), and \( \varepsilon_{H, \hat{a}} \) and \( \varepsilon_{F, \hat{a}} \) are respectively the steady state productivity elasticity of \( H \) and \( F \) at the threshold \( \hat{a} \). Because current production now depends on past rather than present matching activity, the expected value of a match depends on expected future real marginal cost in the job creation condition (2.48). The log-linearized equilibrium conditions are now given by (2.44)–(2.50), the consumption Euler equation (2.17), the Phillips curve (2.18) and interest rate policy (2.19).

With the alternative labor market specification, we numerically investigate determinacy of REE. Following Walsh (2005), it is assumed that idiosyncratic productivity shocks are drawn from a log-normal distribution and are serially uncorrelated with zero mean and a variance of 0.13². The exogenous job destruction rate is assumed equal to \( \rho_x = 0.068 \) following den Haan, Ramey and Watson (2000). With these assumptions the value of the vacancy posting cost is determined by the model’s steady state relationships and is equal to \( \gamma = 0.06 \). Under the baseline calibration, the current-looking and the backward-looking policies guarantee determinacy of REE under similar conditions to those obtained with the benchmark model. The forward-looking policy generates a determinate REE for any inflation coefficient in the interval of \( 1 < \phi_\pi < 1.21 \), which changes little from the one obtained with the benchmark model. Thus, the indeterminacy problem with the forward-looking policy remains.

Intuitively, this problem arises because changes in employment and output are persistent, such that the monetary transmission mechanism contains a vacancy channel in addition to the demand channel as in the benchmark model. Although the labor market specification examined here is more complex than the one in the bench-
mark model, we can see from (2.44) and (2.45) that contemporaneous adjustment of production to a decrease in consumption demand occurs via a rise in the separation rate. But this also destroys employment available for production in the subsequent period in (2.47). To meet an expected recovery of future consumption demand, firms expand current vacancy creation to have new matches in production in the following period. Thus, the rise in the separation rate and the rise in vacancies have opposing effects on the labor market tightness in (2.46) and on future employment. However, to the extent that a strong recovery of future consumption is expected, the current labor market tightens. This is associated via the job creation condition (2.48) with a rise in expected future real marginal cost. Thus, initial inflation expectations can become self-fulfilling.

Regarding the prescriptions for the indeterminacy problem induced by the forward-looking policy, each policy response to current or expected future output, the unemployment rate, or the lagged interest rate, in addition to expected future inflation, yields a very similar determinacy region as in the benchmark model illustrated in Figures 2.1–2.4 respectively. Thus, these prescriptions remain effective for overcoming the indeterminacy problem. When considering E-stability as an REE selection criterion, the forward-looking policy with an inflation coefficient in the intervals of $1 < \phi_\pi < 14.83$ and $\phi_\pi > 22.41$ generates a unique E-stable fundamental REE.

In sum, even when we introduce habit formation in consumption preferences or even when we consider the more conventional labor market specification, the findings obtained with the benchmark model remain unchanged.

2.7 Concluding remarks

We have examined implications of search and matching frictions in the labor market for inflation targeting interest rate policy in terms of equilibrium stability. Such labor market frictions cause sluggish adjustment of production capacity to changes in demand. As a consequence, a rise in the real interest rate increases expected future real marginal cost and hence expected future inflation. Therefore, indeterminacy is likely under forward-looking policy, which adjusts the interest rate in response solely to expected future inflation. However, this indeterminacy can be overcome if the policy adjusts the interest rate in response also to output or the unemployment rate or if it contains interest rate smoothing. Further, if E-stability is adopted as an equilibrium selection criterion, the forward-looking policy generates a unique E-stable fundamental rational expectations equilibrium under active, but not so strong or extremely strong, responses to expected future inflation. These findings are robust even when we introduce habit formation in consumption preferences or when we consider a more conventional labor market specification used in previous studies.
Chapter 3

On-the-Job Search, Sticky Prices, and Persistence

3.1 Introduction

There is a widespread view in the monetary business cycle literature that sticky price models need to be augmented with real rigidities in order to replicate the dynamic responses observed in the data.\(^1\) A theory of real rigidity explains why real wages or real prices are unresponsive to changes in economic activity. Labor market search in the spirit of Mortensen and Pissarides (1994) may be one way of introducing real wage rigidity and has recently been embedded in sticky price models by a number of authors. But Krause and Lubik (2007) find that labor market search frictions do not induce real wage rigidity, which echoes the finding of Shimer (2005a) and Hall (2005) that productivity increases are mostly absorbed by higher wages if those are determined as the outcome of Nash bargaining. Moreover, they report that imposing real wage rigidity only weakly affects real marginal cost and hence inflation. This surprising result arises because labor market search frictions generate a surplus from a match which gives rise to long-term employment relationships. The value of such a relationship to the firm drives a wedge between the real marginal cost and the real wage, and becomes more sensitive to demand shocks under exogenous real wage rigidity due to the larger share of the surplus that accrues to the firm. Therefore, their results suggest that search and matching frictions do not help explain the persistent effects of monetary shocks.

This chapter shows that if workers can search on the job, the response of real wages and the value of a long-term employment relationship to a monetary shock is dampened, such that on-the-job search induces rigidity of real marginal cost and acts as a strong propagation mechanism that results in a persistent output response.

\(^1\)Ball and Romer (1990) show that nominal rigidities can be explained by a combination of real rigidities and small nominal frictions. See also Chari, Kehoe and McGrattan (2000), Dotsey and King (2005), and Jeanne (1998).
Job-to-job mobility is a quantitatively important phenomenon in U.S. labor markets. Moreover, it has the ability to propagate productivity shocks in real models as has been shown in a number of recent studies. The chapter studies a sticky price model economy with money in the utility function where the labor market is characterized by search frictions, job heterogeneity and job-to-job mobility. In particular, high wage and low wage jobs are created through a costly matching process and low wage workers search on the job for high wage jobs as in Krause and Lubik (2006). Following an expansionary monetary shock, consumption demand increases which prompts increased vacancy posting. As a result low wage workers raise their search intensity and increase the effective pool of searchers for high wage jobs, while unemployed searchers redirect their search efforts to low wage jobs. The increased search for both types of jobs prevents the labor market from tightening rapidly and thus dampens the increase in real wages. At the same time, the additional channel of employment growth provides a shift toward the more productive high wage jobs, which relaxes the economy’s resource constraint. The combination of stronger employment growth and increased output per worker prevents the value of a long-term employment relationship in both types of jobs from rising rapidly. The combined effect is a smaller increase in real marginal cost.

Quantitative analysis of the model shows that on-the-job search substantially reduces the volatility of real wages, real marginal cost and inflation relative to output. As a result, output responds strongly to monetary shocks: several measures indicate that the propagation of monetary shocks is at least three times stronger than in a sticky price model without on-the-job search or in a model with a frictionless labor market. The latter two models yield quantitatively similar results. These findings emphasize that recent improvements in real models of labor market search, which concentrate on the volatility of real wages, do not necessarily imply an improvement for models with nominal rigidities. They do so only insofar the mechanism that underlies real wage stickiness also affects the real marginal cost.

Among related literature, Cooley and Quadrini (1999) study the role of labor market search frictions in a limited participation model of money, and Walsh (2003) introduces search frictions in a cash-in-advance model with sticky prices. Walsh (2005) finds that labor market search frictions strengthen the output and inflation persistence in a sticky price model with habit formation in consumption preferences and price indexation to the past inflation rate. These features can substantially influence the

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2Responding to Shimer’s (2005a) criticism of the lack of propagation and the role of wage bargaining in real search and matching models a literature has ensued that aims at remedying this challenge. Hall (2005) proposes a model of equilibrium wage stickiness on which some of the recent sticky price literature draws. Other research derives low wage volatility from fundamentals without invoking wage stickiness. One way of achieving this is by allowing for job-to-job transitions as is done by Krause and Lubik (2006), Nagypal (2006) and Tasci (2007).
dynamic response of real marginal cost to a nominal demand shock, and therefore make the role of search frictions in determining real marginal cost difficult to assess. Trigari (2005) focuses on the role of the intensive and extensive margins to explain output and inflation dynamics, but adds the same inertial features to preferences and price adjustment. On the contrary, Krause and Lubik (2007), discussed above, find that labor market search frictions influence the real effects of nominal price rigidity only weakly.

The chapter proceeds as follows. A sticky price model with on-the-job search is presented and discussed in Section 3.2. In Section 3.3 the model is analyzed quantitatively and the results compared to those from two alternative sticky price models: one with search frictions but no on-the-job search, the other with a competitive labor market. Section 3.4 adds some concluding remarks.

3.2 A sticky price model with on-the-job search

The model economy is inhabited by four types of agents: households, final good producers, intermediate good producers, and a monetary authority. Each of these agents is discussed in turn.

3.2.1 Households

Each period the infinitely-lived representative household supplies one unit of labor inelastically, and chooses paths for consumption \( c_t \), real money balances \( M_t/P_t \), and bond holdings \( B_t \) that maximize its expected lifetime utility

\[
E_t \sum_{k=0}^{\infty} \beta^k \left[ \frac{C_{t+k}^{1-\sigma} - 1}{1 - \sigma} + \varphi \log \left( \frac{M_{t+k}}{P_{t+k}} \right) \right]
\]

subject to the budget constraint

\[
P_t C_t + M_t + B_t = P_t D_t + M_{t-1} + B_{t-1} R_{t-1} + T_t
\]

where \( \sigma > 0 \) is the coefficient of relative risk aversion, \( \beta \in (0,1) \) is the intertemporal discount factor, \( P_t \) is the unit price of consumption, \( R_t \) is the nominal interest rate, \( T_t \) is a lump-sum transfer from the monetary authority, and the disutility associated with work effort is normalized to zero. Real income \( D_t \) consists of monopoly profits from the final good firms, rents related to labor market frictions from the intermediate good firms, either a wage \( w_{jt}^t \) from employment in a type \( j \) job or unemployment income \( h \) when unemployed, minus a lump-sum transfer to finance unemployment income.\(^3\)

\(^3\)In order to avoid distributional issues, it is assumed that employed and unemployed households pool consumption. Equivalently, there could be perfect insurance markets, as in Andolfatto (1996), or households could consist of a continuum of members pooling their income, as in Merz (1995).
The different types of jobs are discussed below. Consumption consists of a composite of differentiated goods \( i \in [0, 1] \) defined as \( C_t = \left[ \int_0^1 C_t(i)^{(\epsilon-1)/\epsilon} \, di \right]^{\epsilon/(\epsilon-1)} \), where \( \epsilon > 1 \) is the elasticity of substitution between these goods. Cost minimization of the household’s consumption across goods implies that the demand for each good is given by \( C_t(i) = \left( \frac{P_t(i)/P_t}{P_t(i)^{1-\epsilon}} \right)^{\epsilon-1} P_t(i) \), while the aggregate price index is \( P_t = \left[ \int_0^1 P_t(i)^{\epsilon-1} \, di \right]^{1/(\epsilon-1)} \).

The household’s choices for consumption, bond holdings and real money balances yield the following optimality conditions

\[
\frac{C_t^{1-\sigma}}{P_t} = R_t \beta E_t \frac{C_{t+1}^{1-\sigma}}{P_{t+1}}, \tag{3.2}
\]

\[
md_t \equiv M_t \frac{P_t}{P_t} = \varphi \frac{R_t}{R_t - 1} c \tag{3.3}
\]

Condition (4.2.1) is the consumption Euler equation and (3.3) specifies the household’s money demand.

### 3.2.2 Final good producers

The final good market is characterized by monopolistic competition. There is a continuum of final good producers, and each producer \( i \in [0, 1] \) combines intermediate goods \( b \) and \( g \) into a differentiated good using a Cobb-Douglas production technology \( Y_t(i) = Y_t^b(i)^{\alpha} Y_t^g(i)^{1-\alpha} \). The share parameter \( \alpha \in [0, 1] \) of input \( b \) can be interpreted as a productivity differential: let \( \alpha < 1/2 \) so that productivity of input \( g \) exceeds that of \( b \). Firm \( i \) chooses the cost-minimizing bundle of inputs that leads to the following demand functions for inputs \( b \) and \( g \)

\[
Y_t^b(i) = \alpha \left( \frac{z_t}{z_t^b} \right) Y_t(i) \tag{3.4}
\]

\[
Y_t^g(i) = (1 - \alpha) \left( \frac{z_t}{z_t^g/1} \right) Y_t(i), \tag{3.5}
\]

where \( z_t^b, z_t^g \) denotes the relative price of good \( b, g \) respectively, and \( z_t \) is the real marginal cost of each final good firm, which is given by

\[
z_t = \left( \frac{z_t^b}{\alpha} \right)^{\alpha} \left( \frac{z_t^g}{1 - \alpha} \right)^{1-\alpha}. \tag{3.6}
\]

Final good producers set the price of their product in order to maximize discounted expected real profits subject to demand from households and subject to Calvo (1983) and Yun (1996) style price stickiness. Specifically, a fixed fraction \( \nu \in (0, 1) \) of randomly chosen firms does not reoptimize price and instead adjusts it for steady state inflation \( \pi \), while each remaining firm faces the problem

\[
\max_{P_t^*} E_t \sum_{k=0}^{\infty} \nu^k \beta t_{t+k} \left( \frac{P_t^* \pi^k}{P_{t+k}} - z_{t+k} \right) \left( \frac{P_t^* \pi^k}{P_{t+k}} \right)^{-\epsilon} C_{t+k}.
\]

34
Here $\beta_{t,t+k} = \beta^k(C_{t+k}/C_t)^{-\sigma}$ is the stochastic discount factor by which firms’ expected profits are evaluated in terms of the value attached to them by the households. Profit maximization then requires the first order condition

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty}(\nu \pi_c)^k E_t \beta_{t,t+k} P_{t+k}^e C_t + \sum_{k=0}^{\infty}(\nu \pi_p)^k E_t \beta_{t,t+k} P_{t+k}^e C_t - \sigma}{\epsilon - 1}$$

(3.7)

to be satisfied. If prices are flexible (i.e. $\nu = 0$), this condition reduces to $z_t \epsilon / (\epsilon - 1) = P_t^* / P_t$, which shows that $\epsilon / (\epsilon - 1)$ is the desired markup of each intermediate good firm’s price over its marginal cost. In the presence of price rigidity, the firm’s actual markup will differ from but tend toward the desired markup. The aggregate price index must satisfy

$$P_t = [(1 - \nu)(P_t^*)^{\epsilon - 1} + \nu(\pi P_{t-1})^{\epsilon - 1}]^{\frac{1}{\epsilon - 1}}.$$  

(3.8)

### 3.2.3 Intermediate good producers

Intermediate good production takes place in two perfectly competitive markets. Heterogeneity results from the presence of two types of firms: those with high wage (“good”) jobs and those with low wage (“bad”) jobs. The cost of creating a job by any firm is represented by the flow cost of posting a vacancy: $\gamma^g$ for good jobs and $\gamma^b$ for bad jobs, where $\gamma^g > \gamma^b$. In the presence of search frictions these costs give rise to surpluses, which in turn lead to equilibrium wage differentials. A proportion $\rho \in (0,1)$ of existing matches is exogenously destroyed before production and matching start, and each surviving worker-firm pair produces $a_t$ units of output in period $t$, where $a_t$ is the aggregate state of productivity that evolves according to

$$\log a_t = \rho a_{t-1} + \epsilon a_t, \quad \epsilon a_t \sim N(0, \sigma_a^2).$$

(3.9)

Thus, aggregate production of good $j = b, g$ is

$$y_t^j = (1 - \rho)a_t n_t^j, \quad j = b, g,$$

(3.10)

where $n_t^j$ is employment in a type $j$ firm. Exogenous match destruction is consistent with the evidence of Hall (2006) and Shimer (2005b) that changes in the separation rate play only a minor role in explaining variations in the unemployment rate. The specification of the labor market follows Krause and Lubik (2006).

**Labor market.** While high wage and low wage firms post vacancies, unemployed workers decide ex ante towards which type of job they direct their search effort. However, labor mobility implies that in equilibrium, the value to an unemployed worker of searching for either type of job must be equal in every period. The matching frictions of workers and firms are represented by a constant returns to scale matching.
function that determines the number of type $j$ matches between job searchers and vacancies as

$$m_j^t = \psi(u_j^t)^\xi (v_j^t)^{1-\xi} \quad j = b, g.$$  

(3.11)

Here $u_j^t$ is the measure of unemployed workers searching for type $j$ jobs, $v_j^t$ is the measure of type $j$ vacancies, the scale parameter $\psi$ reflects the efficiency of the matching process, and $\xi \in (0, 1)$ is the unemployment elasticity of matches. Thus, the evolution of employment in each type of job is described by the motion law

$$n_{t+1}^j = (1 - \rho)n_t^j + m_t^j \quad j = b, g.$$  

(3.12)

With a population size normalized to one, the total number of searchers is given by

$$u_t = 1 - (1 - \rho)n_t,$$  

(3.13)

where $u_t = u_t^b + u_t^g$ and $n_t = n_t^b + n_t^g$.

The ratio of vacancies to searchers, $\theta_j^t = v_j^t / u_j^t$, is a measure of the labor market tightness. The probability to fill a vacancy (the firm matching rate) is given by

$$q_j^t = \frac{m_j^t}{v_j^t} = \psi(\theta_j^t)^{-\xi} \quad j = b, g,$$  

(3.14)

and rises when the labor market becomes slack. Similarly, the probability to find a job (the worker matching rate) is

$$p_j^t = \frac{m_j^t}{u_t^j} = \psi(\theta_j^t)^{1-\xi} \quad j = b, g,$$  

(3.15)

so that a worker is more likely to find a job when the labor market is tight.

**Wage determination.** The surplus from a match is divided between the worker and the firm via Nash bargaining. To determine a wage that gives the worker his share of the bargain, it is helpful to describe the asset values of matched and unmatched firms and households in a recursive form. The value of a matched type $j$ firm, $J_j^t$, is the sum of the current real net revenue and the discounted present value of the firm in the next period

$$J_j^t = z_j^t a_t - w_j^t + E_t \beta_{t+1, t+1} \left[ (1 - \rho)J_{t+1}^j + \rho V_{t+1}^j \right] \quad j = b, g,$$  

(3.16)

where $w_j^t$ is the wage received by the worker and $V_j^t$ is the value of an unmatched type $j$ firm in period $t$. Such a firm pays the vacancy cost and is matched with probability $q_j^t$, such that

$$V_j^t = -\gamma_j + E_t \beta_{t+1} \left[ (1 - \rho)q_j^t J_{t+1}^j + (1 - (1 - \rho)q_j^t)V_{t+1}^j \right] \quad j = b, g.$$  

(3.16)
Free entry drives the asset value of unmatched firms to zero in equilibrium, and thus
\[
\frac{\gamma_j}{q_t^j} = E_t \beta_{t+1} (1 - \rho) \left[ z_{t+1}^j a_{t+1} - w_{t+1}^j + \frac{\gamma_j}{q_t^{j+1}} \right] \quad j = b, g. \tag{3.17}
\]
The job creation conditions (3.17) relate the expected cost of filling a vacancy to the expected value of a match. Suppose, for instance, that the real interest rate falls following an expansionary monetary policy shock. The ensuing rise of the stochastic discount rate increases the discounted present value of a job, which stimulates vacancy posting until the firm matching rate has adjusted to eliminate the ex ante rents from an additional vacancy.

The value of an employed (unemployed) worker is the wage (unemployment income) plus the expected present value of this worker’s employment status in the next period
\[
\begin{align*}
W_t^j &= w_t^j + E_t \beta_{t+1} \left[ (1 - \rho) W_{t+1}^j + \rho U_{t+1}^j \right], \\
U_t^j &= h + E_t \beta_{t+1} \left[ (1 - \rho) p_t^j W_{t+1}^j + (1 - (1 - \rho) p_t^j) U_{t+1}^j \right] \quad j = b, g.
\end{align*}
\]
The Nash bargaining outcome, through which the worker receives a share \( \eta \in (0, 1) \) of the surplus in each type of job, results in the wage equations
\[
\begin{align*}
w_t^j &= \eta \left[ z_t^j a_t + p_t^j \frac{\gamma_j}{q_t^j} \right] + (1 - \eta) h, \quad j = b, g. \tag{3.18}
\end{align*}
\]
The worker is compensated for a fraction \( \eta \) of firm revenue and the saving of hiring cost which the firm enjoys thanks to the match. In addition, the worker is compensated for a fraction \( 1 - \eta \) of the forgone unemployment income.

The assumption that unemployed workers direct their search activity between the two types of jobs yields a restriction on the costs of job creation and on the matching rates of both types of jobs. Specifically, \( U_t^g = U_t^b \) in equilibrium because of arbitrage by unemployed workers, which in turn implies that labor market tightness for good and bad jobs is in fixed proportion
\[
p_t^g \frac{\gamma^g}{q_t^g} = p_t^b \frac{\gamma^b}{q_t^b}. \tag{3.19}
\]

**On-the-job search.** Data suggest that job-to-job movements make up an important part of newly formed matches (Nagypal, 2004). As such, they can potentially shed light on various questions about the labor market (see e.g. Pissarides, 1994, and Burdett and Mortensen, 1998). More recently, the role of on-the-job search has been studied in dealing with the issue raised by Shimer (2005a) that in a standard model of labor market search, productivity increases are mostly absorbed by higher wages.
and do not nearly result in the fluctuations of unemployment and vacancies that are observed in the data. Job-to-job mobility can amplify the increase in vacancy creation after a shock while putting less upward pressure on wages.

Such a mechanism is incorporated in the model of Nagypal (2006), where employers prefer to hire employed workers rather than unemployed because the former are less likely to quit the job and firms incur a training cost after a match is made. Tasci (2007) proposes a model of job-to-job flows that arise because of heterogeneity in match quality. The measure of low-quality matches rises during a boom, which gives rise to procyclical transitions from low quality to high quality jobs. Krause and Lubik (2006) propose an alternative mechanism to address the same issue. They use the labor market with job heterogeneity discussed above, where good jobs and bad jobs coexist due to a difference in the cost of vacancy creation. Workers in bad jobs search endogenously for good jobs, and increase their search effort in periods of high productivity because the increased availability of good job vacancies increases the worker matching rate. The increased pool of search in turn encourages firms to post more vacancies, which leads to a gradual increase in labor market tightness and prevents real wages from rising rapidly. This chapter follows Krause and Lubik (2006)’s approach to introduce job-to-job movements.4

The introduction of on-the-job search changes some equations for good jobs and most equations for bad jobs. All workers in bad jobs search on the job with an endogenous search intensity $s_t$ (whereas the search intensity of unemployed workers is always equal to one). Hence, in period $t$, there are $u_t^g + e_t$ workers searching for good jobs, where

$$e_t = (1 - \rho)n_t^b s_t$$

(3.20)

is the effective search intensity. Thus, the matching function of good jobs is given by

$$m_t^g = \psi(u_t + e_t)\xi(v_t^g)^{1-\xi},$$

(3.21)

and the worker matching rate for good jobs becomes

$$p_t^g = m_t^g / (u_t^g + e_t),$$

(3.22)

while the labor market tightness is adjusted correspondingly to $\theta_t^g = v_t^g / (u_t^g + e_t)$. The evolution of bad jobs becomes

$$n_{t+1}^b = (1 - \rho)(1 - p_t^g s_t)n_t^b + m_t^b.$$  

(3.23)

4These authors also cite evidence that jobs created during recessions are of lower average quality and come from a lower part of the wage distribution than jobs created in booms, and that employment in high wage jobs in U.S. manufacturing is more procyclical than low wage jobs, which agrees with their model’s predictions. Shimer (2005b) reports that the actual job-to-job transition rate is strongly procyclical.
To find the wage equation, the value of a matched firm with a bad job is

$$J_b^t = z_b^t \alpha_t - w_b^t + E_t \beta_{t, t+1} \left[ (1 - \rho)(1 - s_t p_t^b) J_{t+1}^b + (\rho + (1 - \rho) s_t p_t^g V_b^t \right], \quad (3.24)$$

where the probability that the worker might quit reduces the continuation value of the match to the firm. The value of an unmatched firm with a vacancy for a bad job remains unchanged, so that free entry implies

$$\frac{\gamma_b^b}{q_t^b} = E_t \beta_{t, t+1} (1 - \rho) \left[ z_{t+1}^b \alpha_{t+1} - w_{t+1}^b + (1 - s_{t+1} p_{t+1}^g) \frac{\gamma_b^b}{q_{t+1}} \right]. \quad (3.25)$$

The value of an employed worker with a bad job is

$$W_b^t = w_b^t - k(s_t) + E_t \beta_{t, t+1} \left[ (1 - \rho)(1 - s_t p_t^g) W_{t+1}^b + (1 - \rho) s_t p_t^g W_{t+1}^g + \rho V_{t+1}^b \right],$$

where $k(s_t)$ denotes the cost of search, which is a strictly convex function of search intensity with $k(0) = 0$ and $k''(s_t) > 0$. For a worker in a bad job, increasing search intensity is costly but makes finding a good job more likely. The value of an unemployed worker looking for a bad job remains unchanged. Thus, the bargaining outcome yields a wage equation for bad jobs

$$w_b^t = \eta \left[ z_t^b \alpha_t + (1 - s_t) p_t^g \frac{\gamma_b^g}{q_t^g} + (1 - \eta) [h + k(s_t)] \right]. \quad (3.26)$$

A higher search intensity has opposing effects on the wage. It tends to raise the wage in order to compensate the worker for the reduction of surplus due to a higher search cost. But it also tends to lower the wage to compensate the firm for the increased chance of separation due to the worker quitting. A worker in a bad job chooses his search intensity to maximize his asset value of the match, which yields the following condition

$$k'(s_t) = \frac{\eta}{1 - \eta} p_t^g \left( \frac{\gamma_g^g}{q_t^g} - \frac{\gamma_b^b}{q_t^b} \right). \quad (3.27)$$

Thus, workers in bad jobs increase their search intensity if the worker matching rate for a good job rises or if firms’ expected value of a good match increases relative to that of a bad match.

### 3.2.4 Monetary authority

The monetary authority is assumed to conduct its policy according to the following money growth rule

$$\log \mu_t = (1 - \rho) \log \mu + \rho \mu \log \mu_{t-1} + \varepsilon_{\mu t}, \quad \varepsilon_{\mu t} \sim N(0, \sigma^2_{\mu}) \quad (3.28)$$

where $\mu_t$ denotes the growth rate of the nominal money supply, $M_t$, and $\mu$ is its steady state value. This formulation of monetary policy follows most of the literature that
An additional reason to specify monetary policy in terms of money supply changes, rather than an interest rate rule, is that the labor market dynamics since the mid-1960s are considered and the empirically relevant values of the policy parameters of an interest rate rule have changed during the course of this period. The government has access to lump-sum taxes and conducts a Ricardian fiscal policy, so that the government budget constraint need not be specified.

3.2.5 Equilibrium and calibration

In equilibrium, market clearing implies that $B_t = B_{t-1} = 0$ and $M_t = M_{t-1} + T_t$ in each period $t$. Intermediate good market clearing requires that

$$Y^j_t = y^j_t - \gamma^j v^j_t, \quad j = b, g$$

(3.29)

where $Y^j_t \equiv \int Y^j_t(i) di$, and final good market clearing requires that $Y_t(i) = C_t(i)$, $\forall i \in [0,1]$, which imply that

$$Y_t \equiv \int_0^1 Y_t(i) di = C_t \Delta_t,$$

(3.30)

where $\Delta_t = \int [P_t(i)/P_i]^{-\epsilon} di$ measures the relative price dispersion. A rational expectations equilibrium consists of initial values for the productivity level, the growth rate of the nominal money supply, and the number of matched workers in both types of job, as well as sequences for $C_t$, $Y_t$, $Y^j_t$, $y^j_t$, $z_t$, $v^j_t$, $R_t$, $P_t^*$, $P_t$, $n_{t+1}$, $n_{t+1}$, $p^j_t$, $q^j_t$, $u^j_t$, $u_t$, $m^j_t$, $e_t$, $s_t$, $\mu_t$, and $a_t$, for $j = b, g$ satisfying equations (3.1)–(3.15), (3.17)–(3.23), (3.25)–(3.30).

The ensuing analysis uses a realistic calibration of model parameters to evaluate the model quantitatively. The baseline calibration is summarized in Table 3.1. The discount factor $\beta$ is equal to 0.99, $\sigma = 1$, $\epsilon = 11$ is chosen to yield the steady state markup of 1.1, the interest rate semielasticity of money demand is one, which is the low value considered by Dotsey and King (2005), and $\lambda$ is set to correspond to an average frequency of price adjustment equal to three quarters, i.e. $\nu = 0.67$, which is in line with recent microeconomic evidence on the frequency of price changes. The labor market parameters are $\eta = 0.5$ following most of the literature on search frictions, $\xi = 0.4$ based on the empirical estimates of Blanchard and Diamond (1989), $\rho = 0.1$ and the steady state unemployment rate $1 - n = 0.06$ as in Walsh (2005). The level parameter $\psi = 0.6$ is chosen to imply an average firm matching rate of 0.7, following den Haan, Ramey and Watson (2000). With respect to job heterogeneity, Krause and

5Trigari (2005) and Walsh (2005) instead describe monetary policy in terms of an interest rate rule.
Lubik (2006) are followed in setting the steady state quit rate $q_r = p^g sn^b/n = 0.06$, defining the search cost function as $k(s) = \kappa s^\tau$ and setting the search elasticity equal to $\tau = 1.1$, the share of input $b$ in final output to $\alpha = 0.4$, and the vacancy posting cost for a bad job to $\gamma^b = 0.04$. The vacancy posting cost for a good job is assumed to be three times larger than for a bad job. Finally, the monetary growth process is characterized by an autoregressive parameter $\rho_\mu = 0.57$, as in Chari, Kehoe and McGrattan (2000), and a standard deviation $\sigma_\mu = 0.006$, which is a typical estimate (see e.g. Cooley and Quadrini, 1989, or Wang and Wen, 2006). The autoregressive coefficient of the productivity shock is set to 0.9 and its standard error is set to $\sigma_a = 0.001$. The latter value is chosen such that the benchmark model yields a standard deviation of output that corresponds to that observed in the data. The volatility of the model is then assessed by the ratios of the standard deviation of each variable to that of output.

**Table 3.1: Baseline calibration**

<table>
<thead>
<tr>
<th>Preferences and technology</th>
</tr>
</thead>
</table>
| $\beta$                   | 0.99  
| $\sigma$                  | 1  
| $\epsilon$                | 11  
| $\alpha$                  | 0.4  
| $\nu$                     | 0.67  

<table>
<thead>
<tr>
<th>Labor market</th>
</tr>
</thead>
</table>
| $\xi$                     | 0.4  
| $\psi$                    | 0.6  
| $\gamma^b$                | 0.04  
| $\gamma^g$                | 0.12  
| $\rho$                    | 0.1  
| $\tau$                    | 1.1  
| $\eta$                    | 0.5  
| $1 - n$                   | 0.06  
| $q_r$                     | 0.06  

<table>
<thead>
<tr>
<th>Productivity and monetary policy</th>
</tr>
</thead>
</table>
| $\rho_M$                        | 0.57  
| $\sigma_M$                      | 0.006  
| $\rho_a$                        | 0.9  
| $\sigma_a$                      | 0.001  

With this calibration, the steady state relationships imply values for the typespecific parameters. These are $m^g = 0.06$, $m^b = 0.09$, $q^d = 0.94$, $q^b = 0.61$, $p^g = 0.33$, $p^b = 0.64$, $u^g = 0.02$, $u^b = 0.14$, $v^g = 0.06$, $v^b = 0.15$, $n^g = 0.56$, $n^b = 0.38$, and
s = 0.45. With about 40 percent of the workforce employed in bad jobs and search intensity is 45 percent, more than 18 percent of the working population is effectively searching for a job in the steady state. Furthermore, \(w^g = 0.45\), \(w^b = 0.44\), \(z^g = 0.47\), and \(z^b = 0.46\). The real wage of good jobs is some two percent higher than that of bad jobs, and likewise the relative price of good \(g\) exceeds that of good \(b\). In addition, the scale parameter of the search cost and the unemployment income are implied by the steady state relations and take the value \(\kappa = 0.02\) and \(h = 0.39\) respectively. The model is log-linearized around the steady state and the log-linearized equations, with hatted variables denoting percentage deviations from the steady state, are given in the Appendix.

### 3.2.6 Real wages and real marginal cost

Define the value of a long-term employment relationship to a firm as the difference between the asset value of a match available for current production and its discounted expected future value.\(^6\) Equation (3.16) implies that in equilibrium this value is equal to the current real net revenue from an existing match. This is the revenue that an unmatched firm foregoes if a sudden increase in consumption demand cannot be met because it does not have a worker to increase production. For bad matches equation (3.24) implies that this value is further raised by the expected production loss arising from endogenous quits,

\[
J^g_t - \gamma^g/a_t = z^g_t a_t - w^g_t,
\]

\[
J^b_t - (1 - s_t p^g_t)\gamma^b/a_t = z^b_t a_t - w^b_t.
\]

Writing these equations in terms of the intermediate good relative prices, which determine the real marginal cost of final goods, shows that the value of a long-term employment relationship drives a wedge between the ratio of real wage and marginal labor product, and the relative price,\(^7\)

\[
z^g_t = \frac{w^g_t a_t}{a_t} + \frac{J^g_t - \gamma^g/a_t}{a_t},
\]

\[
z^b_t = \frac{w^b_t a_t}{a_t} + \frac{J^b_t - (1 - s_t p^g_t)\gamma^b/a_t}{a_t}.
\]

With a frictional labor market the value of a long-term relationship can display substantial cyclical behavior that causes the dynamics of real marginal cost to differ from

---

\(^6\)The terminology “long-term employment relationship” follows Goodfriend and King (2001) in their description of most labor transactions in advanced economies.

\(^7\)Filling a vacancy is costless and instantaneous in the absence of labor market search frictions, and hence the value of a long-term relationship is zero. In that case, the current relative prices depend only on the current real wages as \(z^j_t = w^j_t/a_t\), \(j = b, g\). Plugging this into equation (3.6) yields the familiar expression of real marginal cost as a weighted average of real factor prices over the marginal product of labor.
that of real wages.\textsuperscript{8} Intuitively, suppose that current consumption demand rises due to a temporary acceleration of the money growth rate. Then intermediate good firms must employ more labor in order to meet the increased demand for final and hence intermediate goods, prompting these firms to post additional vacancies. The labor market tightens as a result and this pushes up wages. Although employment and production will expand with a lag, they cannot increase initially because the number of matches is predetermined. Because output is initially scarce relative to consumption demand, the current value of a match rises more than the discounted present value of a match. That is, the value of a long-term employment relationship rises and consequently the real marginal cost rises more sharply than the real wage. As Krause and Lubik (2007) show, imposing exogenous real wage rigidity would barely affect the dynamics of real marginal cost. This is because if wages are prevented to rise in the face of the nominal demand shock, the additional surplus generated by an existing match accrues entirely to the firm. Hence, the value of the long-term employment relationship to such a firm rises all the more strongly.

On the contrary, on-the-job search can result in a dampening of both the real wage and the value of a long-term relationship by opening up an additional channel of employment growth. Consider first how good job creation is affected. When workers can search on the job the increase in good vacancies posted after an expansionary money growth shock raises the worker matching rate for a good job and thereby induces workers in bad jobs to increase their search intensity for a higher value job. The increase in effective search for good jobs in turn stimulates the posting of more good job vacancies as it prevents the firms’ matching rate from falling rapidly. Hence, employment growth is strengthened by an increased formation of good matches which takes place without a rapidly tightening labor market. As employment rises the real wage of good jobs is held down as an endogenous result of the subdued labor market tightness. The additional employment growth results in a shift to the more productive good jobs. Thus, while employment becomes relatively abundant via on-the-job search, the number of additional matches required to meet the growth in consumption demand is subdued. This implies that the rise in the value of a match is slowed by on-the-job search. And because job creation increases strongly at a given labor market tightness, the rise in the value of a long-term relationship is dampened as well.

At the same time, rising quits prompt firms to post more bad vacancies. As a result, unemployed workers redirect their search toward the bad sector until the value of both types of unemployed searchers is equalized. Hence, the labor market for bad

\textsuperscript{8}Goodfriend and King (2001) also discuss why the “effective” real marginal cost may be more volatile than the real wage because of long-term relationships between workers and firms.
jobs also experiences stronger employment growth without a rapidly tightening labor market. Hence, real wages and the value of a long-term employment relationship for bad jobs are rigid, even though the latter is raised on impact of a shock by the rise in quits during the boom as shown in equation (3.31). Thus, on-the-job search allows the creation of matches of both types of job to expand while preventing real wages and the value of a long-term relationship to rise rapidly. The net effect is that real marginal cost in the final good sector may be substantially less volatile in a model with job-to-job mobility, reducing the dependence on nominal price stickiness. The importance of these effects is evaluated quantitatively in the next section.

3.3 Quantitative analysis

This section discusses the quantitative evaluation of the sticky price model with on-the-job search (henceforth, OJS) and contrasts these results with those obtained from two alternative models. One is a sticky price model with a competitive labor market (henceforth, NK), where households have a convex disutility from supplying labor hours. This model consists of equations (B.1)–(B.6) in the Appendix after log-linearization, and the real marginal cost is determined by the competitive labor market equilibrium condition

\[ \hat{z}_t = (\sigma + \zeta)\hat{Y}_t - (1 + \zeta)\hat{a}_t \] (3.32)

where \( \zeta^{-1} \) is the labor supply elasticity. The value of this elasticity has been a substantial source of controversy in the literature. In business cycle models, higher values are often chosen than those that are estimated in microeconomic studies. Here, an intermediate value is chosen by setting \( \zeta = 2 \). The second is a model with labor market search and job heterogeneity but no on-the-job search (henceforth, DMP), which is obtained by setting the search intensity \( s_t = 0 \) for all \( t \).

3.3.1 Impulse response analysis

Figure 3.1 displays the output response to a one percent increase in the money growth rate, where the solid line corresponds to the OJS model. On impact, output slightly falls because production is predetermined and the rise in consumption demand prompts intermediate good firms to allocate increased resources toward vacancy creation. Output subsequently rises gradually to reach a peak after four quarters. The hump-shaped output response is very similar to that documented for instance by

---

\(^9\)In this case the steady state equations under the baseline calibration imply the following job type-specific parameter values: \( q^g = 0.73, q^b = 0.47, p^g = 0.49, p^b = 0.95, v^g = 0.08, v^b = 0.08, u^g = 0.11, u^b = 0.04, n^g = 0.55, n^b = 0.39, m^g = 0.06, m^b = 0.04, z^g = 0.47, z^b = 0.45, w^g = 0.45, w^b = 0.44, \) and \( s = 0 \).
Christiano, Eichenbaum and Evans (1999, 2005) and others. A straightforward way to quantify the effect of on-the-job search in terms of propagation is via the contract multiplier. Following Christiano, Eichenbaum and Evans (2005) this is defined as the ratio of the number of periods before the output expansion from a monetary shock is back to zero and the contract length of a typical price contract. The contract multiplier equals 30 quarters. Another measure of propagation is the cumulative output response to a money growth shock, which is 6.4 with on-the-job search. The amplitude of the peak impact is 0.4 percent.

Figure 3.1: Output response to a one percent money growth shock

In contrast, in the NK model output peaks on impact as shown by the dotted line in the figure. The contract multiplier is 6 periods, and the cumulative output response of the shock is 1.7. In terms of amplitude the maximum output response is 0.6 percent. This output response illustrates the persistence problem that is also analyzed by Chari, Kehoe and McGrattan (2000) and a large literature. The output response in the DMP model, where the effect of on-the-job search is shut off, is given by the dashed line in the figure. This response is qualitatively similar to that of the OJS model because output displays a hump-shaped response to a monetary shock. However, the output response peaks after a mere two quarters. Indeed, quantitatively the model is closer to the NK model: the contract multiplier is 10, which is somewhat larger than in the NK model but much smaller than in the OJS model. Moreover, the cumulative output response to the nominal shock is 1.4, which is even smaller than in the NK model. The maximum amplitude in the DMP model is 0.2 percent. Thus, allowing job-to-job transitions in the model with search frictions substantially increases the propagation of nominal shocks in the sticky price model. Specifically,

\footnote{This definition of the contract multiplier differs from that of Chari, Kehoe and McGrattan (2000) in order to take account of the hump-shaped output responses.}
the output response is at least three times more persistent than in the NK model and the DMP model by the measures considered. These results are summarized in Table 3.2.

<table>
<thead>
<tr>
<th></th>
<th>OJS</th>
<th>DMP</th>
<th>NK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak response</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Contract multiplier</td>
<td>30</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Cumulative response</td>
<td>6.4</td>
<td>1.4</td>
<td>1.7</td>
</tr>
</tbody>
</table>

The increase in output persistence generated by on-the-job search is mirrored in the labor market by an amplified increase in vacancy creation and associated reduction of unemployment in comparison to the DMP model. This is illustrated in Figure 3.2.

Figure 3.2: Vacancy and unemployment response to a one percent money growth shock

Figure 3.3 shows that inflation peaks on impact in the model with on-the-job search as well as in the alternative models. The amplitude of the peak inflation response with labor market search frictions exceeds that with a competitive labor market. The reason is that while an expansionary monetary shock increases consumption demand on impact, production only responds with a delay because the search frictions render current production predetermined. As a result, intermediate good market clearing requires a strong increase on impact of the intermediate input price and hence of the final good producers’ real marginal cost. Moreover, the concurrent increase in vacancy posting slightly reduces the available output, thus ex-
acerbating the initial increase in real marginal cost. Therefore, the initial inflation response to a monetary shock is very similar regardless of the possibility of on-the-job search. With a competitive labor market, the impact of such a shock on inflation is smaller because production can be adjusted instantaneously. In subsequent periods, however, the inflation response with a competitive market far exceeds the one with search frictions in the labor market. This is because the real marginal cost rises more strongly with a competitive market as firms have to bid up wages throughout the duration of the consumption boom. With labor market search frictions, on the contrary, the increased vacancy creation results in a rising stock of available matches which gradually relaxes the resource constraint as the boom continues. This results in diminished pressure on the real marginal cost. On-the-job search generates additional employment growth and a shift toward more productive jobs, allowing the consumption boom to be satisfied with a substantially subdued increase in real wages and value of long-term employment relationships. Consequently, as the figure shows, on-the-job search results in an inflation response that is only about half as large as in the DMP model.

![Figure 3.3: Inflation response to a one percent money growth shock](image)

3.3.2 Simulation results

Table 3.3 reports the average standard deviations of 500 histories simulated with the three models driven by money growth shocks and productivity shocks. These numbers can be compared to similar statistics computed from quarterly HP filtered U.S. data covering the period 1964:1–2005:4, which are displayed in the first column of the table. In parenthesis are given the models’ volatilities conditional on money growth.
shocks alone.

Table 3.3: Simulation results: U.S. economy and model economies

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tr>
<td>U.S. economy Benchmark</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Std. Deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>1.55</td>
<td>1.55</td>
<td>(0.73)</td>
<td>0.78</td>
<td>(0.27)</td>
<td>0.50</td>
<td>(0.46)</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.56</td>
<td>0.59</td>
<td>(1.18)</td>
<td>1.15</td>
<td>(3.20)</td>
<td>1.30</td>
<td>(1.41)</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.72</td>
<td>1.64</td>
<td>(3.34)</td>
<td>3.07</td>
<td>(8.49)</td>
<td>2.78</td>
<td>(3.00)</td>
</tr>
<tr>
<td>Real MC</td>
<td>–</td>
<td>3.19</td>
<td>(6.44)</td>
<td>5.84</td>
<td>(15.9)</td>
<td>2.79</td>
<td>(3.00)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>7.15</td>
<td>1.95</td>
<td>(2.49)</td>
<td>1.04</td>
<td>(2.32)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Vacancies</td>
<td>8.58</td>
<td>2.48</td>
<td>(3.36)</td>
<td>1.33</td>
<td>(3.22)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Tightness</td>
<td>15.5</td>
<td>4.30</td>
<td>(5.55)</td>
<td>2.13</td>
<td>(4.85)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.86</td>
<td>0.99</td>
<td>(0.96)</td>
<td>0.91</td>
<td>(0.90)</td>
<td>0.71</td>
<td>(0.67)</td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output, inflation</td>
<td>0.26</td>
<td>-0.01</td>
<td>(0.06)</td>
<td>-0.02</td>
<td>(0.05)</td>
<td>0.90</td>
<td>(1.00)</td>
</tr>
</tbody>
</table>

Notes: Statistics for the U.S. economy are computed using quarterly HP filtered data for 1964:1 – 2005:4. Standard deviations are relative to output. Values in parenthesis are conditional on money growth shocks only.

The OJS model matches well the volatility of inflation, contrary to the alternative models for which inflation volatility is about twice as high. Correspondingly real wage volatility is much lower in the OJS model than in the alternatives even though it is still more than twice as large as its empirical counterpart. This reflects the volatility of real marginal cost across models, which is almost twice as large in the DMP model than in the OJS model. The volatility of inflation relative to that of real marginal cost is much smaller in the models with search than in the NK model (about 1/5 compared to 1/2), consistent with the intuition that in the former, expected inflation during a boom is subdued by the expected expansion of production capacity from increased matching. The conclusions from comparison between models remain unchanged when the volatilities are generated conditional on money growth shocks alone, although these do not match well the unconditional volatilities of the data.

The real marginal cost does not have an empirical counterpart because it is unobservable in the presence of labor market frictions. Marginal cost can be measured by unit labor cost only if the production technology is Cobb-Douglas and in the absence of any frictions that could break the proportionality between marginal and average labor costs (see e.g. Rotemberg and Woodford, 1999). This proportionality is in effect lost in the presence of search frictions as shown in Section 3.2.6.
In the labor market, on-the-job search results in strongly increased vacancy and unemployment fluctuations in comparison to the DMP model, although the data exhibit still larger fluctuations. Conditional on money growth shocks the labor market volatilities are very similar in the OJS and DMP models, suggesting that productivity shocks affect the channel of employment creation via on-the-job search much more than nominal demand shocks do.

The correlation coefficient between output and inflation is positive but low in the data. In the search models, demand shocks generate similarly low correlations. On the contrary output and inflation are perfectly correlated conditional on nominal demand shocks in the NK model. Adding productivity shocks slightly reduces the correlation coefficients in all models. The first order autocorrelation coefficient of output indicates that persistence is largest in the OJS model and smallest in the NK model.

Contrary to the role of on-the-job search, exogenous wage rigidity barely affects the dynamics of real marginal cost and hence inflation, as emphasized by Krause and Lubik (2007). The reason is that the real wage rigidity is compensated by an increased sensitivity of the value of a long-term employment relationship, because firms can now accrue a larger share of the surplus from such a relationship. That is, imposing wage rigidity affects the distribution rather than the size of the surplus between worker and firm. To verify this conclusion, real wage rigidity is introduced in the DMP model as a variant of the wage norm proposed by Hall (2005), following the analysis of Krause and Lubik (2007). Let wages in each sector be determined according to \( w^j_t = (1 - \delta)w^j_{nt} + \delta \bar{w}^j_t \) for \( j = b, g \), where \( w^j_{nt} \) is a notional wage, \( \bar{w}^j_t \) is a wage norm and \( \delta \in [0,1] \). Assume that the notional wage is computed as the Nash bargaining outcome of the model without wage rigidity, and that the previous period’s wage determines the wage norm for each type of job \( j = b, g \). With a high degree of real wage rigidity, \( \delta = 0.7 \), the relative standard deviation of the real wage is even smaller than in the OJS model, but the volatilities of the remaining variables listed in Table 3.3 are close to those from the DMP model without real wage rigidity. Thus, the exogenous real wage rigidity is not effective in increasing the persistence of the output response to a nominal shock.

### 3.4 Concluding remarks

Allowing workers to search on the job in the presence of labor market search frictions improves the sticky price model in terms of amplification and propagation of monetary shocks, both in comparison to an associated search model without on-the-job search and compared to a model with a competitive labor market. With on-the-job search the contract multiplier and the cumulative output response to a monetary shock are
at least three times larger than in these alternative models. The volatility of inflation corresponds closely to the U.S. data. Shutting down the possibility of on-the-job search retains the qualitative features of output dynamics but shows, consistent with the finding of Krause and Lubik (2007), that this model is quantitatively similar to the case of a competitive labor market.
Chapter 4
Permanent and Transitory Investment-Specific Technology Shocks in the Business Cycle

4.1 Introduction

Does it matter whether technology shocks have long-run effects to understand their role in the business cycle? In the structural vector autoregression (SVAR) literature, technology shocks are often identified as the only source of long-run variation in labor productivity (see e.g. Galí, 1999). Fisher (2006) likewise identifies investment-specific technology shocks using long-run zero restrictions on the relative price of investment and finds that such shocks account for a sizeable share of business fluctuations. Recent studies that use a Bayesian approach to estimate dynamic stochastic general equilibrium (DSGE) models attribute varying portions of short-run output fluctuations to shocks to the marginal efficiency of investment, which can be interpreted as investment-specific technology shocks. These models can provide insight into the propagation mechanisms that account for the relative importance of such shocks. However, in contrast to the SVAR literature, these studies almost invariably take the view that shocks to the marginal efficiency of investment are transitory (see e.g. Smets and Wouters (2003, 2007), de Walque, Smets and Wouters (2006), Justiniano and Primiceri (2008), Justiniano, Primiceri and Tambalotti (2008)).

This paper investigates whether the propagation mechanisms of transitory investment-specific technology shocks that are put forward in the DSGE literature are also plausible as an explanation for the importance of a random walk technology process such as that identified by Fisher (2006). It aims to fill this gap between the SVAR

\[ \text{An exception is Altig, Christiano, Eichenbaum and Linde (2005), who study permanent investment-specific technology shocks, but focus on the interaction between investment and the frequency of price adjustment and do not investigate the importance of such shocks as a source of business fluctuations in their model.} \]
and the DSGE literature by introducing investment-specific technology shocks with permanent output effects in a calibrated DSGE model. The model incorporates the propagation mechanisms that play an important role in the literature, namely a variable capital utilization rate, variable markups, and habit persistence in consumption preferences. Simulation results allow assessing the quantitative significance of these frictions in the propagation of a permanent investment-specific technology shock. They confirm the importance of variable capital utilization and habit formation, but suggest that variable markups are not required for the propagation of such a shock.

The importance of variations in the capital utilization rate as a propagation mechanism of investment-specific technology shocks is first analyzed by Greenwood, Hercowitz and Huffman (1988), and this mechanism is also employed by Greenwood, Hercowitz and Krusell (2000). More recently, Dave and Dressler (2007) and Justiniano, Primiceri and Tambalotti (2008) show that such shocks can account for a large share of output variance in a model with endogenous markup variation arising as a result of nominal rigidities. The latter authors emphasize that a rise in investment-specific technology stimulates work effort, which tends to reduce labor productivity and increase the marginal rate of substitution between consumption and leisure. A reduction in markups can reconcile these opposite movements with labor market equilibrium without necessitating a countercyclical decline in consumption, thus paving the way for transitory investment-specific technology shocks to take center stage as an explanation of the business cycle. We find that markup variation is not required for the propagation of a permanent shock, because a positive shock raises the trend component of consumption and reduces its cyclical component. The trend increase can prevent a sharp countercyclical adjustment of consumption. The reduction of its cyclical component implies that a markup adjustment is not required to maintain labor market equilibrium.

The remainder of the paper is organized as follows. In Section 4.2, a model with permanent and one with transitory investment-specific technology shocks is presented. Section 4.3 discusses impulse response analysis and simulation results. Section 4.4 concludes.

4.2 The model economy

The model features a number of frictions that have been shown helpful in reproducing business fluctuations, such as investment adjustment costs, variable capital

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Neutral technology frequently follows a random walk in the DSGE literature; see e.g. Altig, Christiano, Eichenbaum and Linde (2005), Ireland (2004), Justiniano and Primiceri (2008), Justiniano, Primiceri and Tambalotti (2008).
utilization, habit formation in consumption preferences, and nominal price rigidity. The analysis will concentrate on the role of these frictions in the propagation of permanent investment-specific technology shocks (henceforth, I-shocks). There are four types of agents: households, perfectly competitive final-good firms, monopolistically competitive intermediate-good firms, and a government.

4.2.1 Households

Each of many households is infinitely lived and maximizes expected lifetime utility in consumption $C_t$ and labor $n_t$

$$E_t \sum_{j=0}^{\infty} \beta^j \left[ \ln(C_{t+j} - hC_{t+j-1}) - \varphi \frac{n_{t+j}^{1+\nu}}{1+\nu} \right]$$

where $\nu \geq 0$ is the inverse labor supply elasticity and $h \in [0,1]$ is the degree of habit persistence. A household rents its labor services to firms for a real wage $w_t$, saves via bonds $B_t$ that pay off one unit of consumption at a price $R_t^{-1}$ and via capital $K_t$, which accumulates according to the law of motion

$$K_{t+1} = (1 - \delta)K_t + \left[ 1 - S\left( \frac{I_t}{I_{t-1}} \right) \right] I_t Q_t. \tag{4.1}$$

Here $I_t$ is investment, $\delta$ is the rate of deterioration of the capital stock, $S$ is an investment adjustment cost introduced by Christiano, Eichenbaum and Evans (2005) that satisfies $S(\gamma) = S'(\gamma) = 0$ and $S''(\gamma) = \psi > 0$ on the balanced growth path with output growth rate $\gamma$. The exogenous level of investment-specific technology is denoted by $Q_t$, and its time series process is discussed below. Capital is rented to firms at a real rental price $r_t$, and there are economy-wide factor markets such that the household takes factor prices as given. Moreover, the household receives dividends $D_t$ from firms and pays lump-sum taxes $T_t$ to the government. Thus, each period $t$ it faces the budget constraint

$$P_t [r_t u_t K_t + w_t n_t] + B_{t-1} + D_t = P_t \left[ C_t + I_t + a(u_t) \frac{K_t}{Q_t} \right] + \frac{1}{R_t} B_t + T_t, \tag{4.2}$$

where $P_t$ is the unit price of the consumption good, and $a(u_t)$ is the cost of capital utilization $u_t$ per unit of capital, scaled by the level of investment-specific technology to be consistent with balanced growth.

The first order conditions of the household’s problem pertain to the optimal choice of consumption, bonds, capital utilization, and supply of labor and capital. The household’s intertemporal optimality condition for consumption is given by

$$\frac{1}{R_t} = \beta E_t \left[ \frac{A_{t+1}}{A_t} \frac{P_t}{P_{t+1}} \right],$$

53
where the marginal utility of consumption is
\[
\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t \left[ \frac{1}{C_{t+1} - hC_t} \right].
\]
The labor supply is determined by setting the real wage equal to the marginal rate of substitution between leisure and consumption
\[
\varphi \frac{n_t'}{\Lambda_t} = w_t. \tag{4.3}
\]
The optimality condition for capital relates the real rental price of capital to its user cost, which depends on the real interest rate on bonds, the depreciation rate, and the expected rate of investment-specific technical change, in addition to the capital utilization rate and the investment adjustment cost.
\[
1 = \mu_t \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{Q_t}{Q_{t+1}} \left[ r_{t+1}Q_{t+1}u_{t+1} + \frac{1 - \delta}{\mu_{t+1}} - a(u_{t+1}) \right],
\]
where
\[
\mu_t = 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right) + \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\mu_t}{\mu_{t+1}} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right]
\]
is the shadow value of consumption in terms of investment. The capital utilization rate is chosen to satisfy
\[
r_tQ_t = a'(u_t).
\]

### 4.2.2 Firms

To produce the final good \( Y_t \) at time \( t \), the representative final-good producer uses a continuum of intermediate goods, indexed \( i \in [0, 1] \), according to the production technology
\[
\left[ \int_0^1 Y_t(i) i \right] \frac{\epsilon}{\epsilon - 1} \geq Y_t \tag{4.4}
\]
where \( Y_t(i) \) denotes the quantity of intermediate good \( i \) used in the production of the final good and \( \epsilon > 1 \) determines the elasticity of substitution across intermediate goods. The final-good producing firm chooses \( Y_t(i) \) to maximize profits \( P_tY_t - \int_0^1 P_t(i)Y_t(i)di \) subject to (4.4), were \( P_t(i) \) is the price of intermediate good \( i \). The first order conditions for profit maximization give rise to the demand for good \( i \)
\[
Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t. \tag{4.5}
\]
Moreover, perfect competition in the final goods market implies a price index
\[
P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}.
\]
Intermediate-good producing firm $i$ chooses a sequence of prices and inputs to maximize its real profits in a monopolistically competitive market. The firm each period determines its cost minimizing factor allocation. To that end, it takes account of the constraint imposed by the state of production technology, which is described by a constant-returns-to-scale Cobb-Douglas function

$$Y_t(i) = (Z_t K_t(i))^{1-\alpha} (w_t K_t(i))^\alpha.$$ 

The log-level of labor-augmenting technology $Z_t$ is driven by a random walk with a drift and autoregressive innovations,

$$\ln Z_t = \ln \gamma_z + \ln Z_{t-1} + \hat{z}_t$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{zt}, \quad \varepsilon_{zt} \sim N(0, \sigma_z^2) \quad -1 < \rho_z < 1,$$

We refer to $Z_t$ as neutral technology because it affects the production of consumption goods and investment goods equally. The first order conditions of firm $i$’s problem are given by the production function and the input demand functions

$$w_t = mc_t (1-\alpha) Y_t(i) n_t(i)$$

$$r_t = mc_t \alpha Y_t(i) K_t(i).$$

Here $mc_t$ denotes the real marginal cost of production. With economy-wide, perfectly competitive factor markets, all firms choose identical capital labor ratios and face the same marginal cost

$$mc_t = \frac{1}{Z_t^{1-\alpha}} \left( \frac{w_t}{1-\alpha} \right)^{1-\alpha} \left( \frac{r_t}{\alpha} \right)^\alpha.$$ 

Improvements in neutral technology lower each firm’s real marginal cost. However, with stationary nominal output shares of labor and capital, the real marginal cost is mean-reverting unlike the level of technology. Thus, the real wage and the real rental price display trend adjustment to changes in technology.

Intermediate good firms set the price of their product in order to maximize discounted expected real profits subject to demand from households and subject to Calvo (1983) style price stickiness. Specifically, a fixed fraction $\xi \in (0, 1)$ of randomly chosen firms does not reoptimize price and instead indexes it to lagged inflation as in Christiano, Eichenbaum and Evans (2005),

$$P_t(i) = \pi_t P_{t-1}(i),$$

where, $\pi_t = P_t/P_{t-1}$ is the gross inflation rate. The remaining fraction $1-\xi$ of firms maximizes

$$E_t \sum_{j=0}^{\infty} (\xi \beta)^j \frac{\Lambda_{t+j}}{\Lambda_t} \left[ \frac{P_t(i)}{P_{t+j}} \left( \Pi_{s=0}^{j-1} \pi_{t-1-s} \right) - mc_{t+j} \right] Y_{t+j}(i)$$

subject to the demand (4.5) and marginal cost (4.7).
4.2.3 Government and resource constraint

The monetary authority conducts interest rate policy that adjusts the nominal interest rate to changes in inflation and the output gap with interest rate smoothing according to the rule

\[ R_t = R_{t-1}^{\phi_R} \left[ R \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right]^{1-\phi_R}, \]

where \( R \) and \( \pi \) are the steady state gross nominal interest rate and inflation rate, and \( Y_t/Y_{t-1}^{f} \) is the output gap between the actual and the flexible-price output level.\(^3\)

The government has access to lump-sum taxes and its fiscal policy is Ricardian, so it appropriately accommodates consequences of monetary policy for the government budget constraint. The government expenditure is determined exogenously as a fraction of output

\[ G_t = \left( 1 - \frac{1}{g_t} \right) Y_t, \]

where the government spending shock follows an autoregressive process

\[ \ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{gt}, \quad \varepsilon_{gt} \sim N(0, \sigma_g^2), \quad -1 < \rho_g < 1. \]

The aggregate resource constraint

\[ Y_t = C_t + I_t + G_t + a(u_t) \frac{K_t}{Q_t} \]

is found by combining the households’ and government’s budget constraint.

4.2.4 Permanent and transitory investment-specific technology shocks

We study rational expectations equilibrium of two models that share the structure laid out in the previous subsections and that differ only by the stochastic process of the I-shock. In the model with permanent I-shocks, the log-level of investment-specific technology follows a random walk with first order autoregressive innovations

\[ \ln Q_t = \ln \gamma_q + \ln Q_{t-1} + \hat{q}_t \]
\[ \hat{q}_t = \rho_q \hat{q}_{t-1} + \varepsilon_{qt}, \quad \varepsilon_{qt} \sim N(0, \sigma_q^2), \quad -1 < \rho_q < 1, \]

where \( \gamma_q > 0 \) represents the gross rate of investment-specific technical change on the balanced growth path. Because the neutral technology shock and the I-shock each contain a unit root with a drift, they introduce a stochastic trend in output. Therefore, log-linearization around a balanced growth path requires that a number of variables be detrended. Let gross output growth be denoted as

\[ \gamma_t = \frac{Y_t}{Y_{t-1}}. \]

\(^3\)Each model is expanded with a flexible price version in order to calculate the model-consistent output gap.
From the aggregate resource constraint it follows that along the balanced growth path, output, consumption, investment, government expenditure and the cost of capital utilization all grow at the balanced growth rate $\gamma$. However, equation (4.1) implies that $K_t$ grows at a faster balanced growth rate $\gamma_K = \gamma q$. The production function implies that $\gamma = \gamma_z^{1-\alpha} \gamma_K^\alpha$, from which it follows that $\gamma = \gamma_z^{1-\alpha} q^{\alpha}$. Hence, a unique transformation can be applied to detrend the variables. The resulting stationary variables are referred to as “cyclical” and are written with lower case letters defined as

$$ y_t = \frac{Y_t}{Z_t Q_t^{1-\alpha}}, \quad c_t = \frac{C_t}{Z_t Q_t^{1-\alpha}}, \quad \lambda_t = \Lambda_t Z_t Q_t^{\alpha}, \quad i_t = \frac{I_t}{Z_t Q_t^{1-\alpha}}, \quad k_{t+1} = \frac{K_{t+1}}{Z_t Q_t^{1-\alpha}}. $$

The stochastic trend implies that a positive shock to either type of technology will result in a permanent increase in output, consumption, investment, government expenditure and capital stock.

The second model is distinguished by transitory I-shocks that follow a first order autoregressive process given by

$$ \ln Q_t = \rho Q \ln Q_{t-1} + \varepsilon_{Qt}, \quad \varepsilon_{Qt} \sim N(0, \sigma_Q^2), \quad -1 < \rho Q < 1, \quad (4.9) $$

as in Dave and Dressler (2007), Justiniano, Primiceri and Tambalotti (2008), Smets and Wouters (2007) and other recent studies of DSGE models. Thus, neutral technical change is the only source of long-run growth in output and stochastically detrended variables are given by

$$ y_t = \frac{Y_t}{Z_t}, \quad c_t = \frac{C_t}{Z_t}, \quad \lambda_t = \Lambda_t, \quad i_t = \frac{I_t}{Z_t}, \quad k_{t+1} = \frac{K_{t+1}}{Z_t}. $$

The loglinearized equilibrium conditions of the models are given in the Appendix.

### 4.2.5 Calibration

The quarterly calibration of the two models is identical except for the parameters characterizing the stochastic processes of the technology shocks. The discount factor is equal to 0.99, the degree of habit persistence in consumption is 0.8 as suggested by Fuhrer (2000), the inverse labor supply elasticity is 2, the elasticity of substitution is chosen to yield a gross balanced growth markup of price over marginal cost equal to 20 percent, the degree of nominal price rigidity implies that nominal prices remain unchanged for six quarters on average, the capital depreciation rate is 0.025, the cost share of capital is 0.33, and the investment adjustment cost is 2.5 as in Christiano, Eichenbaum and Evans (2005). Furthermore, the steady state elasticity of the stationary marginal product of capital with respect to the utilization rate is 5, consistent with the estimates of Justiniano, Primiceri and Tambalotti (2008) and Smets...
and Wouters (2007). The balanced growth rate of output is 0.43 percent and the balanced growth ratio of government spending to output is 0.18, taken from Smets and Wouters (2007). The balanced growth rate of investment-specific technical change is 0.75 percent, as computed from the relative price of investment goods discussed below. Monetary policy parameters are set in line with the estimates of Justiniano, Primiceri and Tambalotti (2008), with an interest rate smoothing coefficient of 0.8, an inflation response coefficient equal to 2, and an output gap response coefficient of 0.05. Finally, the parameters of the government spending shock process are taken from Smets and Wouters’ (2007) estimates, and the autoregressive parameter of the neutral technology shock process is consistent with the estimates of Justiniano, Primiceri and Tambalotti (2008).

Table 4.1: Baseline calibration

<table>
<thead>
<tr>
<th>Preferences and technology</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$h$ habit persistence in consumption</td>
<td>0.8</td>
</tr>
<tr>
<td>$\nu$ inverse labor supply elasticity</td>
<td>2</td>
</tr>
<tr>
<td>$\epsilon$ elasticity of substitution</td>
<td>6</td>
</tr>
<tr>
<td>$\xi$ probability of not reoptimizing price</td>
<td>0.8333</td>
</tr>
<tr>
<td>$\delta$ capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\alpha$ cost share of capital services</td>
<td>0.33</td>
</tr>
<tr>
<td>$\psi$ investment adjustment cost</td>
<td>2.5</td>
</tr>
<tr>
<td>$\chi$ elasticity of MPK wrt utilization rate</td>
<td>5</td>
</tr>
<tr>
<td>$\gamma$ gross output growth</td>
<td>1.0043</td>
</tr>
<tr>
<td>$\gamma_q$ gross rate of investment-specific technical change</td>
<td>1.0075</td>
</tr>
<tr>
<td>$s_g$ government spending to output ratio</td>
<td>0.18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Monetary policy and shock processes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_R$ interest rate policy inertia</td>
<td>0.80</td>
</tr>
<tr>
<td>$\phi_\pi$ interest rate policy response to inflation</td>
<td>2</td>
</tr>
<tr>
<td>$\phi_x$ interest rate policy response to output gap</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho_g$ autoregressive coefficient spending shock</td>
<td>0.97</td>
</tr>
<tr>
<td>$\sigma_g$ standard deviation spending shock (percent)</td>
<td>0.53</td>
</tr>
<tr>
<td>$\rho_z$ autoregressive coefficient neutral technology shock</td>
<td>0.20</td>
</tr>
<tr>
<td>$\rho_q$ autoregressive coefficient permanent I-shock</td>
<td>0.43</td>
</tr>
<tr>
<td>$\sigma_q$ standard deviation permanent I-shock (percent)</td>
<td>0.68</td>
</tr>
<tr>
<td>$\rho_Q$ autoregressive coefficient transitory I-shock</td>
<td>0.91</td>
</tr>
<tr>
<td>$\sigma_Q$ standard deviation transitory I-shock (percent)</td>
<td>0.83</td>
</tr>
</tbody>
</table>

The stochastic processes for the I-shocks, (4.8) and (4.9), are parameterized based on the relative price of investment. The inverse of this relative price is a measure of investment-specific technology, because such technology shocks are the only fac-

---

4I thank Jason Cummins for kindly providing the investment price data.
tor affecting the relative investment price in the models. Specifically, the level of investment-specific technology is measured by the ratio of the constant-quality price of consumption and the constant-quality price of investment. The quality-adjusted consumption price index is constructed as a Tornqvist index from the prices of non-durable consumer goods and non-housing services obtained from the National Income and Product Accounts (NIPA). Quality-adjusted prices of private nonresidential investment are measured as a Tornqvist index of the NIPA price index for private non-residential structures and the quality-adjusted equipment price series constructed by Gordon (1990) and extrapolated by Cummins and Violante (2002). The latter is used because the quality change of equipment may be poorly measured in the national accounts. The series covers the period 1955:1–2001:4, and we allow for a break because its growth rate accelerates in the early 1980s, as noted by Fisher (2006).

For the model with permanent I-shocks we have

\[ \Delta \ln Q_t = 0.0051 + \hat{q}_t, \quad t = 1955:1, \ldots, 1979:4 \]
\[ \Delta \ln Q_t = 0.0106 + \hat{q}_t, \quad t = 1984:1, \ldots, 2001:4, \]

and

\[ \hat{q}_t = 0.43\hat{q}_{t-1} + \varepsilon_{qt}, \quad \sigma_q = 0.0068 \tag{4.10} \]

for all \( t \). Augmented Dicky-Fuller and Phillips-Perron tests for a unit root in the \( \ln Q_t \) fail to reject the null hypothesis of nonstationarity, lending support to this specification of the I-shock process. For the model with transitory I-shocks the level series is first detrended and an autoregressive process is fitted to the residuals.

\[ \ln Q_t = -1.45 + 0.0054t + \hat{Q}_t, \quad t = 1955:1, \ldots, 1979:4 \]
\[ \ln Q_t = -0.80 + 0.0111t + \hat{Q}_t, \quad t = 1984:1, \ldots, 2001:4, \]

and

\[ \hat{Q}_t = 0.91\hat{Q}_{t-1} + \varepsilon_{qt}, \quad \sigma_q = 0.0083 \tag{4.11} \]

for all \( t \). The calibration is summarized in Table 4.1.

---

5Greenwood et al. (1997) argue that the presence of investment-specific technical change is suggested by the negative comovement between annual price and quantity data of equipment investment, which would result from an outward shifting supply curve. At a quarterly frequency the detrended equipment price and new equipment investment are also negatively correlated, albeit less.

6This measure of prices avoids accounting for quality improvements in consumer durables. The approach follows Cummins and Violante (2002).

7This is an annual series that smooths business cycle dynamics and is therefore not suitable for use at business cycle frequencies. Following Fisher (2006) the quarterly NIPA equipment price index is therefore used to interpolate this series, employing the interpolation-by-related-series approach proposed by Denton (1971). The Denton method amounts to using information in a higher frequency indicator variable to interpolate a better quality but lower frequency variable.
4.2.6 Propagation of investment-specific technology shocks

In a standard neoclassical model, Barro and King (1984) stress that the increase in the rate of return on investment following a positive I-shock leads to postponed consumption and increased work effort. Thus consumption and labor productivity decline while hours and output rise. Greenwood, Hercowitz and Huffman (1988) point out that variable capital utilization works to increase labor productivity and thus wages, which raises both hours and consumption via a substitution effect. Recently, Justiniano, Primiceri and Tambalotti (2008) emphasize the role of variable markups and habit formation in consumption for the propagation of I-shocks. In the presence of nominal rigidities, an increase in investment spending does not require consumption to be postponed, because firms increase production to meet the rise in investment demand. Thus, hours worked rise and the marginal product of labor tends to fall, but the markup also falls. As a result, the marginal rate of substitution between consumption and leisure (and hence consumption) does not need to decrease in order to maintain equilibrium in the labor market. Habit formation also serves to dampen a decline in the marginal rate of substitution by linking it to past and expected future consumption in addition to present consumption.

Justiniano, Primiceri and Tambalotti find that I-shocks account for the largest share of output variance in a DSGE model with multiple frictions and shocks estimated with Bayesian methods. They also find that endogenous markups due to imperfect competition with nominal rigidities, and to a lesser extent habit formation and variable capital utilization are key to this result. Dave and Dressler (2007) likewise report that I-shocks contribute more to the error variance of output than neutral technology shocks in the presence of nominal price rigidity, whereas they find the opposite result with flexible prices. All these conclusions are based on the study of transitory I-shocks. In the next section we analyze the role of these frictions in the propagation of permanent I-shocks.

4.3 Quantitative analysis

4.3.1 Impulse responses analysis

Figure 4.1 shows the responses to a positive one-standard deviation government spending shock in the model with permanent I-shocks (henceforth the PS model, solid line) and the one with transitory I-shocks (henceforth the TS model, dashed line). These plots are not exactly identical because the balanced growth rate of capital is different, which affects in particular the spending shares of consumption and investment implied by the balanced growth relations, as well as the capital accumulation constraint and the capital optimality condition of the two models. However, output and hours
Figure 4.1: Responses to a government spending shock

NOTE: A full (dashed) line gives the response in the PS (TS) model
temporarily rise while consumption, investment and labor productivity are temporarily reduced in both models. Inflation, the nominal interest rate and the marginal cost rise.

Figure 4.2: Responses to a neutral technology shock

![Figure 4.2: Responses to a neutral technology shock](image)

NOTE: A full (dashed) line gives the response in the PS (TS) model

Figure 4.2 displays the responses to a positive one-percent neutral technology shock in the PS model (the solid line) and the TS model (the dashed line). The plots are not identical for the same reason as with the spending shocks. However, in both models output, consumption, investment and labor productivity rise permanently. Employment, real marginal cost, inflation and the nominal interest rate are reduced.
on impact due to the presence of nominal price rigidity.

Figure 4.3 gives the impulse responses to a positive one-standard deviation I-shock. Output, investment and labor productivity rise permanently in the PS model (the solid line), and rise temporarily in the TS model (the dashed line). Consumption rises only after a decline, which is shallow in response to a transitory I-shock but is larger and more persistent after a permanent I-shock. Hours, the inflation rate and the nominal interest rate rise on impact and display a hump-shaped response to each type of I-shock, whereas the marginal cost does not rise on impact of a permanent I-shock but shows a hump-shaped response in both models. The impulse responses of the TS model are consistent with those reported by Justiniano, Primiceri and Tambalotti (2008) and Smets and Wouters (2007). In particular, the I-shock generates positive comovement between real activity and inflation, akin to a textbook demand shock, because the I-shock triggers a rise in investment spending. Apart from the diverging long run effects and the initial decline of real marginal cost, the impulse responses to a permanent and a transitory I-shock are qualitatively quite similar.

Whereas the responses to a transitory I-shock are entirely cyclical, the responses to a permanent I-shock can be decomposed into a trend component and a cyclical (non-trend) component. Figure 4.4 shows the dynamic responses of the cyclical variables to a permanent I-shock. It reveals a striking divergence with the responses to a transitory I-shock, which are added. The cyclical component of output displays a decline that mirrors the rise in output after a transitory I-shock. That is, output rises slower than its trend after a permanent I-shock lifts the trend growth rate. This decline is explained by the very steep and persistent decrease in cyclical consumption after a permanent I-shock. Cyclical investment decreases briefly before rising in a persistent hump-shaped response. Consistent with the cyclical output response, the cyclical labor productivity shows a long and persistent decrease due to a permanent I-shock, whereas it rises after a temporary shock. The cyclical capital stock also declines strongly and persistently after a permanent I-shock, in contrast to the gradual and persistent build-up of capital after a transitory I-shock. Finally, the growth rate of output responds to a permanent or transitory I-shock in a similar fashion: it rises on impact and gradually returns to the balanced growth rate. However, this masks a decline in cyclical output growth on impact of about the same magnitude.

4.3.2 Simulation results

Table 4.2 reports the average standard deviations of 1000 histories simulated with the PS and the TS model, driven by the three shocks. These numbers can be compared

\footnote{A positive (negative) response to a permanent I-shock means that the variable adjusts faster (slower) to the shock than its trend. The figure does not repeat the responses of the stationary variables, hours, the inflation rate, the interest rate, and the marginal cost.}
Figure 4.3: Responses to an investment-specific technology shock

NOTE: A full (dashed) line gives the response to a permanent (transitory) I-shock
Figure 4.4: Responses to an investment-specific technology shock

NOTE: A full (dashed) line gives the response to a permanent (transitory) I-shock
Table 4.2: Simulation results: U.S. economy and model economies

<table>
<thead>
<tr>
<th></th>
<th>U.S. economy</th>
<th>PS model</th>
<th>TS model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All shocks</td>
<td>(I-shocks)</td>
<td>All shocks</td>
</tr>
<tr>
<td><strong>Standard deviations (percent)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output growth</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>0.61</td>
<td>0.37</td>
<td>0.33</td>
</tr>
<tr>
<td>Investment growth</td>
<td>3.50</td>
<td>1.21</td>
<td>1.34</td>
</tr>
<tr>
<td>Hours</td>
<td>4.00</td>
<td>1.46</td>
<td>1.33</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.59</td>
<td>0.20</td>
<td>0.18</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>0.82</td>
<td>0.33</td>
<td>0.29</td>
</tr>
<tr>
<td><strong>Relative standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output growth</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>0.64</td>
<td>0.39</td>
<td>0.35</td>
</tr>
<tr>
<td>Investment growth</td>
<td>3.71</td>
<td>1.29</td>
<td>1.43</td>
</tr>
<tr>
<td>Hours</td>
<td>4.24</td>
<td>1.56</td>
<td>1.42</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.63</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>0.87</td>
<td>0.35</td>
<td>0.31</td>
</tr>
<tr>
<td><strong>Correlation output growth and consumption growth</strong></td>
<td>0.53</td>
<td>0.57</td>
<td>(-0.64)</td>
</tr>
</tbody>
</table>

NOTE: Data for the U.S. economy span the period 1955:1–2007:4. Output is measured as GDP divided by population, consumption is personal consumption of nondurable goods and services divided by population, investment is gross private domestic investment plus personal consumption of durable goods divided by population, hours is the log of hours of all persons in the nonfarm business sector divided by population, population is the civilian noninstitutional population 16 years and older, inflation is the percent change in the GDP deflator, and the nominal interest rate is the effective Fed Funds rate.
to similar statistics computed from quarterly U.S. data, which are displayed in the first column of the table. The standard deviation of neutral technology shocks is chosen such that the baseline models’ predictions match the standard deviation of U.S. GDP growth, which is 0.94 percent. Thus, the models cannot be evaluated along this dimension. The standard deviation of technology is consequently set to $\sigma_z = 1.38$ and $\sigma_z = 1.39$ percent respectively in the PS and the TS model.

The two models yield quantitatively similar results. Compared to the U.S. data, they both underpredict the volatilities, especially those of investment growth and hours worked. However, looking at the middle panel of the table, the relative standard deviations of investment growth and hours conditional on I-shocks are much closer to their empirical counterparts. In particular, transitory I-shocks generate large volatility in investment growth, which is consistent with the relative standard deviations reported by Justiniano, Primiceri and Tambalotti (2008). Interestingly, fluctuations in investment growth are much smaller when they are driven by permanent I-shocks. On the other hand, such shocks result in relatively strong fluctuations in hours. The bottom panel gives the correlation between output growth and consumption growth. Both models generate positive comovement between these growth rates, despite the negative comovement induced by the I-shocks. In the absence of the propagation mechanisms that were outlined above, this conditional correlation would be more negative still.

Table 4.3: Relative importance of investment-specific technology shocks for output growth: the role of frictions

<table>
<thead>
<tr>
<th></th>
<th>PS model</th>
<th>TS model</th>
<th>TS model – No scaling $a(u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{\text{sd}(\gamma</td>
<td>q)}{\text{sd}(\gamma)}$</td>
<td>$\frac{\text{sd}(\gamma</td>
</tr>
<tr>
<td>1 Baseline</td>
<td>0.20 (1.00)</td>
<td>0.20 (1.00)</td>
<td>0.14 (1.00)</td>
</tr>
<tr>
<td>2 Perfect competition ($\gamma_f^r, \varepsilon = 1,000$)</td>
<td>0.20 (0.97)</td>
<td>0.15 (0.74)</td>
<td>0.08 (0.61)</td>
</tr>
<tr>
<td>3 No habit persistence ($h = 0.001$)</td>
<td>0.15 (0.76)</td>
<td>0.15 (0.72)</td>
<td>0.10 (0.72)</td>
</tr>
<tr>
<td>4 No var. capital util. ($\chi = 1,000$)</td>
<td>0.14 (0.68)</td>
<td>0.13 (0.63)</td>
<td>0.13 (0.91)</td>
</tr>
<tr>
<td>5 2,3, and 4</td>
<td>0.12 (0.58)</td>
<td>0.07 (0.34)</td>
<td>0.07 (0.49)</td>
</tr>
</tbody>
</table>

NOTE: For each model, the first column gives the ratio of the standard deviation of output growth conditional on I-shocks and the unconditional standard deviation of output growth. The second column (in parenthesis) expresses that ratio as a percent of the baseline model.
Table 4.3 evaluates the proportion of the standard deviation of output growth that is explained by I-shocks in the two models. In the baseline models on line 1, permanent and transitory I-shocks account for $0.19/0.94 = 20$ percent of output growth fluctuations. This proportion is smaller than the corresponding one reported by Justiniano, Primiceri and Tambalotti (2008), who conclude that I-shocks account for more than 50 percent of the variance of the level and the growth rate of output. The discrepancy can be traced back to the variance of the I-shock process, which is much larger in their estimated model than that implied by the relative price of investment, as they point out. Line 2 of the table shows how the relative importance of I-shocks is affected by shutting down the nominal price rigidity and the monopolistic competition. This is the ratio of conditional and unconditional standard deviations of output growth in the flexible price economy where markups are almost zero. Strikingly, the absence of nominal rigidity and imperfect competition has only a very small influence on the role of permanent I-shocks, while it reduces the relative importance of transitory I-shocks by more than 25 percent. Habit formation in consumption is an important propagation channel of I-shocks in both models, although slightly more so for transitory I-shocks, as shown on line 3. Line 4 shows that variable capital utilization remains an important propagation mechanism in both models. The last line illustrates how much the propagation of I-shocks is weakened in the absence of imperfect competition and nominal rigidity, habit formation, and variable capital utilization.

The importance of endogenous markups for the transmission of transitory shocks is even more prominent if the cost of capital utilization, $a(w_t)K_t$ in equation (4.2), is not scaled by the level of investment-specific technology. In that case, the absence of nominal price rigidity and imperfect competition reduces the role of I-shocks by almost 40 percent, as shown in the last two columns of the table. The endogenous markup adjustment now forms the foremost propagation mechanism of a transitory I-shock, in line with the finding of Justiniano, Primiceri and Tambalotti (2008).

Why is the endogenous variation of markups almost irrelevant for the propagation of a permanent I-shock? We obtain the pure random walk specification for investment-specific technology by setting the trend growth $\gamma_q = 1$ and the autoregressive coefficient of the rate of technical change $\rho_q = 0$ in the model with permanent I-shocks.

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9 Justiniano, Primiceri and Tambalotti also estimate a model with permanent I-shocks as a robustness check to their baseline model with transitory I-shocks and find that the permanent shocks account for a slightly larger share of output variance. However, they do not provide further analysis of the model with permanent I-shocks.

10 The result is almost the same when the ratio of standard deviations of output growth is considered at a very small degree of price rigidity (i.e. $\xi = .001$).

11 As noted in Section 4.2, this scaling is required only to ensure consistency with balanced growth in the case of permanent I-shocks. Omitting the scaling corresponds to the setup of Justiniano, Primiceri and Tambalotti.
I-shocks. Similarly, the random walk is approximated by setting $\rho_Q = 0.99$ in the model with transitory I-shocks. Under these assumptions, the absence of imperfect competition and nominal rigidity reduces the relative importance of permanent and transitory investment-specific technology shocks for output growth variation by 12 percent and 13 percent respectively, compared to the reduction of 3 percent and 26 percent in the baseline models as reported in Table 3. Thus, larger persistence in the level of investment-specific technology explains about half of the difference between the results generated by permanent and transitory I-shock. The other half is explained by the persistence in the rate of change of investment-specific technology.

Intuitively, a positive permanent shock exogenously raises trend output and trend consumption. The equilibrium dynamics of the cyclical components thus determine the dynamic responses of output and consumption. Equations (4.3) and (4.6) imply that

$$\frac{\phi n^\nu}{\lambda t} = mc_t(1 - \alpha) \frac{y_t}{m_t}, \quad (4.12)$$

is satisfied in labor market equilibrium. The rise in hours tends to reduce the cyclical marginal product of labor and increase the cyclical marginal rate of substitution in (4.12) (with finite labor supply elasticity). But the decline in cyclical consumption raises the cyclical marginal utility of consumption and thus allows the cyclical marginal rate of substitution to decrease even as hours rises. These cyclical dynamics are consistent with a rise in total consumption and labor productivity driven by the acceleration in trend growth. As a result, variations in the markup are not required to obtain procyclical consumption while maintaining labor market equilibrium after a permanent I-shock. This contrasts with the case of transitory I-shocks analyzed by Justiniano, Primiceri and Tambalotti (2008). Because the increase in hours raises the marginal rate of substitution and reduces the marginal product of labor, there either the markup must decrease, or consumption must decrease.

### 4.3.3 Robustness to the labor supply elasticity

The above analysis leads to the prediction that a more (less) elastic labor supply entails a smaller (larger) role for endogenous markups in the propagation of I-shocks, because a change in employment will move the marginal rate of substitution less (more) in the opposite direction of the marginal product of labor. To verify this we set $\nu = 5$, corresponding to a low labor supply elasticity that is consistent with estimates based on microeconomic data.\(^{12}\) If conditional on permanent I-shocks, the ratio of the conditional and unconditional standard deviation of output growth is $0.15/0.90 = 0.17$ in the baseline PS model and this drops to $0.14/1.03 = 0.14$ in the

\(^{12}\)Card (1994) surveys microeconomic studies of intertemporal labor supply and finds that the labor supply elasticity is probably no higher than 0.2.
associated flexible price model with perfect competition (an 18 percent decline). If conditional on transitory I-shocks, the same ratio is \(0.17/0.90 = 0.19\) in the baseline TS model and only \(0.12/1.03 = 0.12\) under perfect competition and flexible prices (a 38 percent decline). Thus, the markup plays a larger role in the transmission of both permanent and transitory I-shocks, but its role remains much more important in the case of the transitory shocks.

In the limiting case of a perfectly elastic labor supply \((\nu = 0)\), the permanent I-shocks generate a ratio of conditional and unconditional standard deviations of output growth equal to \(0.36/1.13 = 0.32\) with nominal price rigidity and \(0.41/1.24 = 0.33\) under perfect competition and price flexibility (a 4 percent increase). The decline in cyclical consumption in response to the shock is mild, such that consumption rises on impact of the shock. With the transitory I-shocks, these ratios are \(0.32/1.12 = 0.29\) and \(0.34/1.23 = 0.28\) respectively (a 3 percent decline). Thus, now the role of the markup in the propagation of each type of I-shocks is diminished.

### 4.4 Concluding remarks

Whether investment-specific technology shocks have long-run effects has implications for our understanding of their role in the business cycle. Specifically, I find that permanent investment-specific technology shocks are propagated by variable capital utilization and habit formation in consumption preferences, but price markups are not important for their propagation. Because the assumption about the stochastic process of technology matters, it would be interesting to further explore implications of alternative specifications of technical change. For instance, Lippi and Reichlin (1994) specify the trend component of output as a higher order autoregressive-moving average structure on the innovations to reflect gradual diffusion of technical improvements. Analysis of the transmission mechanisms of an investment-specific technology process specified along these lines is left for future research.
Chapter 5

Investment, Interest Rate Policy, and Equilibrium Stability

5.1 Introduction

This essay is based on Kurozumi and Van Zandweghe (2008).

Since the time of Keynes and Hicks, macroeconomics has stressed the importance of investment dynamics in business fluctuations. In line with this view, recent analyses show that investment activity induces critical implications for forward-looking monetary policy. In the face of the widespread belief that the Taylor principle (i.e. active policy) is an essential condition for equilibrium determinacy, Dupor (2001) finds that in a continuous time model with investment and sticky prices, local determinacy is ensured by passive policy that sets the interest rate in response only to instantaneous inflation, whose discrete time counterpart is future inflation.1 In an associated discrete time model, Carlstrom and Fuerst (2005) (henceforth, CF) show that indeterminacy of equilibrium is induced by forward-looking policy that adjusts the interest rate in response only to future inflation, which is in stark contrast with another result of CF that determinacy is guaranteed by active current-looking policy that responds only to current inflation.2 This indeterminacy problem is also pointed out by Huang and Meng (2007a), although they find that the problem is less severe when the cost share of capital decreases, when the price stickiness or the steady state price markup or inflation rate increases, or when prices are modeled as predetermined variables rather than as non-predetermined ones.3

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1A similar result is obtained by Xiao (2008), who uses a discrete time model with an increasing returns to scale production technology.

2Sveen and Weinke (2005) find that the active current-looking policy is more likely to induce indeterminacy as prices become stickier. Under the current-looking policy, determinacy depends on the effects of a cost channel of monetary policy relative to an aggregate demand channel, as mentioned later. Benhabib and Eusepi (2005) provide an analysis of global determinacy under the current-looking policy.

3While CF use a Calvo (1983) style sticky price model, Huang and Meng (2007a) employ a quadratic price adjustment cost model.
In this chapter we address the question of what prescription for the forward-looking monetary policy overcomes the indeterminacy problem, using a stochastic version of CF’s model. This issue is critical because central banks, inflation-targeting ones in particular, are concerned about expected future inflation rather than actual inflation, as also emphasized by Huang and Meng (2007a). We examine the following two prescriptions. One is whether the problem can be ameliorated if the forward-looking policy adjusts the interest rate in response also to output or contains interest rate smoothing, as empirical studies such as Clarida et al. (2000) and Orphanides (2004) use for a better description of actual monetary policy. Another prescription is: when we adopt E-stability as the criterion for selecting one rational expectations equilibrium (REE) from multiple such equilibria, does the forward-looking policy generate a locally-unique E-stable fundamental REE? As Evans and Honkapohja (2001) show in a broad class of linear stochastic models, if a fundamental REE is E-stable and non-explosive, it is least-squares learnable, i.e. stable under least-squares learning. Therefore, E-stability is an essential condition for any REE to be regarded as plausible, as stressed by McCallum (2003).

As for the first prescription, we show that the indeterminacy problem remains when the forward-looking policy sets the interest rate in response also to expected future output. By contrast, we find that the problem can be overcome if the policy responds to current output or contains sufficiently strong interest rate smoothing. This provides a qualification of CF’s conjecture that “including output in the Taylor rule would have only minor effects on the local determinacy conditions” (footnote 7).

Before presenting an intuition for our result, we consider what makes the forward-looking policy induce the indeterminacy problem. As its cause, CF focus attention on households’ arbitrage activity in bond and capital markets, while Huang and Meng

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4 This model assumes the presence of a competitive rental market for capital. Sveen and Weinke (2005) study firm-specific capital and show that a sticky price model with such capital is equivalent in terms of local equilibrium dynamics to an associated rental-capital-market model with a higher degree of price stickiness.

5 Sveen and Weinke (2005) and Benhabib and Eusepi (2005) find that the current-looking policy ensures determinacy with a wider range of model parameters when it responds also to current output.

6 Throughout the chapter, “fundamental” refers to Evans and Honkapohja’s (2001) minimal state variable (MSV) solutions to linear RE models so as to distinguish them from McCallum’s (1983) original MSV solution. We do not examine E-stability of non-fundamental REE such as sunspot equilibria, which may exist in cases of indeterminacy. For E-stability analysis of these REE, see e.g. Honkapohja and Mitra (2004), Carlstrom and Fuerst (2004), and Evans and McGough (2005), who all use associated models without investment. See also footnote 13. We leave E-stability analysis of non-fundamental REE in our model for future work.

7 McCallum argues that in cases of indeterminacy there may be a unique non-explosive REE that is E-stable and thus least-squares learnable, whereas a determinate REE that is E-unstable and thus not least-squares learnable is arguably not a plausible candidate for equilibrium that could be observed in the actual economy.

8 Xiao (2008) shows that in an associated model with a finite labor supply elasticity and a capital adjustment cost, a mild policy response to expected future output can ameliorate the indeterminacy problem.
stress firms’ price setting behavior in monopolistically competitive good markets. Our position is that both of these two are critical to the indeterminacy problem. Any passive forward-looking policy of course induces indeterminacy, and so does even an active policy due to a cost channel of monetary policy, whereby inflation expectations become self-fulfilling. To see this, consider a sunspot increase in inflation expectations. The active policy then leads to a rise in the real interest rate, so that the expected future real rental price of capital increases via a no-arbitrage condition between bonds and capital. This raises expected future real marginal cost and hence expected future inflation via an aggregate price adjustment equation. Consequently, the inflationary expectations become self-fulfilling and therefore indeterminacy is induced.

With sufficiently strong interest rate smoothing, the active forward-looking policy brings about determinacy. Interest rate smoothing means a policy response to the lagged interest rate and hence makes the forward-looking policy respond also to current and past inflation, so that it guarantees determinacy similarly to the current-looking policy examined by CF. The policy response to current output ameliorates the indeterminacy problem dramatically as long as the policy is active, or more accurately, it satisfies the long-run version of the Taylor principle: in the long run the nominal interest rate should be raised by more than the increase in inflation. This is because the policy responses to both current consumption and investment subdue any change in the real interest rate stemming from inflation expectations: a rise (decline) in the real interest rate decreases (increases) consumption and investment, both of which reduce the real rate rise (decline) by the policy responses to them. These two types of feedback on policy are absent from the policy response to expected future output, so that the indeterminacy problem remains. We also show that the feedback from current investment rather than consumption is crucial to determinacy, since the latter feedback on policy is limited due to consumption smoothing. This demonstrates that

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9 This cost channel is similar to that in the existing literature such as Christiano and Eichenbaum (1992) and Barth and Ramey (2001) in that a rise in the interest rate increases firms’ marginal cost. The difference crucial for equilibrium determinacy is that the real interest rate affects expected future real marginal cost in our cost channel, while the nominal interest rate affects current real marginal cost in the literature.

10 As prices become stickier, the real marginal cost elasticity of inflation decreases, which slightly mitigates the effect of the cost channel and hence the indeterminacy problem. This is in stark contrast with Sveen and Weinke (2005), who obtain the exactly opposite result under the current-looking policy as noted in footnote 2.

11 Kurozumi and Van Zandweghe (2006) obtain a necessary and sufficient condition for determinacy under monetary policy that sets the interest rate in response to a weighted average of future and current inflation in CF’s model, and show that determinacy is more likely with a higher weight on current inflation. They also find that determinacy is likely under interest rate policy that responds only to past inflation.

12 As Woodford (2003), Bullard and Mitra (2002), and Kurozumi (2006) show with associated models without investment, the long-run version of the Taylor principle is an essential condition for Taylor style interest rate policy to ensure determinacy and E-stability of REE. If the policy responds only to inflation, the long-run version is consistent with the usual Taylor principle.
investment dynamics, which have been widely viewed as an important determinant of business fluctuations, are likewise of crucial importance in generating determinacy of REE, suggesting that central banks pay special attention to investment activity.

When we consider our second prescription for the indeterminacy problem, i.e. we adopt E-stability as the criterion for selection from multiple REE, we find that even the forward-looking policy generates a locally-unique non-explosive E-stable fundamental REE if its inflation coefficient lies in either of the following two intervals, both of which satisfy the Taylor principle. One interval is extremely narrow, in which the inflation coefficient exceeds one and is very close to one. This contains all the inflation coefficients that bring about determinacy of REE. Another interval requires that the inflation coefficient be sufficiently greater than one and its lower bound increase with stickier prices. Any inflation coefficient in these two intervals succeeds in guiding temporary equilibria under non-rational expectations toward the unique E-stable REE. Further, if the forward-looking policy adjusts the interest rate in response also to current output, almost every pair of the inflation and output coefficients that meets the long-run version of the Taylor principle generates the unique E-stable REE. Therefore, the indeterminacy problem is not critical from the perspective of E-stability or least-squares learnability of fundamental REE. Our E-stability result is a generalization of Bullard and Mitra (2002), who use an associated model without investment to show that the forward-looking policy yields the unique E-stable REE if and only if it meets the Taylor principle. In the absence of investment activity, monetary policy contains only an aggregate demand channel, whereby any active policy can successfully guide temporarily non-rational expectations toward the rational expectations. In the presence of investment activity, the cost channel emerges and reduces the guiding effect of the demand channel. As a consequence, all non-explosive fundamental REE fail to be E-stable if the inflation coefficient lies in the intermediate interval, if any, between the two intervals of inflation coefficients that generate the unique E-stable REE.

The remainder of the chapter proceeds as follows. Section 5.2 presents a stochastic version of CF’s model. Section 5.3 examines our first prescription for the indeterminacy problem induced by the forward-looking policy. Section 5.4 investigates the second one. Finally, Section 5.5 concludes.

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13The indeterminacy problem may not be critical even if we extend our analysis to non-fundamental REE. Carlstrom and Fuerst (2004) show with an associated model without investment that a sunspot equilibrium is E-stable only if a central bank believes in the sunspot. Clearly, this condition is not practical.
5.2 A stochastic version of Carlstrom and Fuerst’s model

We use the same model as CF except in the following two respects. The utility function is assumed to contain uncertain disturbances $\xi_t$ and to be separable between consumption $C_t$ and real money balances $M_{t+1}/P_t$,\textsuperscript{14} where $M_{t+1}$ is nominal balances held at the end of period $t$ and $P_t$ is the price level. The period utility function with leisure $1 - L_t$ then takes the form

$$U(C_t, M_{t+1}/P_t, 1 - L_t; \xi_t) = V(C_t; \xi_t) + W(M_{t+1}/P_t; \xi_t) - L_t.$$  

Another difference from CF is the specification of monetary policy. CF study a forward-looking policy that sets the nominal interest rate $R_t$ in response only to expected future inflation $E_t \pi_{t+1}$. We generalize this policy so that it responds also to current output $Y_t$ or expected future output $E_t Y_{t+1}$ or contains interest rate smoothing,

$$R_t = (R_{t-1})^{\phi_R} \left[ R \left( \frac{E_t \pi_{t+1}}{\pi} \right)^{\phi_\pi} \left( \frac{E_t Y_{t+j}}{Y} \right)^{\phi_Y} \right]^{1-\phi_R}, \ j \in \{0, 1\}, \ \phi_\pi, \phi_Y \geq 0, \ 0 \leq \phi_R < 1,$$

where $E_t$ is the rational expectation operator conditional on information available in period $t$ and $R$, $\pi$ and $Y$ denote steady state values of the interest rate, inflation and output. This generalization is motivated by empirical studies such as Clarida et al. (2000) and Orphanides (2004), who use it for a better description of actual monetary policy.

The equilibrium conditions log-linearized around a steady state are given by\textsuperscript{15}

$$\begin{align*}
\dot{\pi}_t - E_t \tilde{\pi}_{t+1} &= -\sigma^{-1}[(\dot{C}_t - g_t) - (E_t \dot{C}_{t+1} - E_t g_{t+1})], \\
\dot{Y}_t &= s_c \dot{C}_t + s_I \dot{I}_t, \\
\dot{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \lambda \dot{\pi}_t, \\
\dot{R}_t &= \phi_R \tilde{R}_{t-1} + (1 - \phi_R)(\phi_\pi E_t \tilde{\pi}_{t+1} + \phi_Y E_t \dot{Y}_{t+j}), \ j \in \{0, 1\}.
\end{align*}$$

\textsuperscript{14}This separability assumption implies that our results can also be obtained with an associated cashless economy model. If the utility functions are non-separable between consumption and real money balances as in CF, a higher degree of the non-separability makes equilibrium indeterminacy more likely under monetary policy that sets the interest rate in response not only to inflation but also to output, as Kurozumi (2006) shows in an associated model without investment.

\textsuperscript{15}We omit an equilibrium condition for money balances, since the remaining conditions determine local dynamics of REE. We also assume as in CF that fiscal policy is “Ricardian”, i.e. it appropriately accommodates consequences of monetary policy for the government budget constraint. We thus leave hidden the government budget constraint and fiscal policy. For analysis of equilibrium indeterminacy under interest rate policy and non-Ricardian fiscal policy, see e.g. Benhabib et al. (2001), Benhabib and Eusepi (2005), Linnemann (2006), and Kurozumi (2005).
Eq. (5.2) is the Euler equation for households’ optimal consumption decisions with an intertemporal substitution elasticity $\sigma > 0$ and preference shocks $g_t$, which are assumed to follow a stationary first order autoregressive process with a parameter $|\rho| < 1$ and a white noise $\varepsilon_t$

$$g_t = \rho g_{t-1} + \varepsilon_t. \quad (5.9)$$

Eq. (5.2) presents the Fisher relation between the nominal interest rate, expected future inflation, and the real interest rate. Eq. (5.3) is the no-arbitrage condition between bonds and capital, where $z_t$ is firms’ real marginal cost, $K_{t+1}$ is the capital stock at the beginning of period $t + 1$, $\beta \in (0, 1)$ is a discount factor, and $\delta \in (0, 1)$ is the depreciation rate of capital. The right-hand side of (5.3) can be derived from firms’ cost minimization problem, which implies that the real rental price of capital $r_t$ satisfies $r_t = \alpha z_t Y_t / K_t$ in the presence of a competitive rental capital market and a Cobb-Douglas production technology $Y_t = K_t^\alpha L_t^{1-\alpha}$ with a cost share of capital $\alpha \in (0, 1)$. It also implies that the real wage rate $w_t$ satisfies $w_t = (1 - \alpha) z_t (K_t / Y_t)^{\alpha/(1-\alpha)}$, and thus (5.4) is the labor market condition that matches the wage rate to the marginal rate of substitution between consumption and leisure, where we assume as in CF that the labor supply elasticity is an infinity. Eq. (5.5) describes capital accumulation and (5.6) is the resource constraint with steady state output shares of consumption and investment $s_C, s_I \in (0, 1)$. Eq. (5.7) describes Calvo (1983) style staggered price setting of monopolistically competitive firms with indexation to steady state inflation, where the so-called Calvo parameter $\nu \in (0, 1)$ (i.e. the probability of not optimally setting prices) gives rise to the real marginal cost elasticity of inflation $\lambda = (1 - \nu)(1 - \beta \nu)/\nu > 0$.

Here, it is important to stress that in the system of (5.2)–(5.9) there are two channels of monetary policy, which yield exactly opposite effects on inflation. One is the conventional aggregate demand channel, where Euler equation (5.2) leads a rise in the real interest rate to dampen consumption and hence output, both of which lower real marginal cost $\hat{z}_t$ via labor market condition (5.4), thereby reducing current inflation via Phillips curve (5.7). Another is a cost channel, which is one of the main points of this chapter. No-arbitrage condition (5.3) makes a rise in the real interest rate increase the expected future real rental price of capital $E_t \hat{r}_{t+1}$, which is matched to the expected future marginal product of capital adjusted by expected future real marginal cost, $E_t \hat{Y}_{t+1} - \hat{K}_{t+1} + E_t \hat{z}_{t+1}$, in equilibrium. From (5.4) we have

$$E_t \hat{z}_{t+1} = \alpha (E_t \hat{Y}_{t+1} - \hat{K}_{t+1} + E_t \hat{z}_{t+1}) + \sigma^{-1}(1 - \alpha) E_t [\hat{C}_{t+1} - g_{t+1}].$$

Thus, such an increase in the marginal product of capital raises expected future real marginal cost $E_t \hat{z}_{t+1}$, thereby increasing expected future inflation and hence
current inflation via Phillips curve (5.7). Hence, by this cost channel a rise in the real interest rate increases expected future inflation. This induces a possibility that inflation expectations become self-fulfilling and therefore indeterminacy of REE is induced if monetary policy sets the interest rate in response only to expected future inflation.\footnote{With no rental market, firms accumulate capital and \( r_t \), which denotes the real rental price of capital in the presence of a rental market, represents an average reduction in firms’ labor costs due to an additional unit of capital in place in the next period, as Woodford (2003) indicates. Although it differs from the adjusted marginal product of capital, firms’ investment decisions still have counteracting effects on current and expected future real marginal cost. Therefore, the cost channel exists with firm-specific capital.}

\begin{table}[h]
\centering
\caption{Baseline calibration}
\begin{tabular}{ll}
\hline
\( \beta \) & discount factor \hspace{1cm} 0.99 \\
\( \sigma \) & intertemporal substitution elasticity of consumption \hspace{1cm} 1 \\
\( \lambda \) & real marginal cost elasticity of inflation \hspace{1cm} 1/3 \\
\( \alpha \) & cost share of capital \hspace{1cm} 1/3 \\
\( \delta \) & depreciation rate of capital \hspace{1cm} 0.02 \\
\( s_C \) & steady state output share of consumption \hspace{1cm} 0.7 \\
\( s_I \) & steady state output share of investment \hspace{1cm} 0.3 \\
\( \rho \) & autoregression parameter for preference shocks \hspace{1cm} 0.35 \\
\hline
\end{tabular}
\end{table}

The ensuing analysis uses realistic calibrations of model parameters to illustrate conditions for determinacy and E-stability of REE. Table 5.1 summarizes our baseline calibration. These parameter values are taken from CF so that our results are comparable with theirs. Note that under the baseline calibration the Calvo parameter takes a value of \( \nu = 0.57 \), so that firms reset optimal prices of their products, on average, once every 2.3 quarters. As noted by Sveen and Weinke (2005) and Benhabib and Eusepi (2005), the actual value of \( \nu \) is controversial in the empirical literature. Thus we also examine two alternative values, \( \nu = 0.67, 0.80 \), which imply respectively that \( \lambda = 0.18, 0.052 \) and firms reset optimal prices, on average, once every three or five quarters.\footnote{Because of the limited space, we omit to present sensitivity analysis of the other parameters. The qualitative properties of results obtained with the baseline calibration survive in the sensitivity analysis, but of course, the results differ quantitatively.}

5.3 First prescription for the indeterminacy problem

The forward-looking policy, which responds only to expected future inflation, renders REE indeterminate in the model presented above, as shown by CF. In this section we examine our first prescription for this indeterminacy problem: can a policy response to output or interest rate smoothing overcome the problem?
5.3.1 Policy response to expected future output

We first analyze the policy response to expected future output, i.e. \( j = 1, \phi_R = 0 \) in (5.1). With this policy specification, the system of (5.2)–(5.9) can be reduced to a system of the form
\[
E_t x_{t+1} = Ax_t + B g_t, \tag{5.10}
\]
where \( x_0 = [\hat{\pi}_t \hat{C}_t \hat{Y}_t \hat{K}_t \hat{R}_{t-1}]' \) and the coefficient matrix \( A \) is given in Appendix D.1.\(^{18}\)

In this system the first three variables, \( \hat{\pi}_t, \hat{C}_t, \hat{Y}_t \), are non-predetermined while the remaining two, \( \hat{K}_t, \hat{R}_{t-1} \), are predetermined. Hence, Proposition 1 of Blanchard and Kahn (1980) implies that the forward-looking policy with responses to expected future output generates determinacy of REE if and only if the coefficient matrix \( A \) has exactly two eigenvalues inside the unit circle and the other three outside the unit circle.\(^{19}\)

We thus obtain the following result.

**Proposition 6** Suppose that \( b_3 = a_2 - (1 - a_1) \phi_Y \neq 0 \), where \( a_1 = 1 - \beta(1 - \delta)(1 - \alpha) \) and \( a_2 = 1 - \beta(1 - \delta) \). Then, if the forward-looking policy adjusts the interest rate in response also to expected future output, i.e. \( j = 1, \phi_R = 0 \) in (5.1), it generates local determinacy of REE if and only if either of the following two cases is satisfied:

(Case I)
\[
\phi_Y < \frac{1 + \frac{a_2}{\alpha \lambda} - \frac{s_i(1 - a_i)}{\alpha \beta \lambda [s_i + \delta(1 - s_j)]} \phi_Y}{\phi_Y + \frac{(1 - \beta)[s_i \sigma + s_j(1 - \alpha)]}{\lambda(1 - s_j)(1 - \alpha)} > 1,} \tag{5.11}
\]
\[
\phi_Y < \frac{\lambda(a_1 + \alpha)[2s_i + \delta(1 - s_j)]}{(1 + \beta)[s_i \sigma a_2 + s_j(2 - \delta)(2 - \alpha - a_1)]} \left[ 1 + \frac{2a_2(1 + \beta)}{\lambda(a_1 + \alpha)} - \phi_Y \right] \tag{5.12}
\]
\[
b_0^2 - b_3^2 > |b_0 b_2 - b_1 b_3|, \tag{5.13}
\]

where \( b_i, i = 0, 1, 2, \) are given in Appendix D.2.

(Case II) (5.14) and the three strict inequalities opposite to (5.11)–(5.13) hold.

**Proof** See Appendix D.2. ■

This proposition confirms CF’s result that “the presence of capital makes determinacy essentially impossible” (p. 10) in the case of no policy response to output. While CF present only a necessary condition for determinacy, the following corollary provides a necessary and sufficient condition. Note that Huang and Meng (2007a) also present a necessary and sufficient condition in an associated model with a quadratic price adjustment cost, which yields other features of equilibrium determinacy in sticky

\(^{18}\)The form of the vector \( B \) is omitted since it is not needed in what follows.

\(^{19}\)To be precise, this condition is sufficient for determinacy but only generically necessary. Throughout the chapter, consideration of non-generic boundary cases is omitted.
price models with investment, e.g. indeterminacy is less likely when the steady state price markup or inflation rate increases or when prices are modeled as predetermined variables rather than as non-predetermined ones.

**Corollary 1** The forward-looking policy, i.e. \( \phi_Y = \phi_R = 0 \) in (5.1), brings about local determinacy of REE if and only if its inflation coefficient \( \phi_\pi \) satisfies

\[
1 < \phi_\pi < 1 + \frac{a_2}{\lambda} \min \left\{ \frac{1 - \beta}{\alpha}, \frac{2(1 + \beta)}{a_1 + \alpha} \right\}. 
\]

(5.15)

This interval is extremely narrow, e.g. \( 1 < \phi_\pi < 1.0027 \) under the baseline calibration.\(^{21}\) What is the intuition for this indeterminacy problem? As is the case with no investment, any inflation coefficient less than one (i.e. passive policy) induces indeterminacy due to the weakness of the demand channel of monetary policy presented above. Also, indeterminacy is induced by any inflation coefficient greater than an upper bound, which takes such a large value in the absence of investment activity that indeterminacy is unlikely.\(^{22}\) In our model, this upper bound is given by 
\[
1 + 2a_2(1 + \beta)/[\lambda(a_1 + \alpha)],
\]
whose value is 1.52 under the baseline calibration and thus the presence of investment activity lowers the upper bound greatly. In addition, there is another upper bound given by 
\[
1 + a_2(1 - \beta)/{\lambda}\alpha),
\]
which takes a value extremely close to one, e.g. 1.0027 under the baseline calibration, as noted above. This upper bound arises from the cost channel of monetary policy illustrated above. By this channel an active forward-looking policy makes inflation expectations self-fulfilling and hence induces indeterminacy.

One point of condition (5.15) is that the indeterminacy problem becomes slightly less severe when the degree of price stickiness, \( \nu \), increases, as also indicated by Huang and Meng (2007a).\(^{23}\) In (5.15) we can see that an increase in \( \nu \) reduces only the real marginal cost elasticity of inflation \( \lambda \) and hence raises the upper bound on inflation coefficients that generate determinacy.\(^{24}\) This is in stark contrast to Sveen and Weinke’s (2005) finding that the current-looking policy, which responds only to current inflation, is more likely to induce indeterminacy as prices become stickier.

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\(^{20}\)In Proposition 6, no policy response to output, i.e. \( \phi_y = \phi_R = 0 \), implies that (Case II) never holds and (5.12)–(5.14) can be reduced to (5.15), since (5.11) is implied by (5.14). Corollary 1 holds in a more general case of utility functions that are non-separable between consumption and real money balances as in CF. The proof of this case is provided in Kurozumi and Van Zandweghe (2006).

\(^{21}\)This interval, though we employ the same calibration, differs slightly from that of CF, 1 < \( \phi_\pi < 1.0057 \), since they use approximate values \( a_1 = 0.35 \) and \( a_2 = 0.03 \).

\(^{22}\)When the aggregate capital stock is fixed over time (i.e. \( K_t = K \forall t, \delta = 0 \)), this upper bound is given by 
\[
1 + 2(1 + \beta)/{\lambda}[1 + a_1\alpha/(1 - \alpha)],
\]
which takes a value of 9.0 under the baseline calibration. See also Proposition 4 of Bullard and Mitra (2002) and Proposition 4.5 of Woodford (2003).

\(^{23}\)Further, an increase in the depreciation rate of capital \( \delta \) or a decrease in the cost share of capital \( \alpha \) mitigates the indeterminacy problem.

\(^{24}\)With a higher value of \( \nu = 0.67 \) (0.80), this upper bound takes a slightly larger value of 1.0050 (1.0172).
This contrast stems from the way the cost channel induces indeterminacy. Under the forward-looking policy, indeterminacy is only due to this channel. Thus, stickier prices mitigate the effect of the cost channel and hence the indeterminacy problem. Under the current-looking policy, indeterminacy depends on the effect of the cost channel relative to the demand channel. Stickier prices strengthen this relative effect by reducing the effect of the demand channel more than that of the cost channel and as a consequence, indeterminacy is more likely.25

We now illustrate the determinacy condition in Proposition 6. Note that (5.11)−(5.14) are the empirically relevant one, since (Case II) cannot obtain with realistic calibrations of parameters including the baseline one. Condition (5.12) can be given the following interpretation, which is stressed by Woodford (2003), Bullard and Mitra (2002), and Kurozumi (2006). By (5.2)−(5.7), each percentage point of permanently higher inflation implies permanently higher output of \((1 - \beta)(s_c, \sigma + s_q(1 - \alpha))/[\lambda(1 - s_q)(1 - \alpha)]\) percentage points. The left-hand side of (5.12) then shows the long-run rise in the interest rate by policy (5.8) for each unit permanent increase in inflation. Hence, (5.12) can be interpreted as the long-run version of the Taylor principle: in the long run the nominal interest rate should be raised by more than the increase in inflation.

Figure 5.1 shows a region of inflation and output coefficients of the policy that generate determinacy under the baseline calibration. The lower bound on the inflation coefficient \(\phi_\pi\) is provided by the Taylor principle (5.12), while the upper bound on the output coefficient \(\phi_Y\) is given by (5.13). These two bounds arise basically from the demand channel, since we can see the corresponding ones in Figure 3 of Bullard and Mitra (2002) who analyze determinacy in an associated model without investment, in which monetary policy contains only the demand channel. The presence of investment activity gives rise to the upper bound on the inflation coefficient given by (5.14). The cost channel makes both the upper bounds on inflation and output coefficients severely limit the region of the coefficients that bring about determinacy. As shown in Corollary 1, the forward-looking policy ensures determinacy if and only if its inflation coefficient lies in the extremely narrow interval \(1 < \phi_\pi < 1.0027\). Even with the policy response to expected future output, the determinacy region of the coefficients is only slightly widened. The output coefficient of 0.046 generates determinacy for the widest possible interval of the inflation coefficient \(0.998 < \phi_\pi < 1.008\). For the output coefficient greater than 0.047, determinacy is impossible to obtain. In short,

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25As shown by Sveen and Weinke (2005), a stronger policy response to current inflation increases the effect of the demand channel more than that of the cost channel and thereby weakens the relative effect, so that determinacy is more likely. Also, determinacy is obtained for an interval of inflation coefficients that exceed one and are extremely close to one, regardless of the degree of price stickiness. This interval corresponds to the one given by condition (5.15) of Corollary 1, in which the effect of the cost channel is negligible.
the policy response to expected future output cannot ameliorate the indeterminacy problem.\footnote{A higher degree of price stickiness enlarges the determinacy region of the policy coefficients slightly, as is the case with no policy response to output. For $\nu = 0.67$ (0.80), the widest possible inflation coefficient interval, which is obtained with the output coefficient of 0.046, is given by 0.994 $< \phi_{\pi} < 1.018$ (0.983 $< \phi_{\pi} < 1.058$).}

### 5.3.2 Policy response to current output

We next examine the policy response to current output, i.e. $j = 0, \phi_{R} = 0$ in (5.1). As shown in Appendix D.1, this policy specification yields a system of the same form as (5.10) with a different coefficient matrix $A$, whose five eigenvalues are a zero and four solutions to a quartic equation $P(\mu) \equiv \mu^4 + h_3 \mu^3 + h_2 \mu^2 + h_1 \mu + h_0 = 0$. Determinacy requires that exactly two eigenvalues lie inside the unit circle and the other three be outside the unit circle. To the best of our knowledge, it is hard to analytically examine conditions for the quartic equation to contain exactly one solution inside the unit circle and the other three outside the unit circle.\footnote{We conjecture: if the forward-looking policy adjusts the interest rate in response also to current output, i.e. $j = 0, \phi_{R} = 0$ in (5.1), it generates local determinacy of REE if it satisfies (5.12), (5.16) and (5.17). Note that these three inequalities can be reduced to condition (5.15) in the case of no policy response to output.}

We thus carry out numerical investigations.
Figure 5.2 illustrates a region of inflation and output coefficients of the policy that ensures determinacy under the baseline calibration. From the policy specification, the long-run version of the Taylor principle yields the same inequality as the one with the policy response to expected future output, (5.12), which can also be obtained from $P(1) < 0$. This Taylor principle (5.12) provides the lower bound on the inflation coefficient $\phi_\pi$ and hence it seems to be a necessary condition for determinacy.\footnote{Proposition 4.5 of Woodford (2003) shows that in an associated model without investment, the corresponding condition is a necessary condition under which the policy response to current output ensures determinacy.} This lower bound arises from the demand channel, as noted above. The cost channel imposes the upper bounds on the inflation coefficient $\phi_\pi$, given by

$$P(-1) > 0$$

$$\Leftrightarrow \phi_\pi < 1 + \frac{2a_2(1+\beta)}{\lambda(a_1+\alpha)} + \frac{(1+\beta)(s_2(1-\alpha)(2-\delta)[1+\beta(1-\delta)] + \delta s_2 \sigma a_2)}{\lambda(a_1+\alpha)[2s_2 + \delta(1-s_2)]}$$

$$\Leftrightarrow (1-h_0)(1-h_0^2) - h_2(1-h_0)^2 + (h_3 - h_1)(h_1 - h_3 h_0) < 0.$$  

(5.17)

It seems that these two are necessary conditions for determinacy and the three inequalities (5.12), (5.16) and (5.17) are a sufficient condition (see footnote 27). Figure 5.2 (i.e. $0 \leq \phi_\pi \leq 3$) shows that if the output coefficient $\phi_Y$ exceeds 0.2, the policy
guarantees determinacy as long as it meets the Taylor principle (5.12). Therefore, the active forward-looking policy with responses to current output can overcome the indeterminacy problem.

How does the policy response to current output overcome the indeterminacy problem? The key point is that such a policy response prevents self-fulfilling inflation expectations and thereby the active forward-looking policy generates determinacy. To see this, consider a sunspot increase in inflation expectations. Under an active forward-looking policy, the real interest rate rises and then increases expected future inflation by the cost channel. This induces a possibility that the inflationary expectations become self-fulfilling and indeterminacy is generated. But, once the policy adjusts the interest rate in response also to current output, there is feedback on the real interest rate from movements in current consumption and investment. This subdues the real interest rate rise stemming from the inflationary expectations in the following two ways. A rise in the real interest rate discourages current consumption, so that such a rate rise can be reduced by the policy response to current consumption. Also, a real interest rate rise decreases current investment, which reduces this rate rise as long as the policy responds to current investment. In these two ways, the policy response to current output overcomes the indeterminacy problem.

Here, we address the question of which policy response of these two is crucial to such dramatic amelioration of the problem. We first examine the policy response to current consumption by replacing policy (5.8) with

$$\hat{R}_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_C \hat{C}_t. \quad (5.18)$$

Analyzing the system’s coefficient matrix $A$ given in Appendix D.1 yields the next proposition.

**Proposition 7** If the forward-looking policy sets the interest rate in response also to current consumption as in (5.18), it generates local determinacy of REE if and only if it satisfies

$$1 - \frac{1 - \beta}{\lambda(1 - \alpha)} (\sigma \phi_C) < \phi_\pi < 1 + \frac{a_2}{\lambda} \min \left\{ \frac{1 - \beta + \sigma \phi_C}{\alpha}, \frac{(1 + \beta)(2 + \sigma \phi_C)}{a_1 + \alpha} \right\}, \quad (5.19)$$

where $a_1 = 1 - \beta(1 - \delta)(1 - \alpha)$ and $a_2 = 1 - \beta(1 - \delta)$.

**Proof** See Appendix D.3. ⊡

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29A higher degree of price stickiness enlarges the determinacy region of the policy coefficients. For instance, when the output coefficient is 0.5, the inflation coefficient must exceed 0.971 under the baseline calibration, i.e. $\nu = 0.57$, while with $\nu = 0.67$ (0.80) this lower bound on the inflation coefficient is reduced to 0.947 (0.821). Meanwhile, the upper bound on the inflation coefficient increases as prices become stickier.
Like (5.12), the first inequality in (5.19) can be interpreted as the long-run version of the Taylor principle. Figure 5.3 displays a region of inflation and consumption coefficients of policy (5.18) that ensure determinacy under the baseline calibration. As is the case with policy responses to output, the lower bound on the inflation coefficient $\phi_\pi$ arises from the demand channel, while its upper bound is induced by the cost channel. Figure 5.3 shows that a more vigorous policy response to current consumption widens the interval of inflation coefficients that bring about determinacy. If $\phi_c = 0.5$, this interval is $0.98 < \phi_\pi < 1.14$, which is wider than $1 < \phi_\pi < 1.0027$ in the case of no policy response to consumption. One point of condition (5.19) is that both the lower and upper bounds contain the term $\sigma\phi_c$. This implies that the intertemporal substitution elasticity of consumption, $\sigma$, is a crucial factor in generating determinacy. As $\sigma$ increases, consumption becomes more responsive to changes in the real interest rate and as a consequence, the policy response to current consumption becomes more important for determinacy with the policy response to current output. Under realistic calibrations of $\sigma$, however, this is not the case, as shown in Figure 5.3.\footnote{Figure 5.3 provides the range of the consumption coefficient given by $0 \leq \phi_c \leq 2.1$, which corresponds to that of the output coefficient in Figure 5.2 (i.e. $0 \leq \phi_Y \leq 3$) because $0 \leq \phi_c \phi_Y \leq 0.7 \times 3 = 2.1$.\footnote{A higher degree of price stickiness widens the determinacy region of the policy coefficients. With
We next consider the policy response to current investment by replacing (5.8) with
\[ \hat{R}_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_I \hat{I}_t. \] (5.20)

As is the case with the policy response to current output, it seems hard to analytically examine conditions for determinacy, since the policy specification (5.20) yields a system of the same form as (5.10) with a different coefficient matrix \( A \) whose five eigenvalues are a zero and four solutions to a quartic equation \( Q(\mu) \equiv \mu^4 + j_3 \mu^3 + j_2 \mu^2 + j_1 \mu + j_0 = 0 \), as shown in Appendix D.1. Thus, we study determinacy numerically. Figure 5.4 illustrates a region of inflation and investment coefficients of policy (5.20) that ensure determinacy under the baseline calibration. The policy specification (5.20) implies that the long-run version of the Taylor principle yields
\[ \phi_\pi + \frac{(1 - \beta)(s_\sigma \sigma + 1 - \alpha)}{\lambda(1 - s_\pi)(1 - \alpha)} \phi_I > 1, \] (5.21)
which can also be obtained from \( Q(1) < 0 \). This Taylor principle (5.21) provides the lower bound on the inflation coefficient \( \phi_\pi \) and thus it seems to be a necessary condition for determinacy. This lower bound arises from the demand channel, while the cost channel imposes the upper bounds on the inflation coefficient \( \phi_\pi \), given by
\[ Q(-1) > 0 \Rightarrow \phi_\pi < 1 + \frac{2a_x(1 + \beta)}{\lambda(a_1 + \alpha)} + \frac{(1 + \beta)(2 - \delta)[(1 - \alpha)[1 + \beta(1 - \delta)] - s_\sigma \sigma a_x]}{\lambda(a_1 + \alpha)[2s_\pi + \delta(1 - s_\pi)]} \]
(5.22)
\[ (1 - j_0)(1 - j_0^2) - j_2(1 - j_0^2)^2 + (j_3 - j_1)(j_1 - j_3 j_0) < 0. \] (5.23)

It seems that these two are necessary conditions for determinacy and the three inequalities (5.21)–(5.23) are a sufficient condition (see footnote 32). As is the case with the policy response to current output, we can see that determinacy is likely. In Figure 5.4 (i.e. \( 0 \leq \phi_I \leq 3 \)), if the investment coefficient \( \phi_I \) exceeds 0.1, policy (5.20) generates determinacy as long as it satisfies the Taylor principle (5.21).\(^{34}\)

Therefore, the policy response to current investment ameliorates the indeterminacy problem dramatically.

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The consumption coefficient of 0.5, determinacy with \( \nu = 0.67 \) (0.80) requires the inflation coefficient to lie in the interval \( 0.96 < \phi_\pi < 1.25 \) (0.86 < \( \phi_\pi < 1.84 \)).

\( ^{32} \) We conjecture: if the forward-looking policy adjusts the interest rate in response also to current investment as in (5.20), it ensures local determinacy of REE if it satisfies (5.21)–(5.23). Note that these three inequalities can be reduced to condition (5.15) in the case of no policy response to investment.

\( ^{33} \) Figure 5.4 provides the range of the investment coefficient given by \( 0 \leq \phi_I \leq 0.9 \), which corresponds to that of the output coefficient in Figure 5.2 (i.e. \( 0 \leq \phi_Y \leq 3 \)) because \( 0 \leq s_\pi \phi_Y \leq 0.3 \times 3 = 0.9 \).

\( ^{34} \) With stickier prices, the determinacy region of the policy coefficients enlarges. For instance, when the investment coefficient is 0.5, the inflation coefficient must exceed 0.956 under the baseline calibration, i.e. \( \nu = 0.57 \), while with \( \nu = 0.67 \) (0.80) this lower bound on the inflation coefficient is reduced to 0.920 (0.729). Moreover, the upper bound on the inflation coefficient increases as prices become stickier.
The findings above suggest that the policy response to current investment rather than consumption is crucial for determinacy with the policy response to current output under realistic calibrations. This is because current consumption has a dampened response to changes in the real interest rate due to consumption smoothing, so that determinacy requires a large policy response to consumption. Current investment responds more sharply to real interest rate changes, and hence there is stronger feedback from current investment on interest rate policy, which overcomes the indeterminacy problem. This desirable property is inherited by the policy response to current output. Investment dynamics have been widely viewed as an important determinant of business fluctuations, despite a relatively small share of investment spending in aggregate demand. Our finding shows that investment dynamics are likewise of crucial importance in generating determinacy of REE. This suggests that central banks pay special attention to movements in investment activity.

### 5.3.3 Interest rate smoothing

We proceed to examine whether interest rate smoothing, i.e. $\phi_y = 0$ in (5.1), ameliorates the indeterminacy problem. The system of (5.2)–(5.9) can be reduced to a system of the same form as (5.10) with a different coefficient matrix $A$ given in Appendix D.1. By investigating this coefficient matrix, we obtain the following propo-
Proposition 8 The forward-looking policy with interest rate smoothing, i.e. \( \phi_r = 0 \) in (5.1), generates local determinacy of REE if and only if it satisfies

\[
1 < \phi_\pi < \frac{1 + \phi_R}{1 - \phi_R} \left[ 1 + \frac{2a_2(1 + \beta)}{\lambda(a_1 + \alpha)} \right], \quad (5.24)
\]

\[
|d_2| > 3 \quad \text{or} \quad d_0(d_0 - d_1) + d_1 - 1 > 0, \quad (5.25)
\]

where \( a_1 = 1 - \beta(1 - \delta)(1 - \alpha) \), \( a_2 = 1 - \beta(1 - \delta) \), and \( d_i, i = 0, 1, 2 \), are given in Appendix D.4.\(^{35}\)

Proof See Appendix D.4.

This determinacy condition is illustrated with the baseline calibration in Figure 5.5, which displays a region of coefficients of inflation and interest rate smoothing that satisfy the condition. The first inequality in (5.24) is the Taylor principle and is due to the demand channel of monetary policy. It gives rise to the lower bound on the inflation coefficient \( \phi_\pi \). The cost channel of monetary policy induces the upper bounds on the inflation coefficient, given by the second inequalities in (5.24) and (5.25). For weak interest rate smoothing whose degree is less than a certain threshold given by \( \alpha\beta a_2/[a_1(\alpha\lambda + a_2)] = 0.20 \), the upper bound on the inflation coefficient is provided by the second inequality in (5.25), which severely limits the determinacy region of the coefficients, as is the case with no interest rate smoothing. With \( \phi_R = 0.1 \), determinacy is obtained for an inflation coefficient in the interval of \( 1 < \phi_\pi < 1.010 \). Once interest rate smoothing is sufficiently strong, i.e. \( \phi_R > 0.20 \), the second inequality in (5.24) determines the upper bound. In the case of \( \phi_R = 0.5 \), determinacy is guaranteed by any inflation coefficient in the interval of \( 1 < \phi_\pi < 4.55 \). Hence, determinacy is likely with a range of the inflation coefficient and is more likely with more inertial interest rate policy.

As prices become stickier, the determinacy region of the policy coefficients widens in some direction, as is the case with no interest rate smoothing, while it also narrows in that the threshold of interest rate smoothing for determinacy increases. With a higher degree of price stickiness of \( \nu = 0.67 \), weak interest rate smoothing, in which \( \phi_R \) is less than an increased threshold of 0.33, yields a slightly wider determinacy region (e.g. \( 1 < \phi_\pi < 1.012 \) for \( \phi_R = 0.1 \)), and high interest rate smoothing with \( \phi_R > 0.33 \) generates a wider determinacy region (e.g. \( 1 < \phi_\pi < 6.13 \) for \( \phi_R = 0.5 \)). But, with a much higher degree of price stickiness of \( \nu = 0.80 \), the threshold of interest rate

\(^{35}\)Like Corollary 1, this proposition holds in a more general case of utility functions that are non-separable between consumption and real money balances as in CF. The proof of this case is available upon request.
smoothing increases, i.e. 0.58, so that in the case of $\phi_R = 0.5$ the determinacy region becomes much narrower, $1 < \phi_\pi < 1.10$.

What is the intuition for this determinacy with sufficiently strong interest rate smoothing? Interest rate smoothing means a policy response to the lagged interest rate and hence makes the forward-looking policy respond also to current and past inflation. As shown by CF and Kurozumi and Van Zandweghe (2006), equilibrium determinacy is possible with the current-looking or backward-looking policy, which sets the interest rate in response only to current or past inflation. Also, Sveen and Weinke (2005) show that the current-looking policy is more likely to induce indeterminacy as prices become stickier. Therefore, the forward-looking policy with interest rate smoothing inherits these properties from the current-looking and backward-looking policies. In sum, interest rate smoothing helps the forward-looking policy generate determinacy of REE, although its amelioration of the indeterminacy problem is not so effective as the policy response to current output.

5.4 Second prescription for the indeterminacy problem

We turn next to our second prescription for the indeterminacy problem: when E-stability is adopted as the criterion for selecting one from multiple REE, does the
forward-looking policy generate a locally-unique E-stable fundamental REE? Following the literature, our E-stability analysis is based on the so-called “Euler equation” approach suggested by Honkapohja et al. (2003). Specifically, the rational expectation operator $E_t$ is replaced with a possibly non-rational one $\hat{E}_t$ in the system of (5.2)–(5.9) with $\phi_y = \phi_R = 0$. Also, this system can be reduced to a system of the form

$$F y_t = G \hat{E}_t y_{t+1} + H \hat{K}_t + J g_t,$$

(5.26)

where $y_t = [\hat{\pi}_t \hat{C}_t \hat{Y}_t \hat{K}_{t+1}]'$ and the coefficient matrices $F, G, H$ are given in Appendix D.5. Then, fundamental RE solutions to system (5.26) are given by

$$y_t = \bar{c} + \bar{\Phi} \hat{K}_t + \bar{\Gamma} g_t,$$

(5.27)

where the coefficient matrices are determined by

$$\bar{c} = 0_{1 \times 1}, \quad H = (F - G \bar{\Phi}[0_{1 \times 3} 1]) \bar{\Phi}, \quad \bar{\Gamma} = \{F - G \bar{\Phi}[0_{1 \times 3} 1] - \rho G \}^{-1} J.$$

Note that $\bar{\Gamma}$ is uniquely determined given a $\bar{\Phi}$, but $\bar{\Phi}$ is not generally uniquely determined, which induces multiplicity of fundamental REE.

Following Section 10.5 of Evans and Honkapohja (2001), we analyze E-stability of fundamental REE. Corresponding to fundamental RE solutions (5.27), all agents are assumed to be endowed with a perceived law of motion (PLM) of $y_t$

$$y_t = c + \Phi \hat{K}_t + \Gamma g_t.$$

(5.28)

Using a forecast from the PLM and the relation $\hat{K}_{t+1} = [0_{1 \times 3} 1] y_t$ to substitute $\hat{E}_t y_{t+1}$ out of (5.26) leads to an actual law of motion (ALM) of $y_t$

$$y_t = F^{-1} G (I + \Phi[0_{1 \times 3} 1]) c + F^{-1} (G \Phi[0_{1 \times 3} 1] \Phi + H) \hat{K}_t$$

$$+ F^{-1} \{G (\Phi[0_{1 \times 3} 1] \Gamma + \rho \Gamma) + J \} g_t$$

(5.29)

Recall that in this chapter we refer to Evans and Honkapohja’s (2001) MSV solutions to linear RE models as fundamental and do not undertake E-stability analysis of non-fundamental REE (see footnote 6).

The form of the vector $J$ is omitted, since it is not needed in what follows.

System (5.26) contains a predetermined variable $\hat{K}_t$, so that we can consider two learning environments, which are studied respectively in Section 10.3 and 10.5 of Evans and Honkapohja (2001). One environment allows agents to use current endogenous variables in expectation formation, whereas another does not. In this chapter we present only E-stability analysis with the latter environment, as in Bullard and Mitra (2002). This is because any inflation coefficient that generates a locally-unique non-explosive E-stable fundamental REE in the latter environment does so in the former one, as Kurozumi (2006) shows in an associated model without investment. An intuition for this is that in forming future expectations, agents have more information by the current endogenous variables and hence E-stability is more likely in the former environment than in the latter one. Another reason for our focus on the latter environment is that the former induces a problem with simultaneous determination of the expectations and current endogenous variables, which is critical to equilibrium under non-rational expectations as indicated by Evans and Honkapohja (2001) and Bullard and Mitra (2002).
provided that $F$ is invertible. Here, $I$ denotes a conformable identity matrix. Then, a mapping $T$ from the PLM (5.28) to the ALM (5.29) can be defined by

$$T(c, \Phi, \Gamma) = \left( F^{-1}G(I + \Phi[0_{1\times3}1]), F^{-1}(G\Phi[0_{1\times3}1]\Phi + H), F^{-1}(G(\Phi[0_{1\times3}1]\Gamma + \rho\Gamma) + J) \right).$$

For a fundamental RE solution $(\bar{c}, \bar{\Phi}, \bar{\Gamma})$ to be E-stable, the matrix differential equation

$$\frac{d}{d\tau}(c, \Phi, \Gamma) = T(c, \Phi, \Gamma) - (c, \Phi, \Gamma)$$

must have local asymptotic stability at the solution, where $\tau$ denotes a notional time. Then, we have

$$DT_c(c, \Phi) = F^{-1}G(I + \Phi[0_{1\times3}1]),$$

$$DT_\Phi(\Phi) = F^{-1}G([0_{1\times3}1]I\Phi + \Phi[0_{1\times3}1]),$$

$$DT_\Gamma(\Phi, \Gamma) = F^{-1}G(\rho I + \Phi[0_{1\times3}1]).$$

Therefore, it follows that a fundamental RE solution $(\bar{c}, \bar{\Phi}, \bar{\Gamma})$ is E-stable if and only if all eigenvalues of three matrices, $DT_c(\bar{c}, \bar{\Phi}), DT_\Phi(\bar{\Phi}), DT_\Gamma(\bar{\Phi}, \bar{\Gamma})$, have real parts less than one. We summarize this result in the following lemma.

**Lemma 2** Suppose that the coefficient matrix $F$ is invertible. A fundamental RE solution to the system of (5.2)–(5.9) with the forward-looking policy (i.e. $\phi_y = \phi_r = 0$) is E-stable if and only if all eigenvalues of three matrices, $F^{-1}G(\gamma I + \bar{\Phi}[0_{1\times3}1]), \gamma = 1, \rho, \bar{\Phi}_4$, have real parts less than one, where $\bar{\Phi}_4$ is the fourth element of the RE solution vector $\bar{\Phi}$.

With this lemma, we investigate E-stability of fundamental REE numerically, since it seems impossible to analytically solve the matrix equation for $\bar{\Phi}$ in fundamental RE solutions (5.27) and thus to obtain explicit conditions for the E-stability. To compute (5.27), we use the method of Klein (2000) and McCallum (1998), which is a generalization of Blanchard and Kahn (1980). As pointed out by McCallum, different non-explosive fundamental REE are obtained for different groupings of stable generalized eigenvalues of the matrix pencil for system (5.26).\(^{39}\)

The E-stability analysis shows that in the presence of investment activity, the forward-looking policy generates a locally-unique non-explosive E-stable fundamental REE if its inflation coefficient lies in either of the following two intervals, both of which satisfy the Taylor principle, i.e. $\phi_\pi > 1$. One interval is extremely narrow, where the inflation coefficient exceeds one and is very close to one. This contains the interval of

\(^{39}\)In cases of indeterminacy, the baseline calibration shows order one or two indeterminacy and hence two or three distinct fundamental REE.
inflation coefficients that bring about determinacy of REE, given by condition (5.15) of Corollary 1. Under the baseline calibration, the interval for the unique E-stable REE is $1 < \phi_\pi < 1.0032$, which includes the one for determinacy, $1 < \phi_\pi < 1.0027$. Another interval requires that the inflation coefficient be sufficiently greater than one and its lower bound increase with stickier prices. This interval is $\phi_\pi > 1.26$ under the baseline calibration.\footnote{As prices become stickier, both the upper bound of the narrower interval and the lower bound of the wider interval increase, e.g., $1 < \phi_\pi < 1.0059$ (1.0202), $\phi_\pi > 1.54$ (2.71) for $\nu = 0.67$ (0.80).} Any policy response to expected future inflation in these two intervals succeeds in guiding temporary equilibria under non-rational expectations toward the unique E-stable REE by the demand channel of monetary policy. As noted in footnote 25, the effect of the cost channel of monetary policy is negligible for inflation coefficients extremely close to one, and hence such policy responses yield the unique E-stable REE. Also, when the inflation coefficient is sufficiently greater than one, the effect of the cost channel relative to the demand channel is weak enough to generate the unique E-stable REE. Further, if the forward-looking policy responds also to current output, almost every pair of the inflation and output coefficients that meets the long-run version of the Taylor principle (5.12) generates a unique non-explosive E-stable fundamental REE including the determinate REE, which is also E-stable. These results suggest that the indeterminacy problem induced by the forward-looking policy, which is emphasized by CF, is not critical from the perspective of E-stability or least-squares learnability of fundamental REE.

Our E-stability results are a generalization of Bullard and Mitra (2002), who use an associated model without investment to show that the forward-looking policy generates a locally-unique non-explosive E-stable fundamental REE if and only if it meets the Taylor principle. In the presence of investment activity, the cost channel emerges and reduces the guiding effect of the demand channel. As a consequence, all non-explosive fundamental REE fail to be E-stable if the policy response to expected future inflation lies in the intermediate interval, if any, between the two intervals of the inflation coefficients that generate the unique E-stable REE.

### 5.5 Concluding remarks

In the presence of investment activity and price stickiness, indeterminacy of REE is induced by forward-looking monetary policy that sets the interest rate in response only to expected future inflation, as first shown by CF. This indeterminacy problem is due to a cost channel of monetary policy, whereby inflation expectations become self-fulfilling. We have examined two prescriptions for the problem. The first prescription has shown that the indeterminacy problem can be ameliorated once the forward-looking policy adjusts the interest rate in response also to current output
or contains sufficiently strong interest rate smoothing, as empirical studies use it for a better description of actual monetary policy. In particular, the policy response to current output dramatically overcomes the indeterminacy problem in two ways: via policy responses to current consumption and investment. Both of these policy responses subdue changes in the real interest rate stemming from inflation expectations, thereby preventing self-fulfilling inflation expectations and hence indeterminacy. We have also demonstrated that the policy response to current investment rather than consumption is crucial to the determinacy with the policy response to current output in our model, since feedback from current consumption on interest rate policy is limited due to consumption smoothing. The second prescription has shown that when we adopt E-stability as the REE selection criterion, even the forward-looking policy generates a locally-unique non-explosive E-stable fundamental REE as long as its inflation coefficient is sufficiently strong. Further, if the policy adjusts the interest rate in response also to current output, almost every pair of the inflation and output coefficients that meets the long-run version of the Taylor principle generates the unique E-stable REE.

We use a stochastic version of CF’s model. In the actual economy, aggregate variables such as consumption and investment display more considerable persistence than in our model. In order to fit models to actual data, recent business cycle literature such as Christiano et al. (2005) and Smets and Wouters (2003) allows for habit formation in preferences for consumption, a finite labor supply elasticity, staggered nominal wage setting in monopolistically competitive labor markets, adjustment costs in investment or capital, variable capital utilization, and so forth, which all are absent in our model. A few recent studies incorporate some of these features into our fundamental model to investigate equilibrium determinacy numerically. Xiao (2008) uses such a model incorporating a finite labor supply elasticity and a capital adjustment cost and then shows that a mild policy response to expected future output helps the forward-looking policy ensure determinacy. Huang and Meng (2007b) employ a similar model to Xiao to find that under an empirically reasonable labor supply elasticity, the policy response to current output fails to make the forward-looking policy generate determinacy, but once staggered nominal wage setting is incorporated into their model, the role of such a policy response in guaranteeing determinacy is greatly enhanced. These studies suggest that one topic of our future research is to examine what empirically relevant extension of our fundamental model may or may not help the forward-looking policy bring about equilibrium determinacy.

Another topic is E-stability analysis of non-fundamental REE. In this chapter we have investigated only fundamental REE. Some readers may consider this focus unappealing. Carlstrom and Fuerst (2004), however, show that a sunspot equilib-
rium is E-stable only if a central bank believes in the sunspot, using an associated model without investment. Because this condition is not practical, our focus on fundamental REE might be plausible. To make sure of the validity of our focus, we will examine E-stability of non-fundamental REE in our model, following recent analyses with associated models without investment, such as Honkapohja and Mitra (2004), Carlstrom and Fuerst (2004), and Evans and McGough (2005).
Appendix A

Addendum to Chapter 2

A.1 Coefficient matrices in systems of form (2.21)

In the case of the forward-looking policy, i.e. $j = 1$ in (2.12), the coefficient matrix $A$ of system (2.21) is given by

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{11}(\phi - 1)/\sigma & 1 + A_{12}(\phi - 1)/\sigma & A_{13}(\phi - 1)/\sigma \\ 0 & A_{32} & A_{33} \end{bmatrix}, \quad (A.1)$$

where

$$A_{11} = \frac{\chi}{\beta \Omega_1} [\rho(1 - \xi) - s],$$

$$A_{12} = -\frac{\kappa}{\beta \Omega_1} \left\{ s_\nu \xi - \beta (1 - \rho)(\xi - \eta \nu) \left[ s_e - A_{32} \left( 1 - \frac{\rho}{u} + s_e \frac{1 - u}{u} \right) \right] \right\},$$

$$A_{13} = \frac{\kappa}{\beta \Omega_1} [\xi - \beta (1 - \rho)(1 - \nu \eta)A_{33}], \quad A_{32} = \frac{s_\nu (1 - \xi)}{\rho(1 - \xi) - s},$$

$$A_{33} = \frac{s_\nu \left[ \rho(1 - \xi) - (1 - \rho \xi/u) \right]}{\rho(1 - \xi) - s},$$

$$\Omega_1 = \chi [\rho(1 - \xi) - s] + \Omega_0 (\phi - 1),$$

$$\Omega_0 = \kappa (1 - \rho) \{(1 - \nu \eta) [\rho(1 - \xi) - s] - (\xi - \eta \nu)s/e/\sigma \}.$$
matrix $A$ is the same as (A.1), except $A_{12} = A_{12}^0 - (\Omega_0/\Omega_1)\phi_\nu$ and $A_{22} = A_{22}^0 + \phi_\nu/\sigma$. Finally, when the forward-looking policy responds also to the unemployment rate, i.e. $\phi_R = \phi_Y = 0$ in (2.29), the system’s coefficient matrix $A$ is the same as (A.1), except $A_{12} = A_{12}^0 - (\Omega_0/\Omega_1)A_{32}\phi_\nu$, $A_{13} = A_{13}^0 - (\Omega_0/\Omega_1)A_{33}\phi_\nu$, $A_{22} = A_{22}^0 + A_{32}\phi_\nu$, and $A_{23} = A_{23}^0 + A_{33}\phi_\nu$.

A.2 Proof of Proposition 1

For the system’s coefficient matrix $A$ given in Appendix A.1, we can show that its three eigenvalues are the solutions to the cubic equation

$$
\mu^3 + b_1\mu^2 + b_2\mu + b_3 = 0,
$$

where $b_1 = -1 - A_{11}^0 - A_{33} - A_{12}^0/\sigma$, $b_2 = A_{11}^0 + (1 + A_{11}^0)A_{33} + (A_{12}^0A_{33} - A_{13}^0A_{32})(\phi_\nu - 1)/\sigma$, and $b_3 = -A_{11}^0A_{33}$. Because determinacy of equilibrium obtains if and only if the coefficient matrix $A$ has exactly one eigenvalue inside the unit circle and the other two outside the unit circle, it follows that the necessary and sufficient condition for determinacy is that exactly two solutions to the cubic equation above are outside the unit circle and one is inside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied.

(Case 1) $b_1 + b_2 + b_3 < -1$, $b_1 - b_2 + b_4 > 1$;

(Case 2) $b_1 + b_2 + b_4 > -1$, $b_1 - b_2 + b_4 > 1$, $b_4(b_4 - b_1) + b_2 - 1 > 0$ or $|b_1| > 3$.

Then, because $\Omega_1$ is a common denominator in these inequalities, (Case 2) can be reduced to (2.22)–(2.25) if $\Omega_1 > 0$ and to (2.24) and the strict opposite of (2.22), (2.23) and (2.25) if $\Omega_1 < 0$. Likewise, (Case 1) can be reduced to (2.25) and the strict opposite of (2.22) and (2.23) if $\Omega_1 > 0$ and to (2.22) and (2.23) and the strict opposite of (2.25) if $\Omega_1 < 0$.

A.3 Proof of Proposition 2

For the system’s coefficient matrix $A$ given in Appendix A.1, we can show that its three eigenvalues are the solutions to the cubic equation $\mu^3 + c_1\mu^2 + c_2\mu + c_3 = 0$, where $c_1 = b_1 - (1 + A_{11}^0 - A_{12}^0/\sigma)\Omega_0\phi_\nu/(\Omega_1 - \Omega_0\phi_\nu) + A_{12}^0\phi_\nu/\sigma$, $c_2 = b_2 + (1 + A_{11}^0 - 1)(1 + A_{33})\Omega_0\phi_\nu/(\Omega_1 - \Omega_0\phi_\nu) - (A_{12}^0A_{33} - A_{13}^0A_{32})(1 + \Omega_0/(\Omega_1 - \Omega_0\phi_\nu))\phi_\nu/\sigma - A_{12}^0\Omega_1\phi_\nu/[(\Omega_1 - A_{12}^0\Omega_1\phi_\nu)\sigma]$, $c_3 = b_3 - A_{11}^0A_{33}\Omega_0\phi_\nu/(\Omega_1 - \Omega_0\phi_\nu) + (A_{12}^0A_{33} - A_{13}^0A_{32})\Omega_1\phi_\nu/[(\Omega_1 - \Omega_0\phi_\nu)\sigma]$. $A_{12}^0$ and $A_{33}$ are given in Appendix A.1, and $b_i$, $i = 1, 2, 3$, are given in Appendix A.2. The remainder of the proof proceeds in the same way, mutatis mutandis, as in Appendix A.2.
A.4 Proof of Proposition 3

For the system’s coefficient matrix $A$ given in Appendix A.1, we can show that its three eigenvalues are the solutions to the cubic equation $\mu^3 + d_1 \mu^2 + d_2 \mu + d_3 = 0$, where

\[
d_1 = b_1 + A_{11}^0 \Omega_2 + [1 + A_{12}^0 (\phi_\gamma - 1)/\sigma] (\Omega_2 \sigma - \phi_\gamma)/(\sigma - \phi_\gamma),
\]

\[
d_2 = b_2 - A_{11}^0 A_{33} \Omega_2 - [A_{11}^0 + A_{33} + (A_{12}^0 A_{33} - A_{13}^0 A_{32}) (\phi_\gamma - 1)/\sigma] (\Omega_2 \sigma - \phi_\gamma)/(\sigma - \phi_\gamma),
\]

\[
d_3 = b_3 + A_{11}^0 A_{33} (\Omega_2 \sigma - \phi_\gamma)/(\sigma - \phi_\gamma),
\]

$A_{i,j}$ and $A_{3,j}$ are given in Appendix A.1, and $b_i$, $i = 1, 2, 3$, are given in Appendix A.2. The remainder of the proof proceeds in the same way, mutatis mutandis, as in Appendix A.2.

A.5 Proof of Proposition 4

For the system’s coefficient matrix $A$ given in Appendix A.1, we can show that its three eigenvalues are the solutions to the cubic equation $\mu^3 + e_1 \mu^2 + e_2 \mu + e_3 = 0$, where

\[
e_1 = b_1 - \beta A_{11}^0 \phi_\gamma /\sigma,
\]

\[
e_2 = b_2 + A_{11}^0 (1 + \beta A_{33}) \phi_\gamma /\sigma,
\]

\[
e_3 = b_3 (1 + \phi_\gamma /\sigma),
\]

$A_{11}$ and $A_{33}$ are given in Appendix A.1, and $b_i$, $i = 1, 2, 3$, are given in Appendix A.2. The remainder of the proof proceeds in the same way, mutatis mutandis, as in Appendix A.2.

A.6 Proof of Proposition 5

For the system’s coefficient matrix $A$ given in Appendix A.1, we can show that its three eigenvalues are the solutions to the cubic equation $\mu^3 + f_1 \mu^2 + f_2 \mu + f_3 = 0$, where

\[
f_1 = b_1 - \beta A_{11}^0 A_{32} (1 - U) \phi_u /\sigma,
\]

\[
f_2 = b_2 + A_{11}^0 A_{32} (1 - U) \phi_u /\sigma,
\]

\[
f_3 = b_3, A_{11}$ and $A_{32}$ are given in Appendix A.1, and $b_i$, $i = 1, 2, 3$, are given in Appendix A.2. The remainder of the proof proceeds in the same way, mutatis mutandis, as in Appendix A.2.

A.7 Coefficient matrices in system (2.40)

The coefficient matrices $F, G, H$ of system (2.40) are given by

\[
F = \begin{bmatrix}
A_{11}^0 & A_{12}^0 & 0 \\
0 & 1 & 0 \\
0 & A_{32} & -1 
\end{bmatrix}, \quad G = \begin{bmatrix}
1 & 0 & 0 \\
-(\phi_\xi - 1)/\sigma & 1 & 0 \\
0 & 0 & 0 
\end{bmatrix}, \quad H = \begin{bmatrix}
-A_{13}^0 \\
0 \\
-A_{33} 
\end{bmatrix}.
\]

where $A_{1,j}$ and $A_{3,j}$ are given in Appendix A.1.
Appendix B

Addendum to Chapter 3

1. Consumption Euler equation, (4.2.1):

\[ \sigma \hat{C}_t = \sigma E_t \hat{C}_{t+1} - (\hat{R}_t - E_t \hat{\pi}_{t+1}) \]  

(B.1)

2. New Keynesian Phillips curve, (3.7) and (3.8):

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \hat{z}_t \quad \text{where} \quad \lambda = \frac{(1 - \nu)(1 - \beta \nu)}{\nu}. \]  

(B.2)

3. Money growth, (3.28):

\[ \hat{\mu}_t = \rho \mu \hat{\mu}_{t-1} + \varepsilon_{\mu t} \]  

(B.3)

4. Evolution real money balances:

\[ \hat{m}_d_t = \hat{m}_d_{t-1} + \hat{\mu}_t - \hat{\pi}_t \]  

(B.4)

5. Money demand, (3.3):

\[ \hat{m}_d_t = \sigma \hat{C}_t - \frac{1}{R - 1} \hat{R}_t \]  

(B.5)

6. Productivity, (3.9):

\[ \hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{at} \]  

(B.6)

7. Intermediate good production, (3.10):

\[ \hat{y}^j_t = \hat{a}_t + \hat{n}^j_t, \quad j = b, g \]  

(B.7)

8. Intermediate good clearing, (3.29):

\[ Y^j \hat{Y}^j_t = y^j \hat{y}^j_t - \gamma^j v^j \hat{v}^j_t, \quad j = b, g \]  

(B.8)

9. Relative price intermediate goods, (3.4), (3.5):

\[ \hat{z}^j_t = \hat{z}_t + \hat{C}_t - \hat{Y}^j_t, \quad j = b, g \]  

(B.9)
10. Real marginal cost, (3.6):
\[ \hat{z}_t = \alpha \hat{z}_t^b + (1 - \alpha) \hat{z}_t^g \] (B.10)

11. Good job creation, (3.17):
\[ \dot{q}_t = \left[ 1 - \beta(1 - \rho) \right] \left\{ \left( \frac{z^g}{z^g - u^g} \right) E_t \left[ \hat{z}_t^g + \hat{a}_t + \hat{v}_t + \hat{\pi}_t \right] \right\} 
- \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right) - \beta(1 - \rho) E_t \dot{q}_{t+1}^g \] (B.11)

12. Bad job creation, (3.25):
\[ -\dot{q}_t^b = \left[ 1 - \beta(1 - \rho)(1 - sp^g) \right] \left\{ \left( \frac{z^b}{z^b - u^b} \right) E_t \left[ \hat{z}_t^b + \hat{a}_t + \hat{v}_t + \hat{\pi}_t \right] \right\} 
- \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right) - \beta(1 - \rho) \left\{ \left( 1 - sp^g \right) E_t \dot{q}_{t+1}^b \right\} \] (B.12)

13. Wage good job, (3.18):
\[ w^g \hat{w}_t^g = \eta z^g \left[ \hat{z}_t^g + \hat{a}_t \right] + \eta p^g \frac{\gamma^g}{q^g} \left[ \hat{p}_t^g - \hat{q}_t^g \right] \] (B.13)

14. Wage bad job, (3.26):
\[ w^b \hat{w}_t^b = \eta z^b \left[ \hat{z}_t^b + \hat{a}_t \right] + \eta (1 - s) p^b \frac{\gamma^b}{q^b} \left[ \hat{p}_t^b - \hat{q}_t^b \right] - \eta sp^g \frac{\gamma^b}{q^b} \hat{s}_t \] (B.14)

15. Employment good jobs, (3.12):
\[ \hat{n}_{t+1}^g = (1 - \rho) \hat{n}_t^g + \rho (\hat{q}_t^g + \hat{v}_t^g) \] (B.15)

16. Employment bad jobs, (3.23):
\[ \hat{n}_{t+1}^b = (1 - \rho) (1 - sp^g) \hat{n}_t^b - (1 - \rho) sp^g [\hat{s}_t + \hat{p}_t^g] + \frac{m^b}{n_b} \left( \hat{q}_t^b + \hat{v}_t^b \right) \] (B.16)

17. Employment and unemployment, (3.13):
\[ u^b \hat{u}_t^b + u^g \hat{u}_t^g + n^b \hat{n}_t^b + n^g \hat{n}_t^g = 0 \] (B.17)

18. Optimal search, (3.27):
\[ (\tau - 1) \hat{s}_t = \hat{p}_t^g + \frac{\gamma^b / q^b}{\gamma^g / q^g - \gamma^b / q^b} \hat{a}_t + \frac{\gamma^g / q^g}{\gamma^g / q^g - \gamma^b / q^b} \hat{q}_t^g \] (B.18)
19. Effective search, (3.20):
\[ \hat{e}_t = \hat{s}_t + \hat{n}^b_t \] (B.19)

20. Firm matching rate good job:
\[ \hat{q}^g_t = -\xi (\hat{v}^g_t - \hat{u}^g_t - \hat{e}_t) \] (B.20)

\[ \hat{q}^b_t = -\xi \left( \hat{v}^b_t - \hat{u}^b_t \right) \] (B.21)

22. Worker matching rate good job, (3.22)
\[ \hat{p}^g_t = (1 - \xi) \left[ \hat{v}^g_t - \frac{u^g}{u^g + e} (\hat{u}^g_t + \hat{e}_t) \right] \] (B.22)

23. Worker matching rate bad job, (3.15):
\[ \hat{p}^b_t = (1 - \xi) \left( \hat{v}^b_t - \hat{u}^b_t \right) \] (B.23)

24. Directed search, (3.19):
\[ \hat{p}^g_t - \hat{q}^g_t = \hat{p}^b_t - \hat{q}^b_t \] (B.24)
Appendix C

Addendum to Chapter 4

C.1 Log-linearized model with permanent I-shocks

The log-linearized model with permanent I-shocks consists of the equations

\[ \dot{y}_t = s_c \dot{c}_t + s_i \dot{i}_t + s_a \dot{u}_t + \dot{g}_t \]  
\[ \nu \dot{N}_t - \dot{\lambda}_t = m c_t + \dot{y}_t - \dot{N}_t \]  
\[ (1 + \beta) \dot{\pi}_t = \beta E_t \dot{\pi}_{t+1} + \dot{\pi}_t - 1 + (1 - \xi)(1 - \beta \xi) \dot{c}_t \]  
\[ \dot{R}_t = \phi_R \dot{R}_{t-1} + (1 - \phi_R) \left[ \phi_a \dot{c}_t + \phi_x (\dot{y}_t - \dot{y}_t^f) \right] \]  
\[ \dot{\lambda}_t = E_t \dot{\lambda}_{t+1} - E_t \dot{z}_{t+1} - \frac{\alpha}{1 - \alpha} E_t \dot{q}_{t+1} + \dot{R}_t - E_t \dot{\pi}_{t+1} \]  
\[ (\gamma - h)(\gamma - \beta h) \dot{\lambda}_t = -\left( \gamma^2 + \beta h^2 \right) \dot{c}_t + \gamma h \beta E_t \dot{c}_{t+1} + \gamma h \dot{c}_{t-1} \]  
\[ = \gamma h (1 - \beta \rho_z) \dot{z}_t - \frac{\alpha}{1 - \alpha} \gamma h (1 - \beta \rho_q) \dot{q}_t \]  
\[ \dot{k}_{t+1} = \frac{1 - \delta}{\gamma K} \left( \dot{k}_t - \dot{z}_t - \frac{1}{1 - \alpha} \dot{q}_t \right) + \left[ 1 - \left( \frac{1 - \delta}{\gamma K} \right) \right] \dot{i}_t \]  
\[ \dot{R}_t - E_t \dot{\pi}_{t+1} + \dot{\mu}_t = \beta \left( \frac{1 - \delta}{\gamma K} \right) E_t \dot{u}_{t+1} - E_t \dot{q}_{t+1} \]  
\[ = \left[ 1 - \beta \left( \frac{1 - \delta}{\gamma K} \right) \right] \left( E_t m c_{t+1} + E_t \dot{y}_{t+1} \right) \]  
\[ \dot{\mu}_t = -\psi \gamma^2 \left[ (1 + \beta) \dot{i}_t - \beta E_t \dot{i}_{t+1} - \dot{i}_{t-1} \right] \]  
\[ \chi \dot{u}_t = m c_t + \dot{y}_t - \dot{k}_t + \dot{z}_t + \frac{1}{1 - \alpha} \dot{q}_t \]  
\[ \dot{y}_t = (1 - \alpha) \dot{N}_t + \alpha \left( \dot{u}_t + \dot{k}_t - \dot{z}_t - \frac{1}{1 - \alpha} \dot{q}_t \right) \]
C.2 The log-linearized model with transitory I-shocks

The log-linearized model with transitory I-shocks consists of equations (C.1)–(C.4) and

\[
\dot{\lambda}_t = E_t \dot{\lambda}_{t+1} - E_t \dot{z}_{t+1} + \dot{R}_t - E_t \dot{\pi}_{t+1} \tag{C.12}
\]

\[
(\gamma - h)(\gamma - \beta h) \dot{\lambda}_t = -(\gamma^2 + \beta h^2) \dot{c}_t + \gamma h \beta E_t \dot{c}_{t+1} + \gamma h \dot{c}_{t-1} \tag{C.13}
\]

\[
\dot{k}_{t+1} = \left(1 - \frac{\delta}{\gamma}\right) \left(\dot{k}_t - \dot{z}_t\right) + \left[1 - \left(1 - \frac{\delta}{\gamma}\right)\right] \left(\dot{i}_t + \dot{Q}_t\right) \tag{C.15}
\]

\[
\dot{R}_t - E_t \dot{\pi}_{t+1} + \dot{\mu}_t = \beta \left(1 - \frac{\delta}{\gamma}\right) E_t \dot{\mu}_{t+1} + \left[1 - \beta \left(1 - \frac{\delta}{\gamma}\right) \rho_Q\right] \dot{Q}_t \tag{C.16}
\]

\[
\dot{\mu}_t = -\psi \gamma^2 \left[\left(1 + \beta\right) \dot{i}_t - \beta E_t \dot{c}_{t+1} - \dot{\mu}_{t-1} + (1 - \beta \rho_z) \dot{z}_t\right] \tag{C.17}
\]

\[
\chi \dot{u}_t = \dot{m}c_t + \dot{y}_t - \dot{k}_t + \dot{z}_t + \dot{Q}_t \tag{C.18}
\]

\[
\dot{y}_t = (1 - \alpha) \ddot{N}_t + \alpha \left(\dot{u}_t + \dot{k}_t - \dot{z}_t\right) \tag{C.19}
\]
Appendix D

Addendum to Chapter 5

D.1 Coefficient matrices in systems of form (5.10)

Let \( a_1 = 1 - \beta(1 - \delta)(1 - \alpha) \) and \( a_2 = 1 - \beta(1 - \delta) \). When the forward-looking policy responds also to expected output, i.e. \( j = 1, \phi_R = 0 \) in (5.1), the coefficient matrix \( A \) of system (5.10) is given by

\[
A = \begin{bmatrix}
A_{21} & A_{22} & A_{23} & A_{24} \\
A_{31} & A_{32} & A_{33} & A_{34} \\
0 & 0 & \delta_s & 1 - \delta \\
A_{51} & A_{52} & A_{53} & A_{54}
\end{bmatrix}
\]  

where

\[
A_{21} = \tilde{A}_{21}, \quad A_{22} = \tilde{A}_{22} + 1, \quad A_{23} = \tilde{A}_{23}, \quad A_{24} = \tilde{A}_{24},
\]

\[
A_{2n} = \sigma [A_{1n}(\phi_\pi - 1) + A_{3n}\phi_Y],
\]

\[
A_{31} = \tilde{A}_{31},
\]

\[
A_{32} = \tilde{A}_{32} + \frac{a_2[A_{42} - (1 - \alpha)/\sigma]}{a_2 + (a_1 - 1)\phi_Y}, \quad A_{33} = \tilde{A}_{33} + \frac{a_2A_{43}}{a_2 + (a_1 - 1)\phi_Y},
\]

\[
A_{34} = \tilde{A}_{34} + \frac{a_2A_{44}}{a_2 + (a_1 - 1)\phi_Y}, \quad \tilde{A}_{3n} = \frac{A_{1n}(1 - a_1)(\phi_\pi - 1)}{a_2 + (a_1 - 1)\phi_Y},
\]

\[
A_{51} = \tilde{A}_{51}, \quad A_{52} = \tilde{A}_{52}, \quad A_{53} = \tilde{A}_{53}, \quad A_{54} = \tilde{A}_{54}, \quad \tilde{A}_{5n} = A_{1n}\phi_\pi + A_{3n}\phi_Y.
\]

When the forward-looking policy responds also to current output, i.e. \( j = 0, \phi_R = 0 \) in (5.1), the system takes the same form as (5.10) with the same coefficient
matrix $A$ as (D.1), except

\[
A_{21} = \tilde{A}_{21}, \quad A_{22} = \tilde{A}_{22} + 1, \quad A_{23} = \tilde{A}_{23} + \sigma \phi_y, \quad A_{24} = \tilde{A}_{24},
\]

\[
\tilde{A}_{2n} = \sigma A_{1n}(\phi_\pi - 1), \quad A_{31} = \tilde{A}_{31}, \quad A_{32} = \tilde{A}_{32} + A_{42},
\]

\[
A_{33} = \tilde{A}_{33} + A_{43} + \frac{(1 - \alpha)\phi_y}{a_2}, \quad A_{34} = \tilde{A}_{34} + A_{44},
\]

\[
\tilde{A}_{3n} = (1 - \alpha) \left[ \frac{A_{1n}(\phi_\pi - 1)}{a_2} - \frac{A_{2n}}{\sigma} \right],
\]

\[
A_{51} = \tilde{A}_{51}, \quad A_{52} = \tilde{A}_{52}, \quad A_{53} = \tilde{A}_{53} + \phi_y, \quad A_{54} = \tilde{A}_{54}, \quad \tilde{A}_{5n} = A_{1n}\phi_\pi.
\]

The characteristic equation of the coefficient matrix $A$ is given by

\[
\mu P(\mu) = \mu \left( \mu^4 + h_3 \mu^3 + h_2 \mu^2 + h_1 \mu + h_0 \right) = 0,
\]

where

\[
h_3 = -2 - \frac{1}{\beta} - \frac{\delta(1 - s_j)}{s_j} + \frac{\lambda a_1}{\beta a_2} (\phi_\pi - 1) - \frac{1 - a_1}{a_2} \phi_y,
\]

\[
h_2 = \frac{1}{\beta} + \left( 1 + \frac{1}{\beta} \right) \left[ 1 + \frac{\delta(1 - s_j)}{s_j} \right] - \frac{\lambda}{\beta a_2} \left\{ \alpha + a_1 \left[ 1 + \frac{\delta(1 - s_j)}{s_j} \right] \right\} (\phi_\pi - 1)
\]

\[
+ \left\{ \frac{(1 - a_1)(2 - \delta + \beta(1 - \delta)^2)}{\beta a_2(1 - \delta)} + \frac{\delta s_j \sigma}{s_j} \right\} \phi_y,
\]

\[
h_1 = - \left[ 1 + \frac{\delta(1 - s_j)}{s_j} \right] \left[ \frac{1}{\beta} \frac{\lambda a_1}{\beta a_2} (\phi_\pi - 1) \right]
\]

\[
- \left\{ \frac{(1 - a_1)(1 + \beta(1 - \delta)(2 - \delta))}{\beta^2 a_2(1 - \delta)} + \frac{\delta s_j \sigma}{s_j} \right\} \phi_y,
\]

\[
h_0 = \frac{1 - a_1}{\beta^2 a_2} \phi_y.
\]

When the forward-looking policy responds also to current consumption as in (5.18), the system takes the same form as (5.10) with the same coefficient matrix $A$ as the one with the policy response to current output, except $A_{22} = \tilde{A}_{22} + \sigma \phi_c + 1$, $A_{23} = \tilde{A}_{23}$, $A_{32} = \tilde{A}_{32} + A_{42} + (1 - \alpha) \phi_c/a_2$, $A_{33} = \tilde{A}_{33} + A_{43}$, $A_{52} = \tilde{A}_{52} + \phi_c$, $A_{53} = \tilde{A}_{53}$.

When the forward-looking policy responds also to current investment as in (5.20), the system takes the same form as (5.10) with the same coefficient matrix $A$ as the one with the policy response to current output, except $A_{22} = \tilde{A}_{22} - \sigma \phi_i/s_j + 1$, $A_{23} = \tilde{A}_{23} + \sigma \phi_i/s_j$, $A_{32} = \tilde{A}_{32} + A_{42} - (1 - \alpha) \phi_i/(s_j a_2)$, $A_{33} = \tilde{A}_{33} + A_{43} + (1 - \alpha) \phi_i/(s_j a_2)$, $A_{52} = \tilde{A}_{52} - s_c \phi_i/s_j$, $A_{53} = \tilde{A}_{53} + \phi_i/s_j$. The characteristic equation of the coefficient matrix $A$ is given by

\[
\mu Q(\mu) = \mu \left( \mu^4 + j_3 \mu^3 + j_2 \mu^2 + j_1 \mu + j_0 \right) = 0,
\]
where

\[ j_3 = -2 - \frac{1}{\beta} \delta(1 - s_t) + \frac{\lambda a_1}{\beta a_2} (\phi_{\pi} - 1) - \frac{1 - a_1}{s_t a_2} \sigma a_2 \phi_t, \]

\[ j_2 = \frac{1}{\beta} + \left(1 + \frac{1}{\beta} \right) \frac{\delta(1 - s_t)}{s_t} - \frac{\lambda}{\beta a_2} \left\{ \alpha + a_1 \left[ 1 + \delta(1 - s_t) \right] \right\} (\phi_{\pi} - 1) \]

\[ + \left\{ \frac{(1 - a_1)[2 - \delta + \beta(1 - \delta)^2]}{\beta s_t a_2 (1 - \delta)} - \frac{s_c \sigma [1 + \beta(1 - \delta)]}{\beta s_t} \right\} \phi_t, \]

\[ j_1 = - \left[ 1 + \frac{\delta(1 - s_t)}{s_t} \right] \left\{ \frac{1}{\beta} - \frac{\lambda \alpha}{\beta a_2} (\phi_{\pi} - 1) \right\} \]

\[ - \left\{ \frac{(1 - a_1)[1 + \beta(1 - \delta)(2 - \delta)] - s_c \sigma (1 - \delta)}{\beta^2 s_t a_2 (1 - \delta)} \right\} \phi_t, \]

\[ j_0 = \frac{1 - a_1}{\beta^2 s_t a_2} \phi_t. \]

In the case of the forward-looking policy with interest rate smoothing, i.e. \( \phi_L = 0 \) in (5.1), the system takes the same form as (5.10) with the same coefficient matrix \( A \) as the one with the policy response to current output, except \( A_{23} = \tilde{A}_{23}, A_{25} = \sigma \phi_R, \tilde{A}_{2n} = \sigma A_{1n}[(1 - \phi_R)\phi_{\pi} - 1], A_{33} = \tilde{A}_{33} + A_{43}, A_{35} = (1 - a_1)\phi_R/a_2, \tilde{A}_{3n} = (1 - \alpha)[A_{1n}[(1 - \phi_R)\phi_{\pi} - 1]/a_2 - A_{2n}/\sigma], A_{53} = \tilde{A}_{53}, A_{55} = \phi_R, \tilde{A}_{5n} = A_{1n}(1 - \phi_R)\phi_{\pi}. \)

**D.2 Proof of Proposition 6**

For the system’s coefficient matrix \( A \) given in Appendix D.1, we can show that its five eigenvalues are two zeros and three solutions to the cubic equation

\[ b_3 \mu^3 + b_2 \mu^2 + b_1 \mu + b_0 = 0, \]

where

\[ b_3 = a_2 - (1 - a_1)\phi_L, \quad a_1 = 1 - \beta(1 - \delta)(1 - \alpha), \quad a_2 = 1 - \beta(1 - \delta), \]

\[ b_2 = -a_2 \left[ 2 + \frac{1}{\beta} \frac{\delta(1 - s_t)}{s_t} + \frac{\lambda a_1}{\beta} (\phi_{\pi} - 1) \right] \]

\[ + \left\{ (1 - \alpha)[2 - \delta + \beta(1 - \delta)^2] + \frac{s_c \sigma a_2}{s_t} \right\} \phi_L, \]

\[ b_1 = a_2 \left[ 2 + \frac{1}{\beta} \frac{\delta(1 + \beta)(1 - s_t)}{s_t} \right] - \frac{\lambda}{\beta} \left\{ \alpha + a_1 \left[ 1 + \delta(1 - s_t) \right] \right\} (\phi_{\pi} - 1) \]

\[ - \left\{ (1 - \alpha) \left[ \frac{1}{\beta} + (1 - \delta)(2 - \delta) \right] + \frac{s_c \sigma a_2}{\beta s_t} \right\} \phi_L, \]

\[ b_0 = -a_2 \left[ 1 + \frac{\delta(1 - s_t)}{s_t} \right] + \frac{\lambda a_1}{\beta} \left[ 1 + \frac{\delta(1 - s_t)}{s_t} \right] (\phi_{\pi} - 1) + \frac{1 - a_1}{\beta^2} \phi_L. \]

Because the policy generates local determinacy of REE if and only if the matrix \( A \) has exactly two eigenvalues inside the unit circle and the other three outside the unit circle, it follows that the necessary and sufficient condition for determinacy is that all
three solutions to the cubic equation are outside the unit circle. Hence, determinacy requires that the cubic equation have non-zero solutions. This implies that $b_0 \neq 0$ and the cubic equation can be rewritten as, letting $\tilde{\mu} = 1/\mu$,

$$b_0 \tilde{\mu}^3 + b_1 \tilde{\mu}^2 + b_2 \tilde{\mu} + b_3 = 0.$$ 

The necessary and sufficient condition is now that all three solutions to this new cubic equation are inside the unit circle. Then, from the Schur-Cohn criterion,\(^1\) it follows that determinacy is obtained if and only if either of the following two cases is satisfied.

(Case I) \> $b_0 < 0$, $b_0 + b_1 + b_2 + b_3 < 0$, $b_0 - b_1 + b_2 - b_3 < 0$, $b_0^2 - b_3^2 > |b_0 b_2 - b_1 b_3|$;

(Case II) \> $b_0 > 0$, $b_0 + b_1 + b_2 + b_3 > 0$, $b_0 - b_1 + b_2 - b_3 > 0$, $b_0^2 - b_3^2 > |b_0 b_2 - b_1 b_3|$.

Then, the first three inequalities in (Case I) can be reduced to (5.11)–(5.13), respectively.

## D.3 Proof of Proposition 7

For the system’s coefficient matrix $A$ given in Appendix D.1, we can show that its five eigenvalues are two zeros, $1 + \delta(1/s_I - 1) > 1$, and two solutions to the quadratic equation

$$\mu^2 + c_1 \mu + c_0 = 0,$$

where $c_1 = -1 - 1/\beta + \lambda a_1 (\phi_\pi - 1)/(\beta a_2) - \sigma \phi_C$, $c_0 = 1/\beta - \alpha \lambda (\phi_\pi - 1)/(\beta a_2) + \sigma \phi_C/\beta$, $a_1 = 1 - \beta(1 - \delta)(1 - \alpha)$, and $a_2 = 1 - \beta(1 - \delta)$.

From an analogous argument to that in the proof of Proposition 6, it follows that the necessary and sufficient condition for local determinacy of REE is that both solutions to the quadratic equation are outside the unit circle. By Proposition C.1 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied.

(Case I) \> $c_0 > 1$, $c_0 + c_1 > -1$, $c_0 - c_1 > -1$;

(Case II) \> $c_0 + c_1 < -1$, $c_0 - c_1 < -1$.

The three inequalities in (Case I) can be reduced to (5.19). To complete the proof, it suffices to show that (Case II) never obtains. To see this, the two inequalities in (Case II) can be reduced to

$$\phi_\pi - 1 < -\frac{\sigma(1 - \beta)}{\lambda(1 - \alpha)} \phi_C, \quad \phi_\pi - 1 > \frac{2 a_2 (1 + \beta)}{\lambda(a_1 + \alpha)} + \frac{\sigma a_2 (1 + \beta)}{\lambda(a_1 + \alpha)} \phi_C.$$

Combining these two yields a contradiction

$$0 < \frac{2 a_2 (1 + \beta)}{\lambda(a_1 + \alpha)} + \frac{\sigma a_2 (1 + \beta)}{\lambda(a_1 + \alpha)} \phi_C < \phi_\pi - 1 < -\frac{\sigma(1 - \beta)}{\lambda(1 - \alpha)} \phi_C < 0.$$

\(^1\)See e.g. Proposition 5.3 of LaSalle (1986).

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D.4 Proof of Proposition 8

For the system’s coefficient matrix $A$ given in Appendix D.1, we can show that its five eigenvalues are $0, 1 + \delta/(s_j - 1) > 1$, and three solutions to the cubic equation

$$
\mu^3 + d_2 \mu^2 + d_1 \mu + d_0 = 0,
$$

where $d_2 = -1 - 1/\beta + \lambda a_1[(1 - \phi_R)/(\beta a_2) - \phi_R], d_1 = 1/\beta - \alpha \lambda [(1 - \phi_R)/(\beta a_2) - 1] + \lambda a_1/\beta a_2 + 1/\beta \phi_R, d_0 = -\alpha \lambda [(\beta a_2) + 1] \phi_R/\beta, a_1 = 1 - \beta (1 - \delta)(1 - \alpha)$, and $a_2 = 1 - \beta (1 - \delta)$.

From an argument similar to that in the proof of Proposition 6, it follows that the necessary and sufficient condition for local determinacy of REE is that one solution to the cubic equation is inside the unit circle and the other two are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied.

(Case I) $1 + d_2 + d_1 + d_0 < 0, -1 + d_2 - d_1 + d_0 > 0$;
(Case II) $1 + d_2 + d_1 + d_0 > 0, -1 + d_2 - d_1 + d_0 < 0, |d_2| > 3$ or $d_0 (d_0 - d_2) + d_1 - 1 > 0$.

The first two inequalities in (Case II) can be reduced to (5.24). To complete the proof, it suffices to show that (Case I) never obtains. To see this, the two inequalities in (Case I) can be reduced to

$$
\phi_\pi > \frac{1 + \phi_R}{1 - \phi_R} \left[ 1 + \frac{2 a_2 (1 + \beta)}{\lambda (a_1 + \alpha)} \right], \quad \phi_\pi < 1.
$$

Combining these two yields a contradiction

$$
1 < \frac{1 + \phi_R}{1 - \phi_R} \left[ 1 + \frac{2 a_2 (1 + \beta)}{\lambda (a_1 + \alpha)} \right] < \phi_\pi < 1.
$$

D.5 Coefficient matrices in system (5.26)

The coefficient matrices $F, G, H$ of system (5.26) are given by, letting $a_2 = 1 - \beta (1 - \delta)$,

$$
F = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{a_{2}}{1-\alpha} & 0 \\
0 & -s_\gamma & 1 & -\frac{a_{2}}{\beta} & 0 \\
1 & -\frac{\lambda}{\sigma} & -\frac{\lambda a_{2}}{1-\alpha} & 0 & 0 \\
\end{bmatrix}, \\
G = \begin{bmatrix}
-\sigma (\phi_\pi - 1) & 1 & 0 & 0 & 0 \\
- (\phi_\pi - 1) & \frac{a_{2}}{\sigma} & \frac{a_{2}}{1-\alpha} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\beta & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \\
H = \begin{bmatrix}
0 \\
0 \\
-\frac{\delta}{s_\gamma (1-\delta)} \\
-\frac{\lambda a_{2}}{1-\alpha} \\
\end{bmatrix}.
$$
Bibliography


