Disclosures and Investments

By

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ABSTRACT

Disclosures and Investments

Xiaoyan Wen

Corporate disclosure plays an important informational role in the functioning of capital markets. The public, value-relevant, information revealed through corporate disclosure influences the capital market pricing of a firm’s shares. Because a firm’s investment incentive is partially provided by the market pricing of its shares, corporate disclosure in turn, affects the efficiency of the firm’s investment decisions. This dissertation uses the economics-based models of disclosure to establish a link between firm financial disclosure and its economic consequences, and investigates how the efficiency of investment decisions responds to various features of corporate disclosure. The dissertation consists of two closely related studies.
First, this dissertation studies mandatory accounting disclosure and its economic consequences by distinguishing two broad bases for accounting measurements: input-based and output-based accounting. The results show that an output-based measure, such as a fair value measure, has a natural advantage in aligning investment incentives because of its comprehensiveness. The (first-) best investment is achieved when the output-based measure is noiseless and manipulation-free, and more accounting noise/manipulation always leads to more inefficient investment choices. On the other hand, an input-based measure, such as a historical cost measure, may induce more efficient investment decisions than an output-based measure even though it is not as comprehensive. In fact, under an input-based measure, the (first-) best result is achieved when the noise/manipulability is small but positive. In other words, for an input-based measure, being less comprehensive makes small but positive accounting noise/manipulability desirable.

Second, this dissertation also investigates voluntary disclosure and focuses on the trade-off for an individual firm when the benefits and costs of voluntary disclosure stem from the consequences of its investment decision. First, by transmitting value-relevant information to the market, voluntary disclosure leads to a more accurate pricing which, in turn, improves investment efficiency. Second, the firm may affect the market pricing of its shares in its favor by strategically disclosing or withholding its private information. This opportunistic use of disclosure has a
feedback effect on investment efficiency because the firm’s disclosure credibility is enhanced when the disclosure is accompanied by a real investment activity. As a result, the opportunistic use may cause the real investment to be distorted at the margin. In particular, when the information asymmetry is large, such propensity to strategically disclose or withhold private information becomes intense and detrimental to investment efficiency. The presence of a separate mandatory accounting report, while not directly useful in the firm’s investment decision, improves the market pricing and may discipline the voluntary disclosure, because the accounting information helps disentangle the firm’s disclosure incentive, which limits the opportunistic behavior. In particular, when the information asymmetry is large, a marginal improvement in the mandatory report quality enhances investment efficiency.
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Chapter 1

Mandatory Disclosure and Investments

1.1 Introduction

In this paper, we investigate how the accounting measurement basis affects the capital market pricing of a firm’s shares, which, in turn, affects the efficiency of the firm’s investment decisions. We distinguish two broad bases for accounting measurements. The first, an input-based accounting measurement, is designed to measure a firm’s activities by recording the estimated factor costs of production. In contrast, the second basis, an output-based accounting measurement, is designed to measure a firm’s activities by recording the estimated financial benefits of production. We argue that the two bases give rise to different informational properties of accounting numbers. In turn, the different properties induce different capital market pricing behaviors, which leads to a systematic difference in the efficiency of the investment decisions. In particular, we show that an output-based measure, such as a fair value measure, has a natural advantage in aligning firm investment incentives. That is, because an output-based measure provides comprehensive information about both the scale and profitability of the investment, it reduces the negative impact of mispricing on investment efficiency through a dampening effect. However, output-based measures suffer a potential disadvantage in that they may inherit more measurement noise and may be subjected to more managerial manipulation. We show that an output-based measure performs (first-) best when it is noiseless and manipulation-free. However,

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1. An article adapted from Chapter 1 has been published under the title "Accounting Measurement Basis, Market Mispricing, and Firm Investment Efficiency" with Professor Pierre Jinghong Liang in the March 2007 issue of Journal of Accounting Research by the Institute of Professional Accounting at the Graduate School of Business, University of Chicago.
in cases where they are highly noisy and easy to manipulate, as they often are in practice, we show that the investment efficiency can be quite low as a result.

On the other hand, an input-based measure, such as a historical cost measure, may induce more efficient investment decisions than output-based measures, despite having a disadvantage of only providing information about the scale of the firm investment. The reason is two-fold. First, input-based measures are typically associated with less noise and limited manipulation. Second and more importantly, we show that with an input-based measure, some low levels of noise and manipulability are tolerated or even preferred in some cases. In fact, the (first-) best result is achieved when the measurement noise is small but positive. In other words, for an input-based measure, being less comprehensive makes small but positive measurement noise desirable.

The debate on measurement bases has a long and varied standing in accounting history. Paton and Littleton (1940) describe accounting numbers as price-aggregates which measure the economic activities of a firm. The debate on the basis of price-aggregates centers on which "price" to use in such measurements. Extensive subsequent writings describe the rationales for using entry prices, exit prices, past and/or future market prices, and simulated prices (see Edwards and Bell 1961, Sterling 1970, Ijiri 1975). We frame the debate within a partial equilibrium investment setting with rational expectations and evaluate the different measurement bases on the scale of induced investment efficiency. In other words, we adopt an information-economic paradigm and evaluate the alternative measurements according to their information content. (See Christensen and Demski 2002 for more on such a paradigm.)

The insights in this paper may be helpful in current discussions of fair value measurements. There has been considerable movement in public accounting policy toward measurements based on fair value. The move is partially motivated by the desire to increase the relevance of accounting, arguably with some sacrifice in reliability. The emerging concern is the introduction of considerably more estimates which are to be used in preparing fair value accounting measures. The Financial Accounting Standards Board (FASB) has recognized the reliability issue and has prescribed different levels of estimates (Level 1, 2, 3 etc.) accordingly. In this paper, we point to the effects of noise and manipulation on investment efficiency. In particular, we show that using fair value makes noise or manipulability a social "bad" to the economy because either of
the two induces inefficient investments. In other words, if fair value is used, one must always be careful about its negative "real" effect on firm investments if the measure brings in an increased level of noise and manipulability. This is because under fair value, investment inefficiency and accounting noise/manipulability are always complements: more accounting noise/manipulation leads to more inefficient investment choices. On the other hand, with historical cost accounting, the (first-) best result is achieved when the measurement noise is small but positive. As a result, our analysis points to a certain attractiveness of a historical cost measurement basis when some measurement noise is unavoidable for either measure. If the measurement noise is small but positive, historical cost accounting is preferable to fair value accounting. Further, under historical cost, investment inefficiency and accounting noise/manipulability may be substitutes: a slight increase in accounting noise/manipulation may lead to more efficient investment choices.

Generally, our results imply that the rational economic choice between a fair value measure and a historical cost measure is not obvious in most cases where varying levels of noise and manipulability exist in both measures.

Specifically, we consider a simple two-period model in which a firm’s investment decision is jointly affected by the total return of the investment and by the short-term capital market pricing of its ownership shares. Conditional on private information about the profitability of the investment, the firm makes an investment decision which generates cash flows in both the first and second periods. The firm’s objective is to maximize a weighted average of the life-time cash flows and the short-term share price. The share price reflects the rational expectation of firm value based on a public report of the first-period aggregate cash flow (i.e., the sum of first-period investment return and the cash flow from the first-period on-going activities). The market’s inability to identify the sources of the first-period cash flow may induce a systematic short-term mispricing of firm value, from the firm’s perspective. In turn, this mispricing induces a suboptimal investment decision, ex ante. We show that the deviation from the first-best investment level may be positive (i.e., over-investment) or negative (i.e., under-investment) depending, in part, on the timing of the investment return.

Within this framework, accounting is introduced as an information system which provides (price-aggregate) measures about the investment. In pricing the firm’s shares, the capital market uses the information in the accounting measure as a supplement to the information contained in
the aggregate cash flow. We consider two accounting measurement bases. First, the accounting measure is modeled as an unbiased estimate of the investment return (which is a function of the actual investment made and the true investment profitability). We think of this as an output-based measure, such as a fair value measurement. Second and alternatively, the accounting measure is modeled as an unbiased estimate of the actual investment made. We think of this as an input-based measure, such as a historical cost measurement. Either accounting signal provides additional information, which may help the capital market improve the pricing of the firm’s shares. The improved pricing induces better firm investment decisions. As the quality of accounting measures (which is inversely related to the measurement noise) improves, one would expect less mispricing, which would lead to better investment decisions. We show that this conjecture is, indeed, true for both the input- and output-based measures when the noise in these measures is high. That is, when accounting measures are highly imprecise, any improvement in precision will lead to more efficient investment decisions.

When the measurement noise is low, however, the two measurement bases are fundamentally different. For the output-based measure, investment efficiency continues to rise with any increase in measurement quality. At the limit, when the output-based accounting measure perfectly reveals the investment return, first-best investment choices are made. However, for the input-based measure with low measurement noise, the investment efficiency is an inverted U-shape function of the measurement noise. As measurement noise decreases, the investment efficiency first increases up to a threshold point and decreases afterwards. The first-best investment choices are made when the noise is small but not zero (at the threshold point).

The reason for the difference, and for the surprising result, is the different mispricing structures that are generated by the two measurement bases. For the output-based measure, both the actual investment made and the actual investment profitability affect the accounting measure. So, as the firm deviates from the first-best investment level, the effect on its share price is dampened, at the margin, by the measure’s built-in profitability estimate. In other words, the firm’s ability to use real investment to change the market perception of its investment profitability is mitigated by the independent profitability estimate built into the output-based measure. In turn, the dampening effect lessens the ex ante incentives to distort the investment decision. The investment efficiency increases monotonically as the measurement noise decreases. For the
input-based measure, the accounting measure relies only on the actual investment made and does not directly rely on a separate estimate of the investment profitability. The dampening effect does not apply to the (mis)pricing of the accounting report; it only applies to the (mis)pricing of a portion of the cash flow measure. As the measurement noise decreases, the dampening effect is diminished as more valuation weight is shifted from the cash flow to the accounting report. This is the reason that a noiseless input-based measure invites severe deviation from the first-best investment.

Finally, we offer two extensions of the model. First, we add accounting manipulation to the mix. Following Dye and Sridhar (2004a), we model accounting manipulation by giving the firm an option to privately modify accounting measurements before they are reported to the capital market. The total cost of manipulation contains a random element. As a result, accounting manipulation introduces additional noise into the reported accounting measures. Based on the existing economic forces, it is shown that accounting manipulation always makes the firm worse off under an output-based accounting regime because adding more noise to an output-based accounting measure always leads to a less efficient investment decision. However, accounting manipulation may make the firm better off under an input-based accounting regime. This is because there are regions in the inverted U-shape relation where a slight increase in measurement noise will improve the investment efficiency. This is most likely when the existing measurement noise is small, and the noise introduced by accounting manipulation to the input-based measure is also small.

In the second extension, we modify the model to allow an option for the date-2 shareholders to utilize the technology in the firm to generate future cash flows. In this modified model, the market price is reflective of two streams of cash flows: those generated by the initial investment and those generated by future investments. As before, the mispricing affects the initial investment choice and the relative performance of the input- vs. output-based accounting measures. However, it is no longer true that the output-based measure performs best when noiseless. Comparing accounting measurement bases is further complicated and requires knowing more details of the context including, in particular, the importance of future cash flow relative to current cash flow.

The antecedent of studies on investment myopia in finance and economics is Stein (1989).
Stein (1989) finds that the managers, facing stock market pressure, forsake good investments so as to boost current earnings, even though the market is efficient and is not fooled in equilibrium. In spite of being unable to fool the market, managers are trapped into behaving myopically in the classic signal-jamming model. Dye and Sridhar (2004a) examine how investment decisions are affected by the reliable and relevant components of an aggregate accounting report. Their study focuses on the reliability-relevance trade-offs in accounting aggregation, the conditions under which aggregation improves efficiency, and on the optimal weights in constructing an optimal accounting report. While Dye and Sridhar (2004a) and our study share the feature of accounting manipulation, our focus is on the comparison of two alternative accounting measurements, each of which is disaggregated from a common cash flow measure. In addition, Demski, Lin, and Sappington (2005) analyze a setting in which entrepreneurs invest before they learn whether they will be forced to sell their assets. They study the optimal design of asset impairment regulations when the assets resale market suffers from the "lemon" problem.

Using a contracting setting, Prendergast (2002) examines input-based and output-based measures and argues that input-based monitoring, coupled with a directed action, performs best in stable settings, while output-based monitoring is best in uncertain environments. The reason is that output-based contracts, coupled with a delegation of decisions, align social and private incentives better in uncertain situations. In contrast, our study focuses on the market incentives produced by input- and output-based measures and finds that the output-based measure performs best when the measurement is precise. Among other accounting studies on investment efficiency, Kanodia, Singh and Spero (2005) analyze the economic consequences of the interaction between noisy accounting measures and information asymmetry regarding the investment profitability. They find that some degree of accounting imprecision can be value-enhancing, which is consistent with our results on the input-based measure. In contrast, we focus on the comparison of input- vs. output-based measures and show that the role of accounting imprecision depends on the accounting basis: under an output-based measure, noiseless accounting is optimal.

The rest of the paper proceeds as follows. Section 2 describes the model details. Section 3 presents the benchmark results of the basic setup where only an aggregate cash flow is reported. In section 4, we introduce accounting measures and analyze the equilibria induced by account-
ing. Section 5 introduces an extension which models accounting manipulation. Section 6 offers another extension where the firm technology is reusable in the future. Section 7 summarizes the paper.

1.2 The Model

Consider an economy with a risk-neutral firm in a competitive risk-neutral capital market. There are two periods (and three dates), representing short- and long-term concerns. The firm’s normal on-going activities generate a pair of cash flows, denoted $x_t$, realized on date-$t$ ($t = 1, 2$). We assume:

$$
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} \sim \mathcal{N}
\begin{bmatrix}
\mu \\
\mu
\end{bmatrix},
\begin{bmatrix}
\sigma^2 & \alpha \sigma^2 \\
\alpha \sigma^2 & \sigma^2
\end{bmatrix}
$$

(1)

On date-0, the firm is faced with an investment opportunity (called a project) and observes a private signal, denoted $\theta \in \mathbb{R}$, about the project’s profitability. The prior distribution of $\theta$ is normal with mean $\theta_0$ and variance $\sigma^2_\theta$ (i.e., $\theta \sim \mathcal{N}[\theta_0, \sigma^2_\theta]$). The firm chooses an investment level, denoted $I \in \mathbb{R}^+$, to invest into the project. The project generates cash flows on date-1 and date-2, denoted by $f_1(\theta, I)$ and $f_2(\theta, I)$ respectively. For tractability, we assume the project returns $f_1(\theta, I) = k\sqrt{I}$ and $f_2(\theta, I) = (2 - k)\sqrt{I}$, where $k \in (0, 2)$. Thus, the total investment return $(2\sqrt{I})$ depends only on the investment level $I$ and the profitability variable $\theta$. The timing of the investment return depends on $k$: higher $k$ indicates that more investment return is realized in the short-term.\(^2\)

The realized periodic cash flows to the firm are denoted by $z_t$ ($t = 1, 2$):\(^3\)

$$
\begin{align}
z_1 &= x_1 + f_1(\theta, I) = x_1 + k\sqrt{I} \\
z_2 &= x_2 + f_2(\theta, I) = x_2 + (2 - k)\sqrt{I}
\end{align}
$$

(2) (3)

Following the literature (Dye 2002, Dye and Sridhar 2004a, 2004b), we assume that the investment is made privately (i.e., not directly observable) and that the firm is unable to directly

\(^2\)All qualitative results remain if we assume that $f_t(\cdot)$ are the expected values of the short- and long-term investment returns and that the noise in the returns is independent of the existing random variables.

\(^3\)Here we assume that $z_1$ is gross of the investment cost ($I$), which is appropriate given that we assume the investment is made privately. If $z_1$ is net of the investment cost, the quantitative results would change while the same qualitative forces would remain. See Dye (2002) and Dye and Sridhar (2004a) for similar assumptions.
communicate the private information \( (\theta) \) to outsiders, including the capital market. We also assume that \( \theta \) is independent of \( x_1 \) or \( x_2 \) and the on-going cash flows \((x_1 \text{ or } x_2)\) are not affected by the investment choice \((I)\).

At the end of date 1, the firm’s shares are priced in a competitive risk-neutral capital market such that the market price, denoted \( P \), is equal to the expected value of the cumulative cash flow on date-2. Denote the publicly available information set on date-1 by \( \Omega \) (assuming no discounting or dividend payments):

\[
P = E[z_1 + z_2 | \Omega]
\]

The information set \( \Omega \) includes public information available for pricing. In the basic setup, the firm publicly reports its aggregate cash flow only, so \( \Omega \) contains \( z_1 \) alone. Later, we consider additional signals created by an accounting information system; so \( \Omega \) may include additional items such as a deferral or an accrual.

The firm is motivated by both the long-term interest (i.e., cumulative cash flows on date-2) and the short-term interest (i.e., date-1 stock price). We assume for life cycle or liquidity reasons, a portion of the firm, denoted \( \beta \in (0, 1) \), must be sold on date-1. The remaining \((1 - \beta)\) portion will be held by the date-0 shareholders.\(^4\) As a result, the firm’s objective is to maximize a weighted average of date-1 market price and date-2 cumulative cash flow (net of the investment costs \( I \)). For a type-\( \theta \) firm with investment function \( I(\theta) \), the objective function is

\[
-I(\theta) + \beta P + (1 - \beta)(z_1 + z_2)
\]

\(^4\)Another reason may be that the firm faces a probability \( \beta \) of takeover on date-1, in which case the firm wishes to maximize its date-1 share price (see Stein 1989 for additional discussions). In the finance literature, incentive to underinvest may be driven by a debt-overhang problem (see Myers 1977).
The sequence of the events is summarized below.

\[ t = 0 \] 
Firm privately Cash flows \( x_1 \) and \( f_1(\theta, I) \) 
observes \( \theta \) and are realized
chooses \( I(\theta) \) 
Firm releases reports

\[ t = 1 \] 
Cash flows \( x_2 \) and \( f_2(\theta, I) \) 
are realized

\[ t = 2 \] 
Firm is liquidated
Market prices \( P \) based on \( \Omega \)

Figure 1: Time Line of Events

As a reference point for what follows, we provide a description of the first-best investment policy in Lemma 1 (proof omitted).

**Lemma 1 (first-best)** The socially optimal investment policy consists of a function \( I^{FB}(\theta) \), where for each \( \theta \), \( I^{FB}(\theta) \) maximizes \( f_1(\theta, I) + f_2(\theta, I) - I = 2\sqrt{\theta I} - I \). So we have \( I^{FB} = 0 \) when \( \theta > 0 \); and \( I^{FB} = 0 \) when \( \theta < 0 \).

We now define the equilibrium where the investment policy is made in the self-interest of the firm.

**Definition 1** An equilibrium relative to \( \Omega \) consists of an investment function \( I^*(\cdot) \), and a market pricing function \( P(\cdot) \), such that:

(i) Given \( P(\cdot) \), optimal investment function \( I^*(\cdot) \) maximizes \( V(\theta | I(\cdot)) = E_{x_1x_2}[-I + \beta P(\cdot) + (1 - \beta)(z_1 + z_2)] \);

(ii) Given \( I^*(\cdot) \), the pricing function \( P(\cdot) \) satisfies \( P = E[z_1 + z_2 | \Omega, I^*(\cdot)] \).

We employ an approximation assumption to calculate the pricing function in closed-form.

**Approximation Assumption:** Let random variables \( x, y, \) and \( z \) be jointly normally distributed. Denote by \( f(x + a | y + az) \) the conditional density function for some known constant \( a \) and denote \( G(z | y + az) \) the conditional cumulative density function of \( z \). We assume for all realizations of \( y + az \),
\[
\int_{x} \int_{z>0} (x+z)f(x+z|y+az)dzdx + G(0|y+az) \int_{x} xf(x|y)dx = \int_{x} \int_{z} (x+z)f(x+z|y+az)dzdx
\]

(6)

In words, the approximation assumes that the error due to censoring the lower tail of a normally distributed random variable is small when calculating the conditional mean of the sum of the censored variable and another normal random variable. In our context, the investment strategy function \( I(\theta) \) censors the underlying profitability parameter \( \theta \) by the fact \( I = 0 \) if \( \theta < 0 \). The market observes some linear function of \( I(\theta) \), not the uncensored \( \theta \). The assumption allows us to calculate the market inference in closed-form by assuming the censored \( \theta \) is close enough to the uncensored \( \theta \). Because the censoring point is always held at zero, it is clear that this approximation becomes increasingly accurate as \( E[z] \) is large, i.e., the probability mass to the left of zero is increasingly small.\(^5\)\(^6\)

1.3 Basic Setup

Except in Section 6, we assume throughout that on date-1 the realized aggregate cash flow \((z_1)\) is always publicly observable. However, outsiders are not able to differentiate the cash flow components. In other words, cash flows generated from on-going activities and from the investment are aggregated. This aggregation feature plays an important role in this setting.

We now analyze the equilibrium behavior of the firm in the basic setup.

**Theorem 1** Using the approximation assumption (6), there exists a unique linear equilibrium relative to \( \Omega = \{z_1\} \). It is given by

\(^5\)Note this approximation assumption pertains to calculating the conditional expectation (i.e., expected \( x+z \) conditional on the realization of \( y + az \)). We acknowledge that this is a global requirement. That is, the approximation applies for each and every realization of \( y + az \). For extremely negative realizations of the censored variable \( z \), the approximation error may be large. However, the approximation error for a particular realization of \( y + az \), denoted \( AE(y + az) \), becomes smaller and approaches zero as the mean of \( z \) increases. In the appendix, we formally prove this claim.

\(^6\)Dye and Sridhar (2004b) use an approximation assumption in their analysis. While their approximation applies to calculating the unconditional mean of an altered normally distributed random variable, ours applies to a series of conditional mean of an altered normally distributed random variable. Stocken and Verrecchia (2004) also uses a similar approximation.
(i) an equilibrium linear pricing function:

\[ P(z_1) = a + b \times z_1, \quad \text{where} \]
\[ b = \frac{(1 + \alpha)\sigma^2 + 2k\delta\sigma^2_\theta}{\sigma^2 + k^2\delta\sigma^2_\theta} \quad \text{(8)} \]
\[ a = (2 - b)\mu + (2 - kb)\sqrt{\delta}\theta_0 \quad \text{(9)} \]

(ii) an equilibrium investment function:

\[ I(\theta) = \begin{cases} 
\delta\theta, & \text{if } \theta \geq 0 \\
0, & \text{if } \theta < 0 
\end{cases} \quad \text{(10)} \]

where \[ \delta = \left(1 + \beta\left(\frac{bk}{2} - 1\right)\right)^2 \quad \text{(11)} \]

**Proof.** All proofs are placed in the appendix.

Compared with the first-best investment, a key observation in Theorem 1 is that the equilibrium investment choice \( I \) is generally a function of the short-term market pressure (parameter \( \beta \)) and the timing of the investment cash flows (parameter \( k \)), not just a function of the investment profitability (variable \( \theta \)). This is because in pricing the firm’s shares, the market is unable to distinguish the individual components of the short-term cash flow \( z_1 \). To illustrate, suppose that the market is able to distinguish the cash flow components; then the appropriate response to every dollar of the on-going cash flows \( x_1 \) would be \( 1 + \alpha \) and the appropriate response to the short-term investment return \( f_1(\theta, I) = k\sqrt{\theta I} \) would be \( \frac{2}{k} \). However, because only the aggregate \( z_1 = x_1 + k\sqrt{\theta I} \) is reported, the market response is the weighted average of \( 1 + \alpha \) and \( \frac{2}{k} \). Therefore, the market may not price the investment efficiently. Because the firm cares about the short-term share price \( (\beta > 0) \), the efficiency of the pricing affects the investment incentives.

To understand the nature of the mispricing, we return to the firm objective function (5),
which may be rewritten as

\[-I + \beta P + (1 - \beta)(z_1 + z_2)\]

\[= -I + z_1 + z_2 + \beta(P - (z_1 + z_2))\]

\[= \left(-I + x_1 + x_2 + 2\sqrt{\theta I}\right) + \beta(P - (x_1 + x_2 + 2\sqrt{\theta I}))\]  

(12)

The first term is the first-best objective function. The second term, when \(\beta > 0\), is an increasing function of the mispricing (equal to price minus the long-term firm value). When making the investment decision \((I)\), the firm does not know \(x_t\) but knows \(\theta\). So, the expected mispricing, given \(\theta\), is

\[E_{x_1x_2}[P - (x_1 + x_2 + 2\sqrt{\theta I})]\]

Substituting the linear pricing function \(P = a + b(x_1 + k\sqrt{\theta I})\) into the expected mispricing, we have

\[E_{x_1x_2}[P - (x_1 + x_2 + 2\sqrt{\theta I})]\]

\[= E_{x_1x_2}[a + b(x_1 + k\sqrt{\theta I}) - (x_1 + x_2 + 2\sqrt{\theta I})]\]

\[= (bk - 2)\sqrt{\theta I} + E_{x_1x_2}[a + bx_1 - (x_1 + x_2)]\]

Given that the second term is not a function of \(I\), the mispricing will only affect the equilibrium investment through the first term.\(^7\) \((bk - 2)\sqrt{\theta I}\). So we have

\[V(\theta|I(\cdot)) = E_{x_1x_2}\left[-I + x_1 + x_2 + 2\sqrt{\theta I}\right] + \beta(P - (x_1 + x_2 + 2\sqrt{\theta I}))\]

\[= -I + E_{x_1x_2}[x_1 + x_2] + 2\sqrt{\theta I} + \beta(bk - 2)\sqrt{\theta I} + \beta E_{x_1x_2}[a + bx_1 - (x_1 + x_2)]\]

\[= -I + 2\sqrt{\theta I} + \beta(bk - 2)\sqrt{\theta I} + (2 + \beta(b - 2))\mu + \beta a\]

When the firm either does not care about the short-term share price \((\beta = 0)\) or the market correctly prices firm investments \((b = \frac{2}{k})\), the firm makes the first-best investment choices. If neither condition is met, the (mis)pricing is affected by the firm’s investment choice, so the firm

\(^7\)Notice that the second term, while not affected by investment \((I)\), is still a mispricing caused by reporting the aggregate cash flow. This mispricing would induce (operating) inefficiencies if the firm is able to control the timing of the on-going cash flows \((x_1 \text{ and } x_2)\). See Stein 1989.
has an incentive to deviate from the first-best level. In fact, the firm faces conflicting incentives when making its investment decision:

- **Incentive to underinvest.** Given $k \neq 2$, some investment returns are realized in the long-run. Given $0 < \beta < 1$, the firm receives only a fraction (i.e., $(1 - \beta)$ share) of the long-run marginal benefit but must bear the full marginal cost. This leads to underinvestment because the marginal return is positive but decreasing in $I$. (See similar economic forces in Dye 2002, Dye and Sridhar 2004a, 2004b.)

- **Incentive to overinvest.** Given $k \neq 0$, some investment returns are realized in the short-run. Given $0 < \beta < 1$, the firm has an incentive to inflate the first period cash flow ($z_1 = x_1 + k\sqrt{\theta I}$) because the pricing function places a positive weight on $z_1$. In our model, the only way to inflate $z_1$ is to increase investment level ($I$). This leads to overinvestment as the short-run marginal benefit from the investment may be inflated. (See a similar tension in Stein 1989.)

We summarize the investment results with the following theorem.

**Theorem 2** In the basic setup $\Omega = \{z_1\}$, there exists a value for $k$, namely, $k^* \equiv \frac{2}{1 + \alpha}$, such that the linear equilibrium produces the first-best investment level $\delta = 1$. If $k > k^*$, then $\delta > 1$ or the firm overinvests; if $k < k^*$, then $\delta < 1$, or the firm underinvests.

Intuitively, when $k = k^*$, the appropriate response to the short-term investment return ($\frac{2}{k}$) happens to be the same as the appropriate response to the on-going cash flow ($1 + \alpha$). That is, the proportion of the short-term investment return perfectly matches the time-series correlation of cash flows from on-going activities. Thus, even with an aggregated cash flows report, the market price motivates the efficient investment decision.

If $k \neq k^*$, the market reaction to the aggregate cash flow is an average of $\frac{2}{k}$ and $(1 + \alpha)$. When $k > k^*$, the average is higher than $\frac{2}{k}$, which places too high of a weight on the short-term investment return, leading to over-investment. When $k < k^*$, the average is lower, leading to under-investment. In other words, the aggregation of information leads to market mispricing of the firm investment, which leads to sub-optimal investment decisions.
Using this basic model, we can show that (1) both the market response coefficient $b$ and the investment coefficient $\delta$ increases as $\alpha$ increases; (2) if $k < k^*$, both $b$ and $\delta$ increases as the on-going cash flows are less noisy (i.e., $\sigma^2$ decreases); and (3) as the short term pressure $\beta$ increases, market response $b$ decreases and firm investment coefficient decreases if $k < k^*$. (See Proposition 1 in the appendix for details.)

### 1.4 Accounting

Now we expand the information set which is available to the market. We are particularly interested in how information contained in the non-cash component of the financial statements affects the market pricing and the induced investment incentives. We model this by introducing a public signal $y$, which is produced by the accounting information system of the firm. We assume that on date-1 both the realized cash flows ($z_1$) and the accounting signal ($y$) are publicly observable. The general idea is that $y$ may provide additional information beyond an aggregate cash flow report ($z_1$) and in particular, $y$ may help differentiate the individual components of cash flows.\(^8\)

#### 1.4.1 Input-based and Output-based Accounting Measures

In accounting practice, two broad accounting measurement bases dominate how accounting deferrals/accruals are prepared. First, under an input-based measurement basis, accounting metrics are prepared to be estimates of the effort (or costs) expended in various firm activities. The historical (exchange) cost principle reflects this approach well. Ready examples are long-lived assets and inventory, where book values are based on acquisition costs. Second, under an output-based measurement basis, accounting metrics are prepared to be estimates of the expected reward in return (for the costly activities). The fair value principle reflects this approach well. Ready examples are market value methods where assets and liabilities are measured at market value or based on an estimate of the expected NPV of future cash flows (which is designed to simulate a would-be market value). There is a current debate between preparing accounts according to the traditional historical cost approach and moving to a fair value approach. We study both

---

\(^8\)This idea is consistent with some recognizable features of certain timing accruals. For example, the unearned revenue accruals help classify the timing properties of cash inflow, and the extraordinary-item category may help distinguish components of realized cash flows with different serial correlations.
accounting systems and examine the effect of alternative accounting reports on the investment efficiency. We assume that the firm can choose either an input-based approach, labeled historical cost accounting (HC), or an output-based approach, labeled fair value accounting (FV).\footnote{Here we have limited our attention to a single, one-time measurement. In practice, accounting measurement systems can be much more complex with an initial measurement, subsequent (date-2) re-valuation, and a final measurement on the disposal of the item in question. A dynamic model with multiple information arrivals would make it possible to model these issues, and it is certainly an interesting extension of the current model.}

Denote the signal produced by an output-based measure $y^{FV}$, and assume that

$$y^{FV} = k\sqrt{\theta I} + \varepsilon^{FV}$$

where $\varepsilon^{FV} \sim N(0, \sigma^2_{FV})$. The fair value accounting report provides a noisy measurement of the short term investment return.\footnote{For simplicity, we choose the short term return as the expected value of a fair value accounting report. Our results do not change if we assume that the report is scaled up to provide a noisy measurement of the total investment return (i.e., $y^{FV} = 2\sqrt{\theta I} + \varepsilon^{FV}$).}

For the input-based measure, denote the report $y^{HC}$, and assume that

$$y^{HC} = I + \varepsilon^{HC}$$

where $\varepsilon^{HC} \sim N(0, \sigma^2_{HC})$. The historical cost accounting report provides a noisy measurement of the investment cost. In the following, to simplify the notation, we denote the accounting policy by $m$, $m \in \{FV, HC\}$.

1.4.2 Equilibria under Alternative Accounting Regimes

We now analyze the equilibrium behavior of the firm under both fair value accounting and historical cost accounting.

**Theorem 3** If $y^{FV} = k\sqrt{\theta I} + \varepsilon^{FV}$ and $y^{HC} = I + \varepsilon^{HC}$, using (6), there exists a unique linear equilibrium relative to $\Omega = \{z_1, y^m\}$ ($m \in \{FV, HC\}$) and it is given by...
(i) an equilibrium linear pricing function:

\[
P(z_1, y^m) = a^m + b^m_z \times z_1 + b^m_y \times y^m, \quad \text{where} \]

\[
b^FV_z = \frac{(1 + \alpha)k^2\delta_{FV} \sigma^2 \sigma^2_{\theta} + (1 + \alpha)\sigma^2 \sigma^2_{FV} + 2k\delta_{FV} \sigma^2_{FV} \sigma^2_{\theta}}{k^2\delta_{FV} \sigma^2 \sigma^2_{\theta} + \sigma^2 \sigma^2_{FV} + k^2\delta_{FV} \sigma^2_{FV} \sigma^2_{\theta}}
\]

\[
b^FV_y = \frac{[2 - (1 + \alpha)k]k\delta_{FV} \sigma^2 \sigma^2_{\theta}}{k^2\delta_{FV} \sigma^2 \sigma^2_{\theta} + \sigma^2 \sigma^2_{FV} + k^2\delta_{FV} \sigma^2_{FV} \sigma^2_{\theta}}
\]

\[
a^FV = (2 - b^FV_z)\mu + (2 - kb^FV_z - kb^FV_y)\sqrt{\delta_{FV}\theta_0}
\]

\[
b^{HC}_z = \frac{(1 + \alpha)\delta^2_{HC} \sigma^2 \sigma^2_{\theta} + (1 + \alpha)\sigma^2 \sigma^2_{HC} + 2k\delta_{HC} \sigma^2_{HC} \sigma^2_{\theta}}{\delta^2_{HC} \sigma^2 \sigma^2_{\theta} + \sigma^2 \sigma^2_{HC} + k^2\delta_{HC} \sigma^2_{HC} \sigma^2_{\theta}}
\]

\[
b^{HC}_y = \frac{[2 - (1 + \alpha)k]\delta^3_{HC} \sigma^2 \sigma^2_{\theta}}{\delta^2_{HC} \sigma^2 \sigma^2_{\theta} + \sigma^2 \sigma^2_{HC} + k^2\delta_{HC} \sigma^2_{HC} \sigma^2_{\theta}}
\]

\[
a^{HC} = (2 - b^{HC}_z)\mu + (2 - kb^{HC}_z - \sqrt{\delta_{HC}b^{HC}_y})\sqrt{\delta_{HC}\theta_0}
\]

(ii) an equilibrium investment function:

\[
I^m(\theta) = \begin{cases} 
\delta_m \theta, & \text{if } \theta \geq 0 \\
0 & \text{if } \theta < 0
\end{cases}, \quad \text{where} \]

\[
\delta_{FV} = \left(1 - \beta + \frac{\beta k (b^FV_z + b^FV_y)}{2}\right)^2, \quad \delta_{HC} = \left(\frac{1 - \beta + \frac{\beta b_{HC}}{2}}{1 - \beta b_{HC}}\right)^2
\]

To gain some insights into the results, we make the following observations.

- If \( k = k^* = \frac{2}{1 + n} \), the linear equilibrium under either accrual accounting system produces the first-best investment level \( \delta_m = 1 \) and the market response coefficients \( b^m_z = \frac{2}{k} \), \( b^m_y = 0 \) (\( m \in \{FV, HC\} \)). In this case, the cash flow \( z_1 \) provides sufficient information for efficient pricing, and the market ignores the accruals completely (\( b^m_y = 0 \)).

- If \( \sigma^2_m \rightarrow +\infty \), the equilibrium is the same as the basic setup (\( \Omega = \{z_1\} \)). The quality of accruals is so poor that the market ignores the accounting signals (\( b^m_y = 0 \)), which is equivalent to a setting without accounting reports.

- The direction of the response to accounting signals (i.e., the sign of \( b^FV_y \) or \( b^HC_y \)), can be
positive or negative depending on the sign of \([2 - (1 + \alpha)k]\).

The following Corollary summarizes the intuitive properties of the equilibrium under either accounting regime.

**Corollary 1** If \(y^{FV} = k\sqrt{\bar{y}t} + \varepsilon^{FV}\) and \(y^{HC} = I + \varepsilon^{HC}\), using (6), for any \(m \in \{FV, HC\}\),

(i) if \(\sigma^2_m \to +\infty\), the investment choice approaches that in the basic setup where \(\Omega = \{z_1\}\);

(ii) \(\delta_m\) increases (decreases) in \(\sigma^2_m\) when \(k > k^*\) (\(k < k^*\)).

(iii) \(b^m_z\) decreases (increases) in \(\sigma^2_m\) when \(k > k^*\) (\(k < k^*\)).

(iv) \(b^m_y\) increases (decreases) in \(\sigma^2_m\) when \(k > k^*\) (\(k < k^*\)).

This corollary confirms an intuitive relation between measurement noise and investment efficiency. Recall that Theorem 2 shows that the aggregation of on-going and investment cash flows induces the sub-optimal investment. The combination of items (i) and (ii) of Corollary 1 indicates that the sub-optimal investment problem is alleviated by the accounting report. For example, when \(k < k^*\) and \(\sigma^2_m \to +\infty\), we know \(\delta_m\) is the same as in the basic setup and too low compared to the first-best. In this case, item (ii) means a lower \(\sigma^2_m\) (than \(+\infty\)) would induce a higher \(\delta_m\), alleviating the under-investment problem.

When the accounting quality is extremely poor (\(\sigma^2_m \to +\infty\)), no valuation weight is placed on the accounting signals (\(b^m_y = 0\)). As the quality improves, more weight is shifted between the cash flow report and the accounting report. For example, when \(k < k^*\), the market under-prices the investment. As \(\sigma^2_m\) is lowered (from \(+\infty\)), the market response to the accounting signal increases (from zero) and the response to the cash report decreases (i.e., the weight shifts from cash flow to accruals).

Overall, the results thus far show that, when the noise level is high enough, an improvement in the quality of the accounting measures (i.e., a drop in the noise) improves the communication between the firm and the market, and this benefits the investment efficiency. That is, the quality of accruals is well-defined and well-behaved: the lower the variance, the higher the accrual quality, the lower the market mispricing of firm investments, and most importantly, the more efficient the investment decision.
1.4.3 Comparing Output-based and Input-based Accounting Measures

When the noise level is low, the output-based and input-based measures exhibit a fundamental difference. We find a monotonic relation between the investment efficiency and the quality of the fair value accounting signal. That is, the investment efficiency under fair value accounting continues to improve when measurement noise decreases. As $\sigma^2_{FV}$ decreases, $\delta_{FV}$ approaches toward the first-best regardless of the magnitude of the $k$.

In the extreme, fair value accounting achieves the first-best result as $\sigma^2_{FV}$ is reduced to zero. If $\sigma^2_{FV} = 0$, the market is able to infer the short-term investment return perfectly from the accrual report. Subtracting the accounting signal from the aggregate cash flow reveals the cash flow from first period on-going activities. That is, the accounting report helps the investors clearly distinguish the cash flow components. In turn, the market response coefficients are $b^F_{z} = 1 + \alpha$ and $b^F_{y} = \frac{2}{k} - (1 + \alpha)$, leading to a combined reaction of $\frac{2}{k}$ to the short-term investment return, which provides the first-best investment incentive.

In contrast, historical cost accounting is another story in which the results are not as straight-forward. The relation between investment efficiency and measurement noise ($\sigma^2_{HC}$) is not monotonic. When $k > k^*$, the over-investment problem exists only when $\sigma^2_{HC}$ is high enough. If $\sigma^2_{HC}$ is very low, the firm underinvests. In the extreme, when $\sigma^2_{HC} = 0$, outsiders are able to infer the actual investment made ($I$); the market response coefficients are $b^H_{z} = 1 + \alpha$, and $b^H_{y} = [2 - (1 + \alpha)k]\delta_{HC}^{-\frac{1}{2}}$, and the investment level $\delta_{HC} = \left(1 - \beta \left(\frac{k(1+\alpha)}{2} - 1\right)\right)^2 < 1$.

The following theorem summarizes and compares the effects of the two accounting systems on the investment efficiency.

**Theorem 4** If $y^FV = k\sqrt{\theta I} + \varepsilon^{FV}$ and $y^{HC} = I + \varepsilon^{HC}$,

(i) if $\sigma^2_{FV} = 0$, investment choice is the first-best level, and

(ii) if $\sigma^2_{HC} = \sigma^2_{\theta}$, investment choice is the first-best level.

(iii) There exists a $\Sigma$ ($0 < \Sigma < \sigma^2_{\theta}$), such that a sufficient condition for historical cost accounting to be more (less) efficient than the cash flow accounting regime (of the basic setup) is $\sigma^2_{HC} > \Sigma$ ($\sigma^2_{HC} < \Sigma$).

(iv) There exists a $\Sigma'$ ($\Sigma < \Sigma' < \sigma^2_{\theta}$), such that a sufficient condition for historical cost accounting to be more efficient than fair value accounting is the combination of (a) $\sigma^2_{HC} \in [\Sigma', \sigma^2_{\theta}]$. 

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\( \sigma_0^2 \) and (b) \( \sigma_{FV}^2 > \Sigma' \).

The two systems achieve the first-best at different noise levels. Under fair value, the first-best is achieved when the accounting measure is noiseless, which is very intuitive. Under the historical cost system, the first-best is achieved when the noise is small but not zero. More importantly, fair value does not dominate historical cost in all situations. Since some noise is unavoidable in practice, it is likely historical cost may be preferable. Consider the following two comparisons. First, suppose an accounting item in question is well-understood and easy to measure. So we assume both fair value and historical cost measures share the same (small) noise level (i.e., less than \( \sigma_0^2 \)). In this case, Theorem 4 predicts historical cost is preferred if the variance of the noise (\( \sigma_m^2 \)) is between \( \Sigma \) and \( \sigma_0^2 \). Alternatively, suppose the accounting item is not well-understood and hard to measure. So we assume the noise level is high for both measures (i.e., greater than \( \sigma_0^2 \)). In this case, it is more likely that fair value accounting may entail a more noisy measure than historical cost.\(^{11}\) As a result, the investment efficiency under a highly noisy fair value measure may be closer to the (benchmark) cash flow setting (by Corollary 1) while the efficiency under a not-so-noisy historical cost measure may be closer to first-best (by Theorem 4). In this case, historical cost dominates fair value as long as their variance difference is large enough.

The key difference between historical cost and fair value has to do with the fundamental difference between the two accounting approaches. The historical cost method (\( y^{HC} = I + \varepsilon^{HC} \)) requires estimating the investment cost (\( I \)) alone, without any explicit attention to the profitability of the investment (\( \theta \)). The fair value method (\( y^{FV} = k\sqrt{\theta I} + \varepsilon^{FV} \)) requires estimating both \( I \) and \( \theta \). This fundamental difference leads to a structural difference in the market mispricing of firm investments.

Returning to the analysis of investment distortion induced by mispricing, substitute the pricing function (\( P = a^{FV} + b^{FV} (x_1 + k\sqrt{\theta I}) + b_y^{FV} (k\sqrt{\theta I} + \varepsilon^{FV}) \)) into the expected mispricing,

\(^{11}\) One way to view the natural relation between \( \sigma_{HC}^2 \) and \( \sigma_{FV}^2 \) is that the accounting system constructs the fair value measure based on two estimates: an estimate of actual investment made (e.g., \( y^{HC} = \hat{I} = I + \text{noise} \)) and an estimate of profitability (e.g., \( \hat{\theta} = \theta + \text{noise} \)). The fair value measure is estimated using the true production function (roughly, \( y^{FV} = 2\sqrt{\theta I} \)). Viewed this way, it is natural that the overall noise in \( y^{FV} \) is likely to be higher than the noise in \( y^{HC} \).
and we have

\[
E_{x_1x_2}[P - (x_1 + x_2 + 2\sqrt{\theta I})|\theta] = E_{x_1x_2}[a^{FV} + b_z^{FV} (x_1 + k\sqrt{\theta I}) + b_y^{FV} (k\sqrt{\theta I} + \varepsilon^{FV}) - (x_1 + x_2 + 2\sqrt{\theta I})|\theta] \\
= [(b_z^{FV} + b_y^{FV}) k - 2] \sqrt{\theta I} + E_{x_1x_2}[a^{FV} + b_z^{FV} x_1 + b_y^{FV} \varepsilon^{FV} - (x_1 + x_2)]
\]

If \((b_z^{FV} + b_y^{FV}) k - 2 \neq 0\), any investment will affect the market pricing, giving the firm an incentive to over- or under-invest. The marginal effect of investment \((I)\) on the mispricing depends on investment profitability (the derivative equals \(\frac{(b_z^{FV} + b_y^{FV}) k - 2}{2} \sqrt{\frac{\theta}{T}}\)). This leads to a dampening effect: the marginal benefit is concave in \(I\), providing a diminishing return to investment deviations. Intuitively, the firm’s ability to use real investment to change the market perception of its investment profitability is mitigated by the independent profitability estimate built into the output-based measure.

With historical cost accounting, the expected mispricing is

\[
E_{x_1x_2}[P - (x_1 + x_2 + 2\sqrt{\theta I})|\theta] = E_{x_1x_2}[a^{HC} + b_z^{HC} (x_1 + k\sqrt{\theta I}) + b_y^{HC} (I + \varepsilon^{HC}) - (x_1 + x_2 + 2\sqrt{\theta I})|\theta] \\
= (b_z^{HC} k - 2) \sqrt{\theta I} + b_y^{HC} I + E_{x_1x_2}[a^{HC} + b_z^{HC} x_1 + b_y^{HC} \varepsilon^{HC} - (x_1 + x_2)]
\]

The mispricing will only affect the equilibrium investment through the first two terms: \((b_z^{HC} k - 2) \sqrt{\theta I} + b_y^{HC} I\). The marginal effect of investment on the first term \((b_z^{HC} k - 2) \sqrt{\theta I}\) depends on the investment profitability (the derivative is \(\frac{b_z^{HC} k - 2}{2} \sqrt{\frac{\theta}{T}}\)) while the marginal effect on the second term \(b_y^{HC} I\) depends only on an equilibrium constant \(b_y^{HC}\). The dampening effect is active only on the first term, not the second term. Notice, from Theorem 3, that we know in equilibrium,

\[
b_z^{HC} k - 2 = \frac{[(1 + \alpha)k - 2] (\sigma^2 \sigma_{HC}^2 + \delta_{HC}^2 \sigma_{\theta}^2)}{\partial_{HC}^2 \sigma_{\theta}^2 + \sigma_{HC}^2 \sigma_{\theta}^2 + k^2 \delta_{HC} \sigma_{HC}^2 \sigma_{\theta}^2} \\
b_y^{HC} = \frac{[2 - (1 + \alpha)k] \delta_{HC}^2 \sigma_{\theta}^2}{\partial^2_{HC} \sigma_{\theta}^2 + \sigma^2_{HC} \sigma_{\theta}^2 + k^2 \delta_{HC} \sigma_{HC}^2 \sigma_{\theta}^2}
\]
As $k \neq k^*$, the sign of $b_z^{HC} k - 2$ is always the opposite of the sign of $b_y^{HC}$, which indicates that the marginal effects on the two terms are in the opposite direction. The total effects on mispricing depend on which item outweighs the other.

For example, if $k > k^*$, $b_z^{HC} k - 2$ is positive which indicates that a higher investment would increase market mispricing. However, the dampening effect provides a diminishing return to over-investment which makes over-investment less attractive. On the other hand, the firm is also motivated to under-invest because $b_y^{HC}$ is negative. Notice here that the marginal effect is a constant and is independent of the private information $\theta$ and investment level $I$; no dampening is in effect. This hurts the economy when the accounting report is too precise. If $\sigma_{HC}^2$ is too small, the absolute value of $b_y^{HC}$ is too large, which motivates the firm to under-invest by a large amount (Corollary 1). This motivation outweighs the over-investment motivation by the first item because a small $\sigma_{HC}^2$ reduces the market response to the aggregate cash flows report ($b_z^{HC}$).

Consider the limiting case, when $\sigma_{HC}^2 = 0$. Unlike the case for fair value accounting, the first-best investment is not achieved when the historical cost measure is noiseless. Suppose the firm invests the first-best amount (e.g., $I = \theta$), then the market’s best responses are $b_z^{HC} = 1 + \alpha$ and $b_y^{HC} = 2 - (1 + \alpha)k < 0$. These responses invite the firm to under-invest because at $I = \theta$, the marginal benefit of additional investment is $\frac{b_z^{HC} k - 2}{2} + b_y^{HC} = 1 - \frac{(1 + \alpha)k}{2}$, which is less than the marginal cost of additional investment ($= 1$).\(^{12}\)

In another knife-edge case, the two opposite effects exactly offset each other where the first-best is achieved. That is, if $\sigma_{HC}^2 = \sigma_\theta^2$ and we propose that $\delta_{HC} = 1$, then we find that $\frac{b_z^{HC} k - 2}{2} + b_y^{HC} = 0$ (i.e., the marginal effect of any investment deviation is zero). In equilibrium, there is no incentive to distort investment. Thus, with the input-based measure, the first-best is achieved when measurement noise is small but not zero. Table 1 summarizes the performance

\(^{12}\)Further, in this limiting case of perfect knowledge of the actual investment made ($I$), the induced investment efficiency is worse than that of the basic setup (item iv of the Theorem 4). Alternatively, if the profitability ($\theta$) is perfectly revealed and $z$ is reported, it can be shown that the induced efficiency is not first-best. This is consistent with Kanodia et al (2005) where some imprecision in the accounting measurement is preferred. What is different in the current model is that the conclusion on accounting imprecision depends on the accounting measurement basis. With an output-based measure, the ideal accounting is noiseless.
of three accounting regimes

Table I: The performance of three accounting regimes

<table>
<thead>
<tr>
<th>Accounting Regimes</th>
<th>$\beta = 0$</th>
<th>Parameter Regions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k = k^* \equiv \frac{2}{1+\alpha}$</td>
<td>$0 &lt; \beta &lt; 1$</td>
</tr>
<tr>
<td></td>
<td>$k &gt; k^*$</td>
<td>$k &lt; k^*$</td>
</tr>
<tr>
<td>Cash Flow</td>
<td>$\delta = 1$</td>
<td>$\delta = 1$</td>
</tr>
<tr>
<td></td>
<td>$\delta &gt; 1$</td>
<td>$\delta &lt; 1$</td>
</tr>
<tr>
<td>Fair Value</td>
<td>$\delta_{FV} = 1$</td>
<td>$\delta_{FV} = 1$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{FV}^2 = 0$</td>
<td>$\sigma_{FV}^2 &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>$\delta_{FV} &gt; 0$</td>
<td>$\delta_{FV} &lt; 1$</td>
</tr>
<tr>
<td>Historical Cost</td>
<td>$\delta_{HC} = 1$</td>
<td>$\delta_{HC} = 1$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{HC}^2 = \sigma_{\theta}^2$</td>
<td>$\sigma_{HC}^2 \leq \sigma_{\theta}^2$</td>
</tr>
<tr>
<td></td>
<td>$\delta_{HC} \leq 1$</td>
<td>$\delta_{HC} = 1$</td>
</tr>
</tbody>
</table>

Figure 1 illustrates the effect of accounting reports on the efficiency. Here, the expected net project return represents the efficiency of the investment. The FB line stands for the net project return when the investment level is the first-best ($I^{FB} = \theta$). The basic setting results ($I^{SB} = \delta\theta$) are denoted by the SB-CF line. The performance of historical cost accounting ($I^{HC} = \delta_{HC}\theta$) and fair value accounting ($I^{FV} = \delta_{FV}\theta$) is described by the IP and OP solid curves respectively. From the figure, it is easy to see that fair value accounting is dominated by historical cost accounting in the region when the common noise is between $\Sigma$ and $\sigma_{\theta}^2$.13

---

13When the noise is high (i.e., greater than $\sigma_{\theta}^2$), we can show that there exist sufficient conditions that the IP curve and the OP curve will cross at least once. That is, when the noise is high, historical cost does not necessarily dominate fair value when both share a common measurement noise.
1.5 Extension I: Accounting Manipulation

In this section we expand the model to consider managerial manipulation of the accounting measurement. A robust feature of any accrual measurement is that firms have varying degrees of influence (or discretion) on how accruals are prepared. However, other economic factors (e.g., auditing or managerial reputation) prevent the use of complete discretion. We capture this partial discretion by considering a simple model of cost-benefit calculus on the part of the firm.

1.5.1 Equilibrium under Accounting Manipulation

Suppose that the accounting signal \( y^m \) \((m \in \{FV, HC\})\) is subjected to managerial manipulation. A firm can prepare its accounting report \( w^m \) differently from the unmanipulated \( y^m \), at a cost. We assume the cost as \( c(w^m) = c^m \frac{1}{\Sigma} (w^m - y^m - \xi^m)^2 \), where \( \xi^m \) is independent of all other random variables and follows a normal distribution with mean zero and variance of \( \eta^2_m \). Variable \( \xi^m \) captures the random component of manipulation costs.\(^{14}\) From the earlier results, the market response to the accounting report can be positive or negative. Then, with accounting manipulation, the firm can benefit from adjusting the accounting report upwards or downwards, at the margin. We next introduce the definition of an equilibrium for the setup with accounting manipulation.

\(^{14}\)See a similar assumption and more discussions of the cost structure in Dye and Sridhar 2004a.
Definition 2 An equilibrium relative to $\Omega = \{z_1, w^m\}$ consists of an investment function $I^m(\cdot)$, a reporting policy $w^m(\cdot)$, and a market pricing function $P(\cdot)$, such that:

(i) Given $P(\cdot)$, the optimal investment function $I^m(\cdot)$ and the reporting policy $w^m(\cdot)$ maximize $V(\theta|I^m(\cdot), w^m(\cdot)) = E[-I^m + \beta P(\cdot) + (1 - \beta)(z_1 + z_2) - c(w^m)]$

(ii) Given $I^m(\cdot)$ and $w^m(\cdot)$, the pricing function $P(\cdot)$ satisfies $P = E[z_1 + z_2|\Omega, I^m(\cdot), w^m(\cdot)]$

We now analyze the equilibrium behavior of the firm under the accrual basis accounting with manipulation.

Theorem 5 If $y^{FV} = \sqrt{\beta I} + \varepsilon^{FV}$ and $y^{HC} = I + \varepsilon^{HC}$, and $c(w^m) = \frac{c^m}{2}(w^m - y^m - \xi^m)^2$ ($m \in \{FV, HC\}$), where $k \in (0, 2)$ and using (6), there exists a unique linear equilibrium relative to $\Omega = \{z_1, w^m\}$. It is given by

(i) an equilibrium linear pricing function:

$$P(z_1, w^m) = a^m_w + b^m \times z_1 + d^m \times w^m,$$

where

$$b^{FV} = \frac{(1 + \alpha)k^2\gamma_{FV}\sigma^2_\theta^2 + (1 + \alpha)\sigma^2v^2_{FV} + 2k\gamma_{FV}v^2_{FV}\sigma^2_\theta}{k^2\gamma_{FV}\sigma^2_\theta^2 + \sigma^2v^2_{FV} + k^2\gamma_{FV}v^2_{FV}\sigma^2_\theta}$$

$$d^{FV} = \frac{[2 - (1 + \alpha)k] k^2\gamma_{FV}\sigma^2_\theta^2 + \sigma^2v^2_{FV} + k^2\gamma_{FV}v^2_{FV}\sigma^2_\theta}{k^2\gamma_{FV}\sigma^2_\theta^2 + \sigma^2v^2_{FV} + k^2\gamma_{FV}v^2_{FV}\sigma^2_\theta}$$

$$a^{FV}_w = (2 - b^{FV})\mu + (2 - kb^{FV} - kd^{FV})\sqrt{\gamma_{FV}\theta_0} - \frac{\beta(b^{FV})^2}{c^{FV}}$$

$$b^{HC} = \frac{(1 + \alpha)\gamma^2_{HC}\sigma^2_\theta^2 + (1 + \alpha)\sigma^2v^2_{HC} + 2k\gamma_{HC}v^2_{HC}\sigma^2_\theta}{\gamma^2_{HC}\sigma^2_\theta^2 + \sigma^2v^2_{HC} + k^2\gamma_{HC}v^2_{HC}\sigma^2_\theta}$$

$$d^{HC} = \frac{[2 - (1 + \alpha)k] \gamma^3_{HC}\sigma^2_\theta^2}{\gamma^2_{HC}\sigma^2_\theta^2 + \sigma^2v^2_{HC} + k^2\gamma_{HC}v^2_{HC}\sigma^2_\theta}$$

$$a^{HC}_w = (2 - b^{HC})\mu + (2 - kb^{HC} - \sqrt{\gamma_{HC}d^{HC}})\sqrt{\gamma_{HC}\theta_0} - \frac{\beta(d^{HC})^2}{c^{HC}}$$

$$v^2_m = \sigma^2_m + \eta^2_m$$

(22)
(ii) an equilibrium investment function and an equilibrium reporting policy:

\[ I_w^m(\theta) = \begin{cases} \gamma_m \theta, & \text{if } \theta \geq 0 \\ 0 & \text{if } \theta < 0 \end{cases} \]  \hspace{1cm} \text{where}  \hspace{1cm} \phi \leq \psi \leq \theta \leq \phi

\[ \gamma_{FV} = \left(1 - \beta + \frac{\beta k (b_{FV} + d_{FV})}{2}\right)^2, \quad \gamma_{HC} = \left(1 - \beta + \frac{\beta k b_{HC}}{2} \right)^2 \]

\[ w^m = y^m + \xi^m + \frac{\beta d^m}{e^m} \]

According to the equilibrium reporting policy (the \( w^m \) expression in equation 25), the equilibrium accounting report varies from the one without accounting manipulation: the additional noise from a random variable \( \xi^m \) is added as well as a fixed constant \( \frac{\beta d^m}{e^m} \).

The market can perfectly calculate the fixed constant \( \frac{\beta d^m}{e^m} \). (Parameters \( \beta \) and \( e^m \) are common knowledge, and \( d^m \) is the equilibrium market response to the accounting report.) The intercept of the pricing function \( a^m_w \) is adjusted accordingly to "undo" the expected manipulation. The random component of manipulation injects noise \( \xi^m \) into the accounting report. As a result, accounting manipulation worsens the quality of accounting reports. The pricing function is adjusted in a way that the variance of the measurement noise (\( \sigma^2_m \)) in the previous equilibrium pricing function is replaced by \( v^2_m \), which equals to the sum of \( \sigma^2_m \) and \( \eta^2_m \).

1.5.2 Value of Accounting Manipulation

The accounting manipulation leads to a more noisy accounting measure and a dead-weight loss (the manipulation cost to the firm). The latter cost is incorporated into the following cost-benefit analysis of accounting manipulation.

**Corollary 2** Under fair value, accounting manipulation always makes the firm worse off; under historical cost accounting, a sufficient condition for accounting manipulation to be value-enhancing is the combination of (i) \( v^2_{HC} < \sigma^2_0 \) and (ii) \( e^{HC} : \theta_0 \) is sufficiently high.

As we have shown, the performance of fair value accounting monotonically decreases as the quality of the accounting report worsens. Therefore, under fair value, accounting manipulation
reduces efficiency for two reasons. First, a more noisy accounting report induces less efficient investment. Second, the firm incurs a manipulation cost $c(w^m)$.

However, with historical cost accounting, the non-monotonicity feature makes it possible that the incremental noise due to manipulation may benefit the firm. Recall that in Figure 1, making the accruals more noisy can improve efficiency when the accruals are "too" precise ($\sigma_{HC}^2 < \sigma_{\theta}^2$). Thus, if $v_{HC}^2 < \sigma_{\theta}^2$, the expected net project return strictly increases as a result of accounting manipulation, provided that $\eta_{HC}$ is not too large.

Since the expected net project return is $(2\sqrt{T_m} - \gamma_m) \cdot \theta_0$, the higher $\theta_0$ can magnify the gain from more efficient investment choices. The cost of manipulation is

$$c(w^{HC}) = \frac{c_{HC}}{2} \left( w^{HC} - y^{HC} - \xi^{HC} \right)^2 = \frac{(\beta d^{HC})^2}{2c^{HC}}$$

A higher $c^{HC}$ can reduce the dead-weight loss from the firm’s myopic decision. Thus, if $c^{HC} \cdot \theta_0$ is sufficiently high, the cost of earnings management is outweighed by the increase of the expected investment returns.\(^{15}\)

Intuitively, when the actual investment made is measured too precisely, the market pricing places too much valuation weight on the accounting report, providing an unmitigated incentive to over- or under-invest. By allowing accounting manipulation, more noise is injected into accounting reports and the market reacts by reducing the valuation weight. As a result, less pressure leads to less inefficient investment choices.\(^{16}\) Table 2 summarizes the effect of accounting

---

\(^{15}\)It might also be interesting to compare the efficiency of discretionary accounting reports with the basic cash flow setting. This takes the view that allowing either fair value or historical cost accounting measurement implicitly grants the firm the ability to manipulate their accounting performance. In other words, accrual accounting and the manipulation option are a bundle.

From Theorem 4, under fair value accounting, the investment is always more efficient than in the cash flow setup, but manipulation always incurs positive costs $c(w^{FV})$. Whether fair value accounting with manipulation is preferable to the cash flow setup depends on the cost incurred and on benefits derived from the investment improvement. The same tension exists under historical cost accounting with manipulation. Specifically, historical cost accounting with manipulation is more efficient than the cash flow setup when (i) $v_{HC}^2 > \Sigma'$ and (ii) $c \cdot \theta_0$ is sufficiently high. On the other hand, historical cost accounting with manipulation is less efficient than the cash flow setup when $v_{HC}^2 < \Sigma'$.

\(^{16}\)This intuition does not follow when fair value is used because when the measurement noise is low, actual investment only partially affects the accounting signal (recall $y^{FV} = k \sqrt{\theta I} + \varepsilon^{FV}$).
Table 2: The effect of accounting manipulation for different accounting regimes

<table>
<thead>
<tr>
<th>Accounting Regimes</th>
<th>Parameter Region</th>
<th>( k &gt; k^* )</th>
<th>( k &lt; k^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fair Value</td>
<td>( \gamma_{FV} &gt; 1, \frac{\partial}{\partial \gamma_{FV}} \gamma_{FV} &gt; 0 )</td>
<td>( \gamma_{FV} &lt; 1, \frac{\partial}{\partial \gamma_{FV}} \gamma_{FV} &lt; 0 )</td>
<td></td>
</tr>
<tr>
<td>Historical Cost</td>
<td>( \frac{\gamma_{HC}^2}{\sigma_{\theta}^2} )</td>
<td>( \frac{\gamma_{HC}}{\sigma_{\theta}} )</td>
<td>( \frac{\gamma_{HC}}{\sigma_{\theta}} )</td>
</tr>
<tr>
<td>( \gamma_{HC} &gt; 1, \frac{\partial}{\partial \gamma_{HC}} \gamma_{HC} &gt; 0 )</td>
<td>( \gamma_{HC} &lt; 1, \frac{\partial}{\partial \gamma_{HC}} \gamma_{HC} &lt; 0 )</td>
<td>( \gamma_{HC} &gt; 1, \frac{\partial}{\partial \gamma_{HC}} \gamma_{HC} &gt; 0 )</td>
<td></td>
</tr>
</tbody>
</table>

1.5.3 "Real" versus "Accounting" Manipulation

The above analysis points to a link between the so-called "real" earnings management and "accounting" earnings management. Real management typically refers to the firm’s discretionary choices which affect the firm’s cash flow for the sole purpose of inflating reported performance. These choices are not in the best interest of the shareholders. Accounting management typically refers to the firm’s discretionary choices which affect the firm’s reported performance by altering the accounting measurement process. In our model, we interpret investment deviations from the first-best as an example of real earnings management and accounting manipulation of \( y \) into \( w \) as accounting earnings management.

Under the output-based measure, our results indicate that accounting earnings management always leads to more real management. They are complements. This is because "accounting" management leads to more mispricing. Under an input-based measure, a similar result is obtained when the measurement noise is high. However, when noise is low, (especially in the region where investment efficiency increases in noise,) accounting earnings management leads to less mispricing, and more efficient investment choices are made. Here accounting management is a substitute for real management. The intuition is that the accounting management introduces additional noise, which leads to less market reaction to the accounting report. Less pressure on the accounting numbers mitigates over- and under-investment incentives.
1.6 Extension II: Technology for Future Investments

In this section we modify the model to include a situation where the firm technology may be re-used in the future. That is, from date-2 onwards, the owners of the firm may generate future cash flow by investing in the firm technology they have acquired on date-1. In other words, the owners have an option to invest and will exercise the option if the technology turns out to be profitable. In turn, on date-1, the capital market not only prices the cash flows generated from the date-0 investment, it also prices the value of owning the technology that produces future cash flows.

1.6.1 Model Modifications

To simplify the problem, we change the model as the following. On date-0, the firm chooses an investment level, denoted $I_1 \in \mathbb{R}^+$, based on the private signal $\theta$, same as before.

On date-1, shares of the firm are traded in a competitive capital market. However, we assume there are no on-going activities\(^{17}\) ($x_1$ and $x_2$) and only one public signal $\lambda^m$ ($m \in \{FV, HC\}$), which is produced by the accounting information system, is observable to outsiders. Similarly, we assume that the firm can choose either an input-based approach ($HC$), or an output-based approach ($FV$) as the accounting measurement bases. For the output-based measure,

$$\lambda^{FV} = 2\sqrt{\theta I_1} + \varepsilon^{FV}$$ \hspace{1cm} (26)

where $\varepsilon^{FV} \sim N(0, \sigma_{FV}^2)$. The fair value accounting reports provide a noisy measurement of the total investment return. For input-based measure,

$$\lambda^{HC} = I_1 + \varepsilon^{HC}$$ \hspace{1cm} (27)

where $\varepsilon^{HC} \sim N(0, \sigma_{HC}^2)$. The historical cost accounting reports provide a noisy measurement of the investment costs.

Finally, on date-2, the owners observe the return of the initial investment $2\sqrt{\theta I_1}$, and choose additional investments into the existing technology. We assume the present value of future cash flows

\(^{17}\)This assumption is made for simplicity only. All results in this section survive if a stochastic on-going activities are introduced as long as, as before, they are independent of other random variables in the model.
flows is $r\theta, r \in \mathbb{R}^+$. A simple interpretation of this representation is that the future profitability of the technology is the same as $\theta$, and the future owners make optimal investment decision, knowing the true $\theta$. In this case, in every period this technology is viable, the owners choose optimal $I_t$ to maximize $2\sqrt{\theta I_t} - I_t$ and generate periodic profits equal to $\theta$, assuming a positive $\theta$. As a result, future profits can be represented by an annuity (or perpetuity). Parameter $r$ summarizes the importance of these future cash flows relative to the cash flow generated by the initial investment. If the technology is long-lived or if the owners’ discount rate is low, more firm value comes from future investments (or "growth opportunities"), leading to a higher $r$.

A more complex, perhaps more realistic, interpretation involves a non-stationary technology, or future firm owners subjected to additional market frictions, or that the true $\theta$ is revealed to the owners gradually through learning-by-doing. However, under these scenarios, it may be reasonable to assume that the value of this re-investment option is proportional to the past profitability $\theta$. As a result, we believe our characterization is a reasonable approximation to capture the idea of this option without bringing in additional complexity to the model.

1.6.2 Equilibrium under Modified Model

Now return to the pricing problem on date-1, without the on-going cash flows, the capital market must estimate the value of the date-2 cash flow generated by past investment and the value of the potential future cashflows generated by future investments. As a result, the market price is equal to the expected value of the cash flow from existing project plus the present value of the reinvestment option, that is

$$P = E[2\sqrt{\theta I_1} + r\theta|\Omega].$$

As before, when making the initial investment on date-0, the firm is motivated by both the long-term interest and the short term interest. The convex combination of these concerns is the same as the basic setup, that is $-I_1 + \beta P + (1 - \beta)(2\sqrt{\theta I_1} + r\theta)$.

We now analyze the equilibrium behavior of the firm with technology for future investments.

**Theorem 6** If $\lambda^{FV} = 2\sqrt{\theta I_1} + \varepsilon^{FV}$ and $\lambda^{HC} = I_1 + \varepsilon^{HC}$, and using (6), there exists a unique linear equilibrium relative to $\Omega = \{\lambda^m\}$. It is given by
(i) an equilibrium linear pricing function:

\[
P(\lambda^m) = a^m_\lambda + b^m_\lambda \times \lambda^m, \quad \text{where} \\
b^F_{\lambda} = \frac{(4\gamma'_{FV} + 2r\sqrt{\gamma'_{FV}})\sigma^2_{\eta}}{4\gamma'_{FV}\sigma^2_{\eta} + \sigma^2_{FV}} \\
a^F_{\lambda} = (2\sqrt{\gamma'_{FV} + r - 2\gamma'_{FV}b^F_{\lambda}})\theta_0 \\
b^H_{\lambda} = \frac{(2\sqrt{\gamma'_{HC} + r})\gamma'^2_{HC}}{\left(\gamma'_{HC}\right)^2\sigma^2_{\eta} + \sigma^2_{HC}} \\
a^H_{\lambda} = (2\sqrt{\gamma'_{HC} + r - \gamma'_{HC}b^H_{\lambda}})\theta_0
\]  \tag{28}

(ii) an equilibrium investment function:

\[
I^m_1(\theta) = \begin{cases} 
\gamma'_m \theta, & \text{if} \quad \theta \geq 0 \\
0 & \text{if} \quad \theta < 0 
\end{cases} \quad \text{where} \\
\gamma'_{FV} = (1 - \beta + \beta b^F_{\lambda})^2, \quad \gamma'_{HC} = \left(\frac{1 - \beta}{1 - \beta b^H_{\lambda}}\right)^2 \tag{29}
\]

With a valuable technology for future investments, the share price includes the market estimate of how much the firm will benefit from the technology ever after. Higher \(r\) indicates the firm is able to generate more future cashflows for given a positive \(\theta\). Thus, the market response coefficient \(b^m_{\lambda}\) is strictly increasing in \(r\). With higher market response coefficient, the firm has incentive to inflate the perceived profitability. This can be achieved by inflating initial investment, which increases the mean of either the input-based measure or the output-based measure. Therefore, the investment decision \(I^m_1\) is strictly increasing in the parameter \(r\).

Since the accounting measurements may not be perfectly precise, the market also responds to the measurement noise. As the variance of measurement noise \(\sigma^2_{m}\) gets higher, the accounting reports are less informative, leading to a less responsive market price to the accounting report and the firm has less incentive to over-invest. Therefore, the initial investment \((I^m_1)\) is strictly decreasing in the measurement noise \(\sigma^2_{m}\). Because the two exogenous parameters \((r \text{ and } \sigma^2_{m})\) provide opposite incentives of investment choices, there exists a knife-edge case that the two opposite effects exactly offset each other, leading to the first-best initial investment.
1.6.3 Comparing Output-based and Input-based Accounting Measures

The following Corollary summarizes the properties of the equilibrium.

**Corollary 3** If \( \lambda^{FV} = 2\sqrt{\theta I_1} + \varepsilon^{FV} \) and \( \lambda^{HC} = I_1 + \varepsilon^{HC} \),

(i) if \( r^2 \sigma_{FV}^2 = 2r \sigma_\theta^2 \), investment choice is the first-best level, and

(ii) if \( \sigma_{HC}^2 = (r+1) \sigma_\theta^2 \), investment choice is the first-best level.

(iii) If \( r > 1(r < 1) \), there exists a \( \Sigma^* \) which lies in the interval between \((r+1) \sigma_\theta^2 \) and \( 2r \sigma_\theta^2 \), such that a sufficient condition for historical cost accounting to be more efficient than fair value accounting is \( \sigma_{HC}^2 = \sigma_{FV}^2 \in [(r+1) \sigma_\theta^2, \Sigma^*] \) \( (\sigma_{HC}^2 = \sigma_{FV}^2 \in [\Sigma^*, (r+1) \sigma_\theta^2]) \).

The presence of the reinvestment option changes the market pricing and (thus) initial investment decisions. Compared with corresponding results in section 4 (see Theorem 4), the results are different in the following regards. First, even with output-based measure, investment efficiency is no longer monotonic in measurement noise. In particular, the output-based accounting measure does not perform best when the measure is noiseless. Second, the parameter \( r \), a growth potential index so to speak, is important in determining economic efficiency, in addition to accounting rules and measurement errors. Intuitively, as \( r \) increases, the market imposes more pressure on the accounting reports, and the firm is motivated to inflate the reports.

To see the trade-off precisely, we briefly review the mispricing structure. Substituting the pricing function \( P = a^{FV}_\lambda + b^{FV}_\lambda \lambda^{FV} \) into the mispricing expression, we have

\[
E_{\varepsilon^{FV}} \left[ P - (2\sqrt{\theta I_1} + r\theta)\theta \right] = E_{\varepsilon^{FV}} \left[ a^{FV}_\lambda + b^{FV}_\lambda \left( 2\sqrt{\theta I_1} + \varepsilon^{FV} \right) - (2\sqrt{\theta I_1} + r\theta)\theta \right] = \left[ b^{FV}_\lambda - 1 \right] 2\sqrt{\theta I_1} - r\theta + a^{FV}_\lambda + E_{\varepsilon^{FV}} [b^{FV}_\lambda \varepsilon^{FV}] \theta
\]

The investment choice is first-best when the mispricing does not depend on the investment choice.

Under the output-based measure, the mispricing is not a function of the initial investment only if the market response to accounting measure is equal to unity (i.e., \( b^{FV}_\lambda = 1 \)). However, in this modified model, a noiseless output-based measure will no longer lead to a unity market response.

---

\(^{18}\)When \( r = 1 \), we can show that fair value accounting is (weakly) more efficient than historical cost accounting once \( \beta \) is small enough.

\(^{19}\)When \( r = 0 \), the model revert back to the basic model with output-based model performs best when noiseless.
This is because, in this modified model, the market is pricing two streams of cashflows. First, for cashflows due to the initial investment, a unity response is needed with a noiseless measure. Second, for cashflows due to future investments, a non-zero response is needed. Combined the total response would be greater than unity in the noiseless case, thus providing ex ante incentive to deviate from the first-best investment.

Figure 2 provides an illustration of results in Corollary 3. Compared to Figure 1, the main difference is that the efficiency under the output-based measure peaks when the variance of the measure is not zero. Furthermore, when \( r > 1 \), the peak occurs to the right of the peak under the input-based measure. This is because, relative to the input-based measure regime, a higher \( r \) imposes more market pressure on the accounting measure and thus leading a more "distorted" investment choice.

![Figure 3: Investment efficiency in the modified model (\( r = 2 \))](image)

Finally, the extension leads us to rethinking the subtleties of output-based accounting when (re-investment) option value is important. From an accounting measurement perspective, \( \lambda^{FV} \) can be viewed as measure of "value in use," ignoring the option value of future use (through future investments). These measures do exist in accounting practices such as the re-valuation exercise in accounting for asset impairment. However, one may argue the option value would be impounded in a would-be exchange price of the asset in question. Fair value accounting measures, as proposed by the recent FASB exposure draft, may be close to having this characteristic. One
can even argue these measures already exist in accounting practices such as the use of market value in initial and re-valuation of certain assets and in the initial recording of goodwill (provided the market prices are reflective of various option values).

1.7 Summary

In this paper, we explore the trade-offs between two dominant accounting measurement bases: input-based measures, such as the historical cost measurement and output-based measures, such as the fair value measurement. We discover that the trade-offs go beyond relevance and reliability issues commonly mentioned in accounting debates. We show that these two measures affect investment incentives in fundamentally different ways. The fair value measures have a natural advantage in aligning the firm and social investment incentives through a dampening effect, which limits over- and under-investment tendencies. However, high levels of noise and accounting manipulation, which are typically associated with fair value measures, may make fair value accounting far from a perfect solution to all accounting problems.

With an input-based accounting measurement basis, such as historical cost measures, accounting numbers are less comprehensive, but the low levels of noise and accounting manipulation typically associated are its advantage. In fact, being less comprehensive makes small but positive noise and/or manipulation desirable. Based on our analysis, the move toward fair value may not be beneficial and requires more care and more extensive debates.

Our model is simple. Future works may benefit from including operating and financial choices and from analyzing more general settings with heterogeneous firms where accounting standards are central to an economic analysis of accounting.
Appendix

Proof. (of Limit Properties of the Approximation Assumption) In this part of the appendix, we show that the approximation error, denoted $AE$, gets smaller and approaches zero as the mean of $z$ increases, for every realized value of the conditioning variable, $y + az$. To prove this formally, notice for every $W = y + az$,

$$AE(W) = \int_{z>0} \int (x+z)f(x, z | y + az)dxdz + G(0 | y + az) \int x f(x | y)dx$$

$$- \int \int (x+z)f(x, z | y + az)dxdz$$

$$= G(0 | y + az) \int x f(x | y)dx - \int \int (x+z)f(x, z | y + az)dxdz \quad (30)$$

and we need to prove both components of the $AE(W)$ expression approach zero as $E[z]$ increases to every realization of $y + az$.

By assumption, the joint distribution of $x$ and $y$ is

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim N \left( \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} \right)$$

and $z \sim N[\mu_z, \sigma_z^2]$ is independent of $x$ and $y$. Hence, the jointly distribution of $x$, $z$ and $y + az$ is

$$\begin{bmatrix} x \\ z \\ y + az \end{bmatrix} \sim N \left( \begin{bmatrix} \mu_x \\ \mu_z \\ \mu_y + a\mu_z \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & 0 & \sigma_{xy} \\ 0 & \sigma_z^2 & a\sigma_z^2 \\ \sigma_{xy} & a\sigma_z^2 & \sigma_y^2 + a^2\sigma_z^2 \end{bmatrix} \right)$$

By the property of normal density function, the conditional distribution of $z$ given any realization of $W = y + az$ is

$$z | W \sim N[\mu_z + \frac{a\sigma_z^2}{\sigma_y^2 + a^2\sigma_z^2} (W - \mu_y - a\mu_z), \frac{\sigma_z^2\sigma_y^2}{\sigma_y^2 + a^2\sigma_z^2}] \quad (31)$$

To simplify the notation, we denote the above by $z | W \sim N[\mu_{z'}, \sigma_{z'}^2]$. And the jointly conditional
distribution of \( x \) and \( z \) given any realization of \( W \) is

\[
\begin{bmatrix}
  x \\
  z \\
\end{bmatrix} | W \sim N \left( \begin{bmatrix}
  \mu_x + \frac{\sigma_{xy}}{\sigma_y^2 + \alpha^2 \sigma_z^2} (W - \mu_y - a \mu_z) \\
  \mu_z + \frac{a^2 \sigma_z}{\sigma_y^2 + \alpha^2 \sigma_z^2} (W - \mu_y - a \mu_z) \\
\end{bmatrix}, \begin{bmatrix}
  \sigma_x^2 - \frac{\sigma_{xy}^2}{\sigma_y^2 + \alpha^2 \sigma_z^2} - \frac{\sigma_{xy} a^2 \sigma_z^2}{\sigma_y^2 + \alpha^2 \sigma_z^2} \\
  \frac{\sigma_{xz}^2}{\sigma_y^2 + \alpha^2 \sigma_z^2} \\
\end{bmatrix} \right)
\]

(32)

Again to simplify, we denote the above by

\[
\begin{bmatrix}
  x \\
  z \\
\end{bmatrix} | W \sim N \left( \begin{bmatrix}
  \mu_x' \\
  \mu_z' \\
\end{bmatrix}, \begin{bmatrix}
  \sigma_{x'}^2 & \sigma_{xz} \\
  \sigma_{xz} & \sigma_{z'}^2 \\
\end{bmatrix} \right)
\]

Using (31), the conditional cumulative density function of \( z \), \( G(z|W) = \Phi \left( \frac{z - \mu_z'}{\sigma_z'} \right) \), where \( \Phi(\cdot) \) is the standard normal cumulative distribution function. Hence, \( G(0|W) = \Phi \left( \frac{-\mu_z'}{\sigma_z'} \right) \). By definition, \( \Phi \left( \frac{-\mu_z'}{\sigma_z'} \right) \rightarrow 0 \) as \( \frac{-\mu_z'}{\sigma_z'} \rightarrow -\infty \). Since \( \mu_z' = \frac{\sigma_{xz}^2}{\sigma_z'} \mu_z + \frac{a^2 \sigma_z^2}{\sigma_z'} (W - \mu_y) \), for any given \( W \), as \( \mu_z \) increases, \( \frac{-\mu_z'}{\sigma_z'} \) goes to negative infinity. That is \( G(0|W) \int_x x f(x|y)dx \), the first component of (30) goes to zero.

Using (32), the second component of (30) is

\[
\int_{z<0} \int_x (x + z) f(x, z | W \equiv y + az) dx dz
\]

\[
= \int_{z<0} \int_x (x + z) f(z|W) f(x|z, W) dx dz
\]

\[
= \int_{z<0} \left[ \int_x x f(x|z, W) dx + z \int_x f(x|z, W) dx \right] f(z|W) dz
\]

\[
= \int_{z<0} \left[ \mu_x' + \frac{\sigma_{xz}}{\sigma_z'} (z - \mu_z') + z \right] f(z|W) dz
\]

\[
= \left[ \mu_x + \frac{\sigma_{xy}}{\sigma_y^2} (W - \mu_y) \right] G(0|W) + \left( 1 - \frac{a \sigma_{xy}}{\sigma_y^2} \right) \int_{z<0} z f(z|W) dz
\]

Similar to the first component, the first part of second component, \( \left[ \mu_x + \frac{\sigma_{xy}}{\sigma_y^2} (W - \mu_y) \right] G(0|W) \), also approaches zero when \( \mu_z \) increases.

Finally, we need to show the second part of the second component, \( \int_{z<0} zf(z|W)dz \) goes to zero as \( \mu_z \) increases.

\[
\int_{z<0} zf(z|W)dz = \frac{1}{\sigma_z' \sqrt{2\pi}} \int_{-\infty}^{0} z \exp \left( -\frac{(z - \mu_z')^2}{2\sigma_z'^2} \right) dz
\]

\[
= \frac{1}{\sigma_z' \sqrt{2\pi}} \int_{-\infty}^{0} z \exp \left( -\frac{z^2}{2\sigma_z'^2} \right) \cdot \exp \left( \frac{z \mu_z'}{\sigma_z'} \right) \cdot \exp \left( -\frac{\mu_z'^2}{2\sigma_z'^2} \right) dz
\]

If \( z \) is always negative, \( \exp \left( \frac{z \mu_z'}{\sigma_z'} \right) \) must be positive and always smaller than one. Then the absolute
value of above formula must be smaller than the absolute value of \( \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{0} z \exp \left( -\frac{z^2}{2\sigma^2} \right) \cdot \exp \left( -\frac{\mu^2}{2\sigma^2} \right) \, dz \), which is equal to \( \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{0} z \exp \left( -\frac{z^2}{2\sigma^2} \right) \cdot \exp \left( -\frac{\mu^2}{2\sigma^2} \right) \, dz \). As the mean of \( \mu_z \) increases, \(-\frac{\mu^2}{2\sigma^2}\) goes to negative infinity. Hence the absolute value of \( \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{0} z \exp \left( -\frac{z^2}{2\sigma^2} \right) \cdot \exp \left( -\frac{\mu^2}{2\sigma^2} \right) \, dz \) goes to zero, and that is \( \int_{z<0} zf(z|W) \, dz \) approaches zero.

To summarize, we have shown for every \( W \equiv y + az \), as \( \mu_z \) increases, \( AE(W) \) approaches zero. ■

**Proof. (of Theorem 1)** We begin with the linear pricing conjecture:

\[
P(z_1) = a + bz_1
\]

The manager’s maximization program becomes:

\[
\text{Choose } I(\theta) \text{ to max } \int_{\theta} V(\theta|I(\cdot)) G(\theta) \, d\theta
\]

\[
= \int_{\theta} E_{z_1 z_2} [-I + \beta P + (1 - \beta)(z_1 + z_2)] G(\theta) \, d\theta
\]

\[
= -I + \beta(a + b(\mu + \int_{\theta} k\sqrt{\theta} I G(\theta) \, d\theta)) + (1 - \beta) \left( 2\mu + \int_{\theta} 2\sqrt{\theta} I G(\theta) \, d\theta \right)
\]

The point-wise first-order condition with respect to \( I \) is, for \( \theta > 0 \),

\[
0 = -1 + \frac{(1 - \beta + \frac{\beta k}{2})\sqrt{\theta}}{\sqrt{I}}
\]

\[
I = \left( 1 - \beta + \frac{\beta k}{2} \right)^2 \theta
\]

\[
= \left( 1 + \beta \left( \frac{bk}{2} - 1 \right) \right)^2 \theta
\]

and for \( \theta < 0, I = 0 \). So, it must be the case that

\[
\delta = \left( 1 + \beta \left( \frac{bk}{2} - 1 \right) \right)^2
\]
Given that $\delta$ and $k$ are constants,

\[
\begin{pmatrix}
x_1 + x_2 + 2\sqrt{\delta} \\
x_1 + k\sqrt{\delta}
\end{pmatrix}
\sim 
N\left(
\begin{pmatrix}
2\mu + 2\sqrt{\delta}\theta_0 \\
\mu + k\sqrt{\delta}\theta_0
\end{pmatrix},
\begin{bmatrix}
2(1 + \alpha)\sigma^2 + 4\delta\sigma^2_\theta & (1 + \alpha)\sigma^2 + 2k\delta\sigma^2_\theta \\
(1 + \alpha)\sigma^2 + 2k\delta\sigma^2_\theta & \sigma^2 + k^2\delta\sigma^2_\theta
\end{bmatrix}
\right)
\]

So, we have the approximate pricing function, using (6):

\[
P = E[z_1 + z_2 | z_1] = E[x_1 + x_2 + 2\sqrt{\delta}|x_1 + k\sqrt{\delta}]
\]

\[
= \frac{2\mu + 2\sqrt{\delta}\theta_0 + (1 + \alpha)\sigma^2 + 2k\delta\sigma^2_\theta}{\sigma^2 + k^2\delta\sigma^2_\theta} (z_1 - \mu - k\sqrt{\delta}\theta_0)
\]

\[
= \frac{(1 - \alpha)\sigma^2 + 2(k^2 - k)\delta\sigma^2_\theta}{\sigma^2 + k^2\delta\sigma^2_\theta} \mu + \frac{(2 - (1 + \alpha)k)\sigma^2}{\sigma^2 + k^2\delta\sigma^2_\theta} \sqrt{\delta}\theta_0
\]

\[
+ \frac{(1 + \alpha)\sigma^2 + 2k\delta\sigma^2_\theta}{\sigma^2 + k^2\delta\sigma^2_\theta} z_1
\]

So, it must be the case that

\[
a = (2 - b)\mu + (2 - kb)\sqrt{\delta}\theta_0
\]

\[
b = \frac{(1 + \alpha)\sigma^2 + 2k\delta\sigma^2_\theta}{\sigma^2 + k^2\delta\sigma^2_\theta}
\]

To show the existence of $\delta$ and $b$, substituting $b$ into $\delta$, and simplifying, we have

\[
\left(\sqrt{\delta} - 1\right) \cdot k^2\delta\sigma^2_\theta \left(\frac{\sigma^2}{\sigma^2 + k^2\delta\sigma^2_\theta} + 1\right) = \beta \left[\frac{k(1 + \alpha)}{2} - 1\right]
\]

As $\beta \in [0, 1]$, $k \in [0, 2]$, and $\alpha \in (-1, 1)$, the range of the right-hand-side is from -1 to 1. As $\delta > 0$, and all of the functions are continuous, the left-hand-side covers the range from -1 to $+\infty$. Therefore, there is at least one positive root of $\delta$.

Proof. (of Theorem 2) Suppose the manager chooses the first-best investment $\delta = 1$. Corre-

\[\text{Technically, in some rare cases (when the right-hand-side is very close to -1), the function could have three positive roots of } \delta. \text{ Then, in our study, we only consider the root that is closest to one. That is, in multiple equilibrium cases, the economy chooses the most efficient one.} \]

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spondingly, using (7), (8), and (9) the market pricing function would be

\[
P(z_1) = a + b \delta_1 \times z_1, \text{ where} \\
b \delta_1 = \frac{(1 + \alpha)\sigma^2 + 2k\sigma^2}{\sigma^2 + k^2\delta^2} \times a = (2 - b \delta_1) \mu + (2 - kb \delta_1) \theta_0
\]

To sustain the equilibrium, the manager’s reaction to \(b \delta_1 = (1 + \alpha)\sigma^2 + 2k\sigma^2 \) must indeed be to set \(\delta = 1\), which is the first-best investment level. That is, it must be the case that

\[
\delta|_{b \delta_1 = 1, k = k^*} = 1
\]

Using (11), we have

\[
\delta|_{b \delta_1 = 1, k = k^*} = \left(1 + \beta \left(\frac{b \delta_1 k^*}{2} - 1\right)\right)^2 \equiv \left(1 + \beta \left(\frac{(1 + \alpha)\sigma^2 + 2k\sigma^2}{\sigma^2 + k^2\delta^2} \times \frac{1}{2} - 1\right)\right)^2 \equiv 1
\]

Generally, we re-write \(bk\) as the following

\[
bk = \frac{(1 + \alpha)\sigma^2 + 2k\delta\sigma^2}{\sigma^2 + k^2\delta\sigma^2} \cdot k \\
= 2 + \frac{[(1 + \alpha)k - 2]\sigma^2}{\sigma^2 + k^2\delta\sigma^2}
\]

So

\[
\delta = \left(1 + \beta \left(\frac{bk}{2} - 1\right)\right)^2 \equiv \left(1 + \frac{\beta}{2} \cdot \frac{[(1 + \alpha)k - 2] \sigma^2}{\sigma^2 + k^2\delta\sigma^2}\right)^2
\]

Now it is clear that if \(k > k^*, \delta > 1\), and \(k < k^*, \delta < 1\).

**Proposition 1** *Comparative Statics in the Basic Setup*
1. $\delta$ and $b$ strictly increase in $\alpha$;

2. When $k > k^*(k < k^*)$, $\delta$ and $b$ strictly decrease in $\frac{\sigma_2^2}{\sigma^2}$ (strictly increase in $\frac{\sigma_2^2}{\sigma^2}$); and

3. When $k > k^*(k < k^*)$, $\delta$ strictly increases in $\beta$ (strictly decreases in $\beta$). Also $b$ strictly decreases in $\beta$ for any $k \neq k^*$.

\section*{Proof. (of the proposition)}

Using the pricing function (8), substituting $b$ into the manager's investment decision (11), we have the following equation which has to hold in equilibrium.

$$
(\sqrt{\delta} - 1) \cdot (k^2 \sigma_2^2 + 1) = \beta \left[ \frac{k(1 + \alpha)}{2} - 1 \right]
$$

The left-hand-side increases monotonically in $\delta^{21}$, and is independent of $\alpha$. The right-hand-side increases monotonously in $\alpha$, and is independent of $\delta$. It is easy to see that $\delta$ strictly increases in $\alpha$. By (11), $b$ strictly increases in $\delta$, therefore, $b$ strictly increases in $\alpha$. From Theorem 2, $\delta > 1$ when $k > k^*$. Given the equation above, the right-hand-side is positive as $k > k^*$. As $\frac{\sigma_2^2}{\sigma^2}$ increases, $\delta$ has to decrease to let the equation sustain. Similarly if $k < k^*$, $\delta$ increases in $\frac{\sigma_2^2}{\sigma^2}$. By (11), $b$ changes in the same way as $\delta$. As we show above, the left-hand-side increases monotonously in $\delta$. When $k > k^*$, the right-hand-side increases in $\beta$, so $\delta$ strictly increases in $\beta$. Similarly, $\delta$ strictly decreases in $\beta$, when $k < k^*$, and $\delta$ is equal to one regardless of $\beta$, when $k = k^*$. To analyze the change of $b$, re-write (8) as the following,

$$
b = \frac{2}{k} + \frac{(1 + \alpha) - \frac{2}{k}}{1 + k^2 \sigma_2^2 \sigma^2}
$$

If $k > k^*$, $(1 + \alpha) - \frac{2}{k} > 0$. As $\beta$ increases, $\delta$ increases from the above analysis. Therefore, $b$ decreases in $\beta$. Similarly, $b$ also strictly decreases in $\beta$, when $k < k^*$, and is equal to $1 + \alpha$ regardless of $\beta$, when $k = k^*$.

\section*{Remarks}

The first result is quite straight-forward. As $\alpha$ increases, the short-term and long-term cash flows from the on-going activities are more correlated. Therefore, the market

\footnote{The monotonicity is based on the previous assumption that in the case of multiple roots for the equilibrium $\delta$, the economy chooses the root with which the investment is most efficient. See proof of Theorem 1.}
response coefficient \( b \) increases, and the firm has an incentive to invest more to inflate the market price. That is, \( \delta \) increases.

Combined with Theorem 2, the second comparative static result shows that a higher \( \frac{\sigma^2}{\sigma^2 - b} \) can induce a more efficient investment level (i.e., pushing \( \delta \) closer to one). As \( \sigma^2 \) gets lower, the on-going cash flows are less noisy, making the aggregate cash flow report \( (z_1 = x_1 + k\sqrt{\theta I}) \) more informative about the short-term investment return \( (k\sqrt{\theta I}) \). Intuitively, it is easier for the market to identify the first-period investment return. (Technically, the market response coefficient is closer to \( \frac{2}{k} \) as \( \frac{\sigma^2}{\sigma^2} \) increases.) As a result, the short-term investment return is less mispriced while the on-going cash flow is more mispriced. However, given that the mispricing of the on-going cash flow does not have any negative effect on the real investment decision, an increase in \( \frac{\sigma^2}{\sigma^2} \) improves the investment efficiency. In the extreme, when \( \sigma^2 \) approaches zero, the equilibrium achieves first-best results.

With short-term pressure \( (\beta > 0) \), the firm investment deviates from the first-best (by Theorem 2). The magnitude of the deviation increases in market pressure \( (\beta) \). As the firm makes less efficient investment under more market pressure, the market responds less to the cash flow report \( (b \) decreases in \( \beta) \) because the value lost due to inefficient investment increases in \( \beta \).

**Proof. (of Theorem 3)** Suppose the firm chooses a historical cost accounting system, based on the linear conjecture:

\[
P(z_1, y^{HC}) = a^{HC} + b_z^{HC} \times z_1 + b_y^{HC} \times y^{HC}
\]

The manager maximizes the expected payoff:

\[
\text{Choose } I(\theta) \text{ to max } \int_\theta V(\theta|I(\cdot))G(\theta)d\theta
\]

\[
= \int_\theta E_{x_1x_2}[-I + \beta P + (1 - \beta)(z_1 + z_2)]G(\theta)d\theta
\]

\[
= -I + \beta(a^{HC} + b_z^{HC} \mu + \int_\theta k\sqrt{\theta I}G(\theta)d\theta) + b_y^{HC}I) + (1 - \beta)\left(2\mu + \int_\theta 2\sqrt{\theta I}G(\theta)d\theta\right)
\]
The point-wise first-order condition with respect to $I$ is for $\theta > 0$, and we have

$$0 = -1 + \frac{(1 - \beta + \frac{\beta b y^{HC}}{2})\sqrt{\theta}}{\sqrt{I}} + \beta b y^{HC}$$

$$I^{HC} = \left(\frac{1 - \beta + \frac{\beta b y^{HC}}{2}}{1 - \beta b y^{HC}}\right)^2$$

and for $\theta < 0, I^{HC} = 0$.

So it must be the case that

$$\delta^{HC} = \left(\frac{1 - \beta + \frac{\beta b y^{HC}}{2}}{1 - \beta b y^{HC}}\right)^2$$

Using (20), (21), we have

$$\begin{pmatrix} x_1 + x_2 + 2\sqrt{\delta^{HC}\theta} \\ x_1 + k\sqrt{\delta^{HC}\theta} \\ \varepsilon^{HC} + \delta^{HC}\theta \end{pmatrix}$$

is normally distributed with mean

$$\begin{pmatrix} 2\mu + 2\sqrt{\delta^{HC}\theta_0} \\ \mu + k\sqrt{\delta^{HC}\theta_0} \\ \delta^{HC}\theta_0 \end{pmatrix},$$

and variance

$$\begin{bmatrix} 2(1 + \alpha)\sigma^2 + 4\delta^{HC}\sigma^2_{\theta} & (1 + \alpha)\sigma^2 + 2k\delta^{HC}\sigma^2_{\theta} & 2\delta^{\frac{3}{2}}^{HC}\sigma^2_{\theta} \\ (1 + \alpha)\sigma^2 + 2k\delta^{HC}\sigma^2_{\theta} & \sigma^2 + k^2\delta^{HC}\sigma^2_{\theta} & k\delta^{\frac{3}{2}}^{HC}\sigma^2_{\theta} \\ 2\delta^{\frac{3}{2}}^{HC}\sigma^2_{\theta} & k\delta^{\frac{3}{2}}^{HC}\sigma^2_{\theta} & \delta^{HC}\sigma^2_{\theta} + \sigma^2_{HC} \end{bmatrix}. $$
Using the approximation assumption (6), the pricing function becomes

\[
P = E[z_1 + z_2, y^{HC}] = E[x_1 + x_2 + 2\sqrt{\theta I}x_1 + k\sqrt{\theta I}, I + \varepsilon^{HC}] = E[x_1 + x_2 + 2\theta \sqrt{\delta I}x_1 + \delta \theta + \varepsilon^{HC}]
\]

\[
= 2\mu + 2\sqrt{\delta^{HC}}\theta_0 + \left[ (1 + \alpha)\sigma^2 + 2k\delta^{HC}\sigma^2_{\delta} + 2\delta^{\frac{3}{2}}^{HC}\sigma^2_{\delta} \right].
\]

\[
\left[ \sigma^2 + k^2\delta^{HC}\sigma^2_{\delta} \quad k\delta^{\frac{3}{2}}^{HC}\sigma^2_{\delta} \right]^{-1} \cdot \left[ z_1 - \mu - k\sqrt{\delta^{HC}}\theta_0 \right] \cdot \frac{y^{HC} - \delta^{HC}\theta_0}{\delta^{HC}\sigma^2_{\delta} + \sigma^2_{HC} + 2k\delta^{HC}\sigma^2_{HC}\sigma^2_{\delta}}.
\]

\[
= 2\mu + 2\sqrt{\delta^{HC}}\theta_0 + \left[ (1 + \alpha)\delta^{2}\sigma^2_{\delta}^2 + (1 + \alpha)\sigma^2_{HC} + 2k\delta^{HC}\sigma^2_{HC}\sigma^2_{\delta} \right].
\]

\[
\left[ \delta^{2}\sigma^2_{\delta}^2 + \sigma^2_{HC} + 2k\delta^{HC}\sigma^2_{HC}\sigma^2_{\delta} \right] \cdot \left( z_1 - \mu - k\sqrt{\delta^{HC}}\theta_0 \right)
\]

\[
+ \frac{[2 - (1 + \alpha)k]}{\delta^{HC}\sigma^2_{\delta}^2 + \sigma^2_{HC} + 2k\delta^{HC}\sigma^2_{HC}\sigma^2_{\delta}} \cdot (y^{HC} - \delta^{HC}\theta_0)
\]

So it must be the case that

\[
b^{HC}_z = \frac{(1 + \alpha)\delta^{2}\sigma^2_{\delta}^2 + (1 + \alpha)\sigma^2_{HC} + 2k\delta^{HC}\sigma^2_{HC}\sigma^2_{\delta}}{\delta^{2}\sigma^2_{\delta}^2 + \sigma^2_{HC} + 2k\delta^{HC}\sigma^2_{HC}\sigma^2_{\delta}}
\]

\[
b^{HC}_y = \frac{[2 - (1 + \alpha)k]}{\delta^{HC}\sigma^2_{\delta}^2 + \sigma^2_{HC} + 2k\delta^{HC}\sigma^2_{HC}\sigma^2_{\delta}}
\]

\[
a^{HC} = (2 - b^{HC}z)\mu + (2 - kb^{HC}y - \sqrt{\delta^{HC}b^{HC}_y})\sqrt{\delta^{HC}}\theta_0
\]

To show the existence of \(\delta^{HC}, b^{HC}_z\) and \(b^{HC}_y\), substituting \(b^{HC}_z\) (18) and \(b^{HC}_y\) (19) into the investment decision (21), we obtain

\[
\left( \sqrt{\delta^{HC}} - 1 \right) \cdot \left( k^2\delta^{HC}\sigma^2_{\delta} + 1 + \frac{\delta^2_{HC}\sigma^2_{\delta}}{\sigma_{HC}^2} \right) = \beta \left[ \frac{k(1 + \alpha)}{2} \right] \cdot \left( 1 - \frac{\delta^2_{HC}\sigma^2_{\delta}}{\sigma_{HC}^2} \right)
\]

As \(\delta^{HC} \in (0, +\infty)\), and the all functions are continuous, the left-hand-side at least covers the range from \(-1\) to \(+\infty\). The range of the left-hand-side could be larger depending on the parameter space. The right-hand-side (a parabola) at least covers the range \((\beta \left[ \frac{k(1 + \alpha)}{2} \right], -\infty)\).

We showed earlier that the range of \(\beta \left[ \frac{k(1 + \alpha)}{2} \right]\) is from \(-1\) to \(1\). So the left-hand-side curve and the right-hand-side curve must intersect at least once in \(\delta^{HC} \in (0, +\infty)\). Therefore, there is at least one positive root of \(\delta^{HC}\).

Using a similar method, we obtain the linear equilibrium under fair value accounting.
Proof. (of Corollary 1) From Theorem 3, using the pricing function (15) and , substituting $b^F_V$ (16) and $b^F_V$ (17) into the manager’s investment decision (21), we obtain the following equation, which has to hold in equilibrium under the fair value accounting,

$$
\left( \sqrt{\delta_{FV}} - 1 \right) \cdot \left( k^2 \delta_{FV} \frac{\sigma^2}{\sigma^2_{FV}} + 1 + k^2 \delta_{FV} \frac{\sigma^2}{\sigma^2_{FV}} \right) = \beta \left[ \frac{k (1 + \alpha)}{2} - 1 \right]
$$

The second parenthesis on the left-hand-side of the equation is always positive and decreases in $\sigma^2_{FV}$. So, the absolute value of $\left( \sqrt{\delta_{FV}} - 1 \right)$ must increase in $\sigma^2_{FV}$, (given that the right-hand-side is a constant,) which indicates better investment choices as $\sigma^2_{FV}$ is lower. As the sign of $\left( \sqrt{\delta_{FV}} - 1 \right)$ is the same as the right-hand-side, $\delta_{FV}$ increases (decreases) in $\sigma^2_{FV}$ when $k > k^*$ ($k < k^*$).

For historical cost accounting, we substitute $b^H_C$ (18) and $b^H_C$ (19) into the manager’s investment decision (21), and we obtain the following equation:

$$
\left( \sqrt{\delta_{HC}} - 1 \right) \cdot \left( k^2 \delta_{HC} \frac{\sigma^2}{\sigma^2_{HC}} + 1 + \frac{\delta^2_{HC} \sigma^2}{\sigma^2_{HC}} \right) = \beta \left[ \frac{k (1 + \alpha)}{2} - 1 \right] \cdot \left( 1 - \frac{\delta^2_{HC} \sigma^2}{\sigma^2_{HC}} \right)
$$

Suppose $k > k^*$. The right-hand-side must be positive when $\sigma^2_{HC}$ is sufficiently high, which means the left-hand-side is positive or $\delta_{HC} > 1$ or the firm overinvests. As $\sigma^2_{HC}$ is reduced, the right-hand-side decreases and the second parenthesis on the left-hand-side increases. Thus, to maintain equation (34), $\delta_{HC}$ has to be reduced, mitigating the investment inefficiency. Once $\sigma^2_{HC}$ is reduced to $\sigma^2_{\theta}$, the equilibrium reaches the first-best, $\delta_{HC} = 1$. However, as $\sigma^2_{HC}$ continues to decrease past $\sigma^2_{\theta}$, the right-hand-side turns negative, and $\delta_{HC}$ has to be lower than one to sustain equation (34). The investment $\delta_{HC}$ becomes less as $\sigma^2_{HC}$ continues to decrease. We get similar results in the case of $k < k^*$. Thus, $\delta_{HC}$ increases (decreases) in $\sigma^2_{HC}$ when $k > k^*$ ($k < k^*$).

From Theorem 3, using (16), (17), (18), and (19), we can readily find the effect of $\sigma^2_m$ on the market response coefficients $b^m_z$ and $b^m_y$.

Proof. (of Theorem 4) To prove claim (i), substituting $\sigma^2_{FV} = 0$ into (16) and (17), we have $b^F_V = 1 + \alpha$, and $b^F_V = \frac{2}{k} - (1 + \alpha)$. Then using (21), the investment level is first-best ($\delta_{FV} = 1$).

To prove claim (ii), suppose that the manager chooses the first-best investment level $I = \theta$.
under historical cost accounting. Correspondingly, using (15), (18), and (19), the market pricing function would be

\[ P^{HC}(z_1, y^{HC}) = a^{HC} + b^{HC}_{z,a=1} \times z_1 + b^{HC}_{y,a=1} \times y^{HC}, \]

where

\[ b^{HC}_{z,a=1} = \frac{(1 + \alpha)\sigma^2(\sigma^2 + \sigma^2_{HC}) + 2k\sigma^2_{HC}\sigma^2_{\theta}}{\sigma^2(\sigma^2 + \sigma^2_{HC}) + k^2\sigma^2_{HC}\sigma^2_{\theta}}, \]

\[ b^{HC}_{y,a=1} = \frac{[2 - (1 + \alpha)k] \sigma^2\sigma^2_{\theta}}{\sigma^2(\sigma^2 + \sigma^2_{HC}) + k^2\sigma^2_{HC}\sigma^2_{\theta}} \]

To sustain the equilibrium, the manager’s reaction to \( b^{HC}_{z,a=1} \) and \( b^{HC}_{y,a=1} \) must, indeed, be to set the investment at the first-best level. That is, it must be the case that

\[ \delta_{HC}|_{b^{HC}_{z,a=1}, b^{HC}_{y,a=1}} = 1 \]

By assuming \( \sigma^2_{HC} = \sigma^2_{\theta} \), we have

\[ b^{HC}_{z,a=1} = \frac{2(1 + \alpha)\sigma^2\sigma^2_{\theta} + 2k\sigma^4_{\theta}}{2\sigma^2\sigma^2_{\theta} + k^2\sigma^4_{\theta}}, \]

\[ b^{HC}_{y,a=1} = \frac{2 - (1 + \alpha)k}{} \frac{\sigma^2\sigma^2_{\theta}}{2\sigma^2\sigma^2_{\theta} + k^2\sigma^4_{\theta}} \]

Using (21), we have

\[ \delta_{HC} = \left(1 - \beta + \frac{\beta b^{HC}_{y}}{2} \right)^2 \]

Substituting \( b^{HC}_{z,a=1} \) and \( b^{HC}_{y,a=1} \) into the above expression yields:

\[ \delta_{HC}|_{b^{HC}_{z,a=1}, b^{HC}_{y,a=1}} = \left(1 + \beta \frac{2(1 + \alpha)k - 2\sigma^2\sigma^2_{\theta}}{2\sigma^2\sigma^2_{\theta} + k^2\sigma^4_{\theta}} \right)^2 \]

\[ = 1 \]

Claim (iii) and (iv) compare the investment efficiency under different accounting bases. Recall the expected net return of the project is, using (20)

\[ E[R^m] = E[f_1(\theta, I^m) + f_2(\theta, I^m) - I^m] = E[2\sqrt{\theta I^m} - I^m] = [1 - (\sqrt{\theta m} - 1)^2] \cdot \theta_0 \]
It is clear that $E[R^m]$ is single peaked at $\delta_m = 1$.

To prove claim (iii), invoke claim (ii) of Theorem 4: $\delta_{HC} = 1$ when $\sigma_{HC}^2 = \sigma_{\theta}^2$; and invoke claim (ii) of Corollary 1: $\delta_{HC}$ is monotonic in $\sigma_{HC}^2$ (increasing or decreasing depending on $k$). Combined, we must have that $E[R^{HC}]$ is single peaked at $\sigma_{HC}^2 = \sigma_{\theta}^2$ regardless of $k$.

Next, we compare $E[R^{HC}]$ in the two extreme cases: (1) the case without historical cost accruals (or the case $\sigma_{HC}^2 = +\infty$ by Corollary 1 claim (i)), and (2) perfectly precise historical cost accruals ($\sigma_{HC}^2 = 0$). Substituting $\sigma_{HC}^2 = +\infty$ into (18), and (19), we have $b_z^{HC} = \frac{(1+\alpha)\sigma^2 + 2k\delta_{HC}\sigma_{\theta}^2}{\sigma^2 + k^2\delta_{HC}\sigma_{\theta}^2}$, and $b_y^{HC} = 0$. Then using (21), it’s easy to have,

$$\sqrt{\delta_{HC}} = 1 + \frac{\beta\left[\frac{k(1+\alpha)}{2} - 1\right]}{k^2\delta_{HC}\sigma_{\theta}^2 + 1}, \text{when } \sigma_{HC}^2 \to \infty$$

Then the expected project return

$$E[R^{HC}|\sigma_{HC}^2 \to \infty] = \left[1 - \beta^2 \left[\frac{k(1+\alpha)}{2} - 1\right]^2 \right] \cdot \theta_0$$

Similarly, for a perfect historical cost accounting, substituting $\sigma_{HC}^2 = 0$ into (18), and (19), we have $b_z^{HC} = 1 + \alpha$, and $b_y^{HC} = 2 - (1 + \alpha)k/\delta_{HC}$. Then using (21),

$$\sqrt{\delta_{HC}} = 1 - \beta\left[\frac{k(1+\alpha)}{2} - 1\right], \text{when } \sigma_{HC}^2 = 0$$

so the expected project return

$$E[R^{HC}|\sigma_{HC}^2 = 0] = \left[1 - \beta^2 \left[\frac{k(1+\alpha)}{2} - 1\right]^2 \right] \cdot \theta_0$$

Because $\left(k^2\delta_{HC}\sigma_{\theta}^2 + 1\right)^2 > 1$, so $E[R^{HC}|\sigma_{HC}^2 \to \infty] > E[R^{HC}|\sigma_{HC}^2 = 0]$. Therefore, the expected project return in the basic setting is higher than the return with precise historical cost reports. Because $\delta_{HC}$ is continuous in $\sigma_{HC}^2$, so that $E[R^{HC}]$ is continuous in $\sigma_{HC}^2$, there must exist a $\Sigma$ ($0 < \Sigma < \sigma_{\theta}^2$), such that for any $\sigma_{HC}^2 > \Sigma$ ($\sigma_{HC}^2 < \Sigma$), the historical cost accounting is more (less) efficient than the basic cash flow reporting system.
To prove claim (iv), a similar argument shows that $E[R^F] \mid R^F V$ is single peaked at $\sigma_{FV}^2 = 0$. Therefore, $E[R^F] \mid R^F V$ monotonically decreases in $\sigma_{FV}^2 \in \{0, +\infty\}$ regardless of $k$. Given the following arguments:

- $E[R^{HC}]$ is strictly increasing in $\sigma_{HC}^2 \in [0, +\infty)$ and reaches $E[R^{HC}] = 1$ (the first-best) when $\sigma_{HC}^2 = \sigma_{g}^2$;

- $E[R^{FV}]$ is strictly decreasing in $\sigma_{FV}^2 \in [0, \sigma_{g}^2]$ and reaches $E[R^{FV}] = 1$ (the first-best) when $\sigma_{FV}^2 = 0$;

- Both $E[R^{HC}]$ and $E[R^{FV}]$ are continuous in $\sigma_m^2$ over $[0, +\infty)$.

There must exist a $\Sigma' \ (\Sigma < \Sigma' < \sigma_g^2)$, such that for any $\sigma_{HC}^2 \in [\Sigma', \sigma_g^2]$ and $\sigma_{FV}^2 > \Sigma'$, $E[R^{HC}] > E[R^{FV}]$, which indicates the historical cost accounting system is more efficient than the fair value accounting system, as claimed by (iv) of Theorem 4. ■

**Proof. (of Theorem 5)** After privately observing the unmanipulated $y^m$, and the realization of cash flow $(z_1)$ and cost parameter $(\xi^m)$, the manager/firm chooses the accounting report $w^m$ following the optimization program below, with an equilibrium conjecture of linear market pricing function $P(z_1, w^m) = a_w^m + b^m \times z_1 + d^m \times w^m$:

$$\max_{w^m} \beta P(z_1, w^m) + (1 - \beta)E[z_2(z_1 + z_2)] - c(w^m)$$

Given the linear pricing and quadratic personal cost function, the solution to the optimization problem is well defined and yields a simple solution that in equilibrium, the following holds:

$$w^m = y^m + \xi^m + \frac{\beta d^m}{c^m}$$

Therefore the reported accounting number is simply a noised-up version of the underlying, unmanipulated number. Precisely, the reported accounting number has the following property:

$$E[w^m] = E[y^m] + \frac{\beta d^m}{c^m}$$

$$Var[w^m] = Var[y^m] + Var[\xi^m] = \sigma_m^2 + \eta_m^2$$

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The rest of the proof follows exactly as the proof of Theorem 3, substituting $v_{m}^{2}$ with $Var[w^{m}] = \sigma_{m}^{2} + \eta_{m}^{2}$. Note the adjustment to $E[y^{m}]$ will only affect the intercept part of the pricing function ($a_{m}^{m}$), which has no incentive effects on the initial investment.

**Proof. (of Corollary 2)** Given Theorem 5, we know adding accounting manipulation option would increase the noise of the reported accounting number. Under the output-based accounting measures, given Corollary 1-(ii) and Theorem 4-(i), this leads to a loss of welfare on two fronts: (1) less efficient investment decisions and (2) non-zero (personal) cost of manipulation activities. so accounting manipulation always makes the …rm worse off.

Under the input-based accounting measure, if $v_{HC}^{2} < \sigma_{\theta}^{2}$, adding noise to the accounting number improves investment efficiency, given Corollary 1-(ii) and Theorem 4-(ii). If $c_{HC} \cdot \theta_{0}$ is high enough, the personal cost is small enough (because $c_{HC}$ is large) and the benefit of improved investment efficiency is large enough (because $\theta_{0}$ is large). So the investment benefit outweights the loss due to manipulation costs. Therefore, accounting manipulation is value-enhancing.

**Proof. (of Theorem 6)** Suppose the firm chooses a historical cost accounting system, based on the linear conjecture:

$$P(\lambda^{m}) = a_{\lambda}^{m} + b_{\lambda}^{m} \times \lambda^{m}$$

the manager maximizes the expected payoff:

$$Choose \ I_{1}(\theta) \ to \ max \ \int_{\theta} V(\theta|I_{1}(\cdot))G(\theta)d\theta$$

$$= \int_{\theta} E[-I_{1} + \beta P + (1 - \beta)(2\sqrt{\theta} I_{1} + r\theta)]G(\theta)d\theta$$

$$= -I_{1} + \beta(a_{HC} + b_{HC} I_{1}) + (1 - \beta) \left(\int_{\theta} (2\sqrt{\theta} I_{1} + r\theta)G(\theta)d\theta\right)$$

The point-wise first-order condition wrt $I_{1}$ is for $\theta > 0$, we have

$$0 = -1 + \frac{(1 - \beta)\sqrt{\theta}}{\sqrt{I}} + \beta b_{HC}^{HC}$$

$$I_{1}^{HC} = \left(\frac{1 - \beta}{1 - \beta b_{HC}^{HC}}\right)^{2} \theta$$
and for $\theta < 0$, $I_1^{HC} = 0$. So it must be the case that

$$\gamma'_HC = \left(\frac{1 - \beta}{1 - \beta b'_HC}\right)^2$$

Using (29), we have

$$\left(2\sqrt{\gamma'_HC\theta + r\theta}ight) \sim N\left(\left(\frac{(2\sqrt{\gamma'_HC + r})\gamma'_HC\theta_0}{\gamma'_HC\theta_0}\right), \left(\frac{(2\sqrt{\gamma'_HC + r})^2\gamma'_HC}{\gamma'_HC}\right)\right)$$

Using the approximation assumption, the pricing function becomes

$$P = E[2\sqrt{\theta I_1 + r\theta}|\lambda^{HC}] = E[2\sqrt{\theta I_1 + r\theta}|I_1 + \varepsilon^{HC}]$$

$$= E[(2\sqrt{\gamma'_HC + r})\theta] + \frac{(2\sqrt{\gamma'_HC + r})\gamma'_HC\sigma^2_{\theta}}{(\gamma'_HC)^2\sigma^2_{\theta} + \sigma^2_{HC}}(\lambda^{HC} - \gamma'_HC\theta_0)$$

So it must be the case that

$$b'_HC = \frac{(2\sqrt{\gamma'_HC + r})\gamma'_HC\sigma^2_{\theta}}{(\gamma'_HC)^2\sigma^2_{\theta} + \sigma^2_{HC}}$$

$$a'_HC = (2\sqrt{\gamma'_HC + r} - \gamma'_HC b'_HC)\theta_0$$

To show the existence of $\gamma'_HC$ and $b'_HC$, substituting $b'_HC$ into $\gamma'_HC$, and simplifying, we have

$$\sqrt{\gamma'_HC} - 1 = \frac{\gamma'_HC(2\sqrt{\gamma'_HC + r} - \gamma'_HC) - \sigma^2_{\theta}}{\gamma'_HC[\gamma'_HC - \beta(2\sqrt{\gamma'_HC + r})] + \frac{\sigma^2_{\theta}}{\sigma^2_{\theta}}}$$

As $\gamma'_HC \in (0, +\infty)$, and the all functions are continuous, the left-hand-side is increasing in $\gamma'_HC$ and covers the range from $-1$ to $+\infty$. The right-hand-side is $-\beta$ as $\gamma'_HC$ is zero, and also converges to $-\beta$ as $\gamma'_HC$ goes to infinity. Since we assume that $\beta$ is from $-1$ to $1$. So the left-hand-side curve and the right-hand-side curve must intersect at least once in $\gamma'_HC \in (0, +\infty)$. Therefore, there is at least one positive root of $\gamma'_HC$.

Using the similar method, we obtain the linear equilibrium under fair value accounting system.

**Proof.** (of Corollary 3) Suppose the manager chooses the first-best investment level $I_1 = \theta$
under historical cost accounting. Correspondingly, using (28), the market pricing function would be

\[ P(\lambda^{HC}) = a_{\lambda}^{HC} + b_{\lambda, \gamma'_{HC}=1}^{HC} \times \lambda^{HC}, \text{ where} \]

\[ b_{\lambda, \gamma'_{HC}=1}^{HC} = \frac{(2 + r)\sigma_{\beta}^{2}}{\sigma_{\beta}^{2} + \sigma_{HC}^{2}}, \]

To sustain the equilibrium, the manager’s reaction to \( b_{\lambda, \gamma'_{HC}=1}^{HC} \) must, indeed, be to set the investment at the first-best level. That is, it must be the case that

\[ \gamma'_{HC}|_{b_{\lambda, \gamma'_{HC}=1}^{HC}} = 1 \]

By assuming \( \sigma_{HC}^{2} = (r + 1)\sigma_{\beta}^{2} \), we have

\[ b_{\lambda, \gamma'_{HC}=1}^{HC} = \frac{(2 + r)\sigma_{\beta}^{2}}{\sigma_{\beta}^{2} + \sigma_{HC}^{2}} = 1 \]

Using (29), we have

\[ \gamma'_{HC} = \left( \frac{1 - \beta}{1 - \beta b_{\lambda, \gamma'_{HC}=1}^{HC}} \right)^{2} = 1 \]

Using the similar method, we get the firm is motivated to choose \( \gamma'_{FV} = 1 \) if \( \sigma_{FV}^{2} = 2r\sigma_{\beta}^{2} \) under fair value accounting. As the investment \( \gamma'_{m} \) is strictly decreasing in the measurement noise \( \sigma_{m}^{2} \), under both accounting measurements, the efficiency of investment is strictly decreasing from the first-best to either direction. As \( \gamma'_{m} \) is continuous in \( \sigma_{m}^{2} \), if \( r > 1(r < 1) \), there must exist a \( \Sigma^{*} \) which lies in the interval of \( (r + 1)\sigma_{\beta}^{2} \) and \( 2r\sigma_{\beta}^{2} \), such that historical cost accounting is more efficient than fair value accounting when \( \sigma_{HC}^{2} = \sigma_{FV}^{2} \in [(r + 1)\sigma_{\beta}^{2}, \Sigma^{*}] \) \( (\sigma_{HC}^{2} = \sigma_{FV}^{2} \in [\Sigma^{*}, (r + 1)\sigma_{\beta}^{2}]) \).
Chapter 2
Voluntary Disclosure and Investments

2.1 Introduction

Corporate voluntary disclosure has become an important element of capital market dynamics. It has many salient features. First, it conveys value-relevant information for market pricing.\textsuperscript{22} Second, it typically contains information related to firm activities which may not be immediately reported in accounting reports. Third, it interacts with other information sources, such as mandatory accounting reports.\textsuperscript{23} This paper examines the determinants and economic efficiency of corporate voluntary disclosure. The focus is on the trade-off for an individual firm when the benefits and costs of voluntary disclosure stem from the consequences of its investment decision. When a firm’s investment incentive is provided by the short-term market pricing of its shares, as well as long-term cash flow payoffs, investment and voluntary disclosure decisions are intertwined.\textsuperscript{24} First, an investment in a project brings about an option to voluntarily disclose information about the project. By transmitting value-relevant information to the market, voluntary disclosure leads to a more accurate pricing which, in turn, improves investment efficiency. Here the voluntary disclosure option plays a positive role and provides an ex ante benefit to the firm. Second, the firm may affect the market pricing of its shares in its favor by strategically

\textsuperscript{22}Substantial empirical evidence confirms the value relevance and information content of corporate voluntary disclosure (e.g., Patell [1976] and Penman [1980]).

\textsuperscript{23}Chen, Defond and Park [2002] provide empirical evidence that managers voluntarily disclose balance sheets when quarterly earnings are less informative. Lennox and Park [2006] find the propensity to issue earnings forecasts is related to the informativeness of financial reports.

\textsuperscript{24}For example, major mergers-and-acquisitions (M&As) and their announcements typically go hand-in-hand. On October 9, 2006, Google Inc. announced a $1.65 billion purchase of YouTube Inc. resulting in a rise of its share price, which The Wall Street Journal interpreted as its expansion into the lucrative online video and social networking markets. Yahoo’s failure to reach a deal for Facebook highlights the slower pace of its efforts to expand. See The Wall Street Journal dated Oct 12, 2006 for more details.
disclosing or withholding its private information. This opportunistic use of voluntary disclosure has a feedback effect on the investment efficiency, which may be distorted at the margin. Here the voluntary disclosure option plays a negative role and incurs an ex ante cost to the firm. In particular, when the investment profitability is low on average, firms may be tempted to implement not-so-profitable projects and disclose in order to distinguish from those of even worse types. When the investment profitability is high on average, firms may withhold information on slightly profitable projects in order to avoid being compared with those of even better types. Such propensity to selectively disclose becomes intense and detrimental to investment efficiency when the firm knows a lot more about its investment cash flow than outsiders (i.e., the information asymmetry is large). The overall efficiency is determined by both positive and negative effects of voluntary disclosure.\footnote{Rajan and Saouma [2006] study how the owner’s payoffs are related to the quality of the manager’s private information in a contract setting. In contrast, this paper investigates how the firm’s investment efficiency is related to the quality of its private information (voluntary disclosure) in a market setting.}

Also playing a role is a mandatory accounting report. While not directly useful in firm’s investment decision, the mandatory report improves the market pricing and may discipline the firm’s voluntary disclosure. This disciplining role limits the opportunistic behavior and, thus, reduces the efficiency loss. In particular, the mandatory report helps the market distinguish among firms that do not disclose and disentangle firms’ disclosure incentives. Marginal improvement in the informational quality of the mandatory report mitigates firms’ distorted disclosure incentives. This disciplining role of the mandatory report becomes effective when the firm’s private signal is precise. This demonstrates an interaction between the economic efficiency induced by the mandatory accounting report and the private signal.\footnote{Kanodia and Lee [1998] study a different setting where investments undertaken are publicly observable, and mandatory reports alleviate the perverse incentives driven by information asymmetry between firm and outsiders.}

The model setup is consistent with findings in archival and survey research. For instance, the model studies how a firm opportunistically affects the market pricing in its favor through voluntary disclosure, which is consistent with the survey evidence from Graham, Havey and Rajgopal [2005] that firms’ voluntary disclosure decisions are motivated by market prices. Archival empirical evidence also shows that corporate voluntary disclosure is motivated by short-term share price interests, such as CEO stock-based compensation (Aboody and Kasznik [2000]; Nagar, Nanda and Wysocki [2003]), insider trading (Noe [1999]; Cheng and Lo [2006]), and equity...
offering (Lang and Lundholm [2000]). In this paper, the accounting report is mandatory and not subject to firms’ discretion. And the firm would rather make not-so-profitable investment and voluntarily disclose, than communicate the relevant information directly to the market. These modeling choices are consistent with the results in Graham, Havey and Rajgopal [2005] that managers would rather take economic actions and make moderate sacrifices in long-term value, than make within-GAAP accounting choices to avoid negative market reactions in the Sarbanes-Oxley Act environment. In addition, this paper studies how disclosure decisions relate to different characteristics of voluntary disclosure and mandatory reporting as suggested by several empirical studies (e.g., Lennox and Park [2006]). It also suggests implications on the market reaction to voluntary disclosure and mandatory reporting, which has been extensively studied in the empirical literature (e.g., Anilowski, Feng and Skinner [2006], and Chen, Matsumoto and Rajgopal [2006]). Beyond that, this paper shows that corporate voluntary disclosure is related to the economic outlook of investment opportunity (possibly driven by industry, business or firm life cycle), which provides the basis of new empirical hypotheses.

The insights in this paper may shed light on the efficiency issues associated with voluntary disclosure. In recent surveys of the literature, Dye [2001] and Verrecchia [2001] emphasize the importance of a theory designed to identify efficient resource allocations in a disclosure setting. Though the effects of mandatory financial reports on the efficiency of managerial behavior have been studied extensively, the literature on how voluntary disclosure affects the economic efficiency has been scant. By focusing on the link between investment and voluntary disclosure, this study addresses the economic relevance of disclosure as well as its determinants. Unlike the existing literature, this paper also points to the interactive effects between private and public information on firms’ investment and disclosure decisions. The link between asset prices and corporate decisions has been also studied in general equilibrium setting in the literature. Kanodia


investigates how the equilibrium paths of asset prices, corporate investment decisions, and consumers’ decisions are determined simultaneously in dynamic general equilibrium.

The disciplining role of accounting information is widely discussed in the literature. Much recent work has been based on the agency model in which accounting systems are viewed as a monitoring device for managers’ self-reporting (Gigler and Hemmer [1998], Liang [2000], Christensen and Demski [2002], and Arya, Glover, Mittendorf and Zhang [2004]). In this paper, by disciplining the firm’s disclosure incentive, in a market setting (as opposed to a contract setting), an accurate accounting report may allow the firm’s voluntary disclosure to be more beneficial for the economy.\textsuperscript{29}

Specifically, this paper considers a model in which a firm is confronted with an investment decision and a voluntary disclosure option created by the investment. The firm’s objective is to maximize a weighted average of the long-term cash flow payoffs and the short-term share price, net of the investment costs. The firm privately learns whether it will have an investment opportunity or not, which generates a future cash flow correlated with the firm’s ongoing activities. If the investment opportunity is available, the firm observes a private signal informative of the project’s cash flow. Conditional on this private signal, the firm chooses whether to invest in the project or not. If the project is implemented, the firm acquires an additional option to make a voluntary disclosure of its private signal. A key assumption is that the implementation of the project is a prerequisite for voluntary disclosure. Firms without the investment opportunity do not have a credible means of disclosure as such, and are indistinguishable from firms with the opportunity that choose not to disclose.\textsuperscript{30}

After the disclosure decision is made, a mandatory accounting report is announced, which is informative about the firm’s ongoing activities. As the investment cash flow is correlated with the firm’s ongoing activities, the mandatory report indirectly conveys information on both the

\textsuperscript{29} In this paper, both the accounting report and the voluntary disclosure are assumed to be credible, and the analysis focuses on the disciplinary role of the accounting report on the distorted disclosure incentive, rather than its credibility. Gigler and Hemmer [1998], Liang [2000], and Arya, Glover, Mittendorf and Zhang [2004] examine the role of accounting in disciplining softer sources of information in contract settings, where disclosure cannot be credibly communicated, but a verifiable public signal creates an environment that enables the agent to credibly convey a private signal.

\textsuperscript{30} The model captures two popular rationales leading to a partial voluntary disclosure. First, building on Dye [1985] and Jung and Kwon [1988], the model introduces an information endowment uncertainty created by the uncertainty of the investment opportunity. Then, building on Verrecchia [1983; 1990], the model introduces a disclosure cost, which is the endogenous inefficiency of the firm’s investment decision.
project (if implemented) and the firm’s private signal (if available). Based on the mandatory report and the possible voluntary disclosure, the capital market prices the firm’s shares competitively. The market’s inability to distinguish among non-disclosure firms may induce distorted disclosures, in that the firm might be motivated to invest in unprofitable projects, or to withhold information on profitable projects. The direction of the distortion is determined by the ex ante expectation of the project’s profitability. When the ex ante expected profitability is low, average firms in the market have less profitable projects and the market average conjecture on non-disclosure firms is also low. If the project has a slightly negative net return, but is still higher than the market average conjecture, the firm may choose to undertake the project and disclose in order to avoid further under-valuation. In other words, exercising the disclosure option has a positive value ex post, which outweighs the investment loss. Alternatively, when average firms have profitable projects, the market average conjecture on non-disclosure firms is high. The firm with a slightly profitable project may choose to undertake the project but withhold information to avoid being compared to other disclosers (who disclose highly profitable projects). In other words, a voluntary disclosure is costly ex post, so the firm chooses not to exercise the disclosure option. In short, this paper identifies investment cycle (due to business/industry cycle or firm life cycle) as a major determinant of voluntary disclosure.

The second determinant is the information asymmetry between the firm and the market. When the private signal is noisy, the firm has less information advantage over the market. The disclosure is not informative and does not significantly revise the market’s perception. As a result, the coarse private information lessens the incentive to distort the disclosure choice. If the private signal is precise, the information asymmetry between the firm and the market widens and the firm’s disclosure is more influential in the market pricing. Ex post, the firm is motivated to take advantage of its private information and to inflate its share price by selective disclosures, which may hurt the investment efficiency ex ante. On the other hand, a precise private signal allows the firm to better distinguish good or bad projects and therefore, the firm is better able to make efficient investment decisions (because it is partially motivated by long-term firm value). So the dominant effect determines the total efficiency effect of the private signal.\footnote{In Verrecchia [1990] and Pae [1999], the firm is, ex ante, best off by choosing to be completely uninformed. In other words, the efficiency effect of the private signal is always negative. This model allows a positive role of firm’s private signal, so the efficiency effect of the private signal could also be positive.}
The mandatory accounting report also plays an important role in the firm’s investment and disclosure decisions.\textsuperscript{32} Since the mandatory report indirectly conveys information on both the project (if implemented) and the firm’s private signal (if available), a favorable report from non-disclosure firms indicates a higher likelihood of a lack of an investment opportunity (and thus no disclosure opportunity). And similarly, an unfavorable report from non-disclosure firms indicates a higher likelihood that the firm purposely withholds bad news. As a result, the mandatory report helps the market better distinguish among firms that do not disclose and disentangle the firm’s disclosure incentive. Thus the firm’s distorted disclosure incentive is alleviated. When the quality of the mandatory report is high enough, a precise private signal improves the efficiency. The results demonstrate an interaction between the mandatory accounting report and the firm’s private signal.\textsuperscript{33}

Finally, this paper investigates how prohibiting voluntary disclosure affects the investment efficiency. Without voluntary disclosure, the distorted disclosure incentive (which represents the endogenous disclosure cost) is eliminated and no disclosure cost is incurred (because firms only implement profitable projects). However, the market is also unable to fully price in the benefit of the project, and thus provides a more muted investment incentive (leading to the traditional under-investment problem). This analysis shows that when the investment profitability is low on average and the information asymmetry is large, prohibiting voluntary disclosure is efficiency enhancing because the gain from eliminating disclosure cost is higher than the loss from under-investment.

The rest of the paper is organized as follows. Section 2 describes the model in details. Section 3 presents the equilibrium investment and disclosure decisions and the market pricing. Section 4 analyzes how the firm’s disclosure choice responds to a private/public information structure and the prior expectation of investment profitability. Section 5 looks further into the firm’s investment decisions and efficiency implications. Section 6 introduces an extension of prohibiting voluntary disclosure. Section 7 summarizes the paper. The appendix contains all

\textsuperscript{32}Dye [1998], Dutta and Trueman [2002] and Suijs [2006] study the manager’s disclosure strategy when he is uncertain of investor response, because he does not know the market participants’ entire information set.

\textsuperscript{33}Einhorn [2005] examines the interaction between mandatory and voluntary disclosure in a setting in which a manager makes a disclosure decision after observing both public and private signals. In this setting, the disclosure decision is made before the mandatory accounting report is announced. As such, we introduce a disciplining role of the accounting report on disclosure decisions.
2.2 The Model

2.2.1 Investment Decision

There are three dates in the model. The cash flow generated by the firm’s ongoing activities is denoted by $y$, where $y$ is normally distributed with mean zero and variance $\sigma_y^2$, i.e., $y \sim N[0, \sigma_y^2]$. We assume that the ongoing activities’ cash flow $y$ is realized at the end of the game and not observable to the firm or outsiders.\(^{34}\)

On date 1, the firm privately learns whether it faces an investment opportunity (called a project). One can think of this project as an expansion of the firm’s ongoing activities through internal growth. For example, the firm may discover a growing market demand or a new technology may allow additional innovations (such as consumer-friendly features) in its products. It is assumed that the investment opportunity occurs with probability $q \in (0, 1)$, and the availability of the project is independent with all other random variables in the setting.\(^{35}\) Outsiders do not know whether the firm has an investment opportunity or not.

The cash flow generated by the project, denoted by $x$, is closely related to the firm’s ongoing activities. The project’s cash flow $x$ and the ongoing activities’ cash flow $y$ follow the jointly normal distribution given by:

$$
\begin{bmatrix}
  x \\
  y
\end{bmatrix} \sim N
\left(
\begin{bmatrix}
  \mu \\
  0
\end{bmatrix},
\begin{bmatrix}
  \sigma_x^2 & \rho \sigma_x \sigma_y \\
  \rho \sigma_x \sigma_y & \sigma_y^2
\end{bmatrix}
\right),
$$

where $\rho \in (0, 1)$ is the positive correlation coefficient between the project and the firm’s ongoing activities. The mean of the project’s cash flow $\mu$ could be interpreted as an indicator of the business/industry cycle, or as the firm life cycle.\(^{36}\)

\(^{34}\)Alternatively, one can assume the ongoing cash flow is generated by an endogenous choice variable of the firm. The economic efficiency of such firm activities are also affected by the upcoming disclosure choice. One can show that all of the qualitative results remain (analysis is available upon request). A more realistic interpretation is that $y$ summarizes all cash flows related with ongoing activities and possible future investments or growth opportunities.

\(^{35}\)This assumption of independence is used for tractability and is common in the literature (e.g., Dye [1985; 1998]).

\(^{36}\)The mean of the cash flow from ongoing activities is immaterial to this model. For simplicity, we normalize the mean of $y$ to zero.
The firm confronted with the investment opportunity observes a private signal $\theta$ which is informative about the project’s cash flow. The private signal $\theta$ is a garbling of $x$:

$$\theta = x + \varepsilon_\theta,$$

where $\varepsilon_\theta$ is normally distributed with mean zero and variance $\sigma_\theta^2$, i.e., $\varepsilon_\theta \sim N(0, \sigma_\theta^2)$, and independent with all other random variables. The variance $\sigma_\theta^2$ represents the informational quality of the private signal $\theta$. If $\sigma_\theta^2$ is lower, $\theta$ conveys more information about the prospect of the project.

On date 2, the firm with the investment opportunity makes the investment decision denoted by $I \in \{0, 1\}$, as an indicator variable that indicates whether or not the project is undertaken. If the project is undertaken, the cash flow $x$ is realized from the investment with an initial capital cost $I = 1$. Following the literature, the firm’s investment decision is made privately.\(^{37}\)

The socially optimal project selection choice is to implement only if the expected cash flow $(x)$ conditional on the private signal $(\theta)$ is higher than the initial capital cost 1. To be precise, the expected $x$ conditional on $\theta$ is equal to $\mu + \frac{\sigma_\theta^2(\theta-\mu)}{\sigma_\theta^2+\sigma_\theta^2}$, so the socially optimal project selection is to implement only if $\theta$ is higher than the threshold denoted by $\theta^{FB} = 1 + \frac{\sigma_\theta^2(1-\mu)}{\sigma_\theta^2}$. When the private signal is more precise, the firm is able to make a more efficient project selection choice.

2.2.2 Disclosure Choice

If a project is implemented, the firm has an option to make a voluntary disclosure about $\theta$.\(^{38}\) It is assumed that the voluntary disclosure is fully credible if a project is implemented, and is fully incredulous if otherwise. That is, the implementation of a project endows the firm with an option to credibly disclose its private signal. One could interpret the assumption by considering a setting where the firm’s private information can be verified by the outsiders in the long run if the project

\(^{37}\)This private investment assumption is common in the literature (e.g., Dye and Sridhar [2004; 2006], Kanodia, Singh, and Spero [2005], and Liang and Wen [2007]). The market has no information about the firm investment decision as long as it is paid by internal funds (e.g., Myers and Majluf [1984]). It can be a scenario where an entrepreneur invests in a risky project and sells a part of the shares to the open market. One could also interpret the investment as the manager’s effort cost as opposed to a monetary cost. For example, in Pae [1999], an entrepreneur provides costly effort that stochastically enhances the future cash flow.

\(^{38}\)All of the qualitative results remain if we define voluntary disclosure as the firm’s rational expected cash flow of the project given $\theta$, which is $\mu + \frac{\sigma_\theta^2(\theta-\mu)}{\sigma_\theta^2+\sigma_\theta^2}$. 

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is undertaken. For example, the firm’s market demand estimate of a new product is easy to verify after the new product is launched. The credibility could also be interpreted from the manager’s reputation perspective. In Stocken [2000], the manager always endogenously truthfully reveals his private information provided the manager is sufficiently patient in a repeated game.

Firms who invest in a project, but choose not to disclose, are indistinguishable from firms without an investment opportunity. In other words, firms cannot credibly communicate their lack of opportunity. Because the disclosure is not mandatory and is subject to the firm’s discretion, the firm can make a strategic disclosure choice to influence investors’ assessment of the firm’s value. Let the function $d \in \{D, ND\}$ describe the disclosure rule adopted by the firm if a project is undertaken, where $D$ stands for disclosure and $ND$ for non-disclosure.

On date 3, the accounting system publicly reports an unbiased estimate of $y$, denoted by $r$,

$$r = y + \varepsilon_r,$$

where $\varepsilon_r$ is normally distributed with mean zero and variance $\sigma^2_r$, i.e., $\varepsilon_r \sim N(0, \sigma^2_r)$, and independently distributed with all other random variables. The scenario that the model describes is common. For example, accounting reports usually provide data on past transactions, and may not be timely enough to include all value-relevant information. As an alternative information source, the disclosure of the firm’s assessment on soft investments generally remains voluntary and timely.\(^{39}\)

### 2.2.3 Firm’s Objective

Finally, the firm value is priced in a competitive capital market such that the market price, denoted by $P$, is determined by the expected total cash flow $V$, conditional on all available information, including the market’s rational conjectures about the firm’s disclosure rule. Denote

\(^{39}\)In Dutta and Reichelstein [2005], the accounting system is imperfect in its ability to measure "softer" investments, like those in product development and personnel training.
the publicly available information set by Ω (assuming no discounting or dividend payments):

\[ P = E[V|Ω], \]

where \( V = \begin{cases} y, & \text{if } I = 0 \\ x + y, & \text{if } I = 1 \end{cases} \)

and \( Ω = \begin{cases} \{r\}, & \text{if } d = ND \\ \{r,θ\}, & \text{if } d = D \end{cases} \).

Following Stein [1989] and Liang and Wen [2007], the firm is motivated by both the long-term and the short-term interests. In particular, the firm’s objective is to maximize a weighted average of the market price \( P \) and the total cash flow \( V \) (net of the investment costs \( I \)). One interpretation is that a portion of the firm, denoted by \( β \), must be sold to the new shareholders at the end of period 3. The remaining \((1 − β)\) portion will be held by the old shareholders. Another interpretation is that the firm may face a probability \( β \) of takeover risk, which would force it to tender the shares at the market price, or liquidity risk, then the firm would also consider both the current market price and the total future cash flow.

For a realization of \( θ \), the objective function is

\[ U(I(\cdot),d(\cdot)|θ) = E[βP + (1 − β)V|θ] − I. \]

The sequence of the events is summarized below.

<table>
<thead>
<tr>
<th>date 1</th>
<th>date 2</th>
<th>date 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>The firm privately learns whether ( I ) is available, and privately observes ( θ ) if ( I ) is available.</td>
<td>If ( I ) is available, the firm makes decisions on ( I ) and privately observes ( θ ) if ( I ) is available.</td>
<td>Accounting report ( r ) is released, and the market prices ( P ).</td>
</tr>
</tbody>
</table>

Figure 4: Time Line
2.3 Equilibrium Characterization

I now define the equilibrium where the investment and disclosure decisions are made in the self-interest of the firm.

**Definition 3** An equilibrium consists of an investment function $I(\cdot)$, a disclosure rule $d(\cdot)$, and a market pricing function $P(\cdot)$, such that:

(i) Given $P(\cdot)$ and the firm’s private signal $\theta$, the optimal investment $I(\cdot)$ and disclosure rule $d(\cdot)$ maximize $U(I(\cdot), d(\cdot) | \theta) = E[\beta P(\cdot) + (1 - \beta)V[\theta] - I$, subject to the constraint that $d = ND$ if $I = 0$;

(ii) Given $I(\cdot)$ and $d(\cdot)$, the market pricing function $P(\cdot)$ satisfies $P = E[V | \Omega, I(\cdot), d(\cdot)]$.

The definition of equilibrium is straightforward. In part (i), taking the market pricing as given and conditioning on its private signal ($\theta$), the firm chooses the optimal investment $I(\cdot)$ and disclosure rule $d(\cdot)$ to maximize the weighted average of the market price ($P$) and the future total cash flow ($V$). In part (ii), the share price $P$ is determined based on the capital market’s assessment of the total cash flow, making a rational expectation of the firm’s investment and disclosure decisions.

2.3.1 Equilibrium Investment and Disclosure Decisions

The following theorem presents the firm’s investment and disclosure decisions, given an endogenous threshold value $\theta^*$.

**Theorem 7** There exists an equilibrium characterized by a threshold value $\theta^*$, where the firm’s investment and disclosure decisions are given by

(i) an equilibrium investment rule:

$$I = \begin{cases} 
1, & \text{if } \theta > \min(\theta^*, \theta^0) \text{ and the investment is available} \\
0, & \text{if otherwise.}
\end{cases} \quad (35)$$
(ii) an equilibrium disclosure rule:

\[
d = \begin{cases} 
D, & \text{if } \theta > \theta^* \text{ and the investment is available} \\
ND, & \text{if otherwise.}
\end{cases}
\]  

(36)

where:

\[
\theta^0 = \frac{\sigma_x^2 + \sigma_g^2(1 - \mu + \beta \mu)}{(1 - \beta)\sigma_x^2}.
\]

**Proof.** All proofs are placed in the appendix. ■

We make two key observations on the firm’s equilibrium investment and disclosure behavior:

- **Disclosure rule:** The firm’s disclosure rule is represented by a simple cut-off (threshold) value \(\theta^*\). If the firm observes a news better than \(\theta^*\), it will invest in the project and disclose such news (i.e., \(d = D\) if \(\theta > \theta^*\)).

- **Project selection choice:** The firm’s project selection choice is also represented by a cut-off value, the lower of \(\theta^*\) and \(\theta^0\) (i.e., \(I = 1\) if \(\theta > \min(\theta^*, \theta^0)\)). \(\theta^0\) is the break-even level where the firm’s expected net payoff from investing in the project is zero without disclosure, i.e., \((1 - \beta) E[x|\theta^0] - 1 = 0\). Since the firm only yields partial long-term cash return, the break-even level is a little bit higher than the socially optimal threshold \(\theta^{FB}\). When \(\theta^* > \theta^0\), it is possible for the firm to invest but not disclose (\(\theta^* > \theta > \theta^0\)); when \(\theta^* < \theta^{FB} < \theta^0\), it is possible for the firm to invest in an unprofitable project and disclose (\(\theta^* < \theta < \theta^{FB}\)).

According to equation (36), the firm chooses to disclose when its private signal is high and refrains from disclosing when its private signal is low. Though this type of disclosure strategy is similar to extant results in the literature,\(^{41}\) the underlying driving force is different. In this model, the firm’s disclosure decision is associated with its investment decision, because the disclosure is only possible if the project is implemented. This analysis endogenizes the disclosure cost as the possible inefficiency of the firm’s investment decision.

\(^{40}\)When the investment cost is zero \((I = 0)\), the socially optimal threshold \(\theta^{FB}\) is exactly the same as the break-even level \(\theta^0\). Thus, once the disclosure threshold \(\theta^*\) is higher than \(\theta^0\) (or \(\theta^{FB}\)), the investment efficiency achieves the first best.

\(^{41}\)In Dye [1985] and Jong & Kwon [1988], the disclosure strategy is driven by the market’s inability to distinguish informed and uninformed firms. In Verrecchia [1983, 1990], Jorgensen and Kirschenheiter [2003], and Einhorn [2005], the driving force of the disclosure threshold is the exogenous cost of disclosing the private signal.
To see this, assuming the ex ante expectation of the profitability is low (i.e., $\mu$ is low, and thus $\theta^* < \theta^{FB} < \theta^0$), average firms have less profitable projects. Suppose the firm’s expected net project return conditional on its private signal ($\theta$) is slightly negative, but its cash flow prospect of the ongoing activities is higher than the market’s average conjecture of non-disclosure firms. If the firm chooses not to disclose, its ongoing activities will be undervalued. If the firm chooses to disclose and distinguish itself (because of the correlation between the disclosure and the ongoing activities), there would be an efficiency loss because the firm must implement such a project. For these firms, the investment reduces the expected cash flow (negative NPV), but allows them to disclose and raise their share prices (i.e., the investment is the cost of disclosure). At a knife-edge case ($\theta = \theta^*$), the two choices lead to the identical amount of losses and the firm is indifferent between disclosure and non-disclosure. For firms with profitable projects ($\theta > \theta^{FB}$), investment and disclosure are both beneficial. For firms choosing not to disclose ($\theta < \theta^*$), either the disclosure itself is not beneficial, or the disclosure is beneficial but too costly.

One can also interpret the above results from a real option perspective.\footnote{Arya and Glover [2001] study a principal-agent model in which a control (incentive) problem makes the option to wait valuable when it would not have been valuable otherwise.} Consider the investment opportunity as a real option for the firm, exercising the real option (i.e., the investment) generates another voluntary disclosure option. This embedded disclosure option makes the real investment option more valuable than a traditional NPV perspective suggests. Figure 5 illustrates the firm’s disclosure strategy for different intervals of $\theta$ when $\mu$ is low.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Disclosure strategy when $\mu$ is low}
\end{figure}

Now assume average firms have profitable projects (i.e., $\mu$ is high, and thus $\theta^* > \theta^0 > \theta^{FB}$). Suppose the firm’s expected net project return conditional on its private signal ($\theta$) is slightly positive, but its cash flow prospect of the ongoing activities is lower than the market’s average.
conjecture on non-disclosure firms. If the firm chooses to disclose, the price effect could be positive because the market recognizes the implementation of the profitable project. However, the disclosure may lower the market assessment of the ongoing activities’ cash flow. So choosing not to disclose is also attractive in that the firm is pooled with non-disclosure firms whose average type is higher. The disclosure decision is determined by which brings more positive effects. Figure 6 illustrates the firm’s disclosure strategy for different intervals of $\theta$ when $\mu$ is high.43

\[ \begin{align*}
\text{NPV} < 0 & \quad \text{I} = 0 \\
\text{NPV} > 0 & \quad \text{I} = 0 \\
\text{NPV} > 0 & \quad \text{I} = 1 \\
\text{NPV} > 0 & \quad \text{I} = 1 \\
\end{align*} \]

\[ \begin{align*}
\theta^{FB} & \quad \theta^0 \\
\theta^* & \quad \theta^0 \\
\end{align*} \]

\[ \theta^* = \theta^0 \]

Figure 6: Disclosure strategy when $\mu$ is high

2.3.2 Market Pricing

Theorem 7 characterizes the firm’s investment and disclosure decisions. To complete the description of the equilibrium, Theorem 8 presents the equilibrium market pricing functions. Let $P^N(r)$ ("N" for no disclosure) denote the equilibrium price function if no voluntary disclosure is made by the firm, and $P(r, \theta)$ denote the equilibrium price function if the firm makes voluntary disclosure.

**Theorem 8** The equilibrium market pricing functions take the following form:

\[ P(r, \theta) = \mu + b_r r + b_\theta (\theta - \mu), \quad (37) \]

\[ P^N(r) = \begin{cases} 
\frac{r \sigma_y^2}{\sigma_y^2 + \sigma_z^2} - \frac{q b \sigma' f(c)}{1 - q + q \Phi(c)}, & \text{if } \theta^* < \theta^0 \\
\frac{r \sigma_y^2}{\sigma_y^2 + \sigma_z^2} - \frac{q d [b_0 f(c) - (b_0 - b) f(g)] - b_\theta' (\Phi(c) - \Phi(g))}{1 - q + q \Phi(c)}, & \text{if } \theta^* > \theta^0 
\end{cases} \quad (38) \]

43When firms without an investment opportunity can credibly disclose that they lack such opportunity, and are therefore distinguished from those with an opportunity but choose not to disclose, the equilibrium illustrated by Figure 6 no longer appears and only the equilibrium illustrated by Figure 5 exists. In that case, all firms without an investment opportunity will disclose so, and only firms with a very bad project will restrain from disclosure.
where

\( \Phi(\cdot) \) is the cumulative distribution function of a standard normal variable, 

\( \phi(\cdot) \) is the probability density function of a standard normal variable, 

\[
\begin{align*}
    b_r &= \frac{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + (\sigma_y + \rho \sigma_x) \sigma_y \sigma^2_r}{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma^2_y \sigma^2_y + \sigma^2_y \sigma^2_r}, \\
    b_\theta &= \frac{(1 - \rho^2) \sigma_x^2 \sigma^2_y + (\sigma_x + \rho \sigma_y) \sigma_x \sigma^2_r}{(1 - \rho^2) \sigma_x^2 \sigma^2_y + \sigma^2_x \sigma^2_y + \sigma^2_x \sigma^2_r}, \\
    b &= \frac{\rho \sigma_x \sigma_y \sigma^2_r}{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma^2_x \sigma_y^2 + \sigma^2_x \sigma^2_r}, \\
    c &= \frac{\theta^* - \mu'}{\sigma'}, \\
    g &= \frac{\theta^0 - \mu'}{\sigma'}, \\
    \mu' &= \mu + \frac{\rho \sigma_x \sigma_y}{\sigma_x^2 + \sigma^2_y} \sigma^2_r, \\
    \sigma' &= \sqrt{\sigma^2_x + \sigma^2_y - \frac{\rho^2 \sigma^2_x \sigma^2_y}{\sigma_x^2 + \sigma^2_y}}.
\end{align*}
\]

Theorem 8 describes the market’s rational expectation of the total future cash flow given all public information available. When the firm discloses, indicating \( I = 1 \), equation (37) shows that the market price is simply the expected total cash flow conditional on the mandatory report \((r)\) and the disclosure \((\theta)\). For any deviation of the signals, \( r \) and \( \theta \), from their means, the market revises its valuation of the firm by response coefficients \( b_r \) and \( b_\theta \) respectively. The magnitude of the response coefficients measures the extent to which the signal \( r \) or \( \theta \) updates the market’s belief about the total cash flow.

When no information is disclosed \((d = ND)\), the market infers that either the firm does not find investment opportunities, or the firm chooses to withhold its private information. Since the market is not able to perfectly distinguish among non-disclosure firms, the price reflects an average assessment based on the mandatory report \((r)\). In equation (38), the first item reflects the market’s updated conjecture on \( y \) conditional on \( r \), and the second item reflects the market’s downward adjustment given a portion of non-disclosure firms must have observed low private signals.

To illustrate the equilibrium characterized by Theorem 7 & 8, consider the following numerical example. Let \( \beta = 0.1, q = 0.5, \rho = 0.5, \sigma^2_r = 10, \sigma^2_\theta = \sigma^2_x = \sigma^2_y = 1, \) and \( \mu = 0 \). The socially optimal project selection threshold \( \theta^{FB} \) is 2. The break-even level \( \theta^0 \) is 2.22 where the firm’s expected net payoff from investment is zero with non-disclosure. The disclosure threshold \( \theta^* \) is 1.91 which is lower than \( \theta^{FB} \). Figure 7(a) illustrates the relation between the firm’s expected net payoff \((E[U|\theta])\) and its private signal \((\theta)\) when the firm chooses to disclose (the steep line) and not to disclose (the less steep line). These two lines cross at \( \theta^* \) where the firm is indifferent.
between disclosure and non-disclosure. Solid parts of these two lines indicate the firm’s expected net payoff in equilibrium, while the dotted lines indicate off equilibrium choices. Figure 7(b) illustrates the relation between the firm’s expected share price ($E[P|\theta]$) and its private signal ($\theta$) given its optimal disclosure choice. The discontinuity at $\theta^*$ confirms the discussion on the disclosure choice when $\mu$ is low, that if the firm does not disclose, the firm is undervalued and if it discloses, the share price goes up, but it incurs an efficiency loss, so the firm is indifferent between disclosure and non-disclosure.

One can also interpret Figure 7(b) as the market average response to the firm’s voluntary disclosure. It suggests that the market pricing is positively related with the disclosure, and the discontinuity suggests a negative average market reaction to a non-disclosure firm which is consistent with the empirical evidence provided in Chen, Matsumoto and Rajgopal [2006] who find the average negative return around the announcement to stop earnings guidance.

![Figure 7: (a) Relation between the firm’s expected net payoff and $\theta$; (b) Relation between the firm’s expected market price and $\theta$.](image-url)
(ii) When $\frac{\sigma^2_r}{\sigma^2_\theta}$ is sufficiently high, $b_r$ ($b_\theta$) increases (decreases) in $\rho$; when $\frac{\sigma^2_r}{\sigma^2_\theta}$ is sufficiently low, $b_r$ ($b_\theta$) decreases (increases) in $\rho$.

The noise term $\varepsilon_r$ ($\varepsilon_\theta$) reduces the value of the mandatory accounting report $r$ (the voluntary disclosure $\theta$) in communicating information about the future cash flows. Any increase in the variance weakens the relationship between the cash flow and the reported value, so the capital market responds less accordingly, which explains part (i) that the market response coefficient $b_r$ ($b_\theta$) decreases in the noise level of the reported value $\sigma^2_r$ ($\sigma^2_\theta$).

Here, we are particularly interested in the interaction between the market responses to $r$ and $\theta$. Corollary 4 shows the market response to the disclosure ($b_\theta$) increases in the noise level of the mandatory report ($\sigma^2_\theta$). In other words, if the mandatory report is noisier, the market transfers some pricing weight from $r$ to $\theta$. This induces a substitution between the two information sources. The pricing weight is allocated between $r$ and $\theta$ depending on their relative noise levels.

The correlation parameter $\rho$ defines the extent to which the market infers the project’s (the ongoing activities') cash flow from the mandatory (voluntary) report. As $\rho$ increases, the interaction effect becomes more intense. When the information asymmetry is small, indicating that $\theta$ is much noisier than $r$, more pricing weight is transferred from $\theta$ to $r$ when $\rho$ increases, i.e., $b_r$ ($b_\theta$) increases (decreases) in $\rho$. Alternatively, when the information asymmetry is large, more pricing weight is transferred from $r$ to $\theta$ when $\rho$ increases, i.e., $b_r$ ($b_\theta$) decreases (increases) in $\rho$.

2.4 Disclosure Strategy Analysis

Having characterized the equilibrium, I now analyze how the firm’s disclosure choice responds to various parameters of the model. In particular, the analysis focuses on the impact of different features of the information structure (the firm’s private signal and the mandatory accounting report) and the prior expectation of the investment profitability.

2.4.1 Effect of Information Structure

We begin with the case in which the informational quality of the private signal ($\theta$) is relatively low compared with the public report ($r$). That is, the information asymmetry between the firm
and outsiders is small. The following corollary presents the sufficient conditions under which the firm’s disclosure threshold approaches the level where the project yields zero expected net return.

**Corollary 5** When \( \frac{\sigma_\theta^2}{\sigma^2} \) is sufficiently high, or \( \rho \) is sufficiently low, or \( \beta \) is sufficiently low, the disclosure threshold \( \theta^* \) approaches \( \theta^{FB} \).

From the discussion in the last section, the firm has an incentive to affect the market’s perception in its favor by undertaking a less profitable project \( (\theta^* < \theta < \theta^{FB}) \), or by withholding information of a profitable project \( (\theta^{FB} < \theta < \theta^*) \). When the private signal is relatively noisy, the firm can barely affect the market’s perception through selective disclosure, since it becomes easier for the market to distinguish among non-disclosure firms and translate the firm’s disclosure choice based on the accurate mandatory report. In this sense, the noisy private signal makes the opportunistic use of voluntary disclosure less effective, which reduces the distortion. The firm chooses to undertake the project and discloses only if \( \theta \) indicates the project yields a positive expected net return.\(^{44}\)

Next, we consider the case in which the informational quality of the private signal is high relative to the mandatory report. That is, the information asymmetry between the firm and outsiders is large. The firm tends to utilize its information advantage to affect the market’s perception in its favor. Intuitively, such a tendency becomes more intense as the information asymmetry widens. The following theorem analyzes how the disclosure threshold responds to the noise level of the mandatory report when the informational asymmetry is large.\(^{45}\)

**Theorem 9** When \( \frac{\sigma_\theta^2}{\sigma^2} \) is sufficiently low, then:

(i) if \( \mu \) is sufficiently low, the disclosure threshold \( \theta^* \) strictly decreases in the noise level of the mandatory report \( \sigma^2 \);

\(^{44}\)The same results exist when either \( \rho \) or \( \beta \) is sufficiently low. When \( \rho \) is sufficiently low, \( \theta \) is uninformative about the ongoing activities, and thus the distorted incentive is eliminated. Similarly, when \( \beta \) is sufficiently low, the firm’s payoff is more consistent with the underlying firm value, which also dampens the distorted disclosure incentive.

\(^{45}\)Here, we do not analyze the effect of the private signal’s noise level \( (\sigma_\theta^2) \) on the disclosure threshold \( (\theta^*) \) because those results are not comparable. Imagine if \( \sigma_\theta^2 \) is changed, the firm adjusts its expected profitability accordingly. Then the results on the change of the threshold are less illuminating, since it could be either driven by the firm’s adjustment on expected profitability, or by the change of information structure.
(ii) if $\mu$ is sufficiently high, the disclosure threshold $\theta^*$ strictly increases in the noise level of the mandatory report $\sigma^2_r$.

As demonstrated by Theorem 7, the firm with a lower private signal generally tends to refrain from disclosure to benefit from pooling. The mandatory report, to some extent, conveys information about the private signal $\theta$ (via $v_1$ and $v_2$). Indirectly, the mandatory report removes some benefits of pooling. A precise mandatory report makes it harder for the firm with a lower private signal to pool with those without an investment opportunity. This disciplining role discourages (distorted) disclosures when $\mu$ is low, and encourages more disclosures when $\mu$ is high.

More specifically, assuming $\mu$ is low, average firms in the market have less profitable projects. The benefit from disclosure motivates firms to make distorted disclosure decision. When the noise level of the mandatory report ($\sigma^2_r$) decreases and the report conveys more value-relevant information, the marginal benefit of the distorted disclosure is lessened. Thus the firm becomes more reluctant to undertake a less profitable project and disclose. The disclosure threshold $\theta^*$ shifts upward, and the project selection choice becomes more efficient. In this case, the mandatory accounting information is a substitute for the firm’s disclosure. Better accounting information discourages detrimental voluntary disclosures. This substitution result is consistent with the empirical evidence from Chen, Defond and Park [2002] that when quarterly earnings are less informative, managers are more likely to voluntarily disclose information (which is related to balance sheets) to supplement information in earnings.

On the other hand, when $\mu$ is high and the mandatory report conveys more information about $\theta$, the benefit of pooling with non-disclosure firms is diminished. More firms choose to disclose and the disclosure threshold ($\theta^*$) shifts downward. In this case, the mandatory accounting information is a complement for the firm’s disclosure. Better accounting information encourages more voluntary disclosures.

### 2.4.2 Effect of Investment Prospects

In this subsection, we examine the effect of investment prospects (represented by the ex ante expectation of the investment profitability $\mu$) on the disclosure threshold ($\theta^*$). Since neither the
mandatory accounting report nor the firm’s private signal perfectly reveals the firm value, the
prior expectation ($\mu$), to some extent, influences both the firm’s and the market’s perception.
The following theorem summarizes how the prior expectation ($\mu$) affects the firm’s disclosure
strategy.

**Theorem 10** If $\frac{\sigma_2^2}{\sigma^2}$ is sufficiently low, the firm’s disclosure threshold $\theta^*$ strictly increases in $\mu$;
if $\frac{\sigma_2^2}{\sigma^2}$ is sufficiently high, the firm’s disclosure threshold $\theta^*$ strictly decreases in $\mu$.

To interpret the results in Theorem 10, recall that the firm’s disclosure threshold ($\theta^*$) is
determined by the cost (or benefit) of making disclosure, and the cost (or benefit) of pooling
with non-disclosure firms. When the prior expectation ($\mu$) changes, both sides of the trade-off
are affected.

How the investment prospect ($\mu$) relates to the firm’s disclosure trade-off depends on the
relative informational quality of the private signal to the mandatory report. For instance, if
$\frac{\sigma_2^2}{\sigma^2}$ is low, the firm’s perception of the cost (or benefit) of disclosure is not affected by $\mu$, while
the market’s conjecture heavily relies on $\mu$. As $\mu$ is influential to the market perception, the
pooling effect is enlarged. When $\mu$ increases, the cost (or benefit) of pooling with non-disclosure
firms is reduced (increased) substantially, and the firm’s perception of the cost (or benefit) of
disclosure does not change. So more firms choose not to disclose, and the disclosure threshold
shifts upward. Alternatively, when $\frac{\sigma_2^2}{\sigma^2}$ is high, the market price is relatively more accurate in
evaluating firm value. The pooling effect is diminished, and the firm’s payoff is more consistent
with the underlying firm value. When $\mu$ increases, the firm’s perception of the project return
is increased substantially, so more firms choose to disclose, and the disclosure threshold shifts
downward. This analysis suggests an empirical prediction that when the information asymmetry
is small, more firms voluntarily disclose if the investment prospect is high on average.

Table 3 summarizes the basic results of the firm’s disclosure strategy analysis.
Table 3: Summary of the firm’s disclosure strategy

<table>
<thead>
<tr>
<th>Effect of $\sigma^2_r$</th>
<th>Effect of $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sigma^2}{\sigma^2_r}$ is sufficiently high</td>
<td>Ambiguous</td>
</tr>
<tr>
<td>$\frac{\sigma^2}{\sigma^2_r}$ is sufficiently low</td>
<td>$\frac{\partial \theta^*}{\partial \sigma^2_r} &lt; 0$ if $\mu$ is low</td>
</tr>
</tbody>
</table>

2.5 Investment Decisions and Efficiency Implications

To analyze the investment efficiency of the equilibrium characterized by Theorem 7 and 8, we need to consider the firm’s project selection choice. In particular, the investment efficiency is decided by how far the firm’s project selection choice deviates from the socially optimal threshold $\theta^{FB}$. When the disclosure threshold is low ($\theta^* < \theta^0$), the selection choice is directly driven by the disclosure choice. When the threshold is high ($\theta^* > \theta^0$), the selection threshold is $\theta^0$, the break-even level where the firm’s expected net payoff from investing in the project is zero with non-disclosure.

We study the efficiency of the firm’s investment decision by analyzing the effects of both the private signal and the mandatory accounting report, and the interaction between these two information sources.

2.5.1 Efficiency Effects of Private Information

Several studies examine the effects of private information on the firm’s ex ante payoff. Verrecchia [1990] and Pae [1999] reach the same conclusion that the firm’s ex ante payoff is monotonically decreasing in the quality of the private signal. That is, the firm is ex ante best off by choosing to be completely uninformed. The argument is that when the firm is more informed, it is more willing to disclose, which incurs more associated costs. A similar force is present in this setting. If the firm’s private signal is precise, it is tempted to selectively make disclosure to maximize the market’s perception on the firm’s value. When the ex ante investment prospect ($\mu$) is low, the firm would like to undertake a less profitable project and disclose for distinction; when $\mu$ is high, the firm would like to withhold private information for pooling. Such a propensity for
selectively making disclosure or withholding information might be detrimental to the efficiency.

However, this setting allows for a positive role of private information. When the firm obtains better knowledge of the project’s cash flow, it is better able to distinguish good from bad projects, therefore, is better able to make efficient investment decisions. As the firm is partially motivated by its long-term value, better information endowment could also improve efficiency. Thus the positive and negative roles of the private information jointly determine whether private information improves efficiency or not. The economy may be better-off, ex ante, by having a privately informed firm. Theorem 11 provides a comparison between the efficiency effects of a precise private signal versus a coarse private signal.

**Theorem 11** Let $V_H$ ($V_L$) denote the ex ante expected net return of the project when $\sigma_\beta^2$ is sufficiently high (low), then:

(i) when $\mu$ is sufficiently low, $V_H > V_L$;

(ii) when $\beta$ is sufficiently low, $V_H < V_L$;

(iii) when $\sigma_\beta^2$ is sufficiently low, $V_H < V_L$.

Theorem 11 implies that an informative private signal can increase or decrease the efficiency. When $\mu$ is low, an informative private signal hurts efficiency. To explain this, recall that by Theorem 10, when $\sigma_\beta^2$ is sufficiently low, the firm’s disclosure threshold increases in $\mu$. That is, a lower $\mu$ leads to a lower disclosure threshold, which implies that the firm tends to invest in a less profitable project and disclose, since the market puts more pressure on those without disclosure. As $\mu$ decreases, the firm incurs a higher cost for disclosure, which leads to a deterioration in project selection choice. Thus, the negative effects of private information are aggravated.

To let a more informative private signal improve efficiency, the firm must take advantage of its private information in an appropriate way. Recall that Corollary 5 shows that a lower $\beta$ dampens the firm’s distorted disclosure incentive, since the firm’s payoff is more consistent with the underlying firm value. As the negative role of the private signal is diminished, a more informative private signal enhances the efficiency because the firm becomes better able to make efficient investment decisions.

Part (iii) of Theorem 11 demonstrates that when the informational quality of the mandatory accounting report is high, the firm is ex ante better off by having a precise private signal. Recall
in Theorem 9, the mandatory report enables the market to imperfectly distinguish among non-disclosure firms, so it plays a disciplining role on the voluntary disclosure. With a precise mandatory report, the firm’s incentive to selectively disclose or withhold private information is easier for the market to disentangle. In other words, the market pricing is more aligned to the underlying firm value, and the firm is motivated to use its private signal to choose better projects and improve efficiency.

### 2.5.2 Efficiency Effects of Mandatory Accounting Report

In the preceding discussion, we showed that the informational quality of the mandatory accounting report influences the firm’s investment and disclosure decisions through a disciplining effect. To gain further insight into the efficiency effects of the mandatory report, we examine how the noise level of the mandatory report affects the investment efficiency when the information asymmetry is large or small. The results appear in the following theorem.

**Theorem 12** When $\frac{\sigma^2_r}{\sigma^2_\ell}$ is sufficiently low, then:

1. if $\mu$ is sufficiently low, a marginal decrease in $\sigma^2_r$ improves the investment efficiency;
2. if $\mu$ is sufficiently high, a marginal change in $\sigma^2_r$ does not change the investment efficiency.

Part (i) of Theorem 12 implies that when the information asymmetry is large and $\mu$ is sufficiently low, a precise mandatory report enhances the efficiency. In this situation, the disclosure threshold $\theta^*$ is lower than the socially optimal level $\theta^{FB}$, that is, some firms tend to invest in a less profitable project and disclose (illustrated by Figure 5). Recall part (i) of Theorem 9 that in the same situation, the disclosure threshold $\theta^*$ strictly decreases in the noise level of the mandatory report ($\sigma^2_r$). If $\sigma^2_r$ decreases, the disclosure threshold $\theta^*$ shifts upward, so less firms choose to invest in a less profitable project, and the project selection choice becomes more efficient. Thus, this disciplining role of the mandatory report discourages (distorted) disclosures and improves the investment efficiency. Alternatively, if $\mu$ is sufficiently high, the disclosure threshold $\theta^*$ is higher than the break-even level $\theta^0$, and some firms would like to withhold private information for pooling. And the project selection choice is only decided by the break-even level (illustrated by Figure 6). Any change in the mandatory report quality can only affect the firm disclosure decision, but not the investment decision.
Overall, Theorem 12 demonstrates that the efficiency effect of the mandatory report is contingent on the information and economy environment. When the private signal is relatively precise and the prior expectation of the investment profitability is low, a precise mandatory report induces higher efficiency. Alternatively, if the prior expectation of the investment profitability is high, the investment efficiency is not affected by the marginal change in the mandatory report quality.

2.6 Voluntary Disclosure Is Prohibited

In this section, we consider the case in which voluntary disclosure is prohibited. That is, a firm who undertakes a project is not allowed to communicate with the market about its private information. The firm may find it difficult to communicate its private information about the investment, since either disclosure regulation has restrictions on certain kinds of sensitive information, or the disclosure itself cannot be credibly communicated. This analysis may be valuable in that it helps us to understand how prohibiting voluntary disclosure influences the firm’s investment decision, and its subsequent effect on economic efficiency.

2.6.1 Equilibrium with Disclosure Prohibited

Suppose that voluntary disclosure is not available at date 2. The firm who is confronted with an investment opportunity is not allowed to issue voluntary disclosure regardless of its investment decision. There is no information about the investment is available to the market and all value relevant information publicly available is the mandatory accounting report \( r \), i.e., \( \Omega = \{r\} \). We now analyze the equilibrium behavior of the firm and the market pricing function \( P_B(r) \) ("B" for prohibited) when voluntary disclosure is prohibited.

**Theorem 13** If voluntary disclosure is prohibited, there exists an equilibrium where the firm’s investment decision and the market pricing are given by

(i) equilibrium investment function:

\[
I = \begin{cases} 
1, & \text{if } \theta > \theta^0 \text{ and the investment is available,} \\
0, & \text{if otherwise.} 
\end{cases}
\]  

(39)
(ii) equilibrium market pricing function:

$$P^R(r) = \frac{r\sigma^2_y}{\sigma^2_y + \sigma^2_r} + q\Phi(-g) \left[ \mu' + (b_s - b) \frac{\sigma'f(-g)}{\Phi(-g)} \right]. \quad (40)$$

Without voluntary disclosure, the market price is irrelevant to the investment decision since the mandatory report only captures information on the ongoing activities. As a result, the firm only yields a marginal return to the investment cost from long-term interests, which leads to the under-investment decision. The firm chooses to undertake the project only if the expected net payoff is positive without disclosure ($\theta > \theta^0$).

In addition, the mandatory report, to some extent, reveals information about the private signal $\theta$ (via $x$ and $y$) which is related to the firm’s investment decision. When the mandatory report is favorable, the market updated conjecture on the private signal ($\theta$) is high, which indicates a higher likelihood of undertaking a profitable project. When the mandatory report is unfavorable, a lower conjecture on $\theta$ indicates an unprofitable project and a lower likelihood of undertaking a project. In the equilibrium market pricing function (40), the first item reflects the market updated conjecture on $y$ conditional on the mandatory report ($r$), and the second item reflects the market’s upward adjustment on $x$, since it is possible that the firm undertakes a profitable project.

2.6.2 The Efficiency Implications of Prohibiting Voluntary Disclosure

Standard intuition suggests that prohibiting voluntary disclosure would result in inferior decisions for the related reasons: (i) the share price is less accurate since the market has less value-relevant information; (ii) the firm, anticipating this less accurate pricing, will severely under-invest in the project, which leads to less efficiency. It is the purpose of this subsection to show that, notwithstanding these intuitive arguments indicating the inferiority of prohibiting voluntary disclosure, it can perform strictly better than allowing voluntary disclosure in certain circumstances. The following theorem analyzes the efficiency implications of prohibiting voluntary disclosure.

**Theorem 14** If $\mu$ and $\frac{\sigma^2_y}{\sigma^2_r}$ are sufficiently low, prohibiting voluntary disclosure improves the
investment efficiency; if $\frac{\sigma_2^2}{\sigma_1^2}$ is sufficiently high, prohibiting voluntary disclosure decreases the investment efficiency.

According to Theorem 14, when the investment prospect is low and the information asymmetry is large, prohibiting voluntary disclosure is efficiency enhancing. This counter-intuitive result is driven by the firm’s distorted disclosure incentive when voluntary disclosure is allowed. As we have illustrated above, with a precise private signal, the firm’s disclosure threshold ($\theta^*$) differs from the break-even level ($\theta^0$). Especially, a lower $\mu$ leads to a lower disclosure threshold, which implies that the firm tends to invest in a less profitable project and disclose since the market puts more pressure on those without disclosure. In other words, making more disclosures leads to a deterioration of investment decision. When voluntary disclosure is prohibited, the firm’s distorted disclosure incentive is eliminated and no endogenous disclosure cost is incurred, which leads to more efficient investment decisions.\(^{46}\)

When the information asymmetry is small, the mandatory report is very precise and let the market easier to distinguish among non-disclosure firms. And this disciplining role limits the opportunistic behavior and, thus, reduces the efficiency loss when voluntary disclosure is allowed. In this case, an accurate accounting report allows the firm’s voluntary disclosure to be more beneficial for the economy.

**Corollary 6** If voluntary disclosure is prohibited, a more informative private signal (lower $\sigma_2^2$) is always efficiency enhancing.

In contrast to Theorem 11, Corollary 6 shows that if voluntary disclosure is prohibited, a more informative private signal always leads to more efficiency. That is, the firm is always better off by choosing to be more informed. To explain this, recall the positive role (better knowledge of investment cash flow) and the negative role (distorted disclosure incentive) of private information when disclosure is allowed. If voluntary disclosure is prohibited, the negative role of private information is completely eliminated, while the positive role is still in effect. Though the firm does not utilize its private information to the fullest extent, the marginal efficiency improvement of private information is still strictly positive.

\(^{46}\)Again, one can consider voluntary disclosure as the firm’s option. The value of this option is related to the economic situation and the information environment.
2.7 Summary

This paper examines the determinants and economic efficiency of corporate voluntary disclosure, where the economic efficiency stems from the consequences of firm investment decisions. Voluntary disclosure plays two roles (positive and negative) with respect to firm investment efficiency. By transmitting value-relevant information to the market, voluntary disclosure leads to a more accurate pricing and improves investment efficiency, which is the positive role. The negative role is that firms use voluntary disclosure opportunistically to affect the market pricing in its favor, which can be detrimental to investment efficiency. This paper shows that the efficiency implication of voluntary disclosure is not obvious and determined by the dominant one of these two roles. In addition, the mandatory accounting report also plays an important role on firm investment and disclosure decisions. This study finds that the mandatory accounting report helps the market distinguish among non-disclosure firms and disentangle the disclosure incentive, and thus plays a disciplining role of voluntary disclosure. The results demonstrate an interaction between the efficiency effects of the mandatory accounting report and of the private signal.

This paper also examines the firm investment efficiency when voluntary disclosure is prohibited. By comparing the economic efficiency induced by prohibiting or allowing voluntary disclosure, this study may provide policy makers with useful knowledge for designing disclosure regulations in light of their overall impact on the firm’s investment efficiency.

Future works may benefit from including more disclosure regulation implications with regard to the efficiency of discretionary disclosure arrangements. The exploration of interaction between disclosure and firm’s additional operating and financial choices is also left for future research.
Proof. (of Theorem 7&8) We begin with the market pricing functions assuming the disclosure threshold at $\theta^*$. Given the jointly normal distribution,

$$
\begin{pmatrix}
  x + y \\
  y + \varepsilon_r \\
  \theta
\end{pmatrix}
\sim N
\begin{pmatrix}
  \mu \\
  0 \\
  \mu
\end{pmatrix},

\begin{pmatrix}
  \sigma_x^2 + \sigma_y^2 + 2\rho \sigma_x \sigma_y & \sigma_y^2 + \rho \sigma_x \sigma_y & \sigma_x^2 + \rho \sigma_x \sigma_y \\
  \sigma_y^2 + \rho \sigma_x \sigma_y & \sigma_y^2 + \rho \sigma_x \sigma_y & \rho \sigma_x \sigma_y \\
  \sigma_x^2 + \rho \sigma_x \sigma_y & \rho \sigma_x \sigma_y & \sigma_x^2 + \sigma_y^2
\end{pmatrix}.
$$

Following the properties of normal distribution, we have the market pricing with disclosure is,

$$P(r, \theta) = E[x + y|r, \theta]$$

$$= \mu + b_r r + b_\theta (\theta - \mu)\quad (41)$$

where

$$b_r = \frac{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + (\sigma_y + \rho \sigma_x) \sigma_y \sigma_\theta^2}{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_\theta^2 + \sigma_x^2 \sigma_\tau^2 + \sigma_\theta^2 \sigma_\tau^2},
\quad b_\theta = \frac{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + (\sigma_x + \rho \sigma_y) \sigma_x \sigma_\tau^2}{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_\theta^2 + \sigma_x^2 \sigma_\tau^2 + \sigma_\theta^2 \sigma_\tau^2}.$$

Firms without disclosure (assuming $\theta^* < \theta^0$) are composed by firms with a low private signal who choose to withhold information and firms without an investment opportunity. For the first group of firms, the market pricing is

$$P_N^*(r, \theta) < \theta^* = E[y|r, \theta < \theta^*]$$

$$= b'_r r + b (E[\theta|r, \theta < \theta^*] - \mu)\quad (42)$$

where

$$b'_r = \frac{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_\theta^2}{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_\theta^2 + \sigma_x^2 \sigma_\tau^2 + \sigma_\theta^2 \sigma_\tau^2},
\quad b = \frac{\rho \sigma_x \sigma_y \sigma_\tau^2}{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_\theta^2 + \sigma_x^2 \sigma_\tau^2 + \sigma_\theta^2 \sigma_\tau^2}.$$

For the second group of firms, the market pricing is

$$P_N^*(r) = E[y|r] = \frac{r \sigma_y^2}{\sigma_y^2 + \sigma_\tau^2}.$$

To find $E[\theta|r, \theta < \theta^*]$ in (42), the updated distribution of $\theta$ conditional on $r$ is

$$\theta|r \sim N[u', (\sigma')^2],$$

where

$$\mu' = \mu + \frac{r \rho \sigma_x \sigma_y}{\sigma_y^2 + \sigma_\tau^2},
\quad \sigma' = \sqrt{\sigma_x^2 + \frac{\rho^2 \sigma_y^2 \sigma_\tau^2}{\sigma_y^2 + \sigma_\tau^2}}.$$
Then we have
\[ E[\theta|r, \theta < \theta^*] = u' - \sigma f(c) \Phi(c), \]
where \( c = \frac{\theta^* - \mu}{\sigma} \), \( f(\cdot) \) and \( \Phi(\cdot) \) are the probability density function and the cumulative distribution function for a standard normal variable. Substituting (45) into (42) and simplifying, we have
\[ P_N(r, \theta < \theta^*) = \frac{r \sigma_y^2}{\sigma^2_y + \sigma^2_r} - \frac{b \sigma f(c) \Phi(c)}{1 - q + q \Phi(c)}. \]  
(46)

Since the market could not distinguish among firms that do not disclose, the market overall pricing on non-disclosure firms is a weighted average of (43) and (46), which is
\[ P_N(r) = \frac{r \sigma_y^2}{\sigma^2_y + \sigma^2_r} - \frac{b \sigma f(c) \Phi(c)}{1 - q + q \Phi(c)}. \]

Using the same method, we obtain the market pricing of non-disclosure firms when \( \theta^* > \theta^0 \).

Provided the market pricing functions (\( P(r, \theta) \) and \( P_N(r) \)), and the disclosure threshold \( \theta^* \), when the private signal \( \theta \) is higher than \( \theta^* \), the firm has to choose to invest \( (I = 1) \) in order to be able to disclose. When \( \theta \) is lower than \( \theta^* \), the firm chooses to invest only if the expected net payoff is higher than zero, which is \( E[-I + (1 - \beta)x|\theta, I = 1] > 0 \). Then we find the break-even level where the firm’s expected net payoff from the investment is zero, at \( \theta^0 = \frac{\sigma_x^2 + \sigma_y^2(1 - \mu + \beta \mu)}{(1 - \beta) \sigma^2_x} \).

To find the disclosure threshold and show the existence of \( \theta^* \), let \( U(\theta, d = D) \) and \( U(\theta, d = ND) \) be the firm’s expected net payoff with the private signal \( \theta \) when the firm chooses to disclose and not to disclose respectively. Using the above results, we have (assuming \( \theta^* < \theta^0 \))
\[ U(\theta, d = D) = \mu - 1 + (\theta - \mu) \frac{\sigma_x^2 + \rho \sigma_x \sigma_y}{\sigma^2_x + \sigma^2_\theta}, \]
\[ U(\theta, d = ND) = (\theta - \mu) \frac{\rho \sigma_x \sigma_y}{\sigma^2_x + \sigma^2_\theta} \left( 1 - \frac{\beta \sigma_r^2}{\sigma^2_y + \sigma^2_r} \right) - \beta \sigma' \cdot h(q, \theta, c), \]
where \( h(q, \theta, c) = E[q f(c) \Phi(c)|\theta] \).

At the disclosure threshold \( \theta^* \), the firm is indifferent between disclosing and not disclosing, which is \( U(\theta^*, d = D) = U(\theta^*, d = ND) \). We obtain the following equation, which has to hold
As the range of $h(q, \theta, c)$ is bounded, the left hand side (LHS) of (47) is strictly increasing in $\theta$ and the first item in the right hand side (RHS) is strictly decreasing in $\theta$, and all of the functions are continuous, there must be at least one root of $\theta^*$ letting the equation hold. For any $\theta > \theta^*$, the LHS is higher than the RHS and the firm chooses to disclose. For any $\theta < \theta^*$, the LHS is lower than the RHS and the firm chooses not to disclose. Using the same method, we show the existence of disclosure threshold when $\theta^* > \theta^0$. ■

Proof. (of Corollary 4) For part (i), it is easy to find $\frac{\partial b_r}{\partial \sigma_r^2} < 0$ $(\frac{\partial b_y}{\partial \sigma_y^2} < 0)$ since the noise variance $\sigma_r^2$ ($\sigma_y^2$) only appears in the denominator. From (41), we have $\frac{\partial b_r}{\partial \sigma_r^2} = \frac{\rho \sigma_x \sigma_y + \rho \sigma_x \sigma_y \sigma_x + \sigma_x \sigma_y}{\frac{1}{(1 - \rho^2) \sigma_y^2 + \sigma_y^2 \sigma_r^2 + \sigma_y^2 \sigma_r^2 + \sigma_y^2 \sigma_r^2}} > 0$. Using the same method, we have $\frac{\partial b_y}{\partial \sigma_y^2} > 0$. For part (ii),

$$\frac{\partial b_r}{\partial \rho} = \sigma_x \sigma_y \sigma_r^2 \left[ (1 + \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_r^2 + \sigma_x^2 \sigma_r^2 + \sigma_y^2 \sigma_r^2 \right] - 2 \rho \sigma_r^2 (\sigma_x^3 + \sigma_x \sigma_y \sigma_r^2).$$

When $\frac{\sigma_r^2}{\sigma_x^2}$ is sufficiently high, the first item in the numerator is higher than the second item which leads to positive derivative $\frac{\partial b_r}{\partial \rho}$. Similarly, when $\frac{\sigma_y^2}{\sigma_r^2}$ is sufficiently low, $\frac{\partial b_r}{\partial \rho} < 0$. And similarly

$$\frac{\partial b_y}{\partial \rho} = \sigma_x \sigma_y \sigma_r^2 \left[ (1 + \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_r^2 + \sigma_x^2 \sigma_r^2 + \sigma_y^2 \sigma_r^2 \right] - 2 \rho \sigma_r^2 (\sigma_y^3 \sigma_x + \sigma_x \sigma_y \sigma_r^2).$$

Using the same way, we have $\frac{\partial b_y}{\partial \rho}$ is negative (positive) when $\frac{\sigma_y^2}{\sigma_r^2}$ is high (low), ■

Proof. (of Corollary 5) From Theorem 7 & 8, the equation (47) must hold in equilibrium.

Rewrite (47) in the following form

$$\mu - 1 + (\theta^* - \mu) \frac{\sigma_r^2}{\sigma_x^2 + \sigma_y^2} = - \beta \rho \left[ \frac{\sigma_x \sigma_y \theta^* - \mu}{\sigma_x^2 + \sigma_y^2} + \frac{\sigma_x \sigma_y \sigma' h(q, \theta^*, c)}{(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_r^2 + \sigma_x^2 + \sigma_y^2} \right].$$

When $\rho$ or $\beta$ approaches zero, the RHS of (48) approaches to zero. When $\frac{\sigma_r^2}{\sigma_x^2}$ is sufficiently high,
both items in the bracket on the RHS go to zero. Then in all the cases, we have

$$\mu - 1 + (\theta^* - \mu) \frac{\sigma_x^2}{\sigma_x^2 + \sigma_y^2} = 0,$$

which is $\theta^*$ approaches $\theta^{FB} = 1 + \frac{\sigma_x^2}{\sigma_x^2}$. ■

Proof. (Theorem 9 & 10) For Theorem 9, we rewrite the equation (47) into

$$\mu - 1 + \frac{\theta^* - \mu}{\sigma_x^2 + \sigma_y^2} \left( \sigma_x^2 + \frac{\rho \sigma_x \sigma_y \beta \sigma_y^2}{\sigma_y^2 + \sigma_y^2} \right) = -\beta \sigma^r \cdot h(q, \theta^*, c). \quad (49)$$

Since the RHS of (49) is bounded, when $\mu$ is sufficiently low, $(\theta^* - \mu)$ must be positive to sustain the balance. Intuitively, as $\mu$ is lower, $(\theta^* - \mu)$ is getting higher. Taking the first derivative of both sides of equation (49) with respect to $\sigma_x^2$, we have

$$\text{LHS} \quad \frac{(\theta^* - \mu) \rho \sigma_x \sigma_y \beta \sigma_y^2}{(\sigma_x^2 + \sigma_y^2)(\sigma_y^2 + \sigma_y^2)^2}, \quad (50)$$

$$\text{RHS} \quad -\beta \sigma^r h^r(\cdot) \frac{\rho \sigma_x \sigma_y \left[(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_y^2\right]}{[(1 - \rho^2) \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_y^2 + \sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_x^2]^2} + \frac{\beta \rho \sigma_x \sigma_y}{(\sigma_y^2 + \sigma_y^2)^2} h^r(\cdot). \quad (51)$$

The LHS of (49) represents the firm’s expected net payoff by choosing to disclose at $\theta^*$, and the RHS represents the expected net payoff by choosing not to disclose at $\theta^*$. Following the above arguments, (50) is positive ($(\theta^* - \mu)$ is positive) when $\mu$ is sufficiently low, and getting higher as $\mu$ decreases. The range of (51) is bounded given both $h(\cdot)$ and $h^r(\cdot)$ are bounded\(^\ddagger\). When $\frac{\sigma_x^2}{\sigma_y^2}$ is sufficiently low, the absolute value of (51) is approaching zero. When $\mu$ is sufficiently low, (50) must be higher than (51), which implies at $\theta^*$, the firm chooses to disclose and the disclosure threshold shifts downward. Use the same method, we have the disclosure threshold $\theta^*$ shifts upward in $\sigma_x^2$, if $\mu$ is sufficiently high.

For Theorem 10, taking the first derivative of both sides of equation (49) with respect to $\mu$,

\(^\ddagger\text{Since both } f(\cdot) \text{ and } \Phi(\cdot) \text{ are functions with global upper and lower bounds, as long as } q \text{ is not infinitely close to one, } h(\cdot) \text{ must be also bounded. } h^r(\cdot) \text{ is also a global bounded function. More specifically, once } q < 0.999, \text{ the value of } h(\cdot) \text{ is bounded between } -1 \text{ and } 2.5.\)
we have

\[
\begin{align*}
\text{LHS} & \quad 1 - \frac{\sigma_x^2 + \frac{\rho\sigma_x\sigma_y\beta\sigma_r^2}{\sigma_y^2 + \sigma_r^2}}{\sigma_x^2 + \sigma_y^2}, \\
\text{RHS} & \quad \frac{\beta\rho\sigma_x\sigma_y\sigma_r^2}{(1 - \rho^2)\sigma_x^2\sigma_y^2 + \frac{\sigma_y^2\sigma_r^2}{\sigma_y^2} + \frac{\sigma_x^2\sigma_r^2}{\sigma_x^2} + \frac{\sigma_y^2\sigma_r^2}{\sigma_y^2} + \frac{2}{\sigma_r^2}h_r'(\cdot)}.
\end{align*}
\]

When \(\frac{\sigma_r^2}{\sigma_x^2}\) is sufficiently high, (52) is positive and approaches one, and (53) is approaching zero since \(h_r'(\cdot)\) is bounded. So we have (52) is higher than (53) when \(\frac{\sigma_r^2}{\sigma_x^2}\) is sufficiently high.

Following the above arguments, the firm chooses to disclose more and the disclosure threshold shifts downward. Similarly when \(\frac{\sigma_r^2}{\sigma_x^2}\) is sufficiently low, (52) is negative and lower than (53), the firm chooses to disclose less and the disclosure threshold shifts upward.

**Proof.** (Theorem 11) When \(\frac{\sigma_r^2}{\sigma_x^2}\) is sufficiently high, \(b_\theta\) approaches zero, and the ex ante expected net return (denoted by \(V_H\)) approaches

\[
V_H = \begin{cases} 
0, & \text{if } \mu < 1 \\
q(\mu - 1) , & \text{if } \mu > 1 
\end{cases}.
\]

Similarly, when \(\frac{\sigma_r^2}{\sigma_x^2}\) is sufficiently low, the ex ante expected net return (denoted by \(V_L\)) approaches

\[
V_L = \frac{q}{\sigma_x} \int_{-\infty}^{+\infty} [\theta - 1] f \left( \frac{\theta - \mu}{\sigma_x} \right) d\theta.
\]

To prove part (i), rewrite (55) into

\[
V_L = q\Phi \left( \frac{\mu - \theta^*}{\sigma_x} \right) [\mu - 1] + q\sigma_x f \left( \frac{\theta^* - \mu}{\sigma_x} \right).
\]

As the second item in (56) is positive and bounded. When \(\mu\) is sufficiency low, the first item in (56) turns negative. So we have \(V_L\) is lower than \(V_H\). To prove part (ii), from Corollary 5, when \(\beta\) is sufficiently low and \(\sigma_r^2\) approaches zero, in (55), \(\theta^*\) approaches 1. That is, \(\theta - 1\) is positive for any \(\theta > \theta^*\). So \(V_L\) approaches

\[
\frac{q}{\sigma_x} \int_{0}^{+\infty} tf \left( \frac{t + 1 - \mu}{\sigma_x} \right) dt = q(\mu - 1) + \frac{q}{\sigma_x} \int_{-\infty}^{0} -tf \left( \frac{t - 1 - \mu}{\sigma_x} \right) dt.
\]
Since the last item in (57) is positive, \( V_L \) is higher than \( V_H \). Similarly, to prove part (iii), recall (47), when \( \sigma^2_r \) is sufficiently low, the RHS of (47) approaches zero, so the disclosure threshold goes to \( \theta^{FB} \). When \( \sigma^2_r \) is low, the disclosure threshold \( \theta^* \) goes to 1. Again, \( V_L \) approaches the expression (57), and we have \( V_L \) is higher than \( V_H \). ■

**Proof. (Theorem 12)** From Theorem 10, if \( \frac{\sigma^2_r}{\sigma^2_y} \) is sufficiently low, the disclosure threshold \( \theta^* \) increases in \( \mu \). In claim (i), when \( \mu \) is sufficiently low, \( \theta^* \) is lower than the socially optimal level \( \theta^{FB} \). So the investment function is

\[
I = \begin{cases} 
1, & \text{if } \theta > \theta^* \text{ and the investment is available} \\
0 & \text{if otherwise}.
\end{cases}
\]

From Theorem 9, in this situation, \( \theta^* \) decreases in \( \sigma^2_r \). Then lower \( \sigma^2_r \) leads to higher threshold \( \theta^* \), so that lets \( \theta^* \) closer to the optimal level \( \theta^{FB} \) and improves the efficiency. Similarly, in claim (ii), when \( \mu \) is sufficiently high, \( \theta^* \) is higher than the break-even level \( \theta^0 \). So the investment function is

\[
I = \begin{cases} 
1, & \text{if } \theta > \theta^0 \text{ and the investment is available} \\
0 & \text{if otherwise}.
\end{cases}
\]

Any marginal change of \( \sigma^2_r \) will affect the disclosure threshold \( \theta^* \), but has no effect on the break-even level \( \theta^0 \), so that has no effect on the firm investment decision. ■

**Proof. (Theorem 13)** Following the properties of normal distribution, we have the market expectation on the ongoing activities’ cash flow is,

\[
E[y|r] = \frac{r \sigma^2_y}{\sigma^2_y + \sigma^2_r}.
\]  

(58)

Due to the possibility of undertaking the project, the expected gross return on the project is

\[
E[x|r, \theta > \theta^0] = \mu + (b_r - b'_r) r + (b_\theta - b) \left( E[\theta|r, \theta > \theta^0] - \mu \right),
\]  

(59)

where \( b_r, b'_r, b_\theta \) and \( b \) are given by (41) and (42). To find \( E[\theta|r, \theta > \theta^0] \) in (59), invoke the
updated distribution of \( \theta \) conditional on \( r \) in (44) is \( \theta | r \sim N[\mu', (\sigma')^2] \). Then we have

\[
E[\theta|r, \theta > \theta^0] = u' + \sigma' \frac{\int (-g) \Phi(-g)}{\Phi(-g)},
\]

(60)

where \( g = \frac{\theta - \mu}{\sigma} \) and \( u', \sigma' \) are given by (44). Substituting (60) into (59) and simplifying, we have

\[
E[x|r, \theta > \theta^0] = \mu' + (b_0 - b) \frac{\sigma f(-g)}{\Phi(-g)}.
\]

(61)

Since the market could not distinguish whether the project is implemented or not without disclosure, the market price is a weighted average of (58) and (61), which is

\[
P^B(r) = \frac{r \sigma_y^2}{\sigma_y^2 + \sigma_r^2} + q \Phi(-g) \left[ \mu' + (b_0 - b) \frac{\sigma f(-g)}{\Phi(-g)} \right].
\]

Provided the market pricing function \( P^B(r) \), and voluntary disclosure is prohibited, the firm chooses to invest only if the expected net payoff is higher than zero, which is \( E[-I + (1 - \beta)x|\theta, I = 1] > 0 \). That is, the firm chooses to invest in the project only when the private signal is higher than the break-even level \( \theta^0 \).

**Proof.** (Theorem 14 & Corollary 6) When voluntary disclosure is prohibited, the ex ante expected net return (denoted by \( V_B \)) is

\[
V_B = \frac{q}{\sqrt{\frac{\sigma_y^2 + \sigma_r^2}{\sigma_r^2}}} \int_{-\infty}^{+\infty} \left[ \mu - 1 + \frac{\sigma_x^2 (\theta - \mu)}{\sigma_x^2 + \sigma_\theta^2} \right] f \left( \frac{\theta - \mu}{\sqrt{\sigma_x^2 + \sigma_\theta^2}} \right) d\theta \quad (62)
\]

From (62), it is evident that \( V_B \) is decreasing in \( \sigma_\theta^2 \). When voluntary disclosure is allowed and \( \mu \) is low, the ex ante expected net return is

\[
E_\theta [x - I] = q \int_{-\infty}^{+\infty} \frac{\sigma_x^2 (\theta - \mu)}{\sqrt{\sigma_x^2 + \sigma_\theta^2}} f(t) \, dt. \quad (63)
\]

From Theorem 10, when \( \mu \) and \( \frac{\sigma_x^2}{\sigma_r^2} \) is sufficiently low, \( \theta^* \) becomes negative. So we have \( E_\theta [z - I] \)
is lower than $V_B$. From Corollary 5, if $\frac{\sigma^2}{\sigma^2}$ is sufficiently high, $\theta^*$ approaches $\theta^{FB}$, which is optimal than the break-even level $\theta^0$, so $E_{\theta} [z - I]$ is higher than $V_B$. ■
References


