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Dedicated to my parents
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CHAPTER 1

Introduction

Coping with the complexity of today’s business environment, companies become more and more connected with each other. This change in business environment forces companies to consider supply chain networks in various decision-making activities. This dissertation studies strategic interactions among companies, including both competition and cooperation, in global supply chain networks using cooperative and non-cooperative game theory as well as theory of social and economic networks. Each of the subsequent three chapters examines such interactions in diverse business environments. Each chapter is overviewed below.

Chapter 2 analyzes the competition between a brand-name company and its counterfeiter. It is motivated by the observation that counterfeit products are of distinct types: non-deceptive counterfeits (e.g., fake luxury bags) that are sold at a substantial discount so their consumers know they are purchasing counterfeits, and deceptive counterfeits (e.g., fake drugs) that are sold by a licit distributor who deceives consumers into believing those goods to be authentic at time of purchase. While brand-name companies are spending millions of dollars in order to stop or at least to reduce the incidence of counterfeits, the anti-counterfeiting strategies found to be useful to one product may not work for another or can even
unintentionally make counterfeits flourish more in the market. Therefore, there is a need for anti-counterfeiting strategies that are tailored to specific products. Yet, due to the limited understanding of relations among the types of counterfeits and the effectiveness of anti-counterfeiting strategies, OECD (2008) calls for research that strengthens the analysis of counterfeiting and says:

"Assessing the factors driving production and consumption of counterfeit and pirated products can generate insights into the types of products that are most likely to be infringed, . . . , and lead to more efficient and effective [anti-counterfeiting] strategies."

This chapter attempts to provide such an analysis by providing insights to the following questions: (Q1) What anti-counterfeiting strategies should a brand-name company use to improve her own profit? (Q2) What is the impact of anti-counterfeiting strategies on the profit of a counterfeiter? (Q3) What is the impact of counterfeits on consumer welfare? Do consumers also benefit from the strategies that are effective in combating counterfeits?

To answer these questions, we develop a normative model of licit and illicit supply chains, in which a brand-name company competes with her potential counterfeiter. The counterfeiter in our model is either non-deceptive or deceptive, and decides the level of functional quality and wholesale price of his goods after observing the quality and price of the brand-name product. Depending on his type, the counterfeiter faces different opportunities and risks. The non-deceptive counterfeiter competes directly with a brand-name company for price and quality. Thus
the counterfeiter may have to invest in improving the quality of his goods, which will increase the risk of losing the investment in case of getting caught by the authorities. Conversely, the deceptive counterfeiter may not need to invest as much in improving the quality as non-deceptive counterfeits (as long as he can deceive consumers successfully at time of purchase), but he has to infiltrate a licit supply chain via a legitimate distributor who sources both brand-name and counterfeit products. The legitimate distributor then faces a trade-off between a greater profit margin and a risk of getting punished for selling counterfeits.

After finding the equilibrium decisions of the counterfeiter and the distributor, we evaluate the following anti-counterfeiting strategies of which the effectiveness depends on the subsequent reaction of the strategic counterfeiter: (i) quality strategy that alters the quality of brand-name products against a counterfeiter, (ii) pricing strategy that alters the price of brand-name products against a counterfeiter, (iii) marketing campaign that educates consumers about the dangers of counterfeits, and (iv) enforcement strategy that increases the chances to seize the production of counterfeits. Our analysis highlights that the optimal strategy of the brand-name company differs depending on whether she faces the non-deceptive or deceptive counterfeiter. Although it is ideal to see the strategies that increase the profit of the brand-name company be also effective in reducing the profit of the counterfeiter and benefit consumers, our analysis shows that this is not the case for most strategies. It is therefore imperative for industries and governments to
understand the type of potential counterfeiters and to carefully consider a trade-off among different objectives in implementing an anti-counterfeiting strategy.

Chapter 3 investigates the cooperation among firms in transshipping their residual inventories. Transshipment of inventories is common in practice: when a retailer does not hold stock of a product a customer demands, the retailer can satisfy the customer’s demand by having it shipped from another retailer who has it in inventory. In a decentralized system, when multiple independent firms cooperate in order to maximize their own profits, a proper mechanism needs to be developed to ensure that firms have incentives to participate in transshipment. The extant literature analyzes the incentives of independent firms using the cooperative game theory based on the concept of coalitions. A coalition is a set of firms, within which firms can share their excess inventories and unmet demands. On the other hand, we analyze this problem using social and economic networks to model the relationships among firms. Our network-based approach is fundamentally different from the coalition-based approach in the following two important aspects. First, in a network-based game, a firm can share its residual demands or supplies with the firms to which it has direct links, and also with the firms to which it has no direct links but is connected via other intermediate firms. In the former case, transshipment between two firms can occur without the cooperation of any other firms. In the latter case, however, transshipment between two firms occurs only when all other intermediate firms on a transshipment path cooperate as well, and the costs of transshipment may vary across different paths between the two firms.
In contrast, in a coalition-based game, transshipment paths are not specified, and the costs of transshipment between two firms are fixed. Second, within a coalition, firms are distinguished by the amount of their residual demands or supplies. In a network, firms are distinguished by their positions within the network as well as their residual amounts. Overall, networks represent richer relationships among firms than coalitions by taking coalitions as special cases.

To analyze a decentralized transshipment system, we develop a two-stage model. In the first stage, firms make their inventory decisions independently under uncertain demands. After the realization of the demands, some firms may have leftover inventories, while others may have unsatisfied demands. In the second stage, the firms cooperate by transshipping residual supplies to satisfy residual demands. Then the additional profits generated from transshipment are allocated among participants according to a predetermined rule. We analyze this model backwards with emphasis on the second-stage network analysis. Our second-stage analysis contains two parts: (1) examining the stability of an existing network, and (2) predicting networks to be established by firms endogenously. Specifically, in the first part, we extend the concept of the core defined in a coalition-based cooperative game of transshipment into a network-based cooperative game. In a coalition-based cooperative game, to determine whether an allocation is in the core, one needs to show that no subset of firms has an incentive to form subcoalitions by seceding from the grand coalition. On the other hand, in a network-based cooperative game, we need to examine the incentives of firms to form subnetworks by seceding from
a given network. In the second part, we derive equilibrium network structures when firms form networks endogenously under the allocation rule based on firms’ marginal contributions to their networks. In this case, any two firms can decide whether or not to set up a link between them. For our first-stage analysis, we investigate the inventory decisions of the firms, and determine the conditions under which the Nash equilibrium inventory levels coincide with the inventory levels in a centralized system (i.e., the first-best inventory levels).

Based on our model of inventory transshipment networks, we construct a dual price allocation that is in the core for any network structures and for any residual amounts. Further, we examine the allocation based on the marginal contribution of each firm to its network (which we call the "MJW value"). Under this allocation, various network structures such as complete, hub-spoke, and chain networks are stable only under certain conditions on residual amounts. Moreover, these conditions differ across network structures, implying that a network structure plays an important role in establishing the stability of a decentralized distribution system. While the previous coalition-based approach examines deviations of subcoalitions from the grand coalition, we investigate deviations of subnetworks from both complete and incomplete networks, and we find that the complete network tends to be less stable than incomplete networks under the MJW value. Finally, we show that pairwise Nash stable networks underperform the corresponding networks in centralized systems in the case when firms establish networks endogenously.
Chapter 4 investigates the cooperation between competing manufacturers in auditing their common suppliers. This chapter is motivated by the recent product safety incidents in various industries. For example, in food industry, the adulterated milk incident happened in China in 2008. Milk products were adulterated with melamine, which caused an estimated 300,000 illnesses. A similar tragedy happened in pharmaceutical industry in US. The ingredient of Heparin, a drug that is used to prevent coagulation of blood, was contaminated, which led to 81 deaths (Blum, 2008). There are many other high profile product safety events that happened in other product categories including automobiles, high-tech products, toys and so on.

In the past, product failures were often attributed to functional errors in product design or manufacturing process. Today, with globalization, many manufacturers procure their raw materials worldwide and also outsource their manufacturing functions to oversea suppliers. This creates new challenges to product safety. For example, opportunistic behaviors of suppliers, such as product adulteration, have become a common cause of product safety incidents. To deal with this challenge, manufacturers invest significant amount of resource to manage product quality of their suppliers through supplier audits. However, supplier audits may not generate perfect information about product quality. It’s not difficult for suppliers with sub-par quality standards to pass an audit with some understanding of what the process entails. Further, it can be very costly for manufacturers to make audits
more informative as well. Facing imperfect information and cost issues in supplier audits, manufacturers start to cooperate in auditing their common suppliers. There are two major approaches to conduct cooperative audit. The first approach is called "joint audit". In this case, several manufacturers conduct a supplier audit together sharing the cost and the result from the audit. The second approach is called "shared supplier information". In this case, manufacturers still conduct their own audits but they share the results with each other.

Although the cooperative approaches may be appealing to manufacturers due to reduced audit costs, whether they benefit consumers have not been studied well. We analyze this issue and the analysis reveals that they do not necessarily benefit consumers by improving product quality. The effectiveness of these approaches depends crucially on the externality of product safety of one manufacturer on other manufacturers. Specifically, the product safety incident of one manufacturer may have a negative effect on the profit of another manufacturer due to the fact that consumers may lose confidence in the whole industry. On the other hand, the externality can be positive if consumers switch from a product with safety issue to other products. Further, we find that, when the risk of product safety failure is high, shared supplier information is more effective than joint audit; otherwise, joint audit is more effective. We also investigate the incentives of competing manufacturers to audit cooperatively, and design an efficient way to motivate the manufacturers who have better information about product quality to share the information. Since product safety has a significant impact on the well-being of consumers, industries
and governments should assess the level of externality and risk of product safety failure in specific market settings in order to design effective and stable cooperative audit programs.

In the setting of supply chain networks, some decisions can be made independently by a firm, e.g., adjusting price or quality level, while other decisions may need consensus of a group of firms, e.g., how to transship inventories. One coherent message from the three chapters is that, when firms make these decisions, it is crucial to consider the reactions of opponents and allies in supply chain networks. In Chapter 2, the reactions of a counterfeiter determine the effectiveness of anti-counterfeiting strategies. In Chapter 3, we find that the networks established by independent firms depend on how firms react to a particular way to allocate the profit from transshipment of leftover inventories. In Chapter 4, both the effectiveness and the incentive of a group of firms to conduct cooperative audits are affected by other firms’ reactions to their audit efforts. Ignoring the reactions of opponents and allies will eventually lead to a loss of a firm’s profit or a loss of social welfare. As supply chain networks become more and more complex, with the risks of various illegal activities, firms need to identify key tradeoffs and consider potential reactions of different types of supply chain members.
CHAPTER 2

Combating Strategic Counterfeiters in Licit and Illicit Supply Chains

ABSTRACT:¹ Counterfeit goods are becoming more sophisticated from shoes to infant milk powder and aircraft parts, creating problems for consumers, firms, and governments. By comparing two types of counterfeiters - deceptive, so infiltrating a licit (but complicit) distributor, or non-deceptive in an illicit channel, we provide insights into the impact of anti-counterfeiting strategies on a brand-name company, a counterfeiter, and consumers. Our analysis highlights that the effectiveness of these strategies depends critically on whether a brand-name company faces a non-deceptive or deceptive counterfeiter. For example, by improving quality, the brand-name company can improve her expected profit against a non-deceptive counterfeiter when the counterfeiter steals an insignificant amount of brand value. However, the same strategy does not work against the deceptive counterfeiter who can get a free ride on the improved quality. Reducing price works well in combating the non-deceptive counterfeiter, but it could be ineffective against the deceptive counterfeiter. Moreover, the strategies that improve the profit of the brand-name

¹This essay is a joint work with Soo-Haeng Cho and Sridhar Tayur.
company may benefit the counterfeiter inadvertently and even hurt consumer welfare. Therefore, firms and governments should carefully consider a trade-off among different objectives in implementing an anti-counterfeiting strategy.

2.1. Introduction

Trademarks, also called brands, represent the most valuable assets of many firms, requiring significant investment in research and development as well as years of efforts in maintaining high product quality and careful brand management. Famous global brands such as GE, Nike and Nestlé are popular because they offer a guarantee of quality, which is vital to consumers when they make purchasing decisions. For those goods for which the mere display of a particular brand confers prestige on their owners, such as luxury watches and fashion apparel, many consumers purchase branded goods to demonstrate that they are consumers of the particular good. These intrinsic values of trademarks create incentives for counterfeiting.

Nowadays counterfeits have developed into a substantial threat to many industries. The OECD estimates that international trade in counterfeit could amount to up to $250 billion or 1.95% of world trade in 2007, up from $105 billion in 2001 (OECD 2009). If including domestically produced and consumed products, the total magnitude could be several hundred billion dollars more (OECD 2008). The problem is no longer limited to prestigious and easy-to-manufacture products, such as designer clothing, branded sportswear, and fashion accessories. It affects nearly all product categories including items that have an impact on personal
health and safety such as pharmaceuticals, food, drink, toys, medical equipment, and automotive parts (OECD 2008).

Counterfeits are broadly categorized into two types: non-deceptive and deceptive (Grossman and Shapiro 1988a). A non-deceptive counterfeit is the counterfeit a consumer can distinguish from the brand-name product at time of purchase. This type of counterfeits tends to be sold at a substantial discount through an unauthorized sales channel. For example, consumers can easily tell that $10 luxury watches sold by street vendors are counterfeit. On the contrary, a deceptive counterfeit is the counterfeit a consumer believes to be authentic at time of purchase even if it is, in fact, counterfeit. In order to deceive consumers, this type of counterfeit goods has to infiltrate licit supply chains; for example, fake auto parts were found in legitimate repair shops, counterfeit pharmaceutical products at chemists, food products on supermarket shelves (OECD 2008), and pirated software products sold by one of the largest re-sellers (Bass 2010). Solomon (2009) notes that counterfeit drugs make their way through the licit supply chain via a distributor who moves a product from a low-cost channel to a high-cost channel. Collusion between counterfeiteers and licit supply chain members occurs due to a higher profit from selling counterfeits (Green and Smith 2002, Bass 2010). For example, a fake version of cancer drug Avastin, produced at a printing house in Istanbul (Taylor 2014), is suspected to be distributed to U.S. by Ozay, an exporter of drugs, cosmetics and medical devices (Whalen 2014). A deceptive counterfeit is usually sold
at the price that is the same as or close to that of its branded product so as to deceiving consumers. Although it appears to function properly at time of purchase, it lacks durability and often involves health and safety risks of consumers. Examples of deceptive counterfeits abound in both developing and developed countries. In China, after the luxury furniture sold by the licit retailer turned out to be deceptive counterfeits, customers (who were previously deceived) have posted details of how their products were shoddily made or reeking of foul-smelling lacquers (Barboza 2011). In the U.S., a licit distributor who bought counterfeit networking cards for $25 each sold them to the Marine Corps for $625 each after repackaging the cards to make them appear to be Cisco products (McKinley 2010).

Brand-name companies are spending millions of dollars in order to stop or at least to reduce the incidence of counterfeits. They hire full-time employees, invest in new technologies, and redesign their products to make counterfeiting more difficult (Balfour 2005). However, the anti-counterfeiting strategies found to be useful to one product may not work for another or can even unintentionally make counterfeits flourish more in the market. For example, Chinese shoe manufacturers successfully addressed their counterfeiting issues by improving the quality of their products (Qian 2008). This is the outcome of the competition in which high-quality authentic products defeat low-quality non-deceptive counterfeits. However, the same strategy backfired against a Scotch whisky company in the Thailand market (Green and Smith 2002). At the peak of the company’s sales in 1988, 42% of its premium Scotch whisky sales was counterfeit; high quality made the products
more popular and attracted more counterfeits. In this case, the counterfeits were sold as the genuine products and commanding the same price, i.e., sold as *deceptive* counterfeits. After the initial attempt to fight counterfeits by improving quality had failed, the company eventually succeeded in radically reducing the incidence of counterfeiting by establishing a system that monitors supply chains: the company focused on identifying members in its supply chain who were selling the counterfeits, facilitating seizure of counterfeits and punishing counterfeiters.

These contrasting results illustrate a need for anti-counterfeiting strategies that are tailored to specific products. Yet, due to the limited understanding of relations among the types of counterfeits and the effectiveness of anti-counterfeiting strategies, OECD (2008) calls for research that strengthens the analysis of counterfeiting and says:

“Asessing the factors driving production and consumption of counterfeit and pirated products can generate insights into the types of products that are most likely to be infringed, . . . , and lead to more efficient and effective [anti-counterfeiting] strategies.”

This paper attempts to provide such an analysis by providing insights to the following questions: (Q1) What anti-counterfeiting strategies should a brand-name company use to improve her own profit? (Q2) What is the impact of anti-counterfeiting strategies on the profit of a counterfeiter? (Q3) What is the impact of counterfeits on consumer welfare? Do consumers also benefit from the strategies that are effective in combating counterfeits?
To answer these questions, we develop a normative model of licit and illicit supply chains, in which a brand-name company competes with her potential counterfeiter. The counterfeiter in our model is either non-deceptive or deceptive, and decides the level of functional quality and wholesale price of his goods after observing the quality and price of the brand-name product. Depending on his type, the counterfeiter faces different opportunities and risks. The non-deceptive counterfeiter competes directly with a brand-name company for price and quality. Thus the counterfeiter may have to invest in improving the quality of his goods, which will increase the risk of losing the investment in case of getting caught by the authorities. Conversely, the deceptive counterfeiter may not need to invest as much in improving the quality as non-deceptive counterfeits (as long as he can deceive consumers successfully at time of purchase), but he has to infiltrate a licit supply chain via a legitimate distributor who sources both brand-name and counterfeit products. The legitimate distributor then faces a trade-off between a greater profit margin and a risk of getting punished for selling counterfeits.

After finding the equilibrium decisions of the counterfeiter and the distributor, we evaluate the following anti-counterfeiting strategies of which the effectiveness depends on the subsequent reaction of the strategic counterfeiter: (i) quality strategy that alters the quality of brand-name products against a counterfeiter, (ii) pricing strategy that alters the price of brand-name products against a counterfeiter, (iii) marketing campaign that educates consumers about the dangers of counterfeits, and (iv) enforcement strategy that increases the chances to seize the
production of counterfeits. Our analysis highlights that the optimal strategy of the brand-name company differs depending on whether she faces the non-deceptive or deceptive counterfeiter. Although it is ideal to see the strategies that increase the profit of the brand-name company be also effective in reducing the profit of the counterfeiter and benefit consumers, our analysis shows that this is not the case for most strategies. It is therefore imperative for industries and governments to understand the type of potential counterfeiters and to carefully consider a trade-off among different objectives in implementing an anti-counterfeiting strategy.

2.2. Literature Review

Traditional supply chain management research is focused on licit supply chains in which members of supply chains interact with each other by exchanging goods and services legally. In this era of globalization, supply chains are no longer confined within one country as more and more companies offshore and outsource their operations to less developed countries. However, this has created a frightening phenomenon: an ever-rising flood of counterfeit items coming into markets (Business Week 2005). This paper is intended to shed light on counterfeit problems in both licit and illicit supply chains and to analyze the effectiveness of anti-counterfeiting strategies.

The majority of studies on counterfeits are conceptual and descriptive. They provide frameworks for fighting counterfeiting usually based on case studies. For
example, Olsen and Granzin (1992) emphasize the importance of dealers’ cooperation for a manufacturer to implement a program to combat counterfeits. Jacobs et al. (2001) investigate a number of counterfeiting incidences, and propose various measures of fighting these illegal activities. Staake and Fleisch (2008) provide an extensive review of this literature.

Marketing researchers have conducted empirical studies on counterfeits. They mainly focus on the demand side of counterfeits, and try to answer questions such as why consumers purchase counterfeits and how to educate consumers not to purchase counterfeits. Eisend and Schuchert-Guler (2006) review this literature and conclude that further investigation is needed to develop a general framework that integrates the existing results consistently. Recently, using data from Chinese shoe companies, Qian (2008) finds that brand-name companies tend to improve their product quality after the entry of non-deceptive counterfeiters.

There are only a handful of analytical studies that present prescriptive models of counterfeits. Grossman and Shapiro (1988a, 1988b) develop equilibrium models of trades between brand-name firms in a home country and low-quality producers in a foreign country. To sell their goods as counterfeits in the home market, foreign producers must pass the goods through the home-country border, hence facing the risk of confiscation. Grossman and Shapiro (1988a) analyze the consequences of deceptive counterfeits in a market where consumers cannot observe the quality of a product, and provide a welfare analysis of border inspection policy. Grossman and
Shapiro (1988b) present a Cournot competition model between brand-name products and non-deceptive counterfeits given their exogenous quality levels. Because non-deceptive counterfeits can contribute positively to consumer welfare due to their lower price, the authors conclude that policies that discourage foreign counterfeiting need not improve welfare, which is consistent with our finding. Scandizzo (2001) views competition between brand-name firms and non-deceptive counterfeiters as a patent race over time. The author finds that counterfeits improve consumer welfare while reducing firms’ profits, and that the more skewed the income distribution within the economy is towards the poor, the greater the welfare effect and the smaller the profit effect.

There have been growing interests in counterfeit research among operations researchers. Liu et al. (2005) study the decision of an inventory manager who can source both genuine and deceptive counterfeit products and sell them to consumers at one price. Sun et al. (2010) study a global firm’s decision of outsourcing the production of its components to a foreign country. The firm faces a trade-off between lower labor cost and increased risk of imitation by a foreign firm. The authors find an optimal strategy in choosing the range of components to transfer. Zhang et al. (2012) analyze the case when a brand-name firm faces non-deceptive counterfeits. They show that a non-deceptive counterfeit lowers the price and profit of the brand-name product, and a brand-name firm has more incentive to improve her own quality rather than reducing that of a counterfeit. They also analyze a
situation in which two brand-name products compete, which we do not consider in this paper.

We draw on and contribute to this stream of research by addressing the following important issues in counterfeiting problems:

(1) Strategic counterfeiters: The common assumption used in the literature is that the quality is fixed a priori. For example, Grossman and Shapiro (1988a, 1988b) assume that foreign producers always choose the lowest quality because they lack capital, resource, and technology for quality improvement and that there are no entry costs of counterfeiters. Today, thanks to outsourcing and offshoring of numerous global firms, counterfeiters benefit greatly from increasingly easy access to modern production facilities (Staake and Fleisch 2008). Schmidle (2010) notes that today’s counterfeiters come in varying levels of quality depending on their intended markets, and diversify their products and distribution channels to manage the risks involved in their criminal activities. In our model, a counterfeiter decides functional quality and wholesale price of his products by considering a trade-off between the benefit from stealing brand value and the risk of confiscation. Our analysis shows that the effectiveness of anti-counterfeiting strategies depends critically on the strategic response of a counterfeiter to those strategies.

(2) Licit and illicit supply chains: The previous analytical papers assume that a counterfeiter is capable of selling his counterfeits directly to consumers regardless of his type. Although this is quite possible for non-deceptive counterfeits, a deceptive counterfeiter has to infiltrate a licit supply chain; today, very few consumers would
be deceived by the counterfeits sold by street vendors or unknown websites. We take into account this fundamental difference in supply chains of non-deceptive and deceptive counterfeits, and demonstrate that an effective strategy against a non-deceptive counterfeiter may not be effective against a deceptive counterfeiter.

(3) Consumer characteristics: As consumers learn more about counterfeit problems from the media, they become more aware of the presence of counterfeits, and some even become more proactive by taking into account the likelihood of receiving deceptive counterfeits unknowingly when they purchase branded products from licit distributors. Our survey (of which the details are presented in §2.3) indicates that the proportion of proactive consumers in the U.S. is substantially lower than that in China. Our analysis provides insights into how this characteristic of consumers affects the effectiveness of anti-counterfeiting strategies.

(4) Evaluation of anti-counterfeiting strategies: We evaluate the aforementioned strategies by examining their impacts on a brand-name company, a counterfeiter, and consumers. Our analysis complements the previous findings (discussed above) of Grossman and Shapiro (1988a, 1988b) and Zhang et al. (2012). Grossman and Shapiro (1988a, 1988b) provide welfare analyses of border inspection policies, which are similar to the enforcement policies we study. Today, however, the confiscation of counterfeits not only occurs on the border when trading goods between countries but also occurs within each country due to the growing demand within its domestic market; in addition, the seizure of equipment is another threat to counterfeitors (Staake and Fleisch 2008). For example, the Chinese authorities, long unconcerned
about counterfeiting, have begun to take actions as Chinese companies create their
own intellectual properties (Business Week 2005). Zhang et al. (2012) focus on
analyzing the effect of altering the quality of a brand-name good or a non-deceptive
counterfeit on the profit of a brand-name firm. In doing so, they consider neither
potential seizure of counterfeits and equipment, nor welfare implication of those
strategies.

Finally, we note that a counterfeiter’s decision of his distribution channel is
analogous to that of a legitimate firm (e.g., Xu et al. 2010), although the benefit
and risk associated with each channel of counterfeits are unique as described above.
Also, a research question similar to counterfeiting arises in the literature of parallel
importing (or gray market) and software piracy. Parallel importing is the prac-
tice of purchasing products in a lower-priced region and shipping them to a higher
priced region (e.g., Ahmadi and Yang 2000, Hu et al. 2011). While the parallel im-
ported goods are authentic but sold at a lower price, counterfeits are not authentic,
possess lower quality, and are sold at a lower price for non-deceptive counterfeits or
at the same price for deceptive counterfeits. Piracy differs from counterfeiting in
that piracy refers to infringement of copyright. In our model, software piracy can
be viewed as a special case of counterfeiting, in which counterfeit products have
almost the same functional quality as authentic ones but their cost of development
and production is very low. Some of our results can be extended to software piracy
problems; for example, consumers could be better off without piracy protection,
which is consistent with Conner and Rumelt (1991).
In summary, the literature considers only one type of counterfeits with fixed quality that are sold directly to consumers. In contrast, our model captures recent changes in counterfeiting supply and demand by noting the fundamental differences between non-deceptive and deceptive counterfeits in consumers’ awareness and distribution channels, and by considering counterfeiters’ strategic decisions regarding price and functional quality in a market with different consumer characteristics. Our analysis provides novel insights into the effectiveness of several anti-counterfeiting strategies.

2.3. Model

We consider a market served by a brand-name company (‘she’) and her potential counterfeiter (‘he’). The type of the counterfeiter is either non-deceptive or deceptive. We use subscript $i = B$ to denote the brand-name product, $i = N$ to denote the non-deceptive counterfeit, and $i = D$ to denote the deceptive counterfeit. A consumer in this market purchases at most one unit of a product. In making a purchasing decision of product $i$, a consumer considers his/her utility $u_i = \theta \phi_i - p_i$, where $\theta$ represents his/her taste, $\phi_i$ represents the quality of the product a consumer perceives at time of purchase, and $p_i$ represents the retail price of the product. All consumers prefer high quality for a given price, but a consumer with a higher $\theta$ is more willing to pay to obtain a high-quality product. We assume that $\theta$ is uniformly distributed over $[0, 1]$ and that the size of the market is one. A consumer purchases a product only if the utility from purchasing the product
is nonnegative in which case he/she selects a product that provides the highest utility. This is the standard vertical differentiation model, which is also used by Qian (2008) and Zhang et al. (2012). We next present our model components that capture the unique aspects of counterfeiting.

Depending on the counterfeit type, the quality of product \( i \) a consumer perceives at time of purchase, \( \phi_i \), may differ from its real quality \( q_i \). (Throughout this paper, unless mentioned specifically as the perceived quality, quality refers to real quality.) For the non-deceptive counterfeit as well as the brand-name product, consumers know what product they are purchasing, so the perceived quality of either product is the same as its real quality; i.e., \( \phi_B = q_B \) and \( \phi_N = q_N \). However, for the deceptive counterfeit, consumers cannot distinguish it from the brand-name product at time of purchase. There are two types of consumers. First, some consumers are not aware of counterfeits, or even if they are aware, they may consider the likelihood of purchasing counterfeits negligible at legitimate stores. They perceive the quality of any product in the market as \( q_B \); i.e., \( \phi_B = \phi_D = q_B \). On the other hand, other consumers may be “proactive” in the sense that they take into account the likelihood of receiving deceptive counterfeits unknowingly even when purchasing products from legitimate stores. Let \( \xi \in [0, 1] \) denote their expectation about the fraction of deceptive counterfeits in the market. Then proactive consumers perceive the quality of a product in the market as a weighted average of the quality of the brand-name product and that of the deceptive counterfeit; i.e., \( \phi_B = \phi_D = (1 - \xi)q_B + \xi q_D \). Let \( \lambda \in [0, 1] \) denote the fraction of proactive
consumers in the market. In practice, $\lambda$ may vary depending on the characteristic of the market. For example, in our survey of 166 consumers over 4 popular product categories for deceptive counterfeits (see Table 1), we have found that 51% of consumers in China are proactive, whereas only 4% of consumers in the U.S. are proactive. The low value of $\lambda$ in the U.S. reflects the view of Rockoff and Weaver (2012), who say: “Most Americans don’t question the integrity of the drugs they rely on. They view drug counterfeiting, if they are aware of it at all, as a problem for developing countries.”

Table 1. Consumer Survey Results in the U.S. and China

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th></th>
<th>China</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aware</td>
<td>Proactive</td>
<td>Aware</td>
<td>Proactive</td>
</tr>
<tr>
<td>Alcohol</td>
<td>14%</td>
<td>4%</td>
<td>94%</td>
<td>56%</td>
</tr>
<tr>
<td>Car Parts</td>
<td>25%</td>
<td>4%</td>
<td>54%</td>
<td>34%</td>
</tr>
<tr>
<td>Medical Drugs</td>
<td>41%</td>
<td>5%</td>
<td>86%</td>
<td>51%</td>
</tr>
<tr>
<td>Food, Drinks</td>
<td>22%</td>
<td>5%</td>
<td>90%</td>
<td>63%</td>
</tr>
<tr>
<td>Average</td>
<td>26%</td>
<td>4%</td>
<td>81%</td>
<td>51%</td>
</tr>
</tbody>
</table>

(Note) Respondents are college students and faculty with ages from 18 to 50. The number of respondents is 86 in the U.S., and it is 80 in China. Two questions were asked in the questionnaire: (1) Are you aware of the sale of counterfeits in each of the above product categories; and (2) For each product category in which you are aware of the sale of counterfeits, do you take into account the risk of getting a counterfeit and therefore discount the value of the product when you purchase a brand-name product at a full price in a legal store? Those customers who answered yes to (1) are considered “Aware”, and those customers who answered yes to both (1) and (2) are considered “proactive.” The absolute numbers may be escalated because respondents may be reminded of counterfeits by the questionnaire. Our survey indicates that being “aware” of the existence of counterfeits differs from being “proactive.” One may explain such difference from cognitive psychology (e.g., Bendoly et al. 2010, Goldsmith and Amir 2010, and references therein); for example, it may be due to a positive-outcome “bias” or “wishful thinking” caused by overestimating the probability of good things happening.
Since the counterfeit bears the trademark of the brand-name product, a consumer enjoys the brand image even when he/she purchases the counterfeit. Thus we may represent the quality of the counterfeit as $q_i = f_i + \beta q_B$ ($i = N$ or $D$), where $f_i (> 0)$ is the functional quality of the counterfeit $i$ and $\beta q_B$ (where $\beta > 0$) is the brand value that the counterfeit steals from the brand-name product. Essentially, we assume that a product has two attributes: functionality and brand value as in multi-attribute models in marketing (e.g., see Lilien et al. 1992). Brand value reflects advertising investments on which a counterfeiter may get a free ride. The parameter $\beta$ captures the following two factors. First, $\beta$ captures a fraction of the brand value in the quality of the brand-name product, $q_B$. For example, this fraction may be high for luxury goods because a brand plays a significant role when consumers purchase such products, whereas it may be low for fast moving consumer goods (which are sold quickly at relatively low cost) because a brand is less of a concern to consumers for such goods. Second, $\beta$ captures a discount factor of the original brand value for the counterfeit because the counterfeit draws only a part of the brand value from the brand-name product.\(^2\) Following the literature,

\(^2\)For some counterfeits, as their functional quality increases, they might look more similar to branded products so that they can steal a higher fraction of the brand quality. This can be modeled by setting $\beta = \beta_1 + \beta_2 f_i$, where $\beta_1$ captures the characteristic of a product category (like $\beta$ in our base model), and $\beta_2$ captures the property mentioned above. All our subsequent results hold under this alternative model.
we assume that the quality of the brand-name product is superior to that of the counterfeit; i.e., \( q_B > q_N \) and \( q_B > q_D \).

Either type of counterfeiter \( i (= N \text{ or } D) \) makes two decisions sequentially to maximize his expected profit: functional quality \( f_i \) and wholesale price \( w_i \) to a distributor. We assume that the counterfeiter makes these decisions after observing the quality \( q_B \) and price \( p_B \) of a brand-name product because counterfeiters always enter a market following a brand-name company, often after the brand-name product becomes popular. Different types of counterfeiters use different distribution channels to sell their goods. The non-deceptive counterfeiter \( (i = N) \) distributes his goods through an illicit distributor, who then decides the retail price of the non-deceptive counterfeit to consumers, \( p_N \). On the other hand, the deceptive counterfeiter \( (i = D) \) has to break into a licit supply chain by distributing his goods through a licit distributor, who then sells both brand-name products and deceptive counterfeits to consumers at the same price \( p_B \). In this case, the licit distributor determines a proportion \( s \in [0, 1] \) of the deceptive counterfeit among all products he sells to consumers. We next describe the details of our model for non-deceptive and deceptive counterfeits, respectively.

When a non-deceptive counterfeiter exists in the market and sells his products through the illicit distributor, consumers will choose between the brand-name
product and the counterfeit. Both products carry the same brand, but have different qualities and prices. The competition between the non-deceptive counterfeiter and the brand-name company is analogous to duopoly in a vertically differentiated market, but it is not the same because the members of the illicit supply chain bear the risks associated with counterfeiting. The non-deceptive counterfeiter and the illicit distributor make their decisions in three sequential stages as follows. In stage 1, the non-deceptive counterfeiter chooses his functional quality $f_N \in [\underline{f}, \bar{f}]$, where $\bar{f} > f \geq 0$, and makes initial investment to develop and produce goods having $f_N$. The upper bound $\bar{f}$ may represent the functional quality of the brand-name product. We assume $\bar{f} < (1 - \beta)q_B$ such that $q_B > q_N$. The lower bound $\underline{f}$ may represent the minimum level of quality at which a product functions or appears to function properly. To produce counterfeits having $f_N$, the counterfeiter needs to invest $t_N f_N^2$ in acquiring technology and setting up production facilities, where $t_N > 0$. This implies that the development of a product with higher quality requires increasingly more investment. The unit production cost of the counterfeit is normalized to zero. After the investment takes place, however, there are some chances that the investment will be confiscated because it is illegal to produce counterfeits. Suppose this occurs with a probability $\gamma \in (0, 1)$. The parameter $\gamma$ captures the monitoring efforts of the government and the brand-name company on counterfeit production. The potential loss of the investment is a major risk to
the counterfeiter that deters him from making large investments to improve the functional quality of his products (OECD 2008). If the counterfeiter’s investment is confiscated, the counterfeiter cannot sell his goods to the market. Otherwise, the game proceeds to stage 2 in which the non-deceptive counterfeiter decides his wholesale price $w_N$ to the illicit distributor. For simplicity, we represent all distributors/retailers in the illicit supply chain as one illicit distributor. In stage 3, the illicit distributor decides the retail price of the non-deceptive counterfeit to consumers, $p_N$. The illicit distributor has to pay a penalty of $l_N$ if getting caught by the authorities with probability $\alpha_N$.

When a deceptive counterfeiter exists in the market and sells his products through the licit distributor, consumers cannot distinguish deceptive counterfeits from brand-name products. Like the non-deceptive counterfeiter, the deceptive counterfeiter determines his functional quality $f_D \in [\underline{f}, \overline{f}]$ in stage 1, while facing the risk of getting his investment $t_D f_D^2$ confiscated. In stage 2, the deceptive counterfeiter decides his wholesale price $w_D$ to the licit distributor, who later sells the counterfeits to consumers at the same price $p_B$ as the brand-name products. In stage 3, the licit distributor determines a proportion $s$ of the deceptive counterfeit among all products he sells to consumers. We model the risk of the licit distributor selling deceptive counterfeits with a likelihood $\alpha_D$ of getting caught and a penalty $l_D$. Since $\alpha_D$ tends to increase with more counterfeits in the market, we set $\alpha_D$
equal to the fraction of deceptive counterfeits, $s$. In §2.7, we consider a more
general case in which $\alpha_D$ is a function of $f_D$ as well as $s$.

We make the following assumptions to simplify our analysis. First, we as-
sume that the licit distributor does not make a profit from selling brand-name
products, while it makes a positive profit from selling deceptive counterfeits. Our
results continue to hold for any fixed margin of the licit distributor from selling
brand-name products. This may represent a setting in which the retail price is
mainly determined by competition or the brand-name company. For example, TV
manufacturers force distributors to keep manufacturer-specified pricing through a
way called Unilateral Pricing Policy (Morrison 2013). In pharmaceutical industry,
there is increasing proliferation of agency and reduced wholesaler models in dis-
tribution of medicines, with which the discounting ability of distributors is very
limited (Kanavos et al. 2011). In appendix C, we also analyze the case where
the licit distributor decides its profit margins endogenously. Second, we normalize
$l_N = 0$, while having $l_D = l > 0$. In practice, a loss of an illicit distributor from
potential seizure is much smaller than that of a licit distributor. Illicit distributors
are usually street vendors or internet sites. Since their potential loss from seizure
is small, they tend to close their stores temporarily when they get caught and then
reopen the same stores or open new ones later. For example, vendors in the Xiang
Yang market in Shanghai, China, which were once famous for their high-quality

counterfeits but closed due to the government’s massive campaigns in 2006, relocated to the Yatai Xinyang market that is now famous among tourists (Naumann 2009). In contrast, the punishment on the licit distributor for illegal distribution of deceptive counterfeits is very severe. For example, the Chinese court sentenced the distributor of fake pills to 17 years in prison, the nation’s longest term for the crime (Bennett 2010) and the U.S. court sentenced a distributor who sold counterfeit networking cards to the military to 51 months in prison, the maximum term recommended by federal prosecutors (McKinley 2010). Third, for both types of counterfeits, we assume that the probability of counterfeits getting confiscated at the production level ($\gamma$) is independent of that at the distribution level ($\alpha_N$ or $\alpha_D$). In practice, it is extremely difficult to trace back the source of counterfeits even after discovering their distributors. For example, counterfeit versions of cancer drugs Faslodex and Avastin were detected at the distribution level in the U.S., but their sources have not been determined while suspecting offshore production (Rockoff et al. 2012, Weaver et al. 2012). Table 2 summarizes the notation of major variables and parameters.

2.4. Equilibrium Analysis

In this section, we present our model formulation and equilibrium analysis. We use backward induction to derive subgame-perfect Nash equilibrium. In §2.4.1 we
present equilibrium (denoted by superscript \( \ast \)) in a market with a non-deceptive counterfeiter. In §2.4.2 we present equilibrium (denoted by superscript \( \ast \ast \)) in a market with a deceptive counterfeiter. All proofs are provided in appendix A.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i )</td>
<td>Brand-name (= B), non-deceptive (= N), deceptive (= D)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Taste of consumers; ( \theta \in U[0, 1] )</td>
</tr>
<tr>
<td>( p_i )</td>
<td>Retail price of product ( i ) to consumers</td>
</tr>
<tr>
<td>( q_i )</td>
<td>(Real) Quality of product ( i )</td>
</tr>
<tr>
<td>( f_i )</td>
<td>Functional quality of counterfeit product ( i ); ( f_i \in [f, \overline{f}] )</td>
</tr>
<tr>
<td>( \pi_i )</td>
<td>Expected profit from selling product ( i )</td>
</tr>
<tr>
<td>( w_i )</td>
<td>Wholesale price of product ( i ) to a distributor</td>
</tr>
<tr>
<td>( t_i )</td>
<td>Cost parameter used in the cost of developing functional quality</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Fraction of the quality that counterfeits steal; ( \beta \in \left(0, 1 - \frac{\overline{f}}{q_B}\right) )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Probability that a counterfeiter’s investment will be confiscated; ( \gamma \in (0, 1) )</td>
</tr>
<tr>
<td>( l )</td>
<td>Loss of the licit distributor if getting caught for selling counterfeits; ( l &gt; 0 )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Fraction of proactive consumers in the market; ( \lambda \in [0, 1] )</td>
</tr>
<tr>
<td>( s )</td>
<td>Fraction of counterfeits among all products the distributor sells; ( s \in [0, 1] )</td>
</tr>
</tbody>
</table>

2.4.1. Non-Deceptive Counterfeits

Suppose the brand-name product and the non-deceptive counterfeit exist in the market. There are three segments of consumers: (i) consumers who value the quality of a product highly and purchase the brand-name product, (ii) consumers who value the quality less and purchase the non-deceptive counterfeit, and (iii) consumers who value the quality the least and do not purchase any product. The consumer who is indifferent between purchasing the brand-name product and the non-deceptive counterfeit has the taste \( \tilde{\theta} = \frac{p_B - p_N}{q_B - q_N} = \frac{p_B - p_N}{(1 - \beta)q_B - q_N} \), which solves
\[ \tilde{\theta}q_N - p_N = \tilde{\theta}q_B - p_B. \]

Similarly, the consumer who is indifferent between purchasing the non-deceptive counterfeit and not purchasing any product has the taste \( \tilde{\theta} = \frac{p_N}{q_N} = \frac{p_N}{f_N + \beta q_B} \). Let \( m_i \ (\in [0, 1]) \) denote the market share of product \( i \ (= B \text{ or } N) \), and let \( m_0 \) denote the proportion of consumers who do not purchase any product, so that \( m_B + m_N + m_0 = 1 \). Then:

\[
(2.1) \quad m_B = 1 - \tilde{\theta} = 1 - \frac{p_B - p_N}{(1 - \beta)q_B - f_N} \quad \text{and} \quad m_N = \tilde{\theta} - \tilde{\theta} = \frac{p_B - p_N}{(1 - \beta)q_B - f_N} - \frac{p_N}{f_N + \beta q_B}.
\]

In stage 3, the illicit distributor determines the retail price to consumers, \( p_N \), by solving:

\[
(2.2) \quad \max_{p_N} (p_N - w_N)m_N = (p_N - w_N) \left\{ \frac{p_B - p_N}{(1 - \beta)q_B - f_N} - \frac{p_N}{f_N + \beta q_B} \right\}.
\]

By noting that the profit of the illicit distributor in (2.2) is concave in \( p_N \), one can easily obtain her optimal retail price \( p_N^* (w_N, f_N) = \frac{(\beta q_B + f_N)p_B + q_B w_N}{2 q_B} \).

In stage 2, the non-deceptive counterfeiter determines his wholesale price \( w_N \). By anticipating the best response of the illicit distributor, the non-deceptive counterfeiter chooses his optimal wholesale price that maximizes his expected profit given by:

\[
(2.3) \quad \pi_N(w_N, f_N) = (1 - \gamma) \left\{ w_N \left( \frac{p_B - p_N^*}{q_B - q_N} - \frac{p_N^*}{q_N} \right) - t_N f_N^2 \right\} - \gamma t_N f_N^2.
\]
In (2.3), \((1 - \gamma)\) represents the likelihood that the counterfeiter is able to sell his goods without being confiscated, and the next term in the bracket represents the profit of the counterfeiter in that case. The initial investment \(t_N f_N^2\) is considered a sunk cost in (2.3). Note that whether the confiscation of investment occurs after stage 1 or stage 2 does not affect the counterfeiter’s decisions. If confiscation occurs after some units are sold, \((1 - \gamma)\) can be interpreted as the fraction of sales the counterfeiter has generated before confiscation. Since \(\pi_N\) is concave in \(w_N\), we can easily obtain the optimal wholesale price \(w_N^*\) and the corresponding expected profit of the non-deceptive counterfeiter \(\pi_N^*\), respectively, as follows:

\[
(2.4) \quad w_N^*(f_N) = \frac{p_B(f_N + \beta q_B)}{2q_B} \quad \text{and} \quad \pi_N^*(f_N) = \frac{p_B^2(1 - \gamma)(f_N + \beta q_B)}{8q_B ((1 - \beta)q_B - f_N)} - t_N f_N^2.
\]

By substituting \(w_N^*\) into \(p_N^*\), one can verify that the illicit distributor charges a lower price than that of the brand-name product; i.e., \(p_N^* < p_B\).

In stage 1, the non-deceptive counterfeiter decides the functional quality \(f_N\) by considering his optimal wholesale price in stage 2 and the best response of the illicit distributor in stage 3. The counterfeiter solves \(\max_{f_N \in [L,T]} \pi_N^*(f_N)\) by considering the following trade-off: a higher level of functional quality will draw more consumers, but it requires more investment, hence increasing a potential loss from seizure.
Lemma 1. For any given \((p_B, q_B)\), the optimal functional quality of non-deceptive counterfeits, \(f^*_N\), is as follows: if \(t_N < \frac{\gamma}{4\{(1-\beta)q_B-f\}}\) and \(\pi^*_N(f) \geq \pi^*_N(\bar{f})\), then \(f^*_N = f\), and otherwise \(f^*_N\) can be \(\bar{f}\) or \(f^*_N \in (f, \bar{f})\) that satisfies \(\frac{\partial \pi^*_N}{\partial f_N}\) \(|_{f_N=f^*_N} = 0\).

Lemma 1 shows that the non-deceptive counterfeiter may not always choose the lowest quality in contrast to the common assumption used in the literature (e.g., Grossman and Shapiro 1988a,b). In the past, non-deceptive counterfeits with low functional quality such as brand-name costumes, footwear and accessories dominated a counterfeit market. Their functional quality is just enough for consumers to use them, but their durability and performance are substandard. Consumers who purchase such counterfeits are those who want to enjoy the snob appeal of brands, but do not want to pay the high price of genuine goods. However, in today’s counterfeit markets, counterfeiters come in varying levels of quality depending on their intended markets (Schmidle 2010). For example, some counterfeit electronic devices such as cell phones include appealing features which are not included even in authentic products. This is called Shan-Zhai phenomenon in China. According to Gartner, Shan-Zhai phones account for more than 20 percent of sales in China (Barboza 2009). These counterfeiters usually face the least pressure from local enforcement agencies and some are likely to turn into licit competitors once intellectual property rights become more strictly enforced (Staake and Fleisch 2008).
Our result stated in Lemma 1 is consistent with this observation of today’s counterfeit markets.

2.4.2. Deceptive Counterfeits

Suppose the brand-name product and the deceptive counterfeit exist in the market. In this case, both brand-name products and deceptive counterfeits are sold at price $p_B$. While proactive consumers with proportion $\lambda$ perceive the quality of a product in the market as $(1 - \xi_s)q_B + \xi_s q_D$, the rest of consumers perceive the quality of a product in the market as $q_B$. Similar to Grossman and Shapiro (1988a), we assume that the expectation of proactive consumers about the fraction of deceptive counterfeits in the market is rational and hence is equal, in equilibrium, to the actual fraction of counterfeits; i.e., $\xi_s = s$. This notion of rational expectations equilibrium is also used in the recent operations management literature (e.g., Su and Zhang 2008, Cachon and Swinney 2009).

Similar to §2.4.1, we can obtain the market share of the brand-name product and that of the deceptive counterfeit, respectively, as follows:

\begin{equation}
(2.5) \quad m_B = (1 - s)(1 - \bar{\theta}) \quad \text{and} \quad m_D = s(1 - \bar{\theta}),
\end{equation}
where \( 1 - \tilde{\theta} \equiv 1 - \frac{\lambda p_B}{(1-s)q_B + s(f_D + \beta q_B)} - \frac{(1-\lambda)p_B}{q_B} \) represents the “aggregate demand” for both brand-name and counterfeit products at price \( p_B \). Among those consumers who purchase products for \( p_B \), a fraction \( s \) of them receives deceptive counterfeits unknowingly.

In stage 3, the licit distributor solves the following problem to determine \( s \):

\[
(2.6) \quad \max_{s \in [0,1]} s(1 - s)(p_B - w_D) \left\{ 1 - \frac{\lambda p_B}{(1-s)q_B + s(f_D + \beta q_B)} - \frac{(1-\lambda)p_B}{q_B} \right\} - sl.
\]

In (2.6), \( (1 - s) \) represents the likelihood that the distributor will not be detected for selling counterfeits and \( s(p_B - w_D) \left\{ 1 - \frac{\lambda p_B}{(1-s)q_B + s(f_D + \beta q_B)} - \frac{(1-\lambda)p_B}{q_B} \right\} \) represents the distributor’s profit in that case. Recall that the distributor’s profit margin from selling brand-name products is assumed zero (see §2.3). The term ‘\(-sl\)’ in (2.6) represents the expected loss from potential seizure. From (2.6), we can show that the profit of the distributor is strictly decreasing in \( s \) for \( s \in [\frac{1}{2} - \epsilon, 1] \), where \( \epsilon \) is a small and positive constant. Moreover, the profit given in (2.6) is concave in \( s \) for \( s < \frac{1}{2} \). Thus, \( s^{**} \) is 0 or it satisfies the first order condition in \((0, 0.5)\). In the remainder of this paper, we only consider the latter case (i.e., \( s^{**} \in (0, 0.5) \)) because there will be no deceptive counterfeits in the market when the counterfeiter fails to break into the licit supply chain (i.e., \( s^{**} = 0 \)).
In stage 2, the deceptive counterfeiter decides his wholesale price \(w_D\) to maximize his expected profit given by:

\[
\pi_D(w_D, f_D) = (1 - \gamma) \left[ w_D s^{**} \left\{ 1 - \frac{\lambda p_B}{(1 - s^{**})q_B + s^{**}(f_D + \beta q_B)} - \frac{(1 - \lambda)p_B}{q_B} \right\} - t_D f_D^2 \right] - \gamma t_D f_D^2.
\]

(2.7)

By noting that \(\pi_D\) is continuous in \(w_D \in [0, p_B]\), we know that the optimal wholesale price \(w^{**}_D\) always exists in \([0, p_B]\). In the case when \(\lambda > 0\), the closed-form expressions for \(s^{**}\) and \(w^{**}_D\) do not exist. In the case when \(\lambda = 0\), we can obtain from the first-order condition of (2.6) that \(s^{**}(w_D, f_D) = \frac{1}{2} - \frac{lq_B}{2(p_B - w_D)(q_B - p_B)}\). By substituting \(s^{**}\) into (2.7) and solving \(\max_{w_D \in [0,p_B]} \pi_D(w_D, f_D)\), we obtain \(w^{**}_D\) and the corresponding expected profit of the deceptive counterfeiter \(\pi^{**}_D\) as follows:

(2.8)

\[
w^{**}_D(f_D) = p_B - \sqrt{\frac{lq_B}{1 - \frac{p_B}{q_B}}} \quad \text{and} \quad \pi^{**}_D(f_D) = \frac{1}{2} (1 - \gamma) \left\{ \sqrt{p_B \left( 1 - \frac{p_B}{q_B} \right)} - \sqrt{l} \right\} \left[ t_D f_D^2 \right].
\]

From (2.8), we can generate the following insights. First, as the risk of the licit distributor selling counterfeits increases with \(l\), the deceptive counterfeiter has to reduce his price \(w^{**}_D\) to compensate for the increased risk, resulting in a decrease in his expected profit \(\pi^{**}_D\). Second, \(\pi^{**}_D\) increases with \(p_B \left( 1 - \frac{p_B}{q_B} \right)\), which is the
revenue of the brand-name company without counterfeits. This is because the deceptive counterfeit gets a free ride on the brand name of the genuine product.

In stage 1, the counterfeiter decides the functional quality $f_D$ by considering his optimal wholesale price in stage 2 and the best response of the licit distributor in stage 3. The following lemma shows that the deceptive counterfeiter may choose a different level of functional quality depending on the fraction of proactive consumers in the market, $\lambda$.

**Lemma 2.** For any given $(p_B, q_B)$, when $\lambda = 0$, the optimal functional quality of deceptive counterfeits $f_D^{**}$ is $\underline{f}$. When $\lambda > 0$, there exists $\overline{t}_D$ ($> 0$) such that if $t_D \geq \overline{t}_D$, $f_D^{**} = \underline{f}$, and otherwise $f_D^{**}$ can be $\overline{f}$ or $f_D^* \in (\underline{f}, \overline{f})$ that satisfies $\frac{\partial \pi_D^*}{\partial f_D} |_{f_D = f_D^*} = 0$.

In the market with no proactive consumers (i.e., $\lambda = 0$), as one would expect, the deceptive counterfeiter always chooses the lower bound $\underline{f}$ for his functional quality because improving quality does not increase counterfeit sales. In this case, although a counterfeit is visually identical to its brand-name product, its low quality may result in a substantial financial loss to consumers or even endanger their health and safety. Consequently, both counterfeiter and distributor often face considerable punishments if they get caught. Typical examples are food, beverage, agricultural products, pharmaceuticals, and automotive spare parts (OECD 2008,
Staake and Fleisch 2008). In the market with proactive consumers (i.e., \( \lambda > 0 \)), although consumers cannot distinguish the deceptive counterfeit from the brand-name product, the deceptive counterfeiter can still find it optimal to improve his functional quality above the minimum level \( f \). The reason is as follows. When \( f_D \) is improved, both aggregate demand for brand-name and counterfeit products,

\[
1 - \frac{\lambda p_B}{(1-s^*)q_B + s^*(f_D + q_B)} - \frac{(1-\lambda)p_B}{q_B},
\]

and the fraction of deceptive counterfeits, \( s^* \), are increased. Thus the marginal benefit of functional quality is positive. If the marginal benefit exceeds the marginal cost, then the deceptive counterfeiter will choose his functional quality \( f_D^* \) above \( f \). In practice, some deceptive counterfeits reveal different levels of functional quality; for example, fake gasoline with different levels of adulteration has been reported (Lee et al. 2011).

2.5. Anti-Counterfeiting Strategies: Quality and Price

Having analyzed the equilibrium decisions of counterfeiters and distributors in licit and illicit supply chains, we examine the effectiveness of anti-counterfeiting strategies: quality and pricing strategies in §2.5, and marketing and enforcement strategies in §2.6. We analyze each strategy separately in order to isolate its effect on firms’ profits and consumer welfare. When a firm implements multiple strategies simultaneously, one needs to aggregate the effect of each strategy to evaluate the overall effect.
We examine the effectiveness of quality and pricing strategies against the non-deceptive counterfeiter in §2.5.1, and against the deceptive counterfeiter in §2.5.2; then, we compare them in §2.5.3. In each of §2.5.1 and §2.5.2, we proceed our analysis as follows. First, we examine whether the brand-name company should choose higher/lower quality or price than the case with no counterfeiter in order to maximize her expected profit against the counterfeiter. Let \( q^m_B \) and \( p^m_B \) denote the optimal quality and price of the brand-name product with no counterfeiter in the market, respectively. Similarly, let \( q^*_B \) and \( p^*_B \) (resp., \( q^{**}_B \) and \( p^{**}_B \)) denote the optimal quality and price of the brand-name product in the presence of the non-deceptive (resp., deceptive) counterfeiter, respectively. Second, knowing that such strategies of choosing \( q^*_B \) and \( p^*_B \) (resp., \( q^{**}_B \) and \( p^{**}_B \)) instead of \( q^m_B \) and \( p^m_B \) improve the expected profit of the brand-name company, we examine how those strategies affect the expected profit of the non-deceptive (resp., deceptive) counterfeiter. Finally, we investigate how those strategies affect expected consumer welfare, which is defined as follows. When only brand-name products exist in the market, we can define consumer welfare as \( CS_B = \int_{\theta_B}^{1} (\theta q_B - p_B) d\theta \). Similarly, using (2.1) and (2.5), we can define \( CS_N \) or \( CS_D \) as consumer welfare in the market where non-deceptive or
deceptive counterfeits co-exist with brand-name products, respectively, as follows:

\[
CS_N = \int_{\hat{\theta}}^{\bar{\theta}} (\theta q_N - p_N) d\theta + \int_{\bar{\theta}}^{1} (\theta q_B - p_B) d\theta;
\]

\[
CS_D = s \int_{\hat{\theta}}^{1} (\theta q_D - p_B) d\theta + (1 - s) \int_{\hat{\theta}}^{1} (\theta q_B - p_B) d\theta.
\]

In (2.9), the first term represents the surplus of those consumers who purchase the non-deceptive counterfeit and the second term represents the surplus of those consumers who purchase the brand-name product. In (2.10), the first term represents the surplus of those consumers who are cheated and receive the deceptive counterfeit although they pay the price of the brand-name product, and the second term represents the surplus of those consumers who purchase and receive the brand-name product. Considering the chances that counterfeits do not reach the market due to seizure, we can further define \( ECS_N \) or \( ECS_D \) as the expected consumer welfare when the counterfeiter is non-deceptive or deceptive, respectively, as follows:

\[
ECS_N = (1 - \gamma)CS_N + \gamma CS_B \quad \text{and} \quad ECS_D = (1 - \gamma)CS_D + \gamma CS_B.
\]

Let \( ECS_N^* \) or \( ECS_D^* \) denote the corresponding expected consumer welfare in equilibrium. We can show that \( ECS_D^{**} < CS_B < ECS_N^* \). Intuition from this result is as follows. When non-deceptive counterfeits exist in the market, a consumer has
a cheap alternative to the brand-name product. In equilibrium, the non-deceptive counterfeiter sets his price and functional quality such that he offers a higher utility to those consumers who enjoy the brand value of the brand-name product but do not appreciate its high quality or cannot afford its high price. Therefore, non-deceptive counterfeits improve consumer welfare. In contrast, when deceptive counterfeits exist, some consumers are cheated to receive low-quality deceptive counterfeits, resulting in a welfare loss.\(^3\)

2.5.1. Non-Deceptive Counterfeits

This subsection examines the brand-name company’s anti-counterfeiting strategies against the non-deceptive counterfeiter. We first examine the brand-name company’s quality strategy to combat the non-deceptive counterfeiter. In the following proposition, we present the results for the case when \(f^*_N = \underline{f} \) or \( \bar{f} \), since the exposition of our results is much simpler in this case, while presenting the results for the case when \(f^*_N \in (\underline{f}, \bar{f})\) in appendix A (which involve complex conditions for parts (a) and (c)).

\(^3\)We do not consider the socio-economic effects of counterfeiting on corruption, criminal activities, employment, environment, innovation, tax revenues, and so on. If taking into account these indirect or long-term effects into account, then non-deceptive counterfeits may also decrease consumer welfare. Moreover, the anti-counterfeiting strategies that reduce the incidence of counterfeits (e.g., marketing campaigns that reduce \(\beta\)) can have more positive benefits by lessening these harmful effects. We can also examine the aggregate effect of anti-counterfeiting strategies on social welfare, which may be defined as \(SW_i = ECS_i + \pi_B - \pi_i\) for \(i = N\) or \(D\), by combining the results of profits and consumer welfare.
Proposition 3. Suppose $f_N^* = f$ or $\overline{f}$. Then:

(a) $q_B^* > q_B^m$ if and only if $\beta < 1 - \left\{ \frac{q_B^m - q_N(q_B^m)}{q_B^m} \right\}^2$.

(b) Suppose $q_B^* > q_B^m$ (resp., $q_B^* < q_B^m$). Then $\pi_N^*$ is lower (resp., higher) at $q_B = q_B^*$ than at $q_B = q_B^m$.

(c) Suppose $q_B^* > q_B^m$ (resp., $q_B^* < q_B^m$). Then $ECS_N^*$ is higher (resp., lower) at $q_B = q_B^*$ than at $q_B = q_B^m$ unless $f_N^*$ is decreased from $\overline{f}$ at $q_B = q_B^m$ to $f$ at $q_B = q_B^*$.

First, consider the case when the non-deceptive counterfeit draws an insignificant amount of brand value from the brand-name product (i.e., $\beta < 1 - \left\{ q_B^m - q_N(q_B^m) \right\}^2 / (q_B^m)^2$). In this case, Proposition 3(a) shows that the brand-name company should set her product quality higher than $q_B^m$. This strategy not only improves the expected profit of the brand-name company (as compared to choosing $q_B = q_B^m$), but also decreases the expected profit of the non-deceptive counterfeiter (Proposition 3(b)). In this case, even though the improved quality of the brand-name product also improves the quality of the non-deceptive counterfeit, the difference in quality between two competing products becomes larger because the counterfeit steals only a small part of the brand value (i.e., low $\beta$). Consequently, the non-deceptive counterfeiter will lose its quality competition against the brand-name company. This result may explain how the shoe manufacturers mentioned in §2.1 successfully addressed their counterfeiting issues by improving the quality.
of their products (Qian 2008). Finally, Proposition 3(c) shows that, although this strategy improves the expected profit of the brand-name company and reduces the expected profit of the non-deceptive counterfeiter, it does not always benefit consumers. The reason is as follows. This strategy can lead the non-deceptive counterfeiter to lower his functional quality as well as his wholesale price in order to compete better against brand-name products with improved quality. Although this reduces the market share of non-deceptive counterfeits, those consumers who purchase non-deceptive counterfeits can suffer from lower quality, resulting a welfare loss. For example, Figure 2.1 illustrates that $ECS_N^*$ falls when $q_B$ is increased from $q_{B}^{\text{m}} = 3.37$ to $q_{B}^{*} = 3.4$. 

Figure 2.1. Expected consumer welfare as a function of $q_B$ when non-deceptive counterfeits are in the market (Base parameters: $t = 0.01$, $p_B = 0.9$, $\beta = 0.1$, $\gamma = 0.58$, $c = 0.05$, $\underline{f} = 0.1$, and $\bar{f} = 2.5$)
Next, consider the case when the non-deceptive counterfeit draws a significant amount of brand value from the brand-name product (i.e., $\beta > 1 - \{q_B^m - q_N(q_B^m)\}^2/(q_B^m)^2$). In this case, Proposition 3(a) shows that it is not cost-effective for the brand-name company to improve her product quality because the non-deceptive counterfeiter gets a free ride on the improved quality of the brand-name product. While this strategy improves the expected profit of the brand-name company, it can also help the non-deceptive counterfeiter earn higher expected profit inadvertently (Proposition 3(b)), and make consumers suffer from the poor quality of the product (Proposition 3(c)). Therefore, in this case, the brand-name company may not use this strategy to combat the non-deceptive counterfeiter, and if she does, she must take special case to curb counterfeits in the market.

The following proposition shows how the brand-name company can combat the non-deceptive counterfeiter through her pricing strategy.

**Proposition 4.** (a) $p_B^* < p_B^m$ for all $\beta$.

(b) $\pi_N^*$ is lower at $p_B = p_B^*$ than at $p_B = p_B^m$ for all $\beta$.

(c) $ECS_N^*$ is higher at $p_B = p_B^*$ than at $p_B = p_B^m$ unless $f_N^*$ is decreased from $f$ at $p_B = p_B^m$ to $\underline{f}$ at $p_B = p_B^*$ or $\frac{\partial f_N^*}{\partial p_B} > \kappa$ (where the expression of $\kappa$ ($> 0$) is presented in the proof).
In contrast to the earlier quality strategy, Proposition 4(a) shows that for any $\beta$, it is always beneficial for the brand-name company to set her price $p_B^*$ lower than $p_B^m$. This is because a lower price enables the brand-name company to compete better against non-deceptive counterfeits which are cheap alternatives of brand-name products. This strategy helps the brand-name company to gain more market share by inducing some consumers to switch from non-deceptive counterfeits to brand-name goods. As a result, this strategy also reduces the expected profit of the non-deceptive counterfeiter (Proposition 4(b)). We further find that the larger $\beta$ is, the faster the expected profit of the non-deceptive counterfeiter will decrease. This is because the brand-name company relies more on price to compete with the non-deceptive counterfeiter when the quality levels of two products are not so distinguished due to the larger $\beta$. However, similar to the quality strategy, Proposition 4(c) shows that reducing $p_B$ can hurt consumers by inducing the non-deceptive counterfeiter to reduce his quality level. This strategy has been used in practice; for example, the distributors of Hollywood films cut their DVD prices in Malaysia and Russia to combat rampant piracy (Whang 2001, Arvedlung 2004).

2.5.2. Deceptive Counterfeits

This subsection examines the brand-name company’s anti-counterfeiting strategies against the deceptive counterfeiter. As we will show below, most effects of these
strategies are monotonic when no proactive consumers exist in the market (i.e., \( \lambda = 0 \)), whereas all effects of these strategies are non-monotonic when proactive consumers exist in the market (i.e., \( \lambda > 0 \)). Thus, we first examine the former case analytically to establish monotonic results, and then conduct a numerical study for the latter case to show non-monotonicity. This approach will enable us to isolate the effect of \( \lambda \), and explore dominant effects of anti-counterfeiting strategies when positive \( \lambda \) creates non-monotonic effects. Note that the results under \( \lambda = 0 \) also bear some practical relevance (asymptotically) because only a small fraction of consumers may be proactive in developed countries; for example, \( \lambda = 0.04 \) in the U.S. in our survey results shown in Table 1.

Let us first analyze the case when \( \lambda = 0 \). The following proposition shows, counter-intuitively, that by setting the quality level lower than the quality level with no counterfeiter in the market, the brand-name company can improve her expected profit, reduce the expected profit of the deceptive counterfeiter, and even improve expected consumer welfare.

**Proposition 5.** Consider the market with \( \lambda = 0 \). In this market, the following results hold:

(a) \( q_B^{**} < q_B^m \).

(b) \( \pi_D^{**} \) is lower at \( q_B = q_B^{**} \) than at \( q_B = q_B^m \).
(c) $EC S_D^{**}$ is higher at $q_B = q_B^{**}$ than at $q_B = q_B^m$ if $q_d^{**} < q_B - \frac{(1 - p^2_B q_B^{**}) (1 - (1 - \gamma) s^{**})}{2 (1 - \gamma)} \left( \frac{p^2_B}{q_B^{**}} s^{**} + \frac{1}{2} \left( 1 - \frac{p^2_B}{q_B^{**}} \right) \frac{\partial s^{**}}{\partial q_B} \right)^{-1}$ for $q_B \in [q_B^{**}, q_B^m]$.

Proposition 5(a) states that it is optimal for the brand-name company to set her quality $q_B^{**}$ lower than $q_B^m$. Since consumers cannot distinguish deceptive counterfeits from brand-name products, this strategy reduces the perceived quality of any product in the market, and thus reduces the aggregate demand for both brand-name and counterfeit goods. However, the reduced aggregate demand discourages the licit distributor from taking the risk of selling deceptive counterfeits, hence resulting in a lower $s^{**}$. The result stated in Proposition 5(a) suggests that the latter (positive) effect dominates the former (negative) effect, so this strategy improves the expected profit of the brand-name company. This result highlights the importance of modeling the incentive of the licit distributor in this supply chain: Without the licit distributor, the positive effect of this strategy (i.e., lower $s^{**}$) would not exist and therefore the result opposite to Proposition 5(a) would be obtained. Since this strategy reduces both the aggregate demand and the proportion of deceptive counterfeits sold by the licit distributor, it will also reduce the expected profit of the deceptive counterfeiter (Proposition 5(b)). (More generally, we show in the proof that $\pi_D^{**}$ is increasing in $q_B$ for any $\lambda$.) Finally, contrary
to our first intuition that lower quality will hurt consumers, Proposition 5(c) suggests that this strategy can improve consumer welfare. To understand this result, note that there are two opposing effects of having lower quality of brand-name products on consumer welfare: Consumers suffer from lower quality and fewer consumers buy products, but at the same time fewer consumers are deceived to buy low-quality counterfeits. Proposition 5(c) shows that when the quality of deceptive counterfeits is sufficiently low, the latter effect outweighs the former effect, benefiting consumers.

We next examine the effectiveness of the pricing strategy against the deceptive counterfeiter in the market with $\lambda = 0$.

**Proposition 6.** Consider the market with $\lambda = 0$. In this market, the following results hold:

(a) $p^*_B > p^m_B$.

(b) $\pi_D^*$ can be higher or lower when $p_B = p^*_B$ than when $p_B = p^m_B$.

(c) $ECS_D^*$ is higher at $p_B = p^*_B$ than at $p_B = p^m_B$ if $q^*_D < q_B - \frac{q_B - p_B}{1-\gamma} \left\{ \frac{p_B}{q_B} s^* - \frac{1}{2} \left( 1 + \frac{p_B}{q_B} \right) (q_B - p_B) \frac{\partial s^*}{\partial p_B} \right\}^{-1}$ for $p_B \in [p^m_B, p^*_B]$.

With no proactive consumers in the market, Proposition 6(a) states that the brand-name company can improve her expected profit by setting her price $p^*_B$ higher than $p^m_B$ (due to the reason similar to Proposition 5(a)). Unlike the earlier
quality strategy, however, this pricing strategy has non-monotonic impact on the expected profit of the deceptive counterfeiter (Proposition 6(b)). To understand this result, note that there are two effects of raising $p_B$: (i) it reduces the aggregate demand for brand-name and counterfeit goods (i.e., $\frac{\partial}{\partial p_B} (1 - \frac{p_B}{q_B}) < 0$); and (ii) it increases the distributor’s margin from selling deceptive counterfeits (i.e., $\frac{\partial}{\partial p_B} (p_B - w^*_D) = \frac{\partial}{\partial p_B} \sqrt{\frac{p_B}{1 - p_B}} > 0$ from (2.8)). Because of the latter effect, the strategy of raising $p_B$ does not always reduce the proportion $s^{**}$ of deceptive counterfeits the licit distributor sells, nor does it always reduce the deceptive counterfeiter’s market share $m^*_D$ and her expected profit $\pi^*_D$. Therefore, in implementing this pricing strategy, a firm or the government should carefully consider these two counterbalancing effects of raising/reducing price. In practice, we observe both instances of raising or reducing prices: Newton et al. (2002) propose reducing drug prices to make counterfeiting less attractive by reducing the profit margins of fake drugs (i.e., opposite effect of (ii)), and Russia will raise vodka prices to put out of business makers of counterfeit alcohol (via effect (i)) although it will also affect licit companies (Reuters 2012). Finally, Proposition 6(c) suggests that this strategy can improve consumer welfare when the quality of deceptive counterfeits is sufficiently low. We can interpret this result similarly to Proposition 5(c).
Next, we analyze the case in which proactive consumers exist in the market (i.e., $\lambda > 0$). As we have mentioned at the beginning of this subsection, this additional factor causes all the effects of the anti-counterfeiting strategies to become non-monotonic. Specifically, the brand-name company’s optimal quality $q_B^{**}$ (resp., $p_B^{**}$), can be higher or lower than her quality $q_B^m$ with no counterfeiter (resp., $p_B^m$); furthermore, the deceptive counterfeiter’s expected profit $\pi_D^{**}$ and the expected consumer welfare $ECS_D^{**}$ are non-monotonic with a change of $q_B$ or $p_B$. Because the closed-form expressions of $s^{**}$, $w_D^{**}$ and $f_D^{**}$ do not exist when $\lambda > 0$, no simple conditions can be derived analytically for monotonic results (see remarks on the proofs of Propositions 5 and 6 in appendix A). For this reason, we conduct a numerical study to compare the results under $\lambda = 0$ with those under $\lambda > 0$, and explore dominant effects. The numerical experiments are conducted with the following settings: for $\lambda = 0, 0.25$ or $0.5$, we constructed 1024 scenarios using the parameter values shown at the bottom of Table 3, so that they cover various possible scenarios and also satisfy positive $s^{**}$ in equilibrium. We present a summary of the results in Table 3, which reads as follows: for example, when $\lambda = 0.5$, $q_B^{**} < q_B^m$ was observed in 97.3% of 1024 scenarios, and choosing $q_B^{**}$ reduced $\pi_D^{**}$ in 97.3% of 1024 scenarios and increased $ECS_D^{**}$ in 5.3% as compared to choosing $q_B^m$.

Table 3. Effects of Quality and Pricing Strategies against Deceptive Counterfeits
Effects of Choosing $q_B^{**}$ vs. $q_B^m$

<table>
<thead>
<tr>
<th>$q_B^{**}$ vs $q_B^m$</th>
<th>$\pi_D^{**}$ ↓</th>
<th>$ECS_D^{**}$ ↑</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_B^{**} &lt; q_B^m$</td>
<td>1</td>
<td>0.097</td>
</tr>
<tr>
<td>$q_B^{**} &gt; q_B^m$</td>
<td>1</td>
<td>0.016</td>
</tr>
</tbody>
</table>

(Note) Each number in the table indicates a percent of scenarios for the corresponding effect. We used the following parameters: $t \in \{0.005, 0.01, 0.015, 0.02\}$, $\beta \in \{0.1, 0.2, 0.3, 0.4\}$, $\gamma \in \{0.1, 0.2, 0.3, 0.4\}$, $l \in \{0.005, 0.01, 0.015, 0.02\}$, $c \in \{0.1, 0.2, 0.3, 0.4\}$, $f = 0.1$, and $\overline{f} = (1 - \beta) \cdot q_B - 0.1$.

From Table 3, we can observe the following:

(1) The results obtained under $\lambda = 0$ continue to hold in most scenarios under $\lambda > 0$. However, in some scenarios, the brand-name company finds it optimal to set $q_B^{**} > q_B^m$ or $p_B^{**} < p_B^m$. We can explain this result as follows. First, recall from our discussions above that setting lower $q_B^*$ or higher $p_B^*$ reduces the aggregate demand for brand-name and counterfeit goods, and that the reduced aggregate demand discourages the licit distributor from taking the risk of selling counterfeits. Propositions 5(a) and 6(a) suggest that the latter (positive) effect always dominates the former (negative) effect when $\lambda = 0$. However, with proactive consumers in the market (i.e., $\lambda > 0$), the deceptive counterfeiter may improve his functional quality $f_D^{**}$ in response to the reduced demand (see Lemma 2). This additional factor makes the licit distributor more willing to sell counterfeits, so that the positive effect does not always dominate the negative effect.

(2) In those scenarios where $q_B^{**} > q_B^m$, the strategy of setting higher $q_B^{**}$ will
increase the deceptive counterfeiter’s expected profit $\pi_D^{**}$ by making counterfeits flourish more in the market. This happens because the improved quality of the brand-name product results in an increase of the aggregate demand of brand-name and counterfeit goods, which in turn incentivizes the licit distributor to procure more deceptive counterfeits. This may be the cause of the initial failure of the Scotch whisky company which improved her quality to combat deceptive counterfeits in the Thailand market (see §2.1). Also, from the table, we confirm that the expected profit of the deceptive counterfeiter is non-monotonic in $p_B$ for any $\lambda \geq 0$, which can be explained similarly to Proposition 6(b).

(3) The expected consumer welfare $ECS_D^{**}$ has increased in more scenarios in the market with $\lambda > 0$ than in the market with $\lambda = 0$. Similar to our explanation given in (1) above, this is because the counterfeiter may improve his functional quality $f_D^{**}$ with proactive consumers. In general, for any $\lambda \in [0, 1]$, we show in appendix A that if an anti-counterfeiting strategy improves the average product quality in the market, then it improves the expected consumer welfare.

(4) The number of scenarios in which $\pi_D^{**}$ is decreased or $ECS_D^{**}$ is increased is not necessarily monotonic in $\lambda$. For example, a change to $q_B^{**}$ from $q_B^m$ decreases $\pi_D^{**}$ in all scenarios when $\lambda = 0$, in 96.1% of scenarios when $\lambda = 0.25$, and in 97.3% when $\lambda = 0.5$. This result indicates that anti-counterfeiting strategies are not necessarily more effective as more consumers are proactive. Similarly, we can
show that more proactive consumers in the market does *not necessarily* benefit the brand-name company (i.e., $\pi_B^*$ is non-monotonic in $\lambda$). The reason is as follows. Proactive consumers purchase products only when their expected utility is non-negative, considering the likelihood of receiving deceptive counterfeits unknowingly. As more consumers are proactive, therefore, a smaller number of consumers will purchase products. This reduced aggregate demand for products discourages the licit distributor from taking the risk of selling deceptive counterfeits. Thus, depending on which of the two effects (i.e., reduced aggregate demand and reduced $s^{**}$) dominates, the expected profit of the brand-name company as well as her market share may increase or decrease with $\lambda$.

2.5.3. Comparison: Non-Deceptive vs. Deceptive

We now compare the effect of each strategy against the *non-deceptive* counterfeiter with that against the *deceptive* counterfeiter. Using the results presented in §2.5.1 and §2.5.2, we summarize in Table 4 whether the brand-name company should choose higher/lower quality or price than the case with no counterfeiter in order to maximize her expected profit, and how such anti-counterfeiting strategies affect the expected profit of the counterfeiter and the expected consumer welfare. (If a dominant effect exists for a non-monotonic case, Table 4 reports only the dominant effect.)
Table 4. Effects of Anti-Counterfeiting Strategies: Non-Deceptive vs. Deceptive

<table>
<thead>
<tr>
<th>Non-Deceptive Counterfeits</th>
<th>Optimal Strategy</th>
<th>( \pi_N^* )</th>
<th>( ECS_N^* )</th>
<th>Deceptive Counterfeits</th>
<th>Optimal Strategy</th>
<th>( \pi_D^{**} )</th>
<th>( ECS_D^{**} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_B^* &gt; q_B^{m*} ) (low ( \beta ))</td>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td>( q_D^{**} &lt; q_D^{m*} )</td>
<td>( \downarrow )</td>
<td>( (low \ q_D^{<strong>}) ) or ( \downarrow ) (high ( q_D^{</strong>} ))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( q_B^* &lt; q_B^{m*} ) (high ( \beta ))</td>
<td>( \uparrow )</td>
<td>( \downarrow )</td>
<td>( p_B^* &lt; p_B^{m*} )</td>
<td>( \downarrow )</td>
<td>( \uparrow ) (low ( q_D^{<strong>} )) or ( \downarrow ) (high ( q_D^{</strong>} ))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_B^* &lt; p_B^{m*} )</td>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td>( p_B^{<em>} &gt; p_B^{m</em>} )</td>
<td>( \downarrow )</td>
<td>( \uparrow ) (low ( q_D^{<strong>} )) or ( \downarrow ) (high ( q_D^{</strong>} ))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table 4, we can draw the following insights:

(1) The optimal strategy of the brand-name company (that maximizes her expected profit) differs depending on whether she faces the non-deceptive or deceptive counterfeiter. For example, reducing price is optimal against the non-deceptive counterfeiter, whereas raising price is optimal against the deceptive counterfeiter.

(2) Even when the optimal strategy of the brand-name company is the same against both non-deceptive and deceptive counterfeiters, its impact on the counterfeiter’s expected profit and the expected consumer welfare may not be the same. For example, when \( \beta \) is high, setting a lower quality level than the case with no counterfeiter improves the brand-name company’s expected profit against either type of the counterfeiter. While this strategy is effective against the deceptive counterfeiter (i.e., reduces \( \pi_D^{**} \)), it does not work well against the non-deceptive counterfeiter (i.e., increases \( \pi_N^* \)). Moreover, its impact on the expected consumer welfare may not be the same across the two types of the counterfeiter, either.
(3) An ideal anti-counterfeiting strategy should improve the brand-name company’s expected profit, reduce the counterfeiter’s expected profit, and improve the expected consumer welfare. The pricing strategy is such an ideal strategy against the non-deceptive counterfeiter. For the other cases, a brand-name company or the government should carefully consider a trade-off among those three objectives in implementing an anti-counterfeiting strategy.

2.6. Anti-Counterfeiting Strategies: Marketing and Enforcement

In this section, we consider two other anti-counterfeiting strategies that are commonly used in practice. The first strategy we will consider is the marketing campaign that educates consumers about the adversity of counterfeit goods. For example, an electronic manufacturer may emphasize the fact that counterfeit electronics lack in safety features. This strategy helps reduce the brand value the counterfeit steals from the brand-name product, i.e., reduce $\beta$. The second strategy we will consider is the direct enforcement efforts to increase the chances to seize counterfeit products, $\gamma$. In executing these strategies, the brand-name company often collaborates with other organizations or the government. For example, French luxury goods association Comite Colbert launched a campaign (using playful slogans such as “real ladies don’t like fake”) in response to the threat of the counterfeit, and the French police raided the clandestine workshops making Hermes counterfeit.
accessories, of which the surveillance was part of an investigation into the international crime ring that robs many brands (Wellman 2012). Since the brand-name company does not have a full control of these parameters $\beta$ and $\gamma$, we do not consider the brand-name company’s optimal choices of these parameters; instead, we examine how reducing $\beta$ or increasing $\gamma$ will affect firms’ expected profits and expected consumer welfare.

First, let us consider the market in which the brand-name company faces the non-deceptive counterfeiter. It is intuitive that both the marketing campaign and the enforcement strategy will improve the expected profit of the brand-name company and reduce the expected profit of the non-deceptive counterfeiter. However, we can show that both strategies hurt expected consumer welfare for the following reasons. The market campaign makes those consumers who purchase non-deceptive counterfeits enjoy the counterfeit brand less, resulting in a welfare loss. The enforcement strategy makes counterfeits less likely to reach the market, and hence it makes the non-deceptive counterfeiter more reluctant to invest in quality improvement. Therefore, consumers will suffer from less availability of non-deceptive counterfeits (which are cheaper substitutes for brand-name goods) as well as from their lower quality.
Next, we examine the effectiveness of two anti-counterfeiting strategies against the deceptive counterfeiter. The following proposition shows that the effectiveness of these strategies differs significantly from that against the non-deceptive counterfeiter.

**Proposition 7.** For any given \( q_B \) and \( p_B \),

(a) (Marketing) When \( \lambda = 0 \), reducing \( \beta \) has no impact on \( \pi_B^{**} \) and \( \pi_D^{**} \), whereas it reduces \( ECS_D^{**} \). When \( \lambda > 0 \), reducing \( \beta \) decreases \( \pi_D^{**} \), but it can increase or reduce \( \pi_B^{**} \) and \( ECS_D^{**} \).

(b) (Enforcement) When \( \lambda = 0 \), increasing \( \gamma \) improves \( \pi_B^{**} \), reduces \( \pi_D^{**} \), and improves \( ECS_D^{**} \). When \( \lambda > 0 \), increasing \( \gamma \) reduces \( \pi_D^{**} \), but it can increase or reduce \( \pi_B^{**} \) and \( ECS_D^{**} \).

Proposition 7(a) suggests that special care must be taken when implementing the marketing campaign against the deceptive counterfeiter. For the case when no consumers are proactive (i.e., \( \lambda = 0 \)), the marketing campaign has no impact on the firms’ expected profits because consumers do not take into account the possibility of receiving counterfeits unknowingly. This result is expected. On the other hand, proactive consumers correctly expect that they will derive less utilities when receiving deceptive counterfeits unknowingly. Thus, when \( \lambda > 0 \), the marketing
campaign can reduce the expected profit of the deceptive counterfeiter by discour-
aging proactive consumers from purchasing products. However, it could backfire
the brand-name company because proactive consumers reduce their consumption
of brand-name products as well. For example, a large beverage company in Korea
suffered from a sales drop of 15% after their counterfeiting problems were broad-
casted in a TV program (Choi 2009). Finally, unlike the case when \( \lambda = 0 \), this
strategy could improve expected consumer welfare when \( \lambda > 0 \) because a smaller
number of proactive consumers purchase products and hence receive low-quality
deceptive counterfeits.

Proposition 7(b) shows that when no proactive consumers exist in the market
(i.e., \( \lambda = 0 \)), the enforcement strategy works well against the deceptive coun-
terfeiter. However, contrary to a common belief, this strategy may reduce the
expected profit of the brand-name company and also hurt expected consumer wel-
fare in the market where proactive consumers exist (i.e., \( \lambda > 0 \)). This result can be
explained as follows. Similar to the impact of this strategy on the non-deceptive
counterfeiter (discussed above), by increasing the risk of counterfeiting, this strat-
ey makes the deceptive counterfeiter reluctant to invest in quality improvement.
While the lower quality of \textit{non-deceptive} counterfeits helps the brand-name com-
pany regain its market share in quality competition, the lower quality of \textit{deceptive}
counterfeits reduces the perceived quality of products in the market with proactive
consumers, hence reducing the aggregate demand for both brand-name goods and deceptive counterfeits. In this case, consumers also suffer from the lower quality of deceptive counterfeits although fewer consumers will receive deceptive counterfeits unknowingly. In appendix B, we further study how different values of \( \lambda \) affect the effectiveness of these strategies.

2.7. Extension: Risk of Counterfeiting

In this section, we extend our base model to the case where the probability of counterfeits getting confiscated is a decreasing function of their functional quality. This is plausible in some situations because those consumers who have suffered due to the low quality of counterfeits can report them to the authorities, which may lead to the raid of counterfeit factories or distributors. For example, if the fake furniture mentioned in Barboza (2011) had functioned as well as its genuine furniture, a consumer might have not discovered that the furniture he/she has purchased is, in fact, counterfeit. Specifically, suppose that a counterfeiter will get caught by the authorities with the probability of \( \gamma - \delta_1 f_i \) for \( i = N \) or \( D \), and that a licit distributor will get caught with the probability of \( s - \delta_2 f_D \). We assume \( \delta_1 > 0 \) and \( \delta_2 > 0 \), so that the lower the quality of counterfeit goods, the higher the detection probabilities become.\(^4\)

\(^4\)We do not consider the case where the probability of the illicit distributor getting caught for selling non-deceptive counterfeits is decreasing with the quality of non-deceptive counterfeits.
When the non-deceptive counterfeiter exists in the market, it is easy to see that the price decisions of the illicit distributor and the counterfeiter in stages 3 and 2, respectively, are unchanged. In stage 1, the counterfeiter chooses his optimal functional quality \( f_N \) to maximize his expected profit, which is modified from (2.4) as follows:

\[
\pi^*_N(f_N) = \frac{p_B^2 (1 - \gamma + \delta_1 f_N)(f_N + \beta q_B)}{8q_B \{(1 - \beta)q_B - f_N\}} - t_N f_N^2.
\]

Similar to Lemma 1, we can show that \( f_N = \overline{f}, \overline{f} \) or \( f_N \in (\overline{f}, \overline{f}) \) that satisfies

\[
\frac{\partial \pi^*_N}{\partial f_N}|_{f_N=f_N} = 0,
\]

depending on the value of \( t_N \) and whether \( \pi^*_N(\overline{f}) \geq \pi^*_N(\overline{f}) \).

When the deceptive counterfeiter exists in the market, in stage 3, the licit distributor chooses its optimal fraction \( s^* \) of counterfeits by solving the following problem (which is modified from (2.6)):

\[
\max_{s \in [0,1]} s \{1 - s + \delta_2 f_D\}(p_B - w_D) \left\{1 - \frac{\lambda p_B}{(1 - s)q_B + s q_D} - \frac{(1 - \lambda)p_B}{q_B}\right\} - (s - \delta_2 f_D)l.
\]

In stages 2 and 1, the counterfeiter decides \( w_D \) and \( f_D \), respectively, to maximize his expected profit given by:

\[
\pi_D(w_D, f_D) = w_D s^* (1 - \gamma + \delta_1 f_D) \left\{1 - \frac{\lambda p_B}{(1 - s^*) q_B + s^*(f_D + \beta q_B)} - \frac{(1 - \lambda)p_B}{q_B}\right\} - t_D f_D^2.
\]

Such a case is unlikely in practice because consumers already know what they purchase. For example, a street vendor who sells $10 fake watches is not more likely to get caught, as the quality of those watches gets worse. Furthermore, this probability does not affect our results due to our assumption that \( t_N = 0 \) (see §3).
When $\lambda = 0$, by following the procedure similar to that in the base model, we obtain the closed-form expressions of $s^{**}$ and $w^{**}$ as follows: 

\[ s^{**} = \frac{1 + \delta_2 f_D}{2} - \frac{lq_B}{2(p_B - w_D)(q_B - p_B)} \quad \text{and} \quad w_D^{**} = p_B - \sqrt{\frac{lp_B}{(1 - \frac{q_B}{q_B})(1 + \delta_2 f_D)}}. \]

In this case, unlike the base model (c.f. Lemma 2), $f^{**}_D > f$ is possible even without proactive consumers. This is because high-quality counterfeits can induce the licit distributor to procure more counterfeits (i.e., increase $s^{**}$) by reducing the probability of the distributor getting caught. When $\lambda > 0$, similar to the base model, we can show the existence of $s^{**}$, $w^{**}$ and $f^{**}_D$, but their closed-form expressions are not available.

Using the equilibrium analysis above, we show in the following corollary that the main results in the base model continue to hold in this extended model.

**Corollary 8.** Suppose the probability of a counterfeiter getting caught is $\gamma - \delta_1 f_i$ for $i = N$ or $D$, and the probability of a licit distributor getting caught is $s - \delta_2 f_D$, where $\delta_1 > 0$ and $\delta_2 > 0$. Then:

(a) Proposition 3 continues to hold.

(b) Propositions 4, 5 and 6 continue to hold except that the conditions in part (c) are different.

(c) Proposition 7 continues to hold except that increasing $\gamma$ can increase or reduce $\pi_B^{**}$ and $ECS_D^{**}$ when $\lambda = 0$. 

Corollary 8 shows that a more general risk model in this section affects only the impact of enforcement strategy (that increases $\gamma$) on $\pi_B^{**}$ and $ECS_D^{**}$. In the base model, Proposition 7 has shown that this strategy always improves $\pi_B^{**}$ and $ECS_D^{**}$ when $\lambda = 0$. However, Corollary 8(c) shows that this strategy can either increase or decrease $\pi_B^{**}$ and $ECS_D^{**}$ even when $\lambda = 0$. The intuition is as follows. In the base model, when $\lambda = 0$, the optimal functional quality $f_{D}^{**}$ of the deceptive counterfeiter is always $\underline{f}$. However, as we have discussed above, $f_{D}^{**} > \underline{f}$ is possible in the extended model. In this case, as the investment for quality improvement becomes more risky with higher $\gamma$, the deceptive counterfeiter may find it optimal to reduce $f_{D}^{**}$. This in turn increases the risk of the licit distributor selling counterfeits (through $\delta_2 f_D$) as well as his own risk of getting caught (through $\delta_1 f_D$). As a result of these two opposing effects, we find that increasing $\gamma$ can increase or decrease $f_{D}^{**}$. When $f_{D}^{**}$ is increased, it will reduce the risk of the licit distributor selling deceptive counterfeits, hence increasing the fraction $s^{**}$ of deceptive counterfeits; consequently, it could hurt the expected profit of the brand-name company, $\pi_B^{**}$. On the other hand, when $f_{D}^{**}$ is decreased, consumers will suffer from the lower quality of deceptive counterfeits; thus, it could reduce $ECS_D^{**}$. 
2.8. Concluding Remarks

Today counterfeit products are being produced and consumed in virtually all economies (OECD 2008). While easy-to-manufacture goods had dominated counterfeit supply until a decade ago, there has been an alarming expansion of product categories being infringed. As a result of outsourcing and offshoring, counterfeiters have easy access to modern technology and equipment, and they are capable of producing high-quality replicas. Consumers are not easily deceived by fake goods that are sold by vendors in open markets and unknown internet sites. These changing business conditions require industry and governments to enhance their understanding of the current and potential counterfeiters they may face and to develop strategies to limit their activities.

To aid the efforts of industry and governments to combat counterfeiting, we have developed a normative model of counterfeiting. Our model captures the recent changes in counterfeiting supply and demand that are not addressed in the previous literature. For example, the previous literature focuses on the pricing decision of a counterfeiter, assuming that the quality level of his goods is fixed, and he is capable of selling his goods, even deceptive ones, directly to consumers. In contrast, our model takes into account the strategic decisions of a counterfeiter regarding his price and functional quality; and the fundamental difference between non-deceptive
and deceptive counterfeits in consumers’ awareness, distribution channels, and penalty on illegal distribution. We have also considered the case when a fraction of consumers are proactive. Modeling these factors explicitly enables us to evaluate several anti-counterfeiting strategies against both types of counterfeiters, and to draw novel managerial insights.

Our analysis highlights that the strategies which are effective in combating the non-deceptive counterfeiter may not work well against the deceptive counterfeiter. Moreover, even if strategies help the brand-name company improve her expected profit, they may not be effective in limiting counterfeit activities, and they can even hurt consumers. For example:

- The strategy of improving the quality of brand-name products is effective in combating the non-deceptive counterfeiter only when the non-deceptive counterfeiter steals an insignificant amount of brand value. This strategy may not be used in combating the non-deceptive counterfeiter in other situations or in combating the deceptive counterfeiter.

- The strategy of reducing the price of brand-name products is an ideal strategy against the non-deceptive counterfeiter. In contrast, when facing the deceptive counterfeiter, it can hurt the brand-name company’s profit as well as consumer welfare, and also benefit the deceptive counterfeiter inadvertently.

- The marketing campaign and the enforcement strategy are effective in combating
the non-deceptive counterfeiter, but they may not benefit the brand-name company or consumers when consumers are proactive toward deceptive counterfeits.

Therefore, industries and governments should understand the type of potential counterfeiters and the characteristics of consumers in order to design effective strategies to combat counterfeits. Without such understanding, anti-counterfeiting strategies could be ineffective and hurt consumer welfare.

Although our model captures the salient features of counterfeiting, we make several assumptions to maintain tractability. First, we do not consider the effect of positive or negative externality of counterfeits on brand-name products. For some product categories, counterfeits help to increase the size of user base of brand-name products, which refers to positive externality. A typical example is software piracy (Conner and Rumelt 1991). The negative externality of counterfeits refers to the negative impact of counterfeits on the value of a brand. More counterfeits in the market, less prestigious the brand becomes. Second, we assume that consumers are risk-neutral. In some situations, consumers show risk-prone or irrational behavior. For example, fraudsters use their phony pharmaceutical websites to take advantage of the recent swine-flu fears. Some consumers who are anxious for their children take risks of buying fake vaccines and bogus remedies from unknown websites (Taylor 2009). Behavioral research would help enrich our understanding of the risk attitudes of consumers. Third, our model does not capture the details of
specific anti-counterfeiting technologies; e.g., technologies to authenticate products such as NanoInk (http://www.nanoink.net) and technologies to track and trace the movement of products through supply chains such as RFID. Yet, their broad use and success has been limited by a variety of factors, including the ability of counterfeiters to adopt or copy the technologies (OECD 2008). Our current model captures the role of these technologies to some degree: the former type of technologies is captured by the marginal cost of developing functional quality of a counterfeit product (i.e., with such technologies installed, a counterfeiter needs to spend more effort to copy authentic goods) and the latter type of technologies is captured by seizure rate (i.e., with RFID installed, the likelihood of seizing counterfeits increases). More detailed cost-benefit analysis of these technologies in specific industrial settings would be interesting future research.
CHAPTER 3

Stability and Endogenous Formation of Inventory

Transshipment Networks

ABSTRACT:¹ This paper studies a cooperative game of inventory transshipment among multiple firms. In this game, firms first make their inventory decisions independently, and then decide collectively how to transship excess inventories to satisfy unmet demands. In modeling transshipment, we use networks of firms as the primitive, which offer a richer representation of relationships among firms by taking the coalitions used in all previous studies as special cases. For any given cooperative network, we construct a dual price allocation under which the network is stable for any residual demands and supplies in the sense that no firms find it more profitable to form subnetworks. Under the allocation based on the marginal contribution of each firm to its network (called the MJW value), we show that various network structures such as complete, hub-spoke, and chain networks are stable only under certain conditions on residual amounts. Moreover, these conditions differ across network structures, implying that a network structure plays

¹This essay is a joint work with Soo-Haeng Cho.
an important role in establishing the stability of a decentralized transshipment system. Finally, we consider the case when firms establish networks endogenously, and show that pairwise Nash stable networks underperform the corresponding networks in centralized systems.

3.1. Introduction

Networks are often used to represent relationships among multiple firms. In this paper, we investigate networks of firms that share inventory through transshipment. When firms are connected in these networks, they can transship excess inventories (also called “residual supplies”) between each other to satisfy unmet demands (also called “residual demands”). The benefit of inventory transshipment is straightforward. By joining a network of transshipment, firms can generate additional revenues by utilizing their otherwise unused inventories and demands. According to Narus and Anderson (1996), transshipment can reduce inventory cost by 15% to 20% and the amount of lost sales by as much as 75%. Due to these benefits of transshipment, many manufacturers promote inventory transshipment in their networks of independent retailers (Shao et al. 2011).

The importance of transshipment has been recognized well in the operations management (OM) literature. In particular, our paper is related to the stream of research that studies transshipment of inventories in decentralized systems. In
these systems, multiple independent firms cooperate in order to maximize their own profits. Thus a proper mechanism needs to be developed to ensure that firms have incentives to participate in transshipment.\textsuperscript{2} The extant literature analyzes the incentives of independent firms using the cooperative game theory based on the concept of coalitions. A \textit{coalition} is a set of firms, within which firms can share their excess inventories and unmet demands, and thus generate additional profits from transshipment. When all firms belong to one coalition, such a coalition is called the \textit{grand coalition}. The profits from transshipment are allocated among firms within a coalition according to a predetermined allocation rule. A central question of this research stream is whether a certain allocation is in the core. When a core allocation is used, no subset of firms has incentives to form subcoalitions by seceding from the grand coalition.

Our paper is distinguished from the previous research stream by analyzing \textit{transshipment networks in decentralized systems}. As compared with a coalition-based cooperative game studied in the literature, we consider a network-based cooperative game. Our network-based approach is fundamentally different from the coalition-based approach in the following two important aspects. First, in a

\textsuperscript{2}A similar incentive problem can occur even among different branches within the same firm. For example, in our industry project with an energy distribution company, we observed that inventory transshipment occurred between two branches that are located in different regions. In this company, each individual manager who operates a local branch is evaluated based primarily on the performance of his/her own branch. Thus, in order for transshipment to occur, two managers have to agree upon how the additional profit generated from transshipment is allocated.
network-based game, a firm can share its residual demands or supplies with the firms to which it has direct links, and also with the firms to which it has no direct links but is connected via other intermediate firms. In the former case, transshipment between two firms can occur without the cooperation of any other firms. In the latter case, however, transshipment between two firms occurs only when all other intermediate firms on a transshipment path cooperate as well, and the costs of transshipment may vary across different paths between the two firms. In contrast, in a coalition-based game, transshipment paths are not specified, and the costs of transshipment between two firms are fixed. To illustrate this difference, consider a market with three firms. Figure 3.1(a) shows the grand coalition in this market. In a coalition-based game, all three firms can share their residuals with one another only in the grand coalition. On the other hand, Figure 3.1(b)-(e) show that all three firms can share their residuals with one another in four different types of networks: the complete network in which all three firms are linked to each other, and three incomplete networks in which there is one link missing with respect to the complete network. For example, in the incomplete network shown in Figure 3.1(c), firm 1 and firm 3 are connected to each other through firm 2, and they can share residuals only when firm 2 cooperates. Moreover, the cost of transshipment between firm 1 and firm 3 in the network shown in Figure 3.1(c) may differ from that in the network shown in Figure 3.1(d). Second, within a coalition,
firms are distinguished by the amount of their residual demands or supplies. In a network, firms are distinguished by their positions within the network as well as their residual amounts. For example, in the network shown in Figure 3.1(c), firm 2 is positioned better than firms 1 and 3 because firms 1 and 3 can share their residuals only through firm 2, while firm 2 can share its residuals directly with firm 1 or 3.

As illustrated in Figure 3.1, networks represent richer relationships among firms than coalitions by taking coalitions as special cases. For this reason, there have been advances in the theory of a network-based cooperative game, starting with Myerson (1977) who notes that “there are many intermediate possibilities between universal cooperation and no cooperation.” Such partial cooperation structures can be captured by networks, but not by coalitions. For example, in the network shown in Figure 3.1(c), firm 1 and firm 3 have partnerships with firm 2, but not between each other. A comprehensive review for the development of the theory and applications of a network-based cooperative game can be found in Jackson.
Our paper is among the first papers in the OM literature that apply this theory to firms’ operational decisions.

The analysis of various networks is also important from a practical viewpoint. The complete network is often too expensive to implement in practice due to the cost of establishing connections (Jackson 2006). Several papers reviewed by Patterson et al. (2011) have analyzed the efficiency of incomplete networks in a centralized system where a central coordinator optimally determines transshipment based on those networks given exogenously. In our paper, we do not assume the existence of such a central coordinator; instead, we examine firms’ individual incentives to join transshipment networks in a decentralized setting, and also analyze a situation in which firms build their networks endogenously. The following examples further motivate our research:

– In a typical regional blood management system (e.g., Prastacos 1984, Fontaine et al. 2009), multiple independent hospital blood banks set their own inventory levels for fresh units of whole blood or red cells. When one blood bank experiences a shortage and has an urgent need for blood, it needs to have blood transshipped from another blood bank who has it in inventory. Such transshipment occurs through a regional blood center (usually a large hospital) who re-distributes the unused units of blood among smaller blood banks. Although a detailed arrangement of transshipment is fairly complex, this system resembles a hub-spoke network depicted in
Figure 3.1(c), in which a regional blood center works as the hub and smaller blood banks work as spokes.

- Many states in the United States require auto manufacturers to sell their cars through independent dealers (Ramsey and Bauerlein 2013). The availability of various models and colors are different across dealers. If a dealer does not have a stock of the specific model desired by a consumer, a common practice is to have it transshipped from another dealer who has it in inventory (Zhao and Atkins 2009). To initiate such transshipment, a salesperson in the dealer shop first needs to find out which dealers have a desired model in inventory. This can be done either through shared web-based inquiry tools or through phone calls by the salesperson. Once the salesperson has found the model available in others’ inventories, he needs to contact those other dealers and agree upon various terms and conditions for its transshipment. This process involves costs for labor and administrative arrangements (Lien et al. 2011).

Similarly, decentralized transshipment networks are also used in industries such as machine tools and repair parts (Narus and Anderson 1996) and trucking industry (Zhao and Atkins 2009). Duvall (2000) reports that companies in a variety of sectors explore combining their inventories in either physical or virtual warehouses.
To analyze a decentralized transshipment system, we develop a two-stage model that is similar to those of Anupindi et al. (2001) and Granot and Sošić (2003), but we consider the formation of networks instead of coalitions in the prior work. In the first stage, firms make their inventory decisions independently under uncertain demands. After the realization of the demands, some firms may have leftover inventories, while others may have unsatisfied demands. In the second stage, the firms cooperate by transshipping residual supplies to satisfy residual demands. Then the additional profits generated from transshipment are allocated among participants according to a predetermined rule. We analyze this model backwards with emphasis on the second-stage network analysis. Our second-stage analysis contains two parts: (1) examining the stability of an existing network, and (2) predicting networks to be established by firms endogenously. Specifically, in the first part, we extend the concept of the core defined in a coalition-based cooperative game of transshipment into a network-based cooperative game. In a coalition-based cooperative game, to determine whether an allocation is in the core, one needs to show that no subset of firms has an incentive to form subcoalitions by seceding from the grand coalition. On the other hand, in a network-based cooperative game, we need to examine the incentives of firms to form subnetworks by seceding from a given network. In the second part, we derive equilibrium network structures when firms form networks endogenously under the allocation rule based on firms’
marginal contributions to their networks. In this case, any two firms can decide whether or not to set up a link between them. For our first-stage analysis, we investigate the inventory decisions of the firms, and determine the conditions under which the Nash equilibrium inventory levels coincide with the inventory levels in a centralized system (i.e., the first-best inventory levels).

Our main contributions are three-fold. First, we present a novel model of inventory transshipment networks, and construct a dual price allocation that is in the core for any network structures and for any residual amounts. Second, we examine the allocation based on the marginal contribution of each firm to its network that is proposed first by Myerson (1977) and is refined later by Jackson and Wolinsky (1996) (which we call the “MJW value”). Under this allocation, various network structures such as complete, hub-spoke, and chain networks are stable only under certain conditions on residual amounts. Moreover, these conditions differ across network structures, implying that a network structure plays an important role in establishing the stability of a decentralized distribution system. While the previous coalition-based approach examines deviations of subcoalitions from the grand coalition, we investigate deviations of subnetworks from both complete and incomplete networks, and we find that the complete network tends to be less stable than incomplete networks under the MJW value. Finally, while the previous OM literature on supply chain networks (as reviewed by Netessine 2009)
commonly assumes that a network of firms is given exogenously, we analyze the case when firms establish networks endogenously, and show that pairwise Nash stable networks underperform the corresponding networks in centralized systems.

3.2. Related Literature

Our paper is related to the stream of research that studies transshipment of inventories in decentralized systems. For a comprehensive survey of the literature on inventory transshipment, the reader is referred to Paterson et al. (2011).

Two pioneering papers in this stream are Anupindi et al. (2001) and Granot and Sošić (2003). Similar to our model (except that we take a network approach), Anupindi et al. (2001) analyze a two-stage game among multiple firms that sell a common product. They show that a dual price allocation is always in the core in the second stage of transshipment. In addition, for the first-stage decisions, they establish conditions under which the Nash equilibrium inventory levels coincide with the inventory levels in a centralized system. While Anupindi et al. (2001) assume that firms share all residual demands and supplies, Granot and Sošić (2003) consider a model in which each firm decides the amount of its residuals it wants to share with other firms. They show that the dual price allocation, although it is in the core, fails to induce the firms to share all of their residuals. Alternatively, the allocation based on the Shapley value (Shapley 1953), which allocates profits
according to firms’ marginal contributions to their coalitions, induces the firms to share all of their residuals, but it is not always in the core.

Several researchers have studied transshipment in vertically decentralized supply chains. Dong and Rudi (2004) and Zhang (2005) consider transshipment in a two-tier supply chain, where a supplier sells to a single downstream firm with several locations. Slikker et al. (2005) consider a newsvendor game in a supply chain with a single supplier and multiple retailers who coordinate their orders to the supplier. They show that the game has a non-empty core. Özen et al. (2008) consider a supply chain in which multiple firms coordinate their orders supplied from multiple warehouses to increase their joint profits. Kemahhoğlu-Ziya and Bartholdi (2011) study the use of the Shapley value when a supplier and multiple retailers whose orders are filled from the common pool can form an inventory-pooling coalition. Shao et al. (2011) analyze the case in which a supplier sells to multiple independent firms, and compare their results with those in the case where the firms are under joint ownership.

There are some papers including Zhao et al. (2005), Rong et al. (2010), and Huang and Sošić (2010a) that analyze transshipment games in multiple periods. In particular, Huang and Sošić (2010a) extend Granot and Sošić (2003) into a repeated game, and show that it is a subgame-perfect Nash equilibrium for firms to share all of their residuals under the dual price allocation when a discount
factor is sufficiently high. Sošić (2006) extends Granot and Sošić (2003) by proving that, although the Shapley value is not always in the core, the grand coalition is farsighted stable in the sense that no firms will secede from the grand coalition if they consider how others would react to their actions.

Another stream of research studies the role of transshipment prices for distribution of residual profits. Using non-cooperative game theory, Rudi et al. (2001) and Hu et al. (2007) analyze a model in which the residual profit from transshipment is allocated between two firms using predetermined prices. Rudi et al. (2001) develop the expressions for transshipment prices that achieve a first-best outcome. Hu et al. (2007) extend Rudi et al. (2001) by considering the case when there are finite and uncertain capacities. When multiple firms cooperate in transshipment, Huang and Sošić (2010b) compare the performance of the two methods: the use of predetermined prices and the dual price allocation. They show that neither allocation method dominates the other.

To our knowledge, Hezarkhani and Kubiak (2010) is the only work that analyzes the decentralized transshipment problem using the concept of pairwise stability. They study a matching problem in a two-sided market in which a link is established only between a firm with residual supply and a firm with residual demand. The core allocation they consider is from the coalition-based cooperative game, which does not take into account any network structure. In contrast, we allow for general
network structures under which any two firms can build links, and transshipment through intermediate firms requires cooperation of these firms. Thus, in our model, firms’ positions in a network play a critical role in establishing stability results.

While our paper focuses on the inventory transshipment problem, cooperative game theory has been applied to several other problems in the OM literature. These include bargaining in supply chains (e.g., Reyniers and Tapiero 1995), alliance formation among competing firms (e.g., Nagarajan and Sošić 2007), decentralized assembly systems (e.g., Granot and Yin 2008, Nagarajan and Sošić 2009, Yin 2010), group buying (e.g., Chen and Yin 2010, Nagarajan et al. 2010), and capacity allocation and scheduling (e.g., Hall and Liu 2010). Readers are referred to Nagarajan and Sošić (2008) for the review of early work. Note that all these papers analyze coalition-based cooperative games.

Our paper contributes to this literature by applying the theory of social and economic networks to the decentralized inventory transshipment problem. Our network-based approach provides a richer form of representing relationships among firms than the previous coalition-based approach, and enables us to establish the stability of partial cooperation structures based on networks.
3.3. Coalition-Based Cooperative Transshipment Games

In this section, we first describe the simplified variant of the model in Anupindi et al. (2001), which excludes the possibility of storing stocks at commonly shared warehouses. This variant has also been used by Granot and Sošić (2003) in comparing their work with Anupindi et al. (2001). We then introduce some concepts of coalition-based cooperative game theory, and summarize the main results of those two papers. This will help understand how the previous coalition-based approach differs from our network-based approach that will be presented in subsequent sections.

Consider a set \( N = \{1, 2, \ldots, n\} \) of firms who sell a common product. The firms make decisions in two stages as follows. In the first stage, each firm \( i \in N \) determines its order quantity \( X_i \) under uncertain demand \( D_i \) by taking into consideration other firms’ inventory decisions as well as transshipment in the following stage. Define inventory profile \( X = (X_1, \ldots, X_n) \) and demand profile \( D = (D_1, \ldots, D_n) \).\(^3\)

Let \( c_i \) denote a unit ordering cost. After the firms receive what they have ordered, demands are realized. Firm \( i \) generates revenue \( r_i \) for each unit of the demand satisfied, and any excess inventory can be salvaged at \( u_i ( < r_i) \) per unit. The model assumes that each firm satisfies its own demand first using its local inventory, and

\(^3\)This model allows the demand of some firm \( i, D_i, \) to be zero. Such firms can be viewed as independent warehouses that are built only for transshipment.
then ships any excess inventories to satisfy unmet demands of other firms. Let 
\[ H_i = \max\{X_i - D_i, 0\} \] and 
\[ E_i = \max\{D_i - X_i, 0\} \] denote the residual supply and demand of firm \( i \), respectively. In the second stage, the firms transship the residual supplies to satisfy the residual demands. Throughout the paper, we limit our interest to the case where at least one firm has residual demand and at least one firm has residual supply because otherwise no transshipment will occur. Let \( Y_{ij} \) denote the number of units shipped from firm \( i \) to firm \( j \), and let \( t_{ij} \) denote the unit transportation cost associated with this shipment. Further, define transshipment pattern \( Y = \{Y_{ij} : i, j = 1, \ldots, n\} \). Since both firms with residual supplies and firms with residual demands contribute to the additional profits generated from transshipment, the profits are allocated among these firms according to a predetermined allocation rule.

To characterize the transshipment in the second stage of the game, Anupindi et al. (2001) and Granot and Sošić (2003) use the concepts from the coalition-based cooperative game theory. A pair \((N, w)\) is called a cooperative game, in which 
\[ w : 2^N \rightarrow \mathbb{R} \] is a characteristic function of the game. A subset \( S \) of \( N \) is called a coalition and \( N \) itself is the grand coalition. The characteristic function \( w(S) \) captures the value generated by a coalition \( S \). An allocation rule in a cooperative game is a function \( \phi : W(N) \rightarrow \mathbb{R}^n \), where \( W(N) \) is a set of all such games on the
set $N$. An allocation rule is efficient if $\sum_{i \in N} \phi_i(w) = w(N)$. An efficient allocation rule specifies how much of the value generated by the grand coalition is attributed to each firm. We consider only efficient allocation rules in this paper. When an allocation rule applies to a particular game $(N, w)$, we call it an allocation, denoted as $\varphi = \{\varphi_1, ..., \varphi_n\}$. The core defines a set of allocations with a stability property. We say that an efficient allocation $\varphi$ is a member of the core of $(N, w)$ if $\sum_{i \in S} \varphi_i \geq w(S)$ for all $S \subseteq N$; i.e., for any subset of firms, the sum of allocations they receive in the grand coalition is at least as large as the value that they can generate by forming a subcoalition. A core allocation leads to a stable outcome of the cooperative game in the sense that no subset of firms has an incentive to secede from the grand coalition.

The allocation mainly considered by Anupindi et al. (2001) is a dual price allocation. To find dual prices to be used in this allocation, they first formulate the centralized transshipment problem in which a single decision-maker optimizes a transshipment pattern of all firms. Specifically, given inventory profile $X$ and
demand profile $D$, the central decision-maker solves the following program:

\[
CTP(X, D) = \max_Y \sum_{i,j \in N} (r_j - u_i - t_{ij})Y_{ij}
\]

(3.1)

\[
s.t. \quad \sum_{j \in N} Y_{ji} \leq E_i \quad \forall i \in N
\]

(3.2)

\[
\sum_{j \in N} Y_{ij} \leq H_i \quad \forall i \in N
\]

(3.3)

\[
Y_{ij} \geq 0 \quad \forall i, j \in N.
\]

(3.4)

In (3.1), $(r_j - u_i - t_{ij})$ represents the profit of transshipping one unit of inventory from $i$ to $j$. The constraints in (3.2) ensure that the number of units transshipped to firm $i$ does not exceed its residual demand $E_i$. The constraints in (3.3) ensure that the number of units transshipped from firm $i$ does not exceed its residual supply $H_i$. Let $\gamma_i$ and $\delta_i$ be the dual prices associated with the constraints in (3.2) and (3.3), respectively. According to the dual price allocation, each firm $i \in N$ is allocated $\gamma_i E_i + \delta_i H_i$. The main result of Anupindi et al. (2001) is that the dual allocation is in the core of the cooperative transshipment game for any residual supplies and demands.

Instead of the dual price allocation, Granot and Sošić (2003) consider the Shapley value (Shapley 1953). The Shapley value has many desirable properties such as symmetry, efficiency, and additivity (see details in Shapley (1953) or Granot and
Sošič (2003)), and hence it is commonly used in the literature. The allocation rule based on the Shapley value is defined as follows: for firm $i \in N$,

$$
\phi_{SV}^i (w) = \sum_{S \subseteq N \setminus \{i\}} \{w(S \cup \{i\}) - w(S)\} \left\{\frac{\#S!(n - \#S - 1)!}{n!}\right\},
$$

where $\#S$ is the number of firms in $S$. The standard interpretation of the Shapley value is as follows (e.g., in Jackson (2006)). Consider all possible orderings of firms. For each ordering, consider building a society by adding one firm at a time into that order. A firm obtains the marginal contribution that it makes to the society when added to the coalition formed by the firms before it in the order. So, a firm $i$ whose place in the order follows a coalition $S$ receives value $w(S \cup \{i\}) - w(S)$. Since there are $\#S!(n - \#S - 1)!$ such orderings, averaging over all possible orderings $n!$ leads to the Shapley value. Granot and Sošič (2003) have shown that the Shapley value is not always in the core of the cooperative transshipment game although it induces the firms to share all their residuals.

### 3.4. Network-Based Cooperative Transshipment Games

This section is organized as follows. In §3.4.1, we present our network model of the decentralized transshipment problem, and describe the concepts of network-based cooperative game theory that we use to analyze our model. In §3.4.2, we examine whether the pair of a given network and a dual price allocation is always in the core
of the transshipment game. Finally, in §3.4.3, we examine the conditions under which the MJW value is in the core for various network structures. Throughout this section, we highlight the difference between our network-based approach and the coalition-based approach reviewed in §3.3.

3.4.1. Model of Transshipment Networks

Our model has the same sequence of the decisions as in the previous coalition-based model described in §3.3. The main difference is that transshipment in the second stage of our model occurs based on a given network \( g \). A node in \( g \) represents a firm in \( N = \{1, 2, \ldots, n\} \). We call a firm with residual supply a supply node, and call a firm with residual demand a demand node. A bidirectional link \( ij \) between nodes \( i \) and \( j \) represents a partnership between firm \( i \) and firm \( j \), so that two firms can transship inventory directly between each other without the cooperation of other firms. We write \( ij \in g \) to indicate that nodes \( i \) and \( j \) are linked in the network \( g \), and describe a network with a set of links. For example, for \( N = \{1, 2, 3\} \), \( g = \{12, 23\} \) represents a network with two links: one link between nodes 1 and 2 and the other link between nodes 2 and 3. In the case of a regional blood management system, we may represent a regional blood center as node 2, and two blood banks as nodes 1 and 3 that share their residual supplies and demands through the regional blood center. We denote the network obtained by adding link
ij to g by g + ij, and denote the network obtained by deleting link ij from g by g – ij. Further, we use \( g^N \) to denote the complete network on \( N \) in which a link exists between any two nodes \( i, j \in N \). A path between firms \( i \) and \( j \) in \( g \) is a sequence of firms \( i_1, i_2, ..., i_\kappa \) such that \( i_ki_{k+1} \in g \) for each \( k \in \{1, 2, ..., \kappa - 1\} \) with \( i_1 = i \) and \( i_\kappa = j \). Transshipment can occur between two firms directly through a link between them or indirectly through a path between them. We use \( Y_{ij} \) to denote the number of units shipped directly from firm \( i \) to firm \( j \). If \( H_i \) units of inventories are shipped indirectly from firm \( i \) to firm \( j \) through a path \( i_1, i_2, ..., i_\kappa \) with \( i_1 = i \) and \( i_\kappa = j \), then we have \( Y_{i_ki_{k+1}} = H_i \) for \( k = \{1, 2, .., \kappa - 1\} \).

We use the solution concepts in a network-based cooperative game proposed by Jackson and Wolinsky (1996). In this game, the value of a coalition not only depends on the amount of residuals of its members, but also on the structure of the network formed by its members. Similar to the characteristic function in the coalition-based approach, the value function in the network setting is defined as \( v : G(N) \rightarrow R \), where \( G(N) \) denotes a set of all possible networks over \( N \). An allocation rule in a cooperative game can be extended naturally into the network setting as a function \( \phi : G(N) \times V(N) \rightarrow R^n \), in which \( V(N) \) represents the set of all possible value functions for a society \( N \). Since allocations depend on the structure of a network, so does the specification of the core. We say that a pair of a network and an efficient allocation, \((g, \varphi)\), is a member of the core of \((N, v)\).
if $\sum_{i \in S} \varphi_i(g) \geq v(g|_S)$ for all $S \subseteq N$, where $g|_S$ means a subnetwork of $g$ restricted to the firms in $S$. For example, Figure 3.2 shows all subnetworks of the network $\{12, 23\}$ (denote as $g'$) shown earlier in Figure 3.1(c). The requirement of the core is that no subnetworks can deviate and generate a higher value than what they are being allocated in the initial network $g$. So, the core allocation is stable to deviations from the network $g$.

Given inventory profile $X$ and demand profile $D$, the value function $v$ in our model specifies the maximum profit that any subnetwork $g|_S$ can generate through
transshipment. Specifically, we define $v$ as follows:

\begin{equation}
(3.6) \\
v(g|_S) = \max_y \sum_{i \in S} \left[ a_i \left( \sum_{j \in B_i(g|_S)} Y_{ji} - \sum_{j \in B_i(g|_S)} Y_{ij} \right) - \sum_{j \in B_i(g|_S)} Y_{ij} (t_{ij} + u_i - u_j) \right]
\end{equation}

\begin{equation}
(3.7) \\
s.t. \sum_{j \in B_i(g|_S)} Y_{ji} - \sum_{j \in B_i(g|_S)} Y_{ij} \leq E_i \quad \forall i \in S
\end{equation}

\begin{equation}
(3.8) \\
\sum_{j \in B_i(g|_S)} Y_{ij} - \sum_{j \in B_i(g|_S)} Y_{ji} \leq H_i \quad \forall i \in S
\end{equation}

\begin{equation}
(3.9) \\
Y_{ij} \geq 0 \quad \forall i, j \in S,
\end{equation}

where $B_i(g)$ denotes the set of nodes that have a link to node $i$ in $g$, and $a_i$ is defined as $r_i - u_i$ for $i \in S$ with $E_i > 0$, and 0 for $i \in S$ with $E_i = 0$. In (3.6), $a_i(\sum_{j \in B_i(g|_S)} Y_{ji} - \sum_{j \in B_i(g|_S)} Y_{ij})$ represents the net revenue from transshipment, and $\sum_{j \in B_i(g|_S)} Y_{ij} (t_{ij} + u_i - u_j)$ represents the cost of transshipment which includes transportation costs and differences in salvage values. The constraints given in (3.7) (resp., in (3.8)) ensure that the difference between the number of units shipped to firm $i$ and that from firm $i$ does not exceed the residual demand (resp., supply) of firm $i$. Note that $v(g|_S)$ depends on inventory profile $X$ and demand profile $D$, but we suppress $(X, D)$ for notational convenience. In the rest of this paper, we refer to the program (3.6)-(3.9) as the transshipment problem within $S$, and denote its optimal transshipment pattern by $Y^{g|_S}$. 
3.4.2. Dual Price Allocation in Transshipment Networks

This subsection examines whether or not a pair of a given network $g$ and the dual price allocation is in the core of our network-based transshipment game. Anupindi et al. (2001) have shown that the dual price allocation is always in the core when transshipment occurs among members of a coalition. Since, similar to the grand coalition, any two firms can share their residual demands or supplies without the cooperation of other firms in the complete network, a pair of the complete network and the dual price allocation is also in the core of our network-based game.

To examine whether or not a pair of an incomplete network and the dual allocation is in the core, we refine the dual price allocation of Anupindi et al. (2001) such that it depends on the structure of a given network $g$. Similar to $CTP(X,D)$ given in (3.1)-(3.4), we define the centralized transshipment problem $CTP(g,X,D)$ for a network $g$ as the program (3.6)-(3.9) with $S = N$. Then the dual price allocation $\varphi^{DP}(g)$ allocates $\gamma_i^g E_i + \delta_i^g H_i$ to firm $i \in N$, where $\gamma_i^g$ and $\delta_i^g$ are the dual prices associated with the constraints (3.7) and (3.8) with $S = N$ in the network $g$, respectively. To illustrate the dual price allocation $\varphi^{DP}(g)$ in the network-based transshipment game, consider the following example from Anupindi et al. (2001):
Example 1: Suppose that \( r_i = 10 \) ($/unit), \( u_i = 5 \) ($/unit), \( t_{ij} = 1 \) ($/unit) \( \forall i, j \in N = \{1, 2, 3, 4\} \), and that \( H_1 = H_2 = H_3 = 2 \) (units) and \( E_4 = 10 \) (units).

When the complete network is formed as shown in Figure 3.3(a), the total profit from transshipment is: \( \min \{H_1 + H_2 + H_3, E_4\} \times (r_i - u_i - t_{ij}) = 6 \) (units) \( \times 4 \) ($/unit) = $24. Anupindi et al. (2001) have shown that the dual price allocation based on the program given in (3.1)-(3.4) is \( \{8, 8, 8, 0\} \). This allocation assigns all profits from transshipment to firms 1, 2, and 3 with residual supply, and none to firm 4 with residual demand because the total residual demand of 10 units are much larger than the total residual supply of 6 units. Next, consider a “chain” network of \( \{12, 23, 34, 14\} \) as shown in Figure 3.3(b). In this network, the maximum profit from transshipment is $22 because firm 2 has to ship its residual supply of 2 units to firm 4 via firm 1 or firm 3, incurring an additional transportation cost of $2. Now suppose that firms 1, 2, and 3 still receive the same allocation, so that each of these firms receives $22/3. Then this network is no longer stable because a subnetwork of firms 1, 3, and 4 can generate a total profit of 4 (units) \( \times 4 \) ($/unit) = $16, which is larger than the sum of allocations, $44/3, they receive under the initial network. To prevent firms 1, 3 and 4 from forming a subnetwork, our dual price allocation based on \( CTP(g, X, D) \) decreases the allocation of firm 2 by $2, so that \( \varphi^{DP}(\{12, 23, 34, 14\}) = \{8, 6, 8, 0\} \). It is important to observe
that firm 2 has a worse position in the network than firms 1 and 3 because firm 2 is not linked directly to firm 4 which is the only demand node in this example.

The above example highlights that the dual price allocation $\varphi^{DP}(g)$ takes into account the position of a firm in the network $g$ as well as the amount of its residual supply or demand. More generally, we can establish the following result about the stability of a network under the dual price allocation. All proofs are provided in Appendix.

**Proposition 9.** For any given network $g$, the pair $(g, \varphi^{DP}(g))$ is in the core of the network-based cooperative game $(N, v)$ for any residual supplies and demands.

Proposition 9 demonstrates that the dual price allocation is an ideal allocation for the stability of incomplete networks as well as the complete network. This result may not be surprising if one could note that the dual price allocation based on $CTP(X, D)$ under the coalition-based approach could yield the same allocation as $\varphi^{DP}$ based on $CTP(g, X, D)$ under our network-based approach. To understand
this, note the following difference between \( CTP(X, D) \) and \( CTP(g, X, D) \). Our network-based approach explicitly models the actual path \( i_1, i_2, \ldots, i_\kappa \) with \( i_1 = i \) and \( i_\kappa = j \) over which inventories are transshipped between any two firms \( i \) and \( j \) in a network. As a result, the solution of \( CTP(g, X, D) \) provides us with the optimal path for transshipment between any two firms \( i \) and \( j \) among a number of potential paths between those two firms. In contrast, the coalition-based approach specifies one transportation cost \( t_{ij} \) of transshipment between any two firms \( i \) and \( j \); this means that a decision-maker needs to choose one path a priori among a number of potential paths between firms \( i \) and \( j \). If we redefine transportation costs under the coalition-based approach such that they reflect actual transshipment paths (i.e., set \( t_{ij} \) equal to a minimum transportation cost among all paths between every pair of nodes \( i \) and \( j \) in a network), then this approach yields the same allocation as \( \varphi^{DP} \) under our network-based approach.\(^4\) Therefore, the dual price allocation is in the core of our cooperative game for any network \( g \).

The implication of Proposition 9 is as follows: If the goal of an allocation mechanism is to maximize the cooperation of independent firms, thereby maximizing the total value generated from transshipment, then the dual price allocation can

\(^4\)For instance, if we set \( t_{13} = t_{24} = t_{31} = t_{42} = 2 \) ($/unit) in Example 1, the coalition-based approach yields the same dual price allocation as the network-based approach. However, the computation of minimum transportation costs may become complicated as the number of firms and the complexity of a network structure increase because one needs to solve an optimization problem for every pair of nodes in the network.
achieve this goal. For example, in a blood management system, the transshipment of unused units of blood can potentially save the lives of people who urgently need those units. The dual price allocation can incentivize independent blood banks to participate in this cooperative network.

Despite the core property of the dual price allocation, the dual price allocation may not be intuitively appealing. For instance, in Example 1, firm 4 is the only demand node in the network (without which no subnetwork can generate any profit from transshipment), but receives zero allocation. In fact, by the definition of dual prices, if the constraints (3.7) and (3.8) associated with a firm are not binding, then the firm receives zero under the dual price allocation. Thus, as shown by Granot and Sošić (2003), if firms can choose how much residual demand or supply to share, then they may not share all of their residuals under the dual price allocation; on the other hand, the allocation based on the Shapley value induces firms to share all of their residuals because firms can receive a larger allocation by doing so when the allocation is determined according to firms’ marginal contributions. (We can easily show that this result continues to hold in our network setting.) For this reason, the allocation based on firms’ marginal contributions is more commonly used in the literature (see §3.2). We next examine the stability of a network under the allocation based on the marginal contribution of each firm to its network, i.e., the MJW value.
3.4.3. Allocation Based on Marginal Contributions in Transshipment Networks

In this subsection, we examine whether or not the pair of a given network $g$ and the MJW value is in the core of our network-based transshipment game. In the rest of this paper, we say that a network $g$ is $MJW$-stable if the pair of $g$ and the MJW value is in the core of the transshipment game. The allocation rule based on the MJW value for a given network $g$ is defined as follows:

\begin{equation}
\phi_{i}^{MJW}(v, g) = \sum_{S \subseteq N \setminus \{i\}} \{v(g|_{S \cup \{i\}}) - v(g|_{S})\} \left\{\frac{\#S!(n - \#S - 1)!}{n!}\right\} \text{ for } i \in N.
\end{equation}

Similar to the Shapley value, the MJW value captures firms’ marginal contributions to the network $g$, and preserves many desirable properties of the Shapley value (Jackson and Wolinsky 1996). The MJW value can be computed using the procedure similar to the Shapley value calculation, but it is based on how the value changes as firm $i$ is added to the network comprising firms in $S$; i.e., $v(g|_{S \cup \{i\}}) - v(g|_{S})$. To illustrate how the MJW value differs from the Shapley value, we consider the following example of three firms from Sošić (2006):
Example 2: Suppose that $r_i = 1$ ($/\text{unit}$), $u_i = 0$ ($/\text{unit}$), and $t_{ij} = 0$ ($/\text{unit}$) for $i, j \in N = \{1, 2, 3\}$, and that firm 1 and firm 2 have residual demands $E_1$ and $E_2$, respectively, with $E_1 \leq E_2$, while firm 3 has the residual supply $H_3$.

In this example, there are four possible scenarios: (1) $H_3 \geq E_1 + E_2$, (2) $E_2 \leq H_3 < E_1 + E_2$, (3) $E_1 \leq H_3 < E_2$, and (4) $H_3 < E_1$. For these scenarios, Table 1 presents the allocations based on the Shapley value as well as the allocations based on the MJW value with the network $g' = \{12, 23\}$ shown earlier in Figure 3.1(c). (See Appendix for the procedure of computing the MJW value.)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Shapley value</th>
<th>MJW value $g' = {12, 23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Firm 1</td>
<td>Firm 2</td>
</tr>
<tr>
<td>(1) $H_3 \geq E_1 + E_2$</td>
<td>$\frac{E_1}{2}$</td>
<td>$\frac{E_2}{2}$</td>
</tr>
<tr>
<td>(2) $E_2 \leq H_3 &lt; E_1 + E_2$</td>
<td>$\frac{E_1 + 2H_3 - 2E_2}{6}$</td>
<td>$\frac{E_2 + 2H_3 - 2E_1}{6}$</td>
</tr>
<tr>
<td>(3) $E_1 \leq H_3 &lt; E_2$</td>
<td>$\frac{E_1}{6}$</td>
<td>$\frac{3H_3 - 2E_1}{6}$</td>
</tr>
<tr>
<td>(4) $H_3 &lt; E_1$</td>
<td>$\frac{H_3}{6}$</td>
<td>$\frac{H_3}{6}$</td>
</tr>
</tbody>
</table>

Observe from Table 1 that the allocations based on the MJW value in $g'$ are different from those based on the Shapley value under all four scenarios. With the Shapley value, the allocation to firm 3 is strictly larger than the allocation to firm 1 or firm 2 in all four scenarios. This is because there exists only one supply
node (firm 3) while there are two demand nodes (firm 1 and firm 2), and thus the marginal contribution of the supply node is more significant. However, under the MJW value, firms 2 and 3 in the network \( g' \) receive the same allocations in all four scenarios. The intuition is that both firms play equally important roles in the transshipment network in the following sense: without firm 2, no transshipment can occur because firm 1 and firm 3 are not connected to each other, and without firm 3, there is no residual supply to transship. As shown in this example, the MJW value takes into account the structure of a network as well as the residual amounts of firms.

In the following, we examine whether or not the pair \((g, \varphi^{MJW}(g))\) is in the core: (1) when \( g \) is the complete network in §3.4.3.1, and (2) when \( g \) is an incomplete network in §3.4.3.2.

3.4.3.1. Complete Network. In the complete network, any two firms can transship inventory directly between each other without the cooperation of other firms. Thus one can expect that our network-based approach yields the same result as the coalition-based approach. For Example 2, Sošić (2006) has shown that the Shapley value is in the core in Scenario 1, whereas it is not in the core in Scenarios 2, 3, and 4. Thus the Shapley value is not always in the core. Because the allocations under the MJW value in the complete network are the same as those under the
Shapley value, in general, the pair \((g, \varphi^{MJJW}(g))\) is not always in the core of our network-based transshipment game.

However, we can show that the pair \((g, \varphi^{MJJW}(g))\) is in the core under a certain condition on the residual supplies and demands. To establish this result, we use the notion of convexity in cooperative game theory (Shapley 1971). Let the cooperative game \((N, v)\) be convex if

\[
v(g|_{S'' \cup \{i\}}) - v(g|_{S''}) \leq v(g|_{S' \cup \{i\}}) - v(g|_{S'})
\]

for any subsets \(S'\) and \(S''\) of \(N\) that satisfy \(S'' \subset S'\) and any firm \(i \notin S'\). Convexity refers to the property that, as the number of firms in the subnetwork the firm joins increases, the marginal contribution of a firm to the subnetwork weakly increases. Since the number of firms is the largest in the complete network, under convexity, the marginal contribution of any firm to the complete network (hence its allocation under the MJW value) is no smaller than its marginal contribution to any of its subnetworks. Thus, if \((N, v)\) is convex, then no subnetworks have incentives to secede from the complete network under the MJW value, and therefore the pair \((g, \varphi^{MJJW}(g))\) is in the core of \((N, v)\) (Jackson 2006). In our transshipment game, suppose there exist only one demand node and multiple supply nodes. The marginal contribution of the demand node is non-decreasing with the number of firms in any subnetwork because there are more residual supplies within the subnetwork having more firms. For the marginal contribution of a supply node to be non-decreasing with the number of firms in a subnetwork, the subnetwork
should have sufficient residual demands to generate additional profits from the supply of the node. The next proposition bears this intuition.

**Proposition 10.** The pair \((g^N, \varphi^{MJW}(g^N))\) is in the core if there is one supply (resp., demand) node \(i\) and its residual supply \(H_i\) (resp., demand \(E_i\)) is greater than or equal to the sum of all residual demands (resp., supplies), i.e. \(H_i \geq \sum_{j \in N \setminus \{i\}} E_j\) (resp., \(E_i \geq \sum_{j \in N \setminus \{i\}} H_j\)).

In Example 2, Scenario 1 satisfies the condition given in Proposition 10, whereas Scenarios 2, 3, and 4 do not satisfy this condition because there is only one supply node in the network but its residual supply \((H_3)\) is not sufficient to satisfy all residual demands \((E_1 + E_2)\). In those scenarios, a subset of firms will form a subnetwork to earn more profit than the allocations they would receive in the initial network \(g^{\{1,2,3\}}\) under the MJW value. For example, in Scenario 3, firms 2 and 3 will form a subnetwork to earn the total profit of \(H_3\) which is greater than the sum of their allocations under the MJW value in \(g^N, H_3 - E_1/6\). For the case where multiple demand nodes and multiple supply nodes exist in \(g^N\), we show in the proof that the game is no longer convex.

By noting that the condition given in Proposition 10 is very restrictive, we can conclude that the complete network is not MJW-stable in most scenarios. The primary reason for such instability is that any two firms can transship inventory
directly between each other in the complete network, and firms often find it more profitable to secede from the complete network. In a regional blood management system, for example, there is often a designated blood center through which other blood banks transship their unused units of blood. One might be tempted to think that a complete network, which allows direct transshipment between blood banks without going through a regional blood center, may facilitate transshipment further because such direct shipment can potentially save the costs of transportation and coordination. However, our result shows that such a complete network may not be MJW-stable because a subset of blood centers may form their own subnetwork.

Because direct transshipment between every pair of firms is not possible in an incomplete network, we may expect that incomplete networks could be more MJW-stable for transshipment among independent firms. We examine this issue formally next.

3.4.3.2. Incomplete Networks. When a network $g$ is incomplete, the contribution of each firm to transshipment depends on the links associated with the firm as well as the amount of its residual supply or demand. As the allocations based on the MJW value vary across different network structures, so do the stability results. We first examine the MJW-stability of various incomplete network structures that appear in the literature: (1) a hub-spoke network, (2) a line network, and (3) a
chain. Then we demonstrate in (4) how a MJW-stable network with a general network structure can be derived from another (known) MJW-stable network.

**Hub-spoke network** To illustrate this network, let us first consider Example 2 in the incomplete network \( g'' = \{13, 23\} \) shown in Figure 3.4(a) in which firm 3 can be viewed as the hub. Observe that even if the link 12 existed in this network, it would not change the profits from transshipment among any subset of firms. This is because transportation costs are zero in this example and both firms 1 and 2 are demand nodes. This suggests that the allocations under \( g'' \) are the same as those under the complete network. Therefore, from the result of the complete network in §3.4.3.1, the pair \( (g'', \varphi^{MJW}(g'')) \) is in the core in Scenario 1, but not in Scenarios 2, 3, and 4.
For a general hub-spoke network $g^H$ with $n$ firms as shown in Figure 3.4(b), the following proposition presents a sufficient condition under which $(g^H, \varphi^{MJW}(g^H))$ is in the core.

**Proposition 11.** Consider a hub-spoke network $g^H = \{i_1i_2, i_1i_3, \ldots, i_1i_n\}$ where $i_1$ is the hub. The pair $(g^H, \varphi^{MJW}(g^H))$ is in the core if $H_{i_1} \geq \sum_{j \in N \setminus \{i_1\}} E_j$ or $E_{i_1} \geq \sum_{j \in N \setminus \{i_1\}} H_j$.

Different from Proposition 10 for the complete network, Proposition 11 shows that the number of supply nodes or demand nodes in a hub-spoke network need not be one in order for $(g^H, \varphi^{MJW}(g^H))$ to be in the core. This difference arises due to the special role of the hub in this network; i.e., any subnetwork that does not include the hub generates no value. Recall from §3.1 that a regional blood management system with one large blood center and multiple smaller blood banks forms a hub-spoke network. Proposition 11 suggests that, when the regional blood center as the hub has sufficient residual supply to satisfy all residual demands of blood banks in the spokes, this network is MJW-stable. This is because a subnetwork having the hub receives at least as large an allocation in its initial network as the profit it can generate alone.

**(2) Line network** To illustrate this network, let us first consider Example 2 in $g' = \{12, 23\}$ shown in Figure 3.4(c). We refer to such a network as $g'$ in which all
demand nodes are placed on one side and all supply nodes are placed on the other side as a “line.” Table 1 (earlier) shows the allocations based on the MJW value for $g'$. Apparently, subnetworks $g'|_{\{1,3\}}$ and $g'|_{\{1,2\}}$ have no incentives to secede from $g'$ because they cannot generate any profit from transshipment. Thus, to determine whether the pair $(g', \varphi^{MJW}(g'))$ is in the core under each scenario, we only need to examine whether a subnetwork $g'|_{\{2,3\}}$ has an incentive to secede from $g'$. If the subnetwork $g'|_{\{2,3\}}$ is formed, then it can generate a profit of $E_2$ in Scenarios 1 and 2, and a profit of $H_3$ in Scenarios 3 and 4. On the other hand, when firms 2 and 3 stay in $g'$, Table 1 shows that the sum of allocations to both firms is $\frac{2E_1 + 3E_2}{3}$, $\frac{2H_3 + E_2}{3}$, $H_3$, or $H_3$ in Scenario 1, 2, 3, or 4, respectively. By comparing these profits under each scenario, we find that firms 2 and 3 do not earn more by forming the subnetwork $g'|_{\{2,3\}}$. Therefore, the pair $(g', \varphi^{MJW}(g'))$ is in the core under all four scenarios.

By comparing the stability result of $g'$ with those of the complete network $g^N$ and the hub-spoke network $g''$, we can generate the following insight. In $g^N$ or $g''$, we have shown that some demand node is excluded from transshipment (e.g., demand node 2 in Scenario 4) when the residual supply is not sufficient to satisfy total residual demand in Scenarios 2, 3, and 4. In contrast, in $g'$, such a demand node is valuable to the network by providing a path without which indirect transshipment between the other nodes is not possible. For example, demand node
2 in $g'$ provides a path for transshipment between demand node 1 and supply node 3. In this case, we say that the node provides “useful links” to the network.

Building on the intuition from three-firm networks above, we can establish a necessary and sufficient condition for the pair $(g, \varphi^{MJW}(g))$ to be \textit{in the core for any residual amounts}. As shown in Proposition 12, this condition requires that every node in the network $g$ provides useful links to the network, hence contributing to the connectivity of the entire network. Furthermore, the only network structure that satisfies this condition is the line, denoted by $g^L$, as shown in Figure 3.4(d).

**Proposition 12.** The only connected network structure, under which the pair $(g, \varphi^{MJW}(g))$ is \textit{in the core for any residual supply} $H_i$, \textit{residual demand} $E_i$, and \textit{(residual revenue)} $(r_i, t_{ij}, u_i)$ \textit{(i, j $\in$ N)}), is the line $g^L = \{i_1i_2, i_2i_3, \ldots, i_ki_{k+1}, \ldots, i_{n-1}i_n\}$ where $E_{ij} = 0$ for $j \in \{1, \ldots, k\}$ and $H_{ij} = 0$ for $j \in \{k + 1, \ldots, n\}$ with $1 \leq k \leq n - 1$.

(3) **Chain** For a network $g$ having a structure other than the line, Proposition 12 suggests that there always exist some amounts of residual supplies and demands with which some subnetwork has an incentive to secede from the initial network $g$ under the MJW value. Let us illustrate this result using chain networks that were studied previously under a centralized system (e.g., Lien et al. 2011). For example, in the chain network $g''' = \{12, 23, 34, 14\}$ shown in Figure 3.4(e), transshipment between firm 2 and firm 4 can occur either through firm 1 or firm 3. Thus, there are
situations in which firm 1 (resp., firm 3) does not provide useful links to the network and a subnetwork $g''|_{\{2,3,4\}}$ (resp., $g''|_{\{1,2,4\}}$) deviates from its initial network. As a result, additional conditions on the residual supplies and demands are required for this chain to be MJW-stable. Specifically, the following corollary shows such conditions when link $i_ni_1$ is added to the line $g^L$ described in Proposition 12 so that the network becomes the chain $g^C$; see Figure 3.4(f).

**Corollary 13.** Consider the chain network $g^C = \{i_1i_2, i_2i_3, ..., i_ki_{k+1}, ..., i_{n-1}i_n, i_ni_1\}$ where $E_{ij} = 0$ for $j \in \{1, ..., k\}$ and $H_{ij} = 0$ for $j \in \{k+1, ..., n\}$ with $1 \leq k \leq n-1$.

The pair $(g^C, \varphi^{MJW}(g^C))$ is in the core if $\sum_{i \in N} H_i = \sum_{i \in N} E_i$ and $\varphi_i^{MJW}(g^C) \leq v(g^C) - v(g^C|_{N\setminus\{i\}}) \quad \forall i \in N$.

Corollary 13 shows that the pair $(g^C, \varphi^{MJW}(g^C))$ is in the core when two conditions are satisfied. The first condition requires that the total residual supply equal the total residual demand. Suppose the total residual supply are greater than the total residual demand. Unlike the line, there are two paths between any two firms in this chain. Thus, some supply nodes are excluded from transshipment because neither their residual supplies nor their links are valuable to the network. The second condition requires the allocation to each firm to be lower than the firm’s marginal contribution when the firm joins the subnetwork of $(n - 1)$ firms. This
condition ensures that firms have no incentives to cut one path for transshipment by excluding one firm.

(4) Other networks Although the line network presented in Proposition 12 guarantees the MJW value to be in the core for any residual amounts, it is not very likely to observe such a network in practice – especially when a large number of firms exist. Then what other connected networks could be stable under the MJW value? In the following proposition, we demonstrate how a MJW-stable network with an arbitrary network structure can be derived from another MJW-stable network (e.g., a line network).

**Proposition 14.** Suppose the pair \((g, \varphi_{MJW}^M(g))\) is in the core. Then the pair \((g + ij, \varphi_{MJW}^M(g + ij))\) is in the core if \(v(g + ij|s) - v(g|s) \leq \sum_{k \in S} \{\varphi_k^{MJW}(g + ij) - \varphi_k^{MJW}(g)\}\) for all \(S \subseteq N\).

We illustrate the result stated in Proposition 14 using an example shown in Figure 3.5. There are four firms in the market: firms 1, 2 and 3 have residual demands \(E_1 = E_2 = E_3 = 1\) (unit), while firm 4 has residual supply \(H_4 = 3\) (units). Suppose that \(r_i = 1\) ($/unit) and \(u_i = 0\) ($/unit) for \(i \in \{1, 2, 3, 4\}\), and \(t_{ij} = 0\) ($/unit) for \(i, j \in \{1, 2, 3, 4\}\) except that \(t_{34} = t_{43} = 0.9\) ($/unit). The line network \(g = \{12, 23, 34\}\) in Figure 3.5(a) is MJW-stable according to Proposition 12. By adding link 24 to this network \(g\), we construct another network \(g + 24 = \{12, 23, 34, 24\}\)
shown in Figure 3.5(b). In the following, we show that the initial network \( g \) and link 24 satisfy the condition given in Proposition 14, so that the new network \( g+24 \) is MJW-stable as well. To begin, consider \( S = \{1, 2, 4\} \). This subset contains both firms 2 and 4 that are connected through the new link 24. The initial network \( g|_{S} \) generates no profits from transshipment because firms 1 and 2 are not connected to firm 4 without firm 3, whereas the new network \( g + 24|_{S} \) generates a profit of 2 (units) \( \times \) 1 (\$/unit) = $2; hence, \( v(g + 24|_{S}) - v(g|_{S}) = 2 \). On the other hand, we can show that the total allocation to \( S \) under \( g|_{S} \) is $23/120, while that under \( g + 24|_{S} \) is $151/60; hence, \( \sum_{k \in \{1, 2, 4\}} \{\varphi_{k}^{MJW}(g+24) - \varphi_{k}^{MJW}(g)\} = 279/120 > v(g + 24|_{S}) - v(g|_{S}) \). This means that link 24 increases the total allocation to \( S \) more than the additional profit it generates for \( S \). This happens because link 24 benefits firms 1, 2, and 4 in subsets other than \( S \), and the total allocation to \( S \) reflects the overall impact of link 24 across all subsets of \( \{1, 2, 3, 4\} \). Similarly, we can show that the condition given in Proposition 14 is satisfied for \( S = \{2, 3, 4\} \) or \( \{2, 4\} \). For any subset \( S \) that does not contain firm 2, firm 4 or both, link 24 does not generate any additional profit from transshipment (i.e., \( v(g + 24|_{S}) - v(g|_{S}) = 0 \)), nor does it decrease the total allocation to \( S \) (i.e., \( \sum_{k \in S} \{\varphi_{k}^{MJW}(g+24) - \varphi_{k}^{MJW}(g)\} \geq 0 \)); hence, satisfying the condition given in Proposition 14 as well. Therefore, by Proposition 14, we can conclude that the network \( g + 24 \) shown in Figure 3.5(b) is MJW-stable.
Figure 3.5. Examples of networks when four firms exist in the market: (a) line \{12, 23, 34\}, (b) network \{12, 23, 34, 24\} obtained by adding link 24 to line \{12, 23, 34\}.

Let us summarize our main findings in this subsection. Under the MJW value, the line network is the only network structure that is MJW-stable for any residual amounts. For other networks, MJW-stability requires certain conditions on residual amounts. Interestingly, the complete network requires more stringent conditions than some incomplete networks. Therefore, the complete network is not only more expensive in establishing all the links among firms, but can also be less stable in a decentralized transshipment system. Finally, note that this section has examined the MJW-stability of an existing network $g$, assuming that the cost of building a link is a sunk cost. In the next section, we examine the endogenous formation of networks, in which the cost of building a link counts.
3.5. Endogenous Formation of Transshipment Networks

Suppose that, between the first stage of inventory decisions and the second stage of transshipment in the model described in §3.4.1, firms build a network endogenously by taking into account subsequent transshipment based on the network. Firms move sequentially according to an ordering over all possible links, \((i_1j_1, i_2j_2, \ldots, i_mj_m)\), where \(m\) is the number of candidate links. Any link may appear multiple times in this ordering. When link \(ij\), which is not yet established in network \(g\), appears in the ordering, firms \(i\) and \(j\) decide on whether to form that link. If both firms \(i\) and \(j\) want to form the link, then link \(ij\) is established in network \(g\), incurring a link cost \(l_{ij}\) to firm \(i\) and \(l_{ji}\) to firm \(j\). The ordering \((i_1j_1, i_2j_2, \ldots, i_mj_m)\) may be generated randomly or may reflect firms’ preferences. For example, firm 1 may first consider establishing a link with firm 2, and if link 12 is not established, then firm 1 considers establishing a link with firm 3, and so on; in this case, link 12 appears before link 13 in the ordering. This procedure of network formation models the repeated interactions between firms who want to share their residuals. In the example of car dealers discussed in §3.1, dealers with residual demand or residual supply contact other dealers, and transshipment can
occur only between two dealers who are mutually interested in sharing their residu-
als. This process usually involves costs for labor and administrative arrangements
(i.e., link costs).\footnote{Our results in this section also hold when firms decide on forming links simultaneously. Specifically, after observing the realized demand, each firm simultaneously announces a set of firms to which it wants to set up a link, and when both firms $i$ and $j$ have announced each other, a link $ij$ is established in the network $g$. In general, Jackson and Wolinsky (1996) note that the notion of pairwise stability is not dependent on any particular formation process.}

Our objective is to find equilibrium network structures. As for the equilibrium
concept, we use \textit{pairwise Nash stability} that refines Nash equilibrium (Jackson and
Wolinsky 1996). Given an allocation $\varphi$ and link costs, a transshipment network $g$
is \textit{pairwise Nash stable} if the following two conditions are satisfied:

\begin{align}
\varphi_i(g) & \geq \varphi_i(g - ij) + l_{ij} \text{ and } \varphi_j(g) \geq \varphi_j(g - ij) + l_{ji} \text{ for all } ij \in g; \\
\text{if } \varphi_i(g + ij) & > \varphi_i(g) + l_{ij}, \text{ then } \varphi_j(g + ij) < \varphi_j(g) + l_{ji} \text{ for all } ij \notin g. \tag{3.11} \tag{3.12}
\end{align}

A network is pairwise Nash stable if no firm wants to sever a link (ensured by
(3.11)) and no two firms want to add a link between them (ensured by (3.12)).

For ease of exposition, we first consider a special case of zero link costs in §3.5.1,
and then analyze a general case of arbitrary link costs in §3.5.2. In both cases,
we assume that the profit generated from transshipment is allocated according
to the MJW value, $\varphi^{MJW}(g)$.\footnote{The MJW value has also been used in establishing pairwise Nash stability of networks in various other applications because it not only has several desirable properties mentioned earlier but also guarantees the existence of a pairwise Nash stable network (Jackson 2006).} In Appendix, we also consider the case in which
firms build a network before making inventory decisions, and show that our main findings in this section continue to hold.

3.5.1. Zero Link Costs

In order to find a pairwise Nash stable network, we choose the complete network as an initial network (which is created when every pair of firms wants to form the link between them), and examine if some firms have incentives to deviate. Alternatively, one can start from any network other than the complete network, and analyze whether any firm has an incentive to sever a link or any two firms have an incentive to establish a link between them.

To begin, we use Example 2 to illustrate some properties of a pairwise Nash stable network under the MJW value. We compare the allocations to firms 1 and 3 in the complete network $g^{1,2,3}$ (shown in the left column of Table 1) with those in the incomplete network $g' = \{12, 23\}$ without link 13 (shown in the right column of Table 1). Table 1 reveals that the allocations to both firms 1 and 3 in $g^{1,2,3}$ with link 13 are greater than or equal to those in $g'$ without link 13 in all four scenarios. This is because link 13 increases the marginal contributions of firms 1 and 3 to the network. Similarly, we can show that the allocations to firms 2 and 3 with link 23 are greater than or equal to those without link 23. When both links 13 and 23 exist as in $g^{1,2,3}$ and $g'' = \{13, 23\}$, whether or not link 12 exists has
no impact on the allocations because transshipment between two demand nodes 1 and 2 does not add value to the network (when transportation cost is zero). Therefore, both complete network $g^{\{1,2,3\}}$ and incomplete network $g''$ are pairwise Nash stable under the MJW value.

We can generalize the above procedure developed for a three-firm network ($n = 3$) to a network with any size ($n \geq 3$). Define the networks derived from the complete network as follows: A network $g$ is derived from the complete network $g^N$ if there exists a sequence of networks $g_1, ..., g_n$ with $g_1 = g^N$ and $g_n = g$ such that $g_{k+1} = g_k - ij$ for each $k \in \{1, ..., \kappa - 1\}$, where link $ij$ satisfies $\varphi_i^{MJW}(g_{k+1}) \geq \varphi_i^{MJW}(g_k) - l_{ij}$ or $\varphi_j^{MJW}(g_{k+1}) \geq \varphi_j^{MJW}(g_k) - l_{ji}$. The existence of this sequence of networks implies that, starting from the complete network, there exists a sequence of firms that find it weakly profitable to sever their links. As illustrated above, multiple pairwise Nash stable networks may exist (e.g., $g^{\{1,2,3\}}$ and $g''$ in Example 2). Using this procedure, we can obtain the following proposition which describes a set of pairwise Nash stable networks for $N = \{1, 2, ..., n\}$.

**Proposition 15.** Suppose that a transshipment network is formed by $n$ firms endogenously with no link costs. Then the complete network $g^N$ is always pairwise Nash stable under the allocation based on the MJW value. Furthermore, any pairwise Nash stable network $g$ derived from the complete network $g^N$ contains
any link $ij$ with $Y^g_{ij} | s > 0$ for some $S \subseteq N$, where $Y^g_{ij} | s$ is the unique optimal transshipment pattern of the transshipment problem within $S$ given in (3.6)-(3.9).

Proposition 15 states that firms have no incentives to sever any link $ij$ which is useful for transshipment within some $S \subseteq N$ (i.e., $Y^g_{ij} | s > 0$). For example, in our discussion of Example 2 above, link 13 is useful for transshipment within $S = \{1,3\}$, and link 23 is useful within $S = \{2,3\}$; consequently, $g'' = \{13,23\}$ is pairwise Nash stable under the MJW value. Furthermore, Proposition 15 implies that there is over-connection in the pairwise Nash stable networks under the MJW value relative to an optimal transshipment pattern of the centralized transshipment problem. Firms have no incentives to sever any link $ij$ which is useful for some $S$ (i.e., $Y^g_{ij} | s > 0$) even if that link is not utilized in the optimal transshipment pattern of the centralized transshipment problem for $N$ (i.e., $Y^g_{ij} = 0$ and $Y^g_{ji} = 0$). For example, one optimal transshipment pattern of the centralized transshipment problem in Example 2 under Scenario 1 is that $Y_{32} = E_1 + E_2$ and $Y_{21} = E_1$; i.e., firm 3 transships its residual supply of $E_1 + E_2$ ($\leq H_3$) to firm 2, and then firm 2 transships the supply of $E_1$ to firm 1 after satisfying its residual demand $E_2$. In this transshipment pattern, link 13 is not utilized. However, the complete network $g^{\{1,2,3\}}$, which is pairwise Nash stable under the MJW value, includes link 13. From a bargaining perspective, firms 1 and 3 have stronger bargaining powers.
with link 13 when negotiating how to allocate profits from transshipment with firm 2. This is because firms 1 and 3 can still transship $E_1$ without firm 2; in other words, they have a better outside option with link 13. In the example of car dealers above, link costs may be negligible when a web-based inventory system as well as an industry protocol for transshipment is well established. In this case, a dealer may attempt to build connections with many other dealers so as to increase his bargaining power against others even though he may need only few partners for transshipment eventually. Because there is no cost to improve one’s bargaining power in this case, the complete network is always pairwise Nash stable under the MJW value.

3.5.2. Positive Link Costs

Suppose that link cost $l_{ij}$ or $l_{ji}$ is positive for at least one pair of firms $i$ and $j$. Following the procedure developed in §3.5.1, we can identify a set of pairwise Nash stable networks. For example, Figure 3.6 shows all pairwise Nash stable networks in the following example under the MJW value:

**Example 3:** Suppose that $r_i = 1$ ($$/\text{unit}$), $u_i = 0$ ($$/\text{unit}$), $t_{ij} = 0$ ($$/\text{unit}$$) $\forall i, j \in N$, and that $E_1 = 5$ (units), $E_2 = 3$ (units), $H_3 = 5$ (units), $H_4 = 3$ (units) and $l_{ij} = l_{ji} = 1$ ($$/\text{link}$$) $\forall i, j \in g$. 
As shown in Figure 3.6, unlike the case with zero link costs, the complete network is not always pairwise Nash stable under the MJW value because firms face trade-offs between better network positions and additional link costs.

For the case with zero link costs, Proposition 15 in §3.5.1 has shown that firms tend to be over-connected in pairwise Nash stable networks under the MJW value. Since a positive link cost discourages firms from establishing unnecessary links, one may wonder if firms are still over-connected in this case. To examine this question, we introduce the notion of efficiency. We say that a network $g$ is efficient if it maximizes the profit from transshipment less the total link cost, $v(g) - \sum_{ij \in g} (l_{ij} + l_{ji})$. Proposition 16 below shows that pairwise Nash stable networks are not, in general, efficient in transshipment under the MJW value. This demonstrates the tension between the stability and efficiency of a network.\textsuperscript{7}

\textsuperscript{7}This tension is first discovered by Jackson (2006) in the setting where any link in the network generates a positive value; i.e., $v(\{ij\}) > 0$ for any $i, j \in N$. In that case, the complete network is the only pairwise Nash stable network with a small link cost. Our model is different in that
Proposition 16. Suppose that there exist some \( S \subset N \) and a link \( ij \) with \( l_{ij} > 0 \) or \( l_{ji} > 0 \) such that: (i) \( v(g^N|s) \) has a unique optimal transshipment pattern \( Y^{g^N|s} \) with \( Y^{g^N|s}_{ij} > 0 \), and (ii) \( v(g^N) \) has a unique optimal transshipment pattern \( Y^{g^N} \) with \( Y^{g^N}_{ij} = 0 \) and \( Y^{g^N}_{ji} = 0 \). Then there exists \( \bar{l} > 0 \) such that, if \( l_{ij} < \bar{l} \) and \( l_{ji} < \bar{l} \) \( \forall ij \in g \), every pairwise Nash stable network \( g \) derived from the complete network \( g^N \) is not efficient under the allocation based on the MJW value because there is a subnetwork \( g^* \) of \( g \) which satisfies \( v(g) - \sum_{ij \in g} (l_{ij} + l_{ji}) < v(g^*) - \sum_{ij \in g^*} (l_{ij} + l_{ji}) \).

Proposition 16 suggests that a pairwise Nash stable transshipment network under the MJW value is still over-connected with positive link costs. The inefficiency of a pairwise Nash stable network arises because individual firms try to maximize their own allocations instead of maximizing the aggregate profit of the entire network. The condition under which a stable network is inefficient requires the existence of a link with a positive link cost, which is utilized for transshipment within at least one subset \( S \subset N \) (i.e., \( Y^{g^N|s}_{ij} > 0 \)), but is not utilized in the optimal transshipment pattern of the centralized problem among all firms (i.e., \( Y^{g^N}_{ij} = 0 \) and \( Y^{g^N}_{ji} = 0 \)).

In the example of a car dealer network, this result means that, even if the cost of establishing an connection is not negligible, dealers still build connections with a larger number of other dealers than necessary so as to improve their positions. Such \( v(\{ij\}) = 0 \) is possible when \( E_i = E_j = 0 \) or \( H_i = H_j = 0 \) for any \( i, j \in N \), so that incomplete networks can also be pairwise Nash stable.
excess connections hurt the efficiency of a decentralized transshipment system.

To improve the efficiency, a third-party organization such as a local automobile dealers association or an auto manufacturer may intervene in a decentralized dealer network to coordinate the incentives of independent dealers.

3.6. Inventory Decisions

In this section, we analyze firms’ inventory decisions in the first stage of the game. Following Anupindi et al. (2001) and Granot and Šošić (2003), we focus our analysis on whether a certain allocation rule used in the second transshipment stage would lead to inventory decisions in the first stage which are optimal for the centralized system (i.e., achieve the first-best). We will first discuss the case when a network \( g \) is given as in §3.4, and then discuss the case when a network \( g \) is formed endogenously as in §3.5.

Given a network \( g \), each firm determines its inventory level independently by considering uncertain demands and ex-post transshipment of its residual demand or supply with other firms in the network \( g \). The ex-post total profit of firm \( i \) can be expressed as follows: given the network \( g \), demand profile \( D \), its own inventory level \( X_i \), and a vector of other firms’ inventory levels \( X_{-i} \),

\[
\pi_i(g, X_i, X_{-i}, D) = [r_i \min\{X_i, D_i\} + u_i H_i - c_i X_i] + \varphi_i(g, X_i, X_{-i}, D),
\]
where the first term in the bracket represents the profit from satisfying its local demand $D_i$, and the second term $\varphi_i$ represents the allocation it receives from subsequent transshipment. A Nash equilibrium inventory profile $X^{NE}$ satisfies:

$$X_i^{NE}(g) = \arg \max_{X_i} E_D[\pi_i(g, X_i, X_{-i}^{NE}, D)] \text{ for all } i.$$ 

In the coalition-based transshipment game, Anupindi et al. (2001) and Granot and Sošić (2003) have shown that the dual price allocation and the Shapley value do not always lead to the first-best inventory levels, respectively. Because coalitions are special cases of networks, the same results apply to our network-based transshipment game, so that our dual price allocation $\varphi^{DP}(g)$ and the MJW value $\varphi^{MJW}(g)$ do not lead to the first-best inventory levels.

Following the lead of Anupindi et al. (2001) and Granot and Sošić (2003), however, we can construct a new allocation $\varphi^{FB}(g)$ that leads to the first-best inventory levels. While $\varphi^{DP}(g)$ and $\varphi^{MJW}(g)$ determine allocations of firms based on the ex-post profit generated from the second transshipment stage, $\varphi^{FB}(g)$ uses the ex-post profit generated from both stages. Let $\Pi(g, X, D)$ denote the sum of ex-post profits of all firms from both stages. We can express $\Pi$ as follows:

$$\Pi(g, X, D) = \sum_{i \in N} [r_i \min\{X_i, D_i\} + u_i H_i - c_i X_i] + CTP(g, X, D).$$

**Corollary 17.** Let $\lambda_i \in (0, 1)$ such that $\sum_{i \in N} \lambda_i = 1$. When a network $g$ is given, the allocation defined by $\varphi_i^{FB}(g) = \lambda_i \Pi(g, X, D) - [r_i \min\{X_i, D_i\} + u_i H_i - c_i X_i]$ for
\[ i \in N \text{ induces the inventory levels in a first-best solution to be a Nash equilibrium profile } X^{NE}. \]

The intuition from Corollary 17 is straightforward as follows. The allocation \( \varphi^{FB}(g) \) gives each firm \( i \) a fixed fraction \( \lambda_i \) of the total profit \( \Pi(g, X, D) \) of all firms in both stages less the firm’s own profit from the first stage of the game. Then the total profit of a firm from both stages is simply a fixed fraction \( \lambda_i \) of the total profit \( \Pi(g, X, D) \) generated by all firms in the network \( g \). Since \( \lambda_i \) is independent of a firm’s inventory decision, the firm chooses the inventory level to maximize the total profit \( \Pi(g, X, D) \) under this allocation.

When a network \( g \) is formed endogenously as in §3.5, a firm needs to consider the strategic formation of transshipment networks in determining its optimal inventory level, in addition to uncertain demands and ex-post transshipment based on the network \( g \). The total profit \( \Pi(g, X, D) \) in this case can be written as

\[
(3.15) \quad \Pi(g, X, D) = \sum_{i \in N} \left[ r_i \min\{X_i, D_i\} + u_i H_i - c_i X_i - \sum_{j \in B_i(g)} l_{ij} \right] + CTP(g, X, D).
\]

Compared with (3.14), \( \Pi(g, X, D) \) in (3.15) contains the total link cost \( \sum_{i \in N} \sum_{j \in B_i(g)} l_{ij} \). Similar to the allocation presented in Corollary 17, we can construct an allocation \( \varphi^{FB}(g) \) that induces firms to choose their first-best inventory levels. Furthermore,
under \( \varphi^{FB}(g) \), there always exists a pairwise Nash stable network that is also efficient. Corollary 18 summarizes the results.

**Corollary 18.** Let \( \lambda_i \in (0,1) \) such that \( \sum_{i \in N} \lambda_i = 1 \). When a network \( g \) is formed endogenously, there exists an efficient network \( g^* \) that is pairwise Nash stable under the allocation defined by \( \varphi^{FB}_i(g) = \lambda_i \Pi(g, X, D) + \sum_{j \in B_i(g)} l_{ij} - [r_i \min\{X_i, D_i\} + u_i H_i - c_i X_i] \) for \( i \in N \). Furthermore, \( \varphi^{FB}_i(g^*) \) induces the inventory levels in a first-best solution to be a Nash equilibrium profile \( X^{NE} \).

Corollary 18 shows that, unlike the MJW value, there is no tension between stability and efficiency under \( \varphi^{FB}_i(g) \). However, \( \varphi^{FB}_i(g) \) may violate some desired properties of the MJW value such as symmetry when \( \lambda_i \) is not chosen properly.

### 3.7. Conclusion

This paper studies a cooperative game of inventory transshipment among multiple firms. As firms try to maximize their own profits, the value generated through transshipment needs to be allocated properly to coordinate the incentives of firms to participate in transshipment. To analyze this problem, the extant literature uses the concept of coalitions in the cooperative game theory, while we use networks of firms as the primitive. Our network-based approach explicitly models the actual paths over which inventories are transshipped, and provides a richer form of representing relationships among firms than the previous coalition-based approach.
This enables us to analyze partial cooperation structures based on networks in which partnership may exist between some but not among all firms. Our results provide the following managerial insights.

First, if the primary objective of a decentralized transshipment network is to make all firms participate in transshipment, then the dual price allocation can achieve this goal by providing firms with proper incentives. This might be the case for the transshipment of blood. The unused units of blood can potentially save the lives of people who urgently need those units. The dual price allocation we construct in the network setting takes into account the positions of firms in their networks as well as the amounts of their residual supplies or demands. By doing so, this allocation prevents a subset of firms from forming their subnetwork, and thus maximizes the value of pooling all residuals.

Second, compared to the dual price allocation, the MJW value provides a more intuitive way of allocating profits from transshipment to individual firms by considering their marginal contributions to a network. However, one should be cautious in implementing this allocation rule because it may induce some firms to form their own subnetwork, hurting the total value that can be generated from transshipment of all residuals. For example, in a regional blood management system, there is often a designated blood center through which other blood banks transship their unused units of blood. If the MJW value is implemented in this
hub-spoke network, this transshipment system is stable when the regional blood center as the hub has sufficient residual supplies to satisfy all residual demands of blood banks in the spokes.

Third, when the profits from transshipment are allocated according to the MJW value, firms’ incentives to participate in transshipment depend crucially on how they are connected in a network. For example, one might be tempted to think that the complete network, which allows direct transshipment between blood banks without going through a regional blood center, may facilitate transshipment further because direct shipments can potentially save the costs of transportation and coordination. However, our result shows that such a transshipment system can be less stable than a hub-spoke network having a regional blood center as the hub. Thus, firms should consider building an incomplete network for transshipment because it is not only cheaper to institute than the complete network, but can also lead to a more stable cooperative transshipment system.

Fourth, an efficient network in a centralized system may not be stable in a decentralized system of independent firms. For example, a chain of retail stores or warehouses is known to be efficient in a centralized system by allowing all members to share their residual supplies and demands with a small number of links (e.g., Gerchak and Kalikhman 2011, Lien et al. 2011). However, our result shows that a chain of independent firms can incentivize firms to participate in
transshipment only under some restrictive conditions. Thus, managers should be careful in applying the insights obtained for a centralized system to a decentralized system.

Finally, when firms are able to establish connections with each other, they tend to build connections with a large number of other firms even though they may need only few partners for actual transshipments. Such over-connection in a network exists because firms try to increase their bargaining powers against other firms, leading to an inefficient transshipment system. For example, when car dealers search for their partners to share their residual demand or supply, they may attempt to negotiate with many other dealers so as to increase their bargaining positions. This may happen even when the cost of search and negotiation is not negligible. To improve the efficiency of a transshipment network, a third-party organization such as a local automobile dealers association or an auto manufacturer may intervene in a decentralized transshipment system to coordinate the incentives of independent firms.

There are several future research avenues. In addition to the concept of pairwise Nash stability we used in this paper, there also exist other stability concepts such as stochastic or farsighted stability. Such concepts can be applied to analyze the inventory transshipment problem in a richer setting. Besides the inventory transshipment networks studied in this paper, one can potentially apply the theory of
economic and social networks to analyze the stability and formation of networks in various other operational problems. For example, in a supply chain with disruption risks, over-connection in a network might be beneficial for firms in terms of risk-hedging. This paper may serve as the first step to many such analyses.
CHAPTER 4

Managing Suppliers: Joint Audit and Shared Supplier Information

ABSTRACT:¹ Product safety incidents in recent years have compelled many manufacturers to rethink approaches to managing product quality of their suppliers. In this paper, we investigate two cooperative approaches that are often used in practice: auditing common suppliers jointly (“joint audit”) and sharing independent audit results with other manufacturers (“shared supplier information”). We develop a model that captures both competitive and cooperative interactions among manufacturers. Our analysis reveals that, although the cooperative approaches may reduce audit costs, they do not necessarily improve product quality. The effectiveness of these approaches depends crucially on the externality of product safety of one manufacturer on other manufacturers. Further, when the risk of product safety failure is high, shared supplier information is more effective than joint audit; otherwise, joint audit is more effective. We also investigate the incentives of competing manufacturers to audit cooperatively, and design an efficient

¹This essay is a joint work with Soo-Haeng Cho.
way to motivate the manufacturers who have better information about product quality to share the information with other manufacturers.

4.1. Introduction

Recently, product safety incidents occurred in various industries. For example, in the food industry, milk products were adulterated with melamine in China in 2008, which caused an estimated 300,000 illnesses (Roth et al. 2008). A similar tragedy happened in the pharmaceutical industry in the U.S. The ingredient of Heparin, a drug that is used to prevent coagulation of blood, was contaminated, and it led to 81 deaths (Blum 2008). Product safety events happened in other product categories, including automobiles, high-tech products, toys, and so on; see various other product safety events in Marucheck et al. (2011).

In the past, product failures were often attributed to functional errors in product design or manufacturing process. Today, with globalization, many manufacturers procure their raw materials worldwide and also outsource their manufacturing functions to overseas suppliers. This creates new challenges in product safety. For example, opportunistic behaviors of suppliers, such as product adulteration, have become a common cause of product safety incidents (Babich and Tang 2012). In the example of milk adulteration mentioned above, milk dealers intentionally sold
contaminated milk to manufacturers to save production costs. In the Heparin incident, the contamination was due to the supplier who processed the ingredient from animals’ intestines.

To deal with these challenges, manufacturers invest significant amount of resource to manage product quality of their suppliers. A supplier audit is a commonly used approach. Suppliers that fail an audit and may lose the manufacturer as a customer (Egels-Zandén 2007, Lee et al. 2012). This creates incentives for suppliers to improve their product quality. However, such audits may not generate perfect information about product quality. It’s not difficult for suppliers with sub-par quality standards to pass an audit with some understanding of what the process entails. In some cases, an audit is a box-ticking exercise instead of an investigative process (CEB 2014). To make audits more informative, it can be very costly for manufacturers. For example, after the massive Chinese-made toy recall in 2007, the operation delays caused by audits and product testing cost Mattel as much as $50 million (Burke 2007). Some suppliers even charge fees for on-site audits (Gordon 2009).

Facing imperfect information and cost issues in supplier audits, manufacturers have started to cooperate in auditing their common suppliers. There are two major approaches to conducting cooperative audits. The first approach is called “joint audit.” In this approach, several manufacturers conduct a supplier audit together
and they share the cost and result from the audit. For example, ten European telecommunication operators have signed a Memorandum of Understanding to conduct joint audits on their supply chains (GeSI 2014). The second approach is called “shared supplier information.” In this approach, manufacturers conduct their audits independently, but they share the results from those audits with each other. For example, Rx-360, a not-for-profit consortium in the pharmaceutical and biotech industry, provides a platform for its members to share audit reports with each other (RX-360 2014).

Although the cooperative approaches may be appealing to manufacturers due to reduced audit costs, it is unclear they benefit consumers. Our analysis reveals that they do not necessarily benefit consumers by improving product quality. The effectiveness of these approaches depends crucially on the externality of product safety of one manufacturer on other manufacturers. Specifically, the product safety incident of one manufacturer may have a negative externality on the profit of another manufacturer due to the fact that consumers may lose confidence in the whole industry as a result of the safety incident. On the other hand, the externality can be positive if consumers switch from a product having a safety issue to other products. Further, we find that, when the risk of product safety failure is high, shared supplier information is more effective than joint audit; otherwise, joint audit is more effective. We also investigate the incentives of competing manufacturers
to audit cooperatively, and design an efficient way to motivate the manufacturers who have better information about product quality to share the information. Since product safety has a significant impact on the well-being of consumers, industries and governments should assess the level of externality and risk of product safety failure in specific market settings in order to design effective and stable cooperative audit programs.

This paper is related to the existing literature that studies product safety and quality in supply chain settings. Reyniers and Tapiero (1995) study the effect of price rebates and after-sales warranty costs on the choice of quality by a supplier. Chao et al. (2009) consider contracts in which product recall costs can be shared between a manufacturer and a supplier to induce quality improvement effort. Babich and Tang (2012) investigate a mechanism with deferred payments to improve product safety. Rui and Lai (2012) extend such a mechanism to broader settings with general distribution of the defection discovery time. In the literature, typical model setup usually includes one supplier and one manufacturer. In our paper, we consider multiple manufacturers and, more importantly, the competition and cooperation among them.

There are a large number of papers that study supply chain competition. We see an increasing number of literature about supply chain cooperation as well. These include alliance formation among competing firms (e.g., Nagarajan and Sošić 2007),
decentralized assembly systems (e.g., Granot and Yin 2008, Nagarajan and Sošić. 2009, Yin 2010), group buying (e.g., Chen and Yin 2010, Nagarajan et al. 2010), and capacity allocation and scheduling (e.g., Hall and Liu 2010). Readers are referred to Nagarajan and Sošić (2008) for the review of early work. The literature in this stream uses a cooperative game in ordinal function form, while we consider a cooperative game in partition function form to model the externalities of product safety incidents.

In the remainder of the paper, we first introduce our base model in Section 4.2. We analyze the model and find the optimal product quality of a supplier in Section 4.3 under: (i) individual audit, (ii) joint audit, and (iii) shared supplier information. In Section 4.4, we analyze the incentives of manufacturers to conduct cooperative audits by considering a cooperative game among manufacturers. We conclude the paper in Section 4.5.

4.2. Model

We consider a market with a set $N = \{1, ..., n\}$ of competing manufacturers that share one common supplier. The supplier may adulterate the product that he provides to all manufacturers. We use $a = 1$ to denote the case in which the supplier adulterates the product and otherwise $a = 0$.\footnote{More generally, the action of the supplier can be interpreted as producing the product of low quality ($a = 1$) or of high quality ($a = 0$).} The production cost for the supplier
is $g_L$ if the product is adulterated and $g_H$ if not. We assume $g = g_H - g_L > 0$ so that the supplier has an incentive to adulterate his product due to cost saving. The adulteration strategy can be pure or mixed. The supplier decides the probability $\theta$ to adulterate his product.

In the base model, each manufacturer $i$ audits the supplier and obtains a private signal $s_i$ about whether the supplier adulterates his product. The audit may not be perfect, so, if the supplier adulterates his product, there is a probability $e_i$ (which does not necessarily equal to one) such that the adulteration is detected by manufacturer $i$. We use $s_i = 1$ to denote the case when the adulteration is detected by manufacturer $i$ and $s_i = 0$ to denote the case when the adulteration is not detected. If the supplier does not adulterate the product, then $s_i = 0$ for all $i \in N$. For every manufacturer who has detected adulteration, the supplier has to pay a penalty of $l$, which may represent a fine or loss of sales. We assume that the probability $e_i = P(s_i = 1|a = 1)$, called audit effort level, is determined by the manufacturer $i$. In another word, the manufacturers can make efforts to improve the accuracy of their audits. A similar assumption is made in the literature about product safety, e.g., Baiman et al. (2000). There is an audit cost $C(e_i)$ associated with audit effort level $e_i$. We assume that $C(e_i) = c(1 - e_i)^{-x}$ in the remaining parts of the paper.\(^3\)

\(^3\)Our analysis continues to hold with a linear or quadratic cost function.
After the audits, the manufacturers engage in competition. We assume that the product of manufacturer $i$ has quality issues ($q_i = 1$) if the supplier adulterates his product and manufacturer $i$ fails to detect the adulteration, i.e., $a = 1$ and $s_i = 0$; otherwise, it has no quality issues ($q_i = 0$). The demand $D_i$ of manufacturer $i$ is given by

$$D_i = \alpha - \beta \frac{\sum_{j \in N} q_j}{n} + \gamma \left( \frac{\sum_{j \in N} q_j}{n} - q_i \right),$$

where $\alpha$ represents fixed baseline demand. The second term, $\beta \frac{\sum_{j \in N} q_j}{n}$, captures the negative externality of product safety incident; i.e., the quality issue of manufacturer $j$ has a negative effect on the demand of manufacturer $i$ due to the fact that consumers may lose confidence in the whole industry due to the incident. The last term, $\gamma \left( \frac{\sum_{j \in N} q_j}{n} - q_i \right)$, captures the positive externality, which is the positive effect of manufacturer $j$'s quality issue on the demand of manufacturer $i$; i.e., consumers may switch from a product with quality issue to other products. We assume that the price of the product is fixed and normalized to 1.

With joint audit, every manufacturer belongs to one coalition in set $B = \{B_1, \ldots, B_m\}$ with $m \leq n$. We denote $i \in B_j$ if manufacturer $i$ is in coalition $B_j$. In Section 4.3, we consider the case in which set $B$, which is called coalition structure, is given. In Section 4.4, coalitions are formed by the manufacturers endogenously. The manufacturers in coalition $B_j$ jointly decide audit effort $e_{B_j} =$
\( P(s_{B_j} = 1|a = 1) \) and share the result from the audit. Therefore, if the supplier adulterates his product and coalition \( B_j \) fails to detect the adulteration (i.e., \( a = 1 \) and \( s_{B_j} = 0 \)) the product of any manufacturer \( i \in B_j \) has quality issues (i.e., \( q_i = a - s_{B_j} = 1 \)). The audit cost for coalition \( B_j \) is \( C(e_{B_j}) \).

With shared supplier information, each manufacturer \( i \) decides his own audit effort \( e_i \) and the manufacturers in a coalition share the results from their individual audits. Specifically, if there exists at least one manufacturer in coalition \( B_j \) who has detected adulteration, the adulteration is detected by all the manufacturers in coalition \( B_j \), and thus \( s_{B_j} = \max_{i \in B_j} s_i \).

4.3. Effectiveness

In this section, we consider the effectiveness of joint audit and shared supplier information in terms of reducing the probability that the supplier adulterates his product. We use superscript \( (0) \) to denote the optimal decisions in the case with individual audit (base case), \( (1) \) with joint audit and \( (2) \) with shared supplier information.
4.3.1. Individual Audit

In the base case, the expected profit of manufacturer $i$ is given by

\begin{equation}
\pi_i = (1 - \theta)\alpha + \theta \left\{ \alpha - \beta \left( 1 - \frac{\sum_{j \in N} e_j}{n} \right) - \gamma \left( \frac{\sum_{j \in N} e_j}{n} - e_i \right) \right\} - c(1 - e_i)^{-x},
\end{equation}

where the first term is the expected profit when the supplier does not adulterate his product and the second term is the expected profit when the supplier adulterates his profit.

The expected profit of the supplier is given by

\begin{equation}
\pi_0 = r - (1 - \theta)g - \theta \sum_{i \in N} le_i,
\end{equation}

where $r$ is the gross profit of the supplier. The second term represents the additional cost if the supplier’s product is not adulterated, and the third term represents the expected punishment from adulteration being detected.

For given adulteration probability $\theta$, according to the first-order condition, we can obtain the optimal audit effort of manufacturer $i$ from (4.1), which is given by

\[ e_i^{(0)} = 1 - \left[ \frac{\theta[\beta + (n - 1)\gamma]}{c x n} \right]^{\frac{1}{\pi_1}}. \]
It is intuitive that the optimal audit effort $e_{i}^{(0)}$ is increasing in negative externality $\beta$ and positive externality $\gamma$ while it is decreasing in audit cost parameters $c$ and $x$.

When the supplier adulterates his product for certain (i.e., $\theta = 1$), the maximum audit effort is given by $\bar{e} = 1 - \left(\frac{\beta+(n-1)\gamma}{cx}\right)^{-\frac{1}{x+1}}$. We assume that, if audit effort of every manufacturer is greater or equal to $\bar{e}$, it is optimal for the supplier not to adulterate his product (i.e., $\theta = 0$). With this assumption, there exists a unique optimal adulteration probability, which can be obtained by equating the last two terms in (4.2) as follows:

\begin{equation}
\theta^{(0)} = \frac{cxn}{\beta + (n-1)\gamma} \left(1 - \frac{g}{nl}\right)^{-\frac{1}{x+1}}.
\end{equation}

One can easily see that the optimal adulteration probability $\theta^{(0)}$ is increasing in $c$, $x$ and $g$ while it is decreasing in $\beta$, $\gamma$ and $l$.

4.3.2. Joint Audit

If manufacturers conduct joint audit, the total expected profit of manufacturers in coalition $B_j$ is given by

\begin{equation}
\pi_{B_j} = (1-\theta)n_j \alpha + \theta n_j \left\{ \alpha - \beta \left(1 - \frac{\sum_{z=1}^{m} n_z e_{B_z}}{n}\right) - \gamma \left(\frac{\sum_{z=1}^{m} n_z e_{B_z}}{n} - e_{B_j}\right) \right\} - c(1-e_{B_j})^{-x},
\end{equation}
where \( n_j \) is the number of manufacturers in \( B_j \). The optimal audit effort can be obtained as follows

\[
e^{(1)}_{B_j} = 1 - \left[ \frac{\theta n_j (\beta n_j + (n - n_j)\gamma)}{c x n} \right]^{-\frac{1}{x+1}}. \tag{4.5}
\]

The expected profit of the supplier is the same as that in the base case. When the number of manufacturers in every coalition is the same, i.e., with symmetric coalition structure, we can obtain the closed-form solution of the optimal adulteration probability

\[
\theta^{(1)} = \frac{c x n}{n_j [\beta n_j + (n - n_j)\gamma]} \left( 1 - \frac{g}{nl} \right)^{-(x+1)}. \tag{4.6}
\]

When coalition structure is asymmetric, there is no closed-form solution for the optimal adulteration probability. But we can still compare it with \( \theta^{(0)} \) in the base case. The following proposition shows the effectiveness of joint audit by comparing the adulteration probabilities in the two cases. All the proofs are provided in Appendix.

**Proposition 19.** When coalition structure is symmetric, joint audit is effective in reducing adulteration, i.e., \( \theta^{(1)} < \theta^{(0)} \), if and only if \( \frac{\beta}{\gamma} > 1 - \frac{mn}{m+n} \). When coalition structure is asymmetric, there exists a threshold \( t_{\text{joint}} \) such that, if \( \frac{\beta}{\gamma} > t_{\text{joint}} \), joint audit is effective in reducing adulteration.
The total number of audits in the case with joint audit is less than that in the base case so one may expect that joint audit is not effective in reducing adulteration. However, Proposition 19 shows that joint audit can reduce adulteration and improve product quality if \( \frac{\beta}{\gamma} \) is large enough. This happens when manufacturers choose higher level of audit efforts in the case with joint audit, which counteracts the less number of audits. To understand this result, let us consider the impacts of joint audit on audit efforts. First, with joint audit, the consequence of audit failures is more significant than that in the base case. The product quality of all manufacturers in one coalition may be compromised if the joint audit in that coalition fails to detect adulteration. This effect of joint audit creates incentives for manufacturers to increase audit efforts. The magnitude of this effect is related to negative externality \( \beta \). On the other hand, joint audit reduces quality competition among manufacturers within a coalition, which leads to decreased audit efforts. The magnitude of this effect is related to positive externality \( r \). Therefore, the overall impact of joint audit is determined by the ratio \( \frac{\beta}{\gamma} \).

4.3.3. Shared Supplier Information

If manufacturers share the information that they obtain from their individual audits, the audit effort of manufacturer \( i \) in coalition \( B_j \) is determined by his expected
profit, which is given by

\[
\pi_{i,B_j} = (1 - \theta)\alpha + \theta(\alpha - \beta) \left\{ 1 - \sum_{z=1}^{m} \frac{1 - \prod_{y \in B_z} (1 - e_y)}{n} n_z \right\}
- \gamma \left\{ \sum_{z=1}^{m} \frac{1 - \prod_{y \in B_z} (1 - e_y)}{n} n_z - \left( 1 - \prod_{y \in B_j} (1 - e_y) \right) \right\} - c(1 - e_i)^{-x},
\]

where \(1 - \prod_{y \in B_z} (1 - e_y)\) is the probability that the adulteration is detected by at least one manufacturer in coalition \(B_z\) assuming audits are independent of each other. Then, the optimal audit effort of manufacturer \(i\) for given adulteration probability and audit efforts of all the other manufacturers is as follows

\[
e^{(2)}_{i,B_j} = 1 - \left[ \frac{\theta \prod_{k \in B_j \setminus i} (1 - e_k) \{\beta n_j + (n - n_j)\gamma\}}{cxn} \right]^{-\frac{1}{1+\gamma}}.
\]

The expected profit of the supplier is the same as that in the base case. With symmetric coalition structure, we can obtain the closed-form solution of the optimal adulteration probability

\[
\theta^{(2)} = \frac{cxn}{\beta n_j + (n - n_j)\gamma} \left( 1 - \frac{g}{nl} \right)^{-(x/n_j+1)}.
\]

The following proposition compares the adulteration probabilities in the case with shared supplier information with that in the base case.
Proposition 20. When coalition structure is symmetric, shared supplier information is effective in reducing adulteration, i.e., $\theta^{(2)} < \theta^{(0)}$, if and only if 

$$\frac{\beta}{\gamma} > \frac{A(mn-m)-mn+n}{n-Am},$$

where $A = \left(1 - \frac{g}{m}\right)\frac{(n-m)x}{n}$. When coalition structure is asymmetric, there exists a threshold $t_{\text{shared}}$ such that, if $\frac{\beta}{\gamma} > t_{\text{shared}}$, shared supplier information is effective in reducing adulteration.

Proposition 20 suggests that, although manufacturers have the access to more information about product quality of the supplier, shared supplier information may not be always effective in reducing adulteration. The intuition is that, with shared supplier information, manufacturers have incentives to free ride on others’ audit efforts, which reduces audit efforts and hence the quality of the information generated from audits. This free-ride effect is significant when the negative externality is high and the positive externality is low.

Lastly, we compare joint audit with shared supplier information in terms of their effectiveness in reducing adulteration. The result is summarized in the following proposition.

Proposition 21. When coalition structure is symmetric, shared supplier information is more effective in reducing adulteration than joint audit, i.e., $\theta^{(1)} > \theta^{(2)}$ if and only if $\frac{\beta}{\gamma} > n \left\{1 - \left(\frac{m}{n}\right)\frac{x}{n-m}\right\}$. When coalition structure is asymmetric, there
exists a threshold $t_{\text{compare}}$ such that, if $\frac{g}{l} > t_{\text{compare}}$, shared supplier information is more effective in reducing adulteration than joint audit.

When the cost saving from adulteration $g$ is high or the punishment $l$ is low, there is a high chance that the supplier adulterates his product. Anticipating this, manufacturers choose high levels of audit efforts to monitor product quality. As a result, audits are informative and thus sharing the information from the audits creates significant value.

4.4. Incentives to Cooperate

In this section, we consider manufacturers’ incentives to cooperate in conducting joint audit or shared supplier information. For this purpose, we introduce the notion of a cooperative game. A cooperative game in ordinal function form is represented by the pair $(N, w)$, in which $w : 2^N \to R$ is a characteristic function of the game. A subset $S$ of $N$ is called a coalition and $N$ itself is the grand coalition. The characteristic function $w(S)$ captures the value generated by a coalition $S$. However, we cannot use this type of a cooperative game to analyze our model because there exist negative and positive externalities; i.e., the value generated by coalition $S$ depends not only on the audit efforts of the manufacturers in $S$ but also on the audit efforts of outsiders. Therefore, we employ a different approach
based on games in partition function form introduced by Thrall and Lucas (1963), which is introduced in the following.

4.4.1. Game in Partition Function Form

A cooperative game in partition function form is defined by \((N, \Pi, \{v_B\}_{B \in \Pi})\), where \(\Pi\) is the set of all coalition structures and \(v_B\) is a partition function that associates each coalition \(B_j\) in \(B\) with the value it can generate, \(v_B(B_j)\). The value of \(v_B(B_j)\) depends on how manufacturers outside \(B_j\) form coalitions; i.e., \(v_B(B_j)\) and \(v_{B'}(B_j)\) may be different if \(B_j \in B\), \(B_j \in B'\) and \(B \neq B'\). Given a coalition structure \(B = \{B_1, \ldots, B_m\}\), an allocation is a payoff vector \(\varphi \in \mathbb{R}^N\), and it is feasible under \(B\) if it satisfies \(\sum_{k \in B_j} \varphi_k \leq v_B(B_j)\) for \(j = 1, \ldots, m\). In order to define the stability concept for our analysis, we introduce a domination relation for two allocations. Consider two allocations \(\varphi\) and \(\varphi'\) and a coalition \(S\) in \(N\). We say \(\varphi\) dominates \(\varphi'\) via \(S\) and denote \(\varphi \ dom_S \varphi'\) if the following two conditions hold:

\[
\sum_{k \in S} \varphi_k \leq v_B(S) \quad \text{for all } B \supseteq S; \quad \text{and}
\]

\[
\varphi_k > \varphi'_k \quad \text{for all } k \text{ with } k \in S.
\]

Each member of \(S\) can get a larger payoff under the feasible allocation \(\varphi\) than under the present allocation \(\varphi'\) independent of coalition formation among outsiders. In
addition, we say that \( \varphi \) dominates \( \varphi' \) and denote \( \varphi \dom \varphi' \) if there exists \( S \subseteq N \) such that \( \varphi \dom_S \varphi' \).

The stability concept we use is called the core, which is the set of feasible allocations that are not dominated by any other allocations. Define \( B^N = \{ N \} \) as the coalition structure that contains only the grand coalition. If \( v_{B^N}(N) > \sum_{j=1}^{m} v_B(B_j) \) for any \( B \), which means that the total value that all the manufacturers generate is the largest in the grand coalition, then any allocation \( \varphi \) in the core also satisfies \( \sum_{i \in N} \varphi_i = v_{B^N}(N) \); otherwise, the allocation is either not feasible or dominated by some feasible allocation via \( N \). Then, a core allocation leads to a stable outcome of the cooperative game in the sense that no subset of firms has an incentive to secede from the grand coalition.

4.4.2. Results

If manufacturers conduct joint audit, the partition function \( v_B^{(1)}(B_j) \) can be obtained by substituting the optimal audit efforts \( e^{(1)} \) into (4.4). Since joint audit occurs before manufacturers learn the results of their audits, manufacturers are symmetric in this case. A natural way to allocate the value is the egalitarian allocation, which gives every manufacturer an equal fraction of the total value, i.e.,

\[
\varphi_i^{Eg} = \frac{v_{B^N}(N)}{n} \quad \text{for all } i \in N.
\]

The following proposition illustrates the conditions under which the egalitarian allocation is in the core with joint audit.
Proposition 22. The core of cooperative game \((N, \Pi, \{v_B^{(1)}\}_{B \in \Pi})\) with joint audit is non-empty and the egalitarian allocation \(\varphi^{Eg}\) is in the core if one of the following conditions is satisfied: (i) there exists a threshold \(t_{\text{cost}}\) such that \(c \geq t_{\text{cost}}\); (ii) \(\frac{\beta}{\gamma} \geq 1\).

The first condition requires audit cost to be sufficiently large. It is intuitive that manufacturers have incentives to cooperate with all others under this condition because the coalition of all manufacturers minimizes the audit cost shared by each manufacturer. The second condition is about the externalities. It says that, if negative externality dominates positive externality, manufacturers have incentives to cooperate with all others. Recall the condition under which joint audit is effective, \(\frac{\beta}{\gamma} > t_{\text{joint}}\), and further we can prove \(t_{\text{joint}} \leq 1\). Two conditions are consistent in the sense that joint audit is effective in reducing adulteration when manufacturers have incentives to cooperate. This result has important practical implications because anti-trust agencies may worry about that cooperation may reduce competition and hurt consumers. Proposition 22 shows that, as long as audit cost is not very large, allowing manufacturers to form coalitions to audit their common supplier will not hurt consumers.

With shared supplier information, manufacturer \(i\) can decide whether or not to share the information about adulteration after he observes the signal \(s_i\). Thus
the partition function \(v_B^{(2)}(B_j)\) is defined as follows\(^4\):

\[
v_B^{(2)}(B_j) = n_j \left\{ \alpha - \beta \left( 1 - \frac{\sum_{k=1}^{m} n_k I(B_k)}{n} \right) - \gamma \left( \frac{\sum_{k=1}^{m} n_k I(B_k)}{n} - I(B_j) \right) \right\} - \sum_{i \in B_j} c(1 - e^{(2)}_{i,B_j})^{-x},
\]

(4.9)

where \(I(B_k) = 1\) if there exists \(i \in B_k\) such that \(s_i = 1\) and \(I(B_k) = 0\) otherwise.

The audit costs are sunk so we normalize \(\sum_{i \in B_j} c(1 - e^{(2)}_{i,B_j})^{-x}\) to zero. We focus on the case in which the supplier adulterates the product because otherwise there is no information to share, and we assume that there exists at least one manufacturer that has detected the adulteration. The following proposition illustrates the core of this game.

**Proposition 23.** If \(\frac{\beta}{\gamma} \geq 1\), the egalitarian allocation \(\varphi^{Eg}\) is in the core of cooperative game \((N, \Pi, \{v_B^{(2)}\}_{B \in \Pi})\) with shared supplier information. If \(\frac{n-2}{2n-2} \leq \frac{\beta}{\gamma} < 1\) and there exists only one manufacturer \(i \in N\) such that \(s_i = 1\), the following allocation is in the core

\[
\varphi_i^{no} = \alpha + \frac{n-1}{n} \left\{ \beta(n - 1) + \gamma \right\} \text{ and } \\
\varphi_{i'}^{no} = \alpha - \frac{n-1}{n} \left\{ \beta + \frac{\gamma}{n-1} \right\} \text{ for } i' \in N \setminus i.
\]

\(^4\)Alternatively, we can define the partition function similar to the one in the case with joint audit, i.e., based on the expected profits of manufacturers, and obtain similar results as in Proposition 22.
In addition to egalitarian allocation, we can consider other allocation \( \varphi^{\text{mo}} \) (the superscript stands for "motivate") with shared supplier information since there exists asymmetry in terms of the information about product quality. Some manufacturers have detected the adulteration while others have not. This is different from the joint audit case, in which the cooperative game is based on the expected profits and manufacturers are symmetric. We find that, when \( \frac{\beta}{\gamma} < 1 \), the core is still non-empty although the egalitarian allocation is not in the core. Specifically, the condition requires that there exists only one manufacturer that has detected adulteration. This manufacturer is compensated by others who fail to detect adulteration.

4.5. Conclusion

Product safety incidents in recent years have compelled many manufacturers to rethink approaches to manage product quality of their suppliers. In this paper, we investigate two cooperative approaches that are often used in practice: auditing common suppliers jointly ("joint audit") and sharing independent audit results with other manufacturers ("shared supplier information"). We develop a model that captures both competitive and cooperative interactions among manufacturers. Our analysis reveals that, although the cooperative approaches may reduce audit costs, they do not necessarily improve product quality. The effectiveness
of these approaches depends crucially on the externality of product safety of one manufacturer on other manufacturers. Further, when the risk of product safety failure is high, shared supplier information is more effective than joint audit; otherwise, joint audit is more effective. We also investigate the incentives of competing manufacturers to audit cooperatively, and design an efficient way to motivate the manufacturers who have better information about product quality to share the information. Since product safety has a significant impact on the well-being of consumers, industries and governments should assess the level of externality and risk of product safety failure in specific market settings in order to design effective and stable cooperative audit programs.
CHAPTER 5

Conclusion

The main focus of this dissertation is on competition and cooperation in global supply chains, and the primary research methodologies used are cooperative and non-cooperative game theory, and the theory of social and economic networks.

Chapter 2 considers counterfeit products of different types: non-deceptive counterfeits (e.g., fake luxury bags) that are sold at a substantial discount so their consumers know they are purchasing counterfeits, and deceptive counterfeits (e.g., fake drugs) that are sold by a licit distributor who deceives consumers into believing those goods to be authentic at time of purchase. We investigate the impacts of such difference on the effectiveness of anti-counterfeiting strategies. The competition between a brand-name firm and a counterfeiter is quite different from the competition between brand-name firms; e.g., a counterfeiter is free riding on brand value while facing the risk of legal punishment. Our analysis reveals that the anti-counterfeiting strategies found to be useful against one type of a counterfeit may not work for the other type, or can even make counterfeits flourish in the market. A recent OECD study estimates that international trade in counterfeit
goods amounts up to $250 billion or 1.95% of world trade. In order to combat counterfeits more effectively, industries and governments should understand the type of potential counterfeiters.

Chapter 3 investigates cooperation among firms focusing on understanding the incentives to transship inventories. Transshipment of inventories is common in practice: when a retailer does not hold stock of a product a customer demands, the retailer can satisfy the customer’s demand by having it shipped from another retailer who has it in inventory. We analyze this traditional problem from a new perspective, i.e., using social and economic networks to model the relationships among firms. This new approach, by allowing one firm to cooperate with selective firms instead of requiring universal cooperation, offers a richer representation of relationships among firms taking the coalitions used in the prior literature as special cases. We analyze multiple methods to allocate the profit from inventory transshipment and various network structures among firms such as complete, hub-spoke, and chain networks. It turns out that a network structure plays an important role in determining the incentives of firms to cooperate in a decentralized distribution system. While the previous coalition-based approach examines only the grand coalition (i.e., complete network), we find, interestingly, that firms are less likely to transship inventories cooperatively under the complete network compared to incomplete networks.
Chapter 4 analyzes recent food safety incidents in global supply chains. One way to improve food safety is to adopt the successful practice in pharmaceutical and automobile industry, which takes advantage of the joint efforts of manufacturers to monitor product safety of their suppliers. Two or more manufacturers can jointly audit their common suppliers or share product safety information of the suppliers with each other. However, such cooperation complicates the relationships among manufacturers: Given that they are competitors, they may not have incentive to cooperate through jointly audit or sharing private supplier information. Further, joint audit or information sharing may not improve the product safety of suppliers. We find that the effectiveness and incentives depend on the externality of product safety of one manufacturer on the rest of manufacturers. Since product safety has a significant impact on the well-being of consumers, industries and governments should assess the level of externality in specific market settings in order to design an effective joint audit or information sharing program.

As firms become more and more connected with each other in supply chain networks, it is crucial for them to consider the reactions of opponents and allies. In Chapter 2, the reactions of a counterfeiter determine the effectiveness of anti-counterfeiting strategies. In Chapter 3, we find that the networks established by independent firms depend on how firms react to a particular way to allocate the
profit from transshipment of leftover inventories. In Chapter 4, both the effectiveness and the incentive of a group of firms to conduct cooperative audits are affected by other firms’ reactions to their audit efforts. Ignoring the reactions of opponents and allies will eventually lead to a loss of a firm’s profit or a loss of social welfare.
References


Appendix

Appendix A. Proofs of Analytical Results in Chapter 2

We use (A1) and (A2) to indicate the following assumptions we have made earlier:

(A1) \( q_B > q_N = f_N + \beta q_B \) and \( q_B > q_D = f_D + \beta q_B \); (A2) \( 1 - \frac{p_B}{q_B} > 0 \) and \( 1 - \frac{p_B - p_N}{q_B - q_N} > 0 \) so that \( m_B > 0 \).

Proof of Lemma 1: From (2.4), we obtain

\[
\frac{\partial^2 \pi^*_N}{\partial f_N^2} = \frac{(1-\gamma)p_B^2}{4((1-\beta)q_B-f_N)^2} - 2t_N
\]

which is positive if \( t_N < \frac{(1-\gamma)p_B^2}{8((1-\beta)q_B-f_N)^2} \). Thus, if \( t_N < \frac{(1-\gamma)p_B^2}{8((1-\beta)q_B-f_N)^2} \), \( \pi^*_N \) is convex in \( f_N \in [\underline{f}, \bar{f}] \), so \( \pi^*_N = \underline{f} \) when \( \pi^*_N(f) \geq \pi^*_N(\bar{f}) \). Otherwise, \( \pi^*_N \) can be \( \bar{f} \) or \( \pi^*_N \in (\underline{f}, \bar{f}) \) that satisfies the first order condition \( \frac{\partial \pi^*_N}{\partial f_N}|_{f_N=\pi^*_N} = 0 \). \( \square \)

Remark A sufficient condition for \( \pi^*_N > \underline{f} \) is if \( t_N < \frac{(1-\gamma)p_B^2}{16\xi((1-\beta)q_B-f_N)^2} \), which can be obtained from \( \frac{\partial \pi^*_N}{\partial f_N}|_{f_N=\underline{f}} = \frac{(1-\gamma)p_B^2}{8((1-\beta)q_B-f_N)^2} - 2t_N \underline{f} > 0 \).

Proof of Lemma 2: When \( \lambda = 0 \), from (2.8), \( \frac{\partial \pi^*_{wD}}{\partial f_D} = -2t_D f_D < 0 \), so \( f_D^* = \bar{f} \).

When \( \lambda > 0 \), we next show that \( f_D^{**} = \underline{f} \) if \( t_D \geq \bar{t}_D \). For any \( f_D \in (\underline{f}, \bar{f}) \),

\[
\pi^*_D(w^*_D(f_D), \underline{f}) \geq \pi^*_D(w^*_D(f_D), f_D) \text{ if } t_D \geq (1-\gamma)w^*_D(f_D)\{m^*_D(w^*_D(f_D), f_D) - m^*_D(w^*_*(f_D), f_D)\}(f_D^2 - \underline{f}^2)^{-1}.
\]

Suppose \( t_D \geq \bar{t}_D \equiv \max_{f_D \in (\underline{f}, \bar{f})} (1-\gamma)w^*_D(f_D)\{m^*_D(w^*_D(f_D), f_D) - m^*_D(w^*_*(f_D), f_D)\}(f_D^2 - \underline{f}^2)^{-1} \). Then, for any \( f_D \in (\underline{f}, \bar{f}) \),

\[
\pi^*_D(w^*_D(f), \underline{f}) \geq \pi^*_D(w^*_D(f_D), f_D) \geq \pi^*_D(w^*_D(f_D), \underline{f}).
\]
\( \pi_D^*(w_D^*(f_D), f_D) \), where the first inequality is due to the optimality of \( w_D^*(f) \) given \( f \), and the second inequality follows from \( t_D \geq \bar{t}_D \). Therefore, \( f_D^* = \_f \).

In the rest of the proof, we show \( \bar{t}_D > 0 \) in two steps: we first show that \( s^{**} \) is increasing in \( f_D \) for given \( w_D \), and then show that the market share of the deceptive counterfeiter, \( m_D^{**} = s^{**} \{ 1 - \bar{\theta}(s^{**}) \} \), is increasing in \( f_D \) for any given \( w_D \). Then from the definition of \( \bar{t}_D \), \( \bar{t}_D > 0 \). Let \( \pi_{LD} \) denote the expected profit of the licit distributor given in (2.6). Then

\[
\frac{\partial \pi_{LD}}{\partial s} = (1 - 2s)(p_B - w_D) \left\{ 1 - \bar{\theta} \right\} - s(1 - s)(p_B - w_D) \frac{\partial \bar{\theta}}{\partial s} - l, \text{ and}
\]

\[
\frac{\partial^2 \pi_{LD}}{\partial s \partial f_D} = (2 - 3s)(p_B - w_D) \frac{\lambda p_B s}{\{(1-s)q_B + s(f_D + \beta q_B)\}} + 2s(1 - s)(p_B - w_D) \frac{\lambda p_B s(q_B - f_D - \beta q_B)}{\{(1-s)q_B + s(f_D + \beta q_B)\}}^2,
\]

where the first term is positive because we know from §2.4.2 that \( s^{**} < 0.5 \) and \( w_D \leq p_B \), and the second term is also positive according to (A1). Therefore, \( \frac{\partial \pi_{LD}}{\partial s} \) is increasing in \( f_D \). Since \( s^{**} \) satisfies \( \frac{\partial \pi_{LD}}{\partial s} \big|_{s=s^{**}} = 0 \) due to the concavity of \( \pi_{LD} \) with respect to \( s \), \( s^{**} \) is increasing in \( f_D \).

Next, we show that \( m_D^{**} \) increases as \( f_D \) increases from \( f_{DL} \) to \( f_{DH} \) for given \( w_D \). Suppose this does not hold. Then, \( \pi_{LD} \) satisfies the following:

\[
\pi_{LD}(s^{**}(f_{DH}), f_{DH}) = s^{**}(f_{DH})(1 - s^{**}(f_{DH}))(p_B - w_D) \left\{ 1 - \bar{\theta}(s^{**}(f_{DH}), f_{DH}) \right\} - s^{**}(f_{DH})l
\]

\[
\leq s^{**}(f_{DL})(1 - s^{**}(f_{DH}))(p_B - w_D) \left\{ 1 - \bar{\theta}(s^{**}(f_{DL}), f_{DL}) \right\} - s^{**}(f_{DH})l
\]

\[
< s^{**}(f_{DL})(1 - s^{**}(f_{DH}))(p_B - w_D) \left\{ 1 - \bar{\theta}(s^{**}(f_{DL}), f_{DH}) \right\} - s^{**}(f_{DH})l
\]
< s**(f_{DL})(1 - s**(f_{DL}))(p_B - w_D) \{ 1 - \theta(s**(f_{DL}), f_{DH}) \} - s**(f_{DL})l = \pi_{LD}(s**(f_{DL}), f_{DH}),

where the first inequality follows from our premise, the second inequality follows from \( \frac{\partial \theta}{\partial f_D} = -\frac{\lambda p_B s}{(1-s)q_B + s(f_{DH} + \beta q_B)} \) < 0 for fixed \( s \), and the last inequality follows from \( \frac{\partial s**}{\partial f_D} > 0 \). However, this contradicts the condition that \( s**(f_{DH}) \) maximizes the licit distributor’s profit \( \pi_{LD} \) given \( f_{DH} \). Therefore, \( m_D^{**} \) is increasing in \( f_D \) for given \( w_D \), and \( t_D > 0 \). \( \Box \)

Remark A sufficient condition for \( f_D^{**} > f \) is \( t_D < t_D = \max_{f_D \in [f, \overline{f}]} (1 - \gamma) w_D^{**}(f) \{ m_D^{**}(w_D^{**}(f), f_D) - m_D^{**}(w_D^{**}(f), f) \} (f_D^2 - \overline{f}^2)^{-1} \). We show this by contradiction.

Suppose \( f_D^{**} = f \) and define \( f_{max} = \arg \max_{f_D \in [f, \overline{f}]} (1 - \gamma) w_D^{**}(f) \{ m_D^{**}(w_D^{**}(f), f_D) - m_D^{**}(w_D^{**}(f), f) \} (f_D^2 - \overline{f}^2)^{-1} \). Then \( \pi_D^{**}(w_D^{**}(f_{max}), f_{max}) \geq \pi_D^{**}(w_D^{**}(f), f_{max}) > \pi_D^{**}(w_D^{**}(f), f) \), where the first inequality is due to the optimality of \( w_D^{**}(f_{max}) \) given \( f_{max} \), and the second inequality follows from \( t_D < t_D \). However, this contradicts our premise that \( f_D^{**} = f \). Therefore, \( f_D^{**} > f \) if \( t_D < t_D \).

Proof of Proposition 3: (a) The proof proceeds as follows: We first obtain \( q_B^m \) and \( q_B^* \), and then derive the condition for \( q_B^* > q_B^m \). When there is no counterfeiter, the expected profit of the brand-name company is given as follows:

\[
\pi_B^{**} = (p_B - c) \left( 1 - \frac{p_B}{q_B} \right) - t_B q_B^2,
\]

(5.1)
where \( c \) \((>0)\) is the marginal cost of the brand-name product. From (5.1), \( \frac{\partial^2 \pi^*_B}{\partial q_B^2} = -\frac{2(p_B-c)p_B}{q_B^3} - 2t_B < 0 \), so we obtain \( q_B^m = \frac{(p_B-c)p_B}{2t_Bq_B^2} \) from the first order condition.

When the non-deceptive counterfeiter exists in the market, we obtain \( \pi^*_B \) after substituting \( p^*_B \) and \( w^*_N \) into \( m_B \) in (2.1) as follows:

\[
(5.2) \\
\pi^*_B = (p_B-c)m_B - t_Bq_B^2 = (p_B-c) \left\{ 1 - \frac{(1-\gamma)p_B}{4((1-\beta)q_B - f^*_N)} - \frac{(3+\gamma)p_B}{4q_B} \right\} - t_Bq_B^2.
\]

From (5.2), when \( f^*_N = f \) or \( \bar{f} \), \( \frac{\partial^2 \pi^*_B}{\partial q_B^2} = -\frac{p_B-c}{2} \left( \frac{p_B(1-\gamma)(1-\beta)^2}{(q_B-q_N)^3} + \frac{p_B(3+\gamma)}{q_B} \right) - 2t_B < 0 \) due to (A1). In this case, from the first order condition of (5.2), \( q_B^* = \frac{p_B-c}{2t_B} \left\{ \frac{(1-\gamma)(1-\beta)p_B}{4((1-\beta)q_B - f^*_N(q_B))^2} + \frac{(3+\gamma)p_B}{4q_B^2} \right\} \).

We next show by contradiction that \( q_B^* > q_B^m \) when \( \beta < 1 - \left( \frac{q_B^m-q_N(q_B^m)}{q_B^m} \right)^2 \). Suppose \( \beta < 1 - \left( \frac{q_B^m-q_N(q_B^m)}{q_B^m} \right)^2 \) and \( q_B^* \leq q_B^m \). For \( f_{NH} > f_{NL} \), from (2.4), we obtain \( \frac{\partial \pi^*_N(f_{NH})}{\partial q_B} - \frac{\partial \pi^*_N(f_{NL})}{\partial q_B} = \frac{(\gamma-1)p_B^2(f_{NH}-f_{NL})(1-\beta)(1-\beta)q_B-f_{NH}+f_{NL})/2}{4((1-\beta)q_B-f_{NH})^2((1-\beta)q_B-f_{NL})^2} < 0 \) due to (A1), so \( f^*_N \) is decreasing in \( q_B \). Then \( q_B^* = \frac{p_B-c}{2t_B} \left\{ \frac{(1-\gamma)(1-\beta)p_B}{4((1-\beta)q_B - f^*_N(q_B))^2} + \frac{(3+\gamma)p_B}{4q_B^2} \right\} \geq \frac{p_B-c}{2t_B} \left\{ \frac{(1-\gamma)(1-\beta)p_B}{4((1-\beta)q_B^m - f^*_N(q_B^m))^2} + \frac{(3+\gamma)p_B}{4q_B^m} \right\} \geq \frac{(p_B-c)p_B}{2t_Bq_B^2} = q_B^m \), where the first inequality follows from \( q_B^* \leq q_B^m \) and \( f^*_N(q_B^*) \geq f^*_N(q_B^m) \), and the second inequality follows from \( \beta < 1 - \left( \frac{q_B^m-q_N(q_B^m)}{q_B^m} \right)^2 \). Thus, there is a contradiction, so \( q_B^* > q_B^m \) when \( \beta < 1 - \left( \frac{q_B^m-q_N(q_B^m)}{q_B^m} \right)^2 \). The case in which \( \beta \geq 1 - \left( \frac{q_B^m-q_N(q_B^m)}{q_B^m} \right)^2 \) can be shown similarly and is hence omitted.
(b) To establish the result in the proposition, it suffices to show that \( \pi_N^* \) is decreasing in \( q_B \). The proof proceeds in two steps: We first show that \( \pi_N^* \) decreases in \( q_B \) for any given \( f_N \), and then show that this result holds even when \( f_N^* \) changes with \( q_B \). First, from (2.4), we obtain
\[
\frac{\partial \pi_N^*}{\partial q_B} = \frac{(1-\gamma)q_B^2}{4q_B^2(1-\beta q_B - f_N)} \left\{ \beta q_B^2 (\beta - 1) + 2q_B f_N (\beta - 1) + f_N^2 \right\},
\]
which is negative by (A1) for any given \( f_N \). Next, we consider the case in which \( f_N^* \) changes from \( f_{N1} \) to \( f_{N2} \) when \( q_B \) is increased from \( q_{BL} \) to \( q_{BH} \). In this case, \( \pi_N^*(f_{N1}, q_{BL}) \geq \pi_N^*(f_{N2}, q_{BL}) > \pi_N^*(f_{N2}, q_{BH}) \), where the first inequality follows from \( f_N^*(q_{BL}) = f_{N1} \) and the second inequality is due to \( \frac{\partial \pi_N^*}{\partial q_B} < 0 \) for any \( f_N \).

(c) We first prove that \( ECS_N^* \) is increasing in \( q_B \) for given \( f_N \), and then prove that \( ECS_N^* \) decreases when \( f_N^* \) is decreased from \( f_{NH} \) to \( f_{NL} \) for any given \( q_B \).

To prove that \( ECS_N^* \) is increasing in \( q_B \), it suffices to show that \( \frac{\partial ECS_N}{\partial q_B} > 0 \) for any given \( f_N \) because \( \frac{\partial CS_B}{\partial q_B} > 0 \) from the definition of \( CS_B \). Now suppose that \( q_B \) is increased from \( q_{BL} \) to \( q_{BH} \). Then \( q_N \) is also increased from \( q_{NL} \) to \( q_{NH} \) given \( f_N \); \( \hat{\theta} \) is decreased from \( \hat{\theta}_L \) to \( \hat{\theta}_H \); and \( \tilde{\theta} \) is decreased from \( \tilde{\theta}_L \) to \( \tilde{\theta}_H \). Using \( p_N^* = \frac{3pq_N^*}{4q_B} \), we can rewrite (2.9) and find
\[
CS_N(q_{BH}) > \int_{\hat{\theta}_L}^{\hat{\theta}_H} \left( \theta - \frac{3pq_B}{4q_{BH}} \right) q_{NH} d\theta + \int_{\tilde{\theta}_L}^{\tilde{\theta}_H} \left( \theta - \frac{3pq_B}{4q_{BH}} \right) q_{NH} d\theta + \int_{\tilde{\theta}_L}^{\tilde{\theta}_H} (\theta q_{BH} - p_B) d\theta > CS_N(q_{BL}).
\]
The first inequality holds because \( \hat{\theta}_L > \hat{\theta}_H \) and \( \left( \theta - \frac{3pq_B}{4q_{BH}} \right) q_{NH} > 0 \) for \( \theta \in (\hat{\theta}_H, \hat{\theta}_L) \). The second inequality holds because \( \theta q_{BH} - p_B > \theta q_{NH} - p_N^* \) for \( \theta \in (\tilde{\theta}_H, \tilde{\theta}_L) \). The third inequality follows from the fact that \( q_{BH} > q_{BL} \) and \( q_{NH} > q_{NL} \).
Next, suppose $f^*_N$ is decreased from $f_{NH}$ to $f_{NL}$ for fixed $q_B$. Then $\hat{\theta}$ remains the same, whereas $\tilde{\theta}$ is decreased from $\tilde{\theta}' \equiv \frac{p_B}{4((1-\beta)q_B-f_{NH})} + \frac{3p_B}{4q_B}$ to $\tilde{\theta}'' \equiv \frac{p_B}{4((1-\beta)q_B-f_{NL})} + \frac{3p_B}{4q_B}$. Then,

$$ECS^*_N(f_{NH}) = (1-\gamma) \left\{ \int_{\tilde{\theta}''}^{\tilde{\theta}'} (\theta - 3p_B) \left( f_{NH} + \beta q_B - p_B \right) d\theta + \int_{\tilde{\theta}'}^{\frac{1}{q_B}} (\theta q_B - p_B) d\theta \right\} + \gamma \int_{\tilde{\theta}''}^{\frac{1}{q_B}} (\theta q_B - p_B) d\theta
$$

$$= (1-\gamma) \left\{ \int_{\tilde{\theta}''}^{\tilde{\theta}'} (\theta - 3p_B) \left( f_{NL} + \beta q_B - p_B \right) d\theta + \int_{\tilde{\theta}'}^{\frac{1}{q_B}} (\theta q_B - p_B) d\theta \right\} + \gamma \int_{\tilde{\theta}''}^{\frac{1}{q_B}} (\theta q_B - p_B) d\theta
$$

In the above, the first inequality holds because $(\theta - 3p_B) (f_{NH} + \beta q_B) > \theta q_B - p_B$ for $\theta \in (\tilde{\theta}'', \tilde{\theta}')$, and the second inequality follows from $f_{NH} > f_{NL}$. □

Remark When $f^*_N \in (\underline{f}, \overline{f})$, assuming $\frac{\partial^2 f^*_N}{\partial q_B^2} \geq 0$, we can still obtain $\frac{\partial^2 f^*_N}{\partial q_B^2} < 0$.

From the first order condition of (5.2), $q^*_B = \frac{p_B}{2} \left\{ \frac{(1-\gamma)(1-\beta-\partial f^*_N/\partial q_B)p_B}{4((1-\beta)q_B-f^*_N(q_B))^2} + \frac{(3+\gamma)p_B}{4q^*_B} \right\}$.

The condition for $q^*_B > q^*_B$ then becomes $\frac{(1-\gamma)(1-\beta-\partial f^*_N/\partial q_B)p_B}{4((1-\beta)q_B-f^*_N(q_B))^2} + \frac{3+\gamma}{4q^*_B} - \frac{1}{q^*_B^2} > 0$.

Unfortunately, this condition cannot be simplified further to the form like $\beta < 1 - \left( \frac{q^*_B-q^*_B(q^*_B)}{q^*_B} \right)^2$ because the closed-form expressions for $f^*_N$ and $\partial f^*_N/\partial q_B$ are not available.

The proof for (b) does not require $f^*_N = \underline{f}$ or $\overline{f}$, so it also holds for $f^*_N \in (\underline{f}, \overline{f})$.

For (c), suppose $q^*_B > q^*_B$. From (2.11), $\frac{\partial ECS^*_N}{\partial q_B} = [q^*_B(1-\beta)\{16(1-\beta)q^*_B + p^2_B(\beta(15+\gamma) - 16)\} + f^*_N \{p^2_B(15+\gamma) - 16q^*_B\} \{2(1-\beta)q_B - f^*_N \} + \frac{p^2_Bq^*_B}{1-\beta}(1-\beta)q_B}$.
Proof of Proposition 4: (a) From (5.1), \( \frac{\partial^2 \pi_B}{\partial p_B^2} = - \frac{2}{q_B} < 0 \), so we obtain \( p_m^B = \frac{q_B + c}{2} \) from the first order condition. Next, consider the market in which the non-deceptive counterfeiter exists. When \( f_N^* = f \) or \( \overline{f} \), from (5.2), \( \frac{\partial^2 \pi_B}{\partial p_B^2} = - \frac{1 - \gamma}{2(q_B - q_N)} - \frac{3 + \gamma}{2q_B} < 0 \) and \( p^* = \frac{q_B q_N (1 - \gamma)}{2(-4q_B + 3q_N + q_B q_N)} + \frac{q_B^2 + c}{2} \); in this case, \( p_B^* < p_B^m \) due to (A1). When \( f_N^* \in (f, \overline{f}) \), we show \( \frac{\partial \pi_B}{\partial p_B} \bigg|_{p_B = q_B + c} < 0 \), which then results in \( p_B^* < p_B^m \). From (5.2), we obtain
\[
\frac{\partial \pi_B}{\partial p_B} \bigg|_{p_B = q_B + c} = \frac{(1 - \gamma)(4f_N^2 + 4(2\beta - 1)f_N q_B + 4(\beta - 1)q_B^2 + (c - q_B)(c + q_B)\partial f_N / \partial p_B)}{16(1 - \beta)q_B - f_N^*}^2,
\]
which is negative because:
\[
4f_N^2 - 4(1 - 2\beta)f_N q_B - 4(1 - \beta)q_B^2 - \frac{\partial f_N}{\partial p_B}(q_B - c)(q_B + c) < 4f_N^2 - 4(f_N^* - \beta q_B)f_N^* - 4f_N^* \beta q_B - \frac{\partial f_N}{\partial p_B}(q_B - c)(q_B + c) = \frac{\partial f_N}{\partial p_B}(q_B - c)(q_B + c) \leq 0,
\]
where the first inequality is based on (A1) and the second inequality holds because \( \frac{\partial f_N}{\partial p_B} \geq 0 \) and \( q_B > p_B \geq c \) by (A2).

(b) The proof is similar to that of Proposition 3(b), and is hence omitted.

(c) The proof for the case in which \( f_N^* = f \) or \( \overline{f} \) is similar to that of Proposition 3(c). When \( f_N^* \in (f, \overline{f}) \), from (2.11), \( \frac{\partial \text{ECS}_N^*}{\partial p_B} = [-2 \{(1 - \beta)q_B - f_N^*\} \{16(1 - \beta)q_B^2 + p_B q_B(\beta(15 + \gamma) - 16) + (p_B(15 + \gamma) - 16q_B)f_N^*\} + p_B q_B(1 - \gamma)\partial f_N^* / \partial p_B] \{(1 - \beta)q_B - f_N^*\}^{-2} \).

Define \( \kappa = \max_{p_B \in [p_B^m, p_B^m]} 2 \{(1 - \beta)q_B - f_N^*\} \{16(1 - \beta)q_B^2 + p_B q_B(\beta(15 + \gamma) - 16) + p_B^2 q_B(1 - \gamma)\partial f_N^* / \partial p_B\} \{(1 - \beta)q_B - f_N^*\}^{-2} \).
(\(p_B(15 + \gamma) - 16q_B\))f_N^* p_B^2 q_B^{-1}(1 - \gamma)^{-1} \{ (1 - \beta)q_B - f_N^* \}^{-2}$. Then \(ECS_N^*\) is increasing in \(p_B \in [p_B^*, p_B^m]\) so that \(ECS_N^*\) is lower at \(p_B^*\) than at \(p_B^m\) if \(\frac{\partial f_N^*}{\partial p_B} > \kappa\).

\(\square\)

**Proof of Proposition 5:** (a) When the deceptive counterfeiter exists in the market with \(\lambda = 0\), we obtain \(\pi_B^*\) after substituting \(s^{**}\) and \(w_D^*\) into \(m_B\) in (2.5) as follows:

\[
\pi_B^{**} = (p_B - c)m_B - t_B q_B^2 = (p_B - c) \left[ (1 - \gamma) \left\{ \frac{1}{2} (1 - \frac{p_B}{q_B}) + \frac{1}{2} \sqrt{\frac{l(1 - \frac{p_B}{q_B})}{p_B}} \right\} + \gamma \left(1 - \frac{p_B}{q_B}\right) \right] - t_B q_B^2.
\]

(5.3)

From (5.3), \(\frac{\partial^2 \pi_B^{**}}{\partial q_B^2} = (p_B - c) \{ -\frac{p_B(1 + \gamma)}{q_B} - \frac{(4q_B - 3p_B)(1 - \gamma)}{8(q_B - p_B)q_B} \sqrt{\frac{4q_B p_B}{q_B - p_B}} \} - 2t_B < 0\) due to (A2), and \(q_B^{**} = \frac{p_B - c}{2t_B} \left[ (1 - \gamma) (1 - \frac{p_B}{q_B}) + \frac{1}{2} \sqrt{\frac{l p_B q_B}{q_B - p_B}} \right] \) from the first order condition. By the same procedure in the proof of Proposition 3(a), we can prove by contradiction that \(q_B^{**} < q_B^m\) if and only if \(l < 4p_B(1 - \frac{p_B}{q_B})\). Since \(s^{**} = \frac{1}{2} - \frac{1}{2} \sqrt{\frac{l}{p_B(1 - \frac{p_B}{q_B})}} > 0\), we find that \(l < p_B(1 - \frac{p_B}{q_B}) < 4p_B(1 - \frac{p_B}{q_B})\), so we always have \(q_B^{**} < q_B^m\).

(b) To establish the result in the proposition, it suffices to show that \(\pi_D^{**}\) is increasing in \(q_B\). When \(\lambda = 0\), it is easy to see \(\frac{\partial \pi_D^{**}}{\partial q_B} = \frac{\partial \pi_D^*}{\partial q_B} \frac{p_B}{q_B^2} > 0\) \(\forall f_D\) from (2.8).

Since \(f_D^{**} = f \forall q_B\), the result follows.

(c) When \(\lambda = 0\), from (2.11), \(\frac{\partial ECS_N^*}{\partial q_B} = -(1 - \gamma) \left\{ \frac{p_B}{q_B} s^{**} + \frac{1}{2} \left(1 - \frac{p_B}{q_B}\right) \frac{\partial \pi_B^{**}}{\partial q_B}\right\} (q_B - q_D^*) +\)
\[
\frac{1}{2} \left(1 - \frac{v^2}{q_B^2}\right) \{1 - (1 - \gamma)s^{**}\}. \text{Using } \frac{\partial s^{**}}{\partial q_B} > 0 \text{ and (A2), we obtain the condition given in Proposition 5(c).} \quad \square
\]

**Remark**  When \(\lambda > 0\), both \(q_B^{**} < q_B^m\) and \(q_B^{**} > q_B^m\) are possible as shown in Table 3. The condition for \(q_B^{**} < q_B^m\) is \(t_B > \frac{(p_B - c)}{q_B^m} \{\frac{\gamma p_B}{q_B^m} - (1 - \gamma)\{1 - \frac{(1 - \lambda)p_B}{q_B^m} - (1 - s^{**}(q_B^m))q_B^m + s^{**}(q_B^m)(\beta q_B^m + f_D^*(q_B^m))\}\} \frac{\partial s^{**}}{\partial q_B} + \frac{\lambda p_B + \lambda p_B + s^{**}(q_B^m)(\beta - 1 + \frac{\partial f^*_D}{\partial q_B}) + (f_D^*(q_B^m) + \beta q_B^m - q_B^m) \frac{\partial s^*}{\partial q_B} \{(1 - s^{**}(q_B^m))q_B^m + s^{**}(q_B^m)(\beta q_B^m + f_D^*(q_B^m))^{-2}\}\},\)

which can be obtained from \(\frac{\partial s^{**}}{\partial q_B} \mid_{q_B=q_B^m} < 0\). In this case, \(\pi^{**}_D\) is increasing in \(q_B\) as in the case when \(\lambda = 0\) shown in the proof of Proposition 5(b). The proof follows the same procedure as in that of Lemma 2, so we provide a sketch of the proof here. For given \(w_D\) and \(f_D\), we can show that \(s^{**}\) and \(m^{**}_D\) are increasing in \(q_B\). Then when \(q_B\) is increased from \(q_B^L\) to \(q_B^H\), the following inequalities hold in equilibrium: \(\pi^{**}_D(w^{**}_D(q_B^H), f^{**}_D(q_B^H), q_B^H) \geq \pi^{**}_D(w^{**}_D(q_B^L), f^{**}_D(q_B^L), q_B^H) > \pi^{**}_D(w^{**}_D(q_B^L), f^{**}_D(q_B^L), q_B^L)\). Finally, when \(\lambda > 0\), the condition for \(\frac{\partial ECS^{**}_D}{\partial q_B} < 0\) given in Proposition 5(c) is modified to the following: \(q_D^{**} \left\{2\bar{\theta}s^{**} \frac{\partial \bar{\theta}}{\partial q_B} + (1 - \bar{\theta}^2) \frac{\partial s^{**}}{\partial q_B}\right\} < \frac{\gamma}{1 - \gamma} \left(1 - \frac{v^2}{q_B^2}\right) + 2 \left\{p_B - \bar{q}_B(1 - s^{**})\right\} \frac{\partial \bar{q}_B}{\partial q_B} + (1 - \bar{\theta}^2) \left\{1 - (1 - \beta - \frac{\partial f^*_D}{\partial q_B}) s^{**} - q_B \frac{\partial s^*}{\partial q_B}\right\} .\)

Unfortunately, this condition cannot be simplified further since the closed-form expressions of \(s^{**}\) and \(f^{**}_D\) do not exist. The non-monotonicity of \(ECS^{**}_D\) is shown in Table 3.
Proof of Proposition 6: (a) From (5.3), \( \frac{\partial^2 \pi_B^{**}}{\partial p_B^2} = -\frac{1+\gamma}{q_B} - \frac{1-\gamma}{2p_B^2} \sqrt{\frac{1}{q_B - p_B}} \left\{ c + \frac{q_B(p_B - c)}{3(q_B - p_B)} \right\} < 0 \), and \( \frac{\partial^2 \pi_B^{**}}{\partial p_B^2} \bigg|_{p_B=p_B^{**}} = \frac{c(1-\gamma)}{2(c+q_B)} \sqrt{\frac{1}{q_B(c+q_B)}} > 0 \) due to \( q_B > p_B \geq c \) by (A2). Therefore, by the concavity of \( \pi_B^{**} \), \( p_B^{**} > p_B^{m} \).

(b) The non-monotonicity of \( \pi_D^{**} \) with respect to \( p_B \) is shown in Table 3.

(c) When \( \lambda = 0 \), from (2.11), \( \frac{\partial E C S_{D}^{**}}{\partial p_B} = (1-\gamma)[s_{**}^{p_B} \left( 1 - \frac{q_B^{**}}{q_B} \right) + \left( 1 - \frac{q_B}{q_B} \right) \left\{ \frac{1}{2} \left( 1 + \frac{q_B}{q_B} \right) \frac{\partial s_{**}^{p_B}}{\partial p_B} (q_D^{**} - q_B) - 1 \right\}] + \gamma \left( \frac{p_B}{q_B} - 1 \right) \). Using \( \frac{\partial s_{**}^{p_B}}{\partial p_B} < 0 \) and (A2), we obtain the condition given in Proposition 6(c). \( \square \)

Remark When \( \lambda > 0 \), both \( p_B^{**} > p_B^{m} \) and \( p_B^{**} < p_B^{m} \) are possible as shown in Table 3. The condition for \( p_B^{**} > p_B^{m} \) is \( \gamma(1 - \frac{p_B^{m}}{q_B}) + (1 - \gamma)(1 - s^{**}(p_B^{m}))Y + (p_B - c)[1 - \gamma - (1 - \gamma)Y \frac{\partial s^{**}}{\partial p_B} + (1 - \gamma)(1 - s^{**}(p_B^{m})) \left\{ Y \frac{1}{p_B^{m}} + \frac{\lambda p_B^{m}}{1 - s^{**}(p_B^{m})} \frac{\partial s^{**}}{\partial p_B} \left( (1 - s^{**}(p_B^{m}))q_B + s^{**}(p_B^{m})(\beta q_B + f_D^{**}(p_B^{m})) \right) \right\} ] > 0 \), where \( Y = 1 - \frac{(1-\lambda)p_B^{m}}{q_B} - \frac{\lambda p_B^{m}}{1 - s^{**}(p_B^{m})q_B + s^{**}(p_B^{m})(\beta q_B + f_D^{**}(p_B^{m}))} \). This can be obtained from \( \frac{\partial^2 \pi_B^{**}}{\partial p_B^2} \bigg|_{p_B=p_B^{**}} > 0 \). The condition for \( \frac{\partial E C S_{D}^{**}}{\partial p_B} > 0 \) given in Proposition 6 is modified to the following: \( q_D^{**} \left\{ \frac{\partial s^{**}}{\partial p_B} - \frac{1 - \gamma}{2} \frac{\partial s^{**}}{\partial p_B} \right\} < \frac{\gamma}{1 - \gamma} \left( \frac{p_B}{q_B} - 1 \right) + \{ p_B - \theta q_B(1 - s^{**}) \} \frac{\partial \theta}{\partial p_B} + \frac{1 - \gamma}{2} \left\{ s^{**} \frac{\partial q_D^{**}}{\partial p_B} - q_B \frac{\partial s^{**}}{\partial p_B} \right\} \right\} - 1 + \theta. \) Unfortunately, this condition cannot be simplified further since the closed-form expressions of \( s^{**} \) and \( f_D^{**} \) do not exist. The non-monotonicity of \( E C S_{D}^{**} \) is established in Table 3.

Proof of Proposition 7: When \( \lambda = 0 \), we observe from (5.3) that \( \pi_B^{**} \) does not change with \( \beta \), and that \( \pi_B^{**} \) is increasing in \( \gamma \). When \( \lambda > 0 \), similar to the proof
of Lemma 2, we can show that the aggregate demand, $(1 - \bar{\theta})$, for the brand-name product and the deceptive counterfeit is increasing in $\beta$ and decreasing in $\gamma$, and that the fraction of the brand-name product, $(1 - s^*)$, is decreasing in $\beta$ and increasing in $\gamma$. The non-monotonicity of $\pi_B^{**}$ is shown in our numerical experiments presented in appendix B. The proofs for $\pi_D^{**}$ and $ECS_D^{**}$ are similar to those of Proposition 5(b)-(c), and hence are omitted. □

Proposition 24. For any $\lambda \in [0, 1]$, if any anti-counterfeiting strategy improves the average product quality in the market, $(1 - s^*)q_B + s^*(f_{D}^{**} + \beta q_B)$, then $ECS_D^{**}$ increases.

Proof of Proposition 24: Let us examine $\frac{\partial ECS_D^{**}}{\partial q_B}$. From the definition of $CS_D$, $\frac{\partial CS_D}{\partial q_B} > 0$ if and only if $\frac{\partial}{\partial q_B}\{(1 - s^*)q_B + s^*q_D\} > 0$. Since $\frac{\partial CS_B}{\partial q_B} > 0$, $\frac{\partial ECS_D^{**}}{\partial q_B} = (1 - \gamma)\frac{\partial CS_D}{\partial q_B} + \gamma\frac{\partial CS_B}{\partial q_B} > 0$ if $\frac{\partial}{\partial q_B}\{(1 - s^*)q_B + s^*q_D\} > 0$. The results for the other parameters can be shown similarly. □

Proof of Corollary 8: (a) Suppose $f_N^* = f$ or $\bar{f}$. We can obtain $\frac{\partial \pi_B^*}{\partial q_B}$ by replacing $\gamma$ in the base model with $\gamma - \delta_1 f_N^*$. To show that Proposition 3(a) continues to hold, we need to prove that $f_N^*$ is decreasing in $q_B$. For $f_{NH} > f_{NL}$, $\frac{\partial \pi_B^*(f_{NH})}{\partial q_B} - \frac{\partial \pi_B^*(f_{NL})}{\partial q_B}$ can be expressed as follows:

$$-\frac{p_2(1-\gamma+\delta_1 f_{NH})(q_B(\beta q_B + f_{NH})(1-\beta) + f_{NH}(q_B + \beta q_B - f_{NH}))}{8q_B^2((1-\beta)q_B - f_{NH})^2}$$
of Proposition 3(a), we can show which is negative due to (A1). Then, following the same procedure as in the proof of Proposition 3(c), to prove that follows from Proposition 3(b), when to $q_B$, $\pi^*_N(f_N, q_B) \geq \pi^*_N(f_{N2}, q_B) > \pi^*_N(f_{N2}, q_B)$, where the first inequality follows from $f_N^*(q_B) = f_{N1}$ and the second inequality is due to $\frac{\partial \pi^*_N}{\partial q_B} < 0$ due to (A1). When $f_N^*$ changes from $f_{N1}$ to $f_{N2}$ as $q_B$ is increased from $q_{BL}$ to $q_{BH}$, $\pi^*_N(f_N, q_B) \geq \pi^*_N(f_{N2}, q_B)$, where the first inequality follows from $f_N^*(q_B) = f_{N1}$ and the second inequality is due to $\frac{\partial \pi^*_N}{\partial q_B} < 0 \forall f_N$. For Proposition 3(c), to prove that $ECS^*_N$ is increasing in $q_B$, it suffices to show that $\frac{\partial ECS^*_N}{\partial q_B} > 0$. Since $\frac{\partial ECS^*_N}{\partial q_B}$ does not depend on $\gamma$ when $f_N$ is given, Proposition 3(c) continues to hold.

(b) We can show that the proof for Proposition 4 also applies to the case with $\gamma - \delta f_N$ similarly to Proposition 3, except part (c) when $f_N^* \in (\underline{f}, \bar{f})$. With the extension, $\frac{\partial ECS^*_N}{\partial p_B} = \{(1 - \beta)q_B - f_N^*\}^2$

$\left[p_B^2 \frac{\partial \pi^*_N}{\partial p_B} \{-\delta f_N^*(2\beta q_B - 2q_B + f_N^*) - q_B((\beta - 1)\beta \delta f_N + \gamma - 1)\} - 2((\beta - 1)q_B + f_N^*)\right] f_N p_B (\beta \delta f_N - \gamma - 15 + \delta f_N^* + \frac{16q_B}{p_B}) + q_B (16(\beta - 1)q_B - p_B \beta (\gamma + 15) + 16p_B)].$

Define $\kappa = \max_{p_B \in [p_B^m, p_B^m]} 2p_B^2 ((\beta - 1)q_B + f_N^*) \{f_N p_B (\beta \delta f_N - \gamma - 15 + \delta f_N^* + \frac{16q_B}{p_B}) +$
\[q_B(16(\beta - 1)q_B - p_B\beta(\gamma + 15 + 16p_B))\{-\delta_1 f_N^*(2\beta q_B - 2q_B + f_N^*) - q_B((\beta - 1)\beta\delta_1 q_B + \gamma - 1)\}^{-1}.\] Then \(ECS_N^*\) is increasing in \(p_B \in [p^*_B, p^m_B]\) so that \(ECS_N^*\) is lower at \(p^*_B\) than at \(p^m_B\) if \(\frac{\partial f_N^*}{\partial p_B} > \kappa\).

To show that Proposition 5(a) continues to hold, when \(\lambda = 0\), we can prove by contradiction that \(q^{**}_B < q^m_B\) if and only if \((1 + \delta_2 f_D)l < 4p_B(1 - \frac{pb}{q_B})\) by the same procedure as in the proof of Proposition 3(a). Since \(s^{**} = \frac{1 + \delta_2 f_D}{2} - \frac{1}{2} \sqrt{\frac{f(1 + \delta_2 f_D)}{p_B(1 - \frac{pb}{q_B})}} > 0\), we obtain \(l < p_B(1 - \frac{pb}{q_B})(1 + \delta_2 f_D) < \frac{4p_B}{1 + \delta_2 f_D}(1 - \frac{pb}{q_B})\), so \(q^{**}_B < q^m_B\) always holds. For Proposition 6(a), \(\frac{\partial s^{**}_D}{\partial p_B}|_{p_B = q_B} = \frac{c(1 - \gamma + \delta_1 f_D)}{2(c + q_B)} \sqrt{\frac{f(q_B - c)(1 + \delta_2 f_D)}{q_B(c + q_B)}} > 0\), so \(p^{**}_B > p^m_B\). For Propositions 5(b) and 6(b), we can show that, with the extension, \(s^{**}\) and \(m^{**}\) are increasing in \(q_B\) and decreasing in \(p_B\) for given \(w_D\) and \(f_D\). Then, when \(q_B\) is increased from \(q_{BL}\) to \(q_{BH}\), the following inequalities hold in equilibrium: \(\pi_{D}^{**}(w_{D}^{**}(q_{BH}), f_{D}^{**}(q_{BH}), q_{BH}) \geq \pi_{D}^{**}(w_{D}^{**}(q_{BL}), f_{D}^{**}(q_{BL}), q_{BH}) > \pi_{D}^{**}(w_{D}^{**}(q_{BL}), f_{D}^{**}(q_{BL}), q_{BL})\). Similarly, when \(p_B\) is decreased from \(p_{BH}\) to \(p_{BL}\), the following inequalities hold in equilibrium: \(\pi_{D}^{**}(w_{D}^{**}(p_{BL}), f_{D}^{**}(p_{BL}), p_{BL}) \geq \pi_{D}^{**}(w_{D}^{**}(p_{BH}), f_{D}^{**}(p_{BH}), p_{BL}) > \pi_{D}^{**}(w_{D}^{**}(p_{BH}), f_{D}^{**}(p_{BH}), p_{BH})\). Propositions 5(c) and 6(c) can be shown similarly to Proposition 4(c).

(c) The proofs of \(\pi_{D}^{**}\) and \(ECS_D^{**}\) are similar to those of Proposition 5(b)-(c). When \(\lambda = 0\), the non-monotonicity of \(\pi_{B}^{**}\) or \(ECS_D^{**}\) with respect to \(\gamma\) can be shown numerically as follows. Set \(q_B = 1\), \(p_B = 0.5\), \(t_B = t_D = 0.01\), \(c = 0.01\), \(\beta = 0.1\), \(l = 0.02\), and \(\delta_1 = \delta_2 = 0.1\). As \(\gamma\) increases from 0.2 to 0.3, \(\pi_{B}^{**}\) increases from 0.174
to 0.181 and $ECS_D^{**}$ increases from 0.070 to 0.076. As $\gamma$ increases from 0.3 to 0.4, $\pi_B^{**}$ decreases from 0.181 to 0.173 and $ECS_D^{**}$ decreases from 0.070 to 0.069. □

Appendix B. Numerical Experiments in Chapter 2

This section contains our numerical study that examines the effectiveness of the marketing campaign and the enforcement strategy against the deceptive counterfeiter. Similar to the numerical study presented in §2.5.2, we have constructed 1024 scenarios for $\lambda = 0, 0.25$ or $0.5$, using the parameter values shown in the bottom of Table 5, so that they cover various possible scenarios and also satisfy positive $s^{**}$ in equilibrium. We computed the difference in firms’ expected profits and expected consumer welfare associated with the adjacent values of $\beta$ or $\gamma$ for a fixed set of other parameter values. There are 3 increments of $\beta$ or $\gamma$ for a set of 256 possible values of $(t_i, c, \gamma, l)$, so there are 768 scenarios for which we can examine the direction of changes with a decrease of $\beta$ or an increase of $\gamma$. The results are summarized in Table 5, which reads as follows: for example, when $\lambda = 0.5$, reducing $\beta$ increased $\pi_B^{**}$ in 33.1% of 768 scenarios, decreased $\pi_D^{**}$ in all scenarios, and increased $ECS_D^{**}$ in 13.9% of 768 scenarios.

Table 5. Effects of Marketing Campaigns and Enforcement against Deceptive Counterfeits


Table 5 confirms the results stated in Proposition 7. In addition, similar to the quality and pricing strategies discussed in §2.5.2, these strategies are not necessarily more effective as more consumers are proactive with higher $\lambda$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & Effects of Reducing $\beta$ & & Effects of Increasing $\gamma$ & \\
 & $\pi_B^{**} \uparrow$ & $\pi_D^{**} \downarrow$ & $ECS_D^{**} \uparrow$ & $\pi_B^{**} \uparrow$ & $\pi_D^{**} \downarrow$ & $ECS_B^{**} \uparrow$ \\
\hline
$\lambda = 0$ & no change & no change & 0 & 1 & 1 & 1 \\
\hline
$\lambda = 0.25$ & 0.374 & 1 & 0.260 & 1 & 1 & 0.952 \\
\hline
$\lambda = 0.5$ & 0.331 & 1 & 0.139 & 0.990 & 1 & 0.927 \\
\hline
\end{tabular}
\end{table}

(Note) Each number in the table indicates a percent of scenarios for the corresponding effect. We used the following parameters: $t \in \{0.005,0.01,0.015,0.02\}$, $\beta \in \{0.1,0.2,0.3,0.4\}$, $\gamma \in \{0.1,0.2,0.3,0.4\}$, $l \in \{0.005,0.01,0.015,0.02\}$, $c \in \{0.005,0.01,0.015,0.02\}$, $f = 0.1$, and $\mathcal{F} = (1 - \beta) \times q_B - 0.01$. $(q_B, p_B)$ is fixed at $(q_B^0, p_B^0)$.

Appendix C. Extension: Price Decision of Licit Distributor

Suppose, instead of the brand-name company, the licit distributor decides the retail price of the brand-name product, $p_B$, when combating the deceptive counterfeiter. The brand-name company instead decides the wholesale price, $w_B$, to the licit distributor. The rest of the decisions remain the same as in the base model. The sequence of decisions is as follows: After observing the quality $q_B$ and wholesale price $w_B$ of the brand-name product, the deceptive counterfeiter decides his functional quality $f_D$ and wholesale price $w_D$ in stages 1 and 2, respectively. In stage 3, the licit distributor decides a fraction of deceptive counterfeits $s$, and
then decides the retail price $p_B$. Note that the licit distributor can source products from the brand-name company at the wholesale price $w_B$, and/or from the deceptive counterfeiter at the wholesale price $w_D$, should counterfeit goods reach the market.

We analyze this model backwards. In stage 3, the licit distributor first solves the following problem to determine $p_B$:

(5.4) \[
\max_{p_B} (1-s) \{ p_B - sw_D - (1-s)w_B \} \left\{ 1 - \frac{\lambda p_B}{(1-s)q_B + s(f_D + \beta q_B)} - \frac{(1-\lambda)p_B}{q_B} \right\} - sl.
\]

One can verify from (5.4) that the profit of the licit distributor is concave in $p_B$. We obtain from the first-order condition the optimal retail price: 

\[
p_B^{**} = \frac{sw_D + (1-s)w_B}{2} + \frac{1}{2} \left( \frac{\lambda}{(1-s)q_B + s(f_D + \beta q_B)} + \frac{1-\lambda}{q_B} \right)^{-1}.
\]

Since $w_D < w_B$ and $f_D + \beta q_B < q_B$, the distributor charges a lower retail price $p_B^{**}$ when the fraction of counterfeits $s$ is larger or the fraction of proactive consumers $\lambda$ is larger.

Next, to find the optimal fraction $s^{**}$, the licit distributor solves the following problem which is obtained by substituting $p_B^{**}$ into (5.4):

(5.5) \[
\max_s \frac{1}{4} (1-s) \left\{ \frac{\lambda}{(1-s)q_B + s(f_D + \beta q_B)} + \frac{1-\lambda}{q_B} \right\}
\times \left[ sw_D + (1-s)w_B - \left\{ \frac{\lambda}{(1-s)q_B + s(f_D + \beta q_B)} + \frac{1-\lambda}{q_B} \right\}^{-1} \right]^2 - sl
\]
Due to complexity, however, it is not possible to find the closed-form expression of 
$s^{**}$ from (5.5). Although the part of our analysis in the base model does not rely on its closed-form expression, the impact of any anti-counterfeiting strategy on $s^{**}$ becomes prohibitively complex to obtain any analytical result. Thus, we conduct extensive numerical experiments to examine the effects of the anti-counterfeiting strategies. We use the same set of parameters as in appendix B. For each case with $\lambda = 0$ and $\lambda = 0.5$, there are 1024 scenarios in which we can investigate the anti-counterfeiting strategies that change quality or price from the case with no counterfeiter to the optimal levels. On the other hand, similar to Table 5, there are 768 scenarios for which we can examine the anti-counterfeiting strategies that reduce $\beta$ or increase $\gamma$. We present a summary of the results in Table 6, which reads similarly to Table 5.

Table 6. Effects of Quality and Pricing Strategies in the Extended Model

<table>
<thead>
<tr>
<th></th>
<th>$\pi_B^{**}$</th>
<th>$\pi_D^{**}$</th>
<th>$ECS_D^{**}$</th>
<th>$\pi_B^{**}$</th>
<th>$\pi_D^{**}$</th>
<th>$ECS_D^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^m_B \rightarrow q^{**}_B$</td>
<td>1</td>
<td>0.320</td>
<td>0.672</td>
<td>1</td>
<td>0.998</td>
<td>0.647</td>
</tr>
<tr>
<td>$w^m_B \rightarrow w^{**}_B$</td>
<td>1</td>
<td>0.508</td>
<td>0.805</td>
<td>1</td>
<td>0.998</td>
<td>0.647</td>
</tr>
<tr>
<td>$\beta$ ↓</td>
<td>no change</td>
<td>no change</td>
<td>0</td>
<td>0.135</td>
<td>1</td>
<td>0.135</td>
</tr>
<tr>
<td>$\gamma$ ↑</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.779</td>
<td>1</td>
<td>0.798</td>
</tr>
</tbody>
</table>

Table 6 shows that the effects of anti-counterfeiting strategies remain directionally true in this extended model. For example, as the price changes from $w^m_B$ to $w^{**}_B$, $\pi_D^{**}$ and $ECS_D^{**}$ can increase or decrease; this is consistent with Proposition 6. Also, as stated in Proposition 7, when $\lambda = 0$, reducing $\beta$ has no impact on $\pi_B^{**}$.
and $\pi_D^{**}$, but reduces $ECS_D^{**}$, whereas increasing $\gamma$ reduces $\pi_D^{**}$ and increases $\pi_B^{**}$ as well as $ECS_B^{**}$; when $\lambda > 0$, reducing $\beta$ or increasing $\gamma$ reduces $\pi_D^*$, but it can increase or reduce $\pi_B^{**}$ and $ECS_D^{**}$. One notable exception is that when $\lambda = 0$, changing $q_B^{m}$ to $q_B^{*}$ can increase $\pi_D^{**}$ although it always reduces $\pi_D^{**}$ in the base model. This happens because of the additional lever (i.e., determining $p_B$ as well as $s$) the licit distributor has in this extended model. In response to the change of $q_B$, the distributor can increase the aggregate demand for both brand-name and counterfeit goods by reducing the retail price $p_B$. As a result, we find that the distributor may increase or decrease the fraction of counterfeits, $s^{**}$, in response to this strategy, which thus creates a non-monotonic effect on $\pi_D^{**}$.

Appendix D. Proofs of Analytical Results in Chapter 3

Proof of Proposition 9: For a given network $g$, the value function $v$ given in (3.6)-(3.9) is obtained by solving a linear program. So, the cooperative transshipment game given a network $g$ can be viewed as the linear production game considered by Owen (1975). Moreover, this game can be modeled as the market game of Shapley and Shubik (1975) in which players trade products with each other. Then the result follows from Shapley and Shubik (1975) who have shown that the dual price allocation is in the core of a market game. Q.E.D.
Remark: Essentially, this proof is the same as the proof of Theorem 4.1 in Anupindi et al. (2001) because the linearity in (3.6)-(3.9) is preserved in our network setting.

Proof of Proposition 10: Consider the case when only one supply node exists in the network. The proof for the case when only one demand node exists follows the same procedure. To prove that the pair \((g^N, \varphi^{MJW}(g^N))\) is in the core of \((N, v)\), it suffices to show that \((N, v)\) is convex (Jackson 2006). We prove that \((N, v)\) is convex by showing that \(v(g|_{S'' \cup \{i\}}) - v(g|_{S' \cup \{i\}}) \leq v(g|_{S'' \cup \{i\}}) - v(g|_{S'})\) for any subsets \(S'\) and \(S''\) of \(N\) that satisfy \(S'' \subset S'\) and any node \(i \notin S'\) in each of the following two cases: (Case 1) node \(i\) is the supply node, and (Case 2) node \(i\) is not the supply node.

(Case 1): If \(i\) is the supply node, then neither \(S'\) nor \(S''\) contains any supply nodes, so \(v(g^N|_{S''}) = v(g^N|_{S'}) = 0\). Thus, if \(v(g^N|_{S' \cup \{i\}}) \geq v(g^N|_{S'' \cup \{i\}})\), then \((N, v)|_{S' \cup \{i\}} - v(g^N|_{S'})) \geq v(g^N|_{S'' \cup \{i\}}) - v(g^N|_{S''})\). To prove that \((N, v)|_{S' \cup \{i\}} \geq v(g^N|_{S'' \cup \{i\}})\), we define \(f(g|_S, Y) \equiv \sum_{i \in S} \{a_i(\sum_{j \in B_i(g|_S)} Y_{ji} - \sum_{j \in B_i(g|_S)} Y_{ij}) - \sum_{j \in B_i(g|_S)} Y_{ij}(t_{ij} + u_i - u_j)\}\), which is the objective function of the program given in (3.6). Then, \((N, v)|_{S' \cup \{i\}} = f(g^N|_{S' \cup \{i\}}, Y^N|_{S' \cup \{i\}}) \geq f(g^N|_{S'' \cup \{i\}}, Y^N|_{S'' \cup \{i\}}) = f(g^N|_{S'' \cup \{i\}}, Y^N|_{S'' \cup \{i\}}) = v(g^N|_{S'' \cup \{i\}})\) because:

- By the definition of optimal transshipment patterns, \(v(g^N|_{S' \cup \{i\}}) = f(g^N|_{S' \cup \{i\}}, Y^N|_{S' \cup \{i\}})\)
and \( v(g^N|_{S'' \cup \{i\}}) = f(g^N|_{S'' \cup \{i\}}, Y^g^N|_{S'' \cup \{i\}}) \);

- Since \( S'' \subset S' \), the transshipment pattern \( Y^g^N|_{S'' \cup \{i\}} \) is feasible to the program (3.6)-(3.9) with \( g = g^N \) and \( S = S' \cup \{i\} \), so that \( f(g^N|_{S' \cup \{i\}}, Y^g^N|_{S'' \cup \{i\}}) = f(g^N|_{S'' \cup \{i\}}, Y^g^N|_{S'' \cup \{i\}}) \);

- Because \( Y^g^N|_{S' \cup \{i\}} \) is the optimal transshipment pattern among all the feasible transshipment patterns given the network \( g^N|_{S' \cup \{i\}} \), \( f(g^N|_{S' \cup \{i\}}, Y^g^N|_{S'' \cup \{i\}}) \geq f(g^N|_{S' \cup \{i\}}, Y^g^N|_{S'' \cup \{i\}}) \).

(Case 2): Suppose \( j \neq i \) is the supply node. There are three possibilities:

(a) If \( j \notin S' \) (hence \( j \notin S'' \)), then \( v(g^N|_{S'' \cup \{i\}}) = v(g^N|_{S''}) = v(g^N|_{S' \cup \{i\}}) = v(g^N|_{S'}) = 0 \) because \( S' \cup \{i\}, S', S'' \cup \{i\}, \) and \( S'' \) have no supply nodes.

(b) If \( j \in S' \) and \( j \notin S'' \), then \( v(g^N|_{S'' \cup \{i\}}) - v(g^N|_{S'}) \geq 0 = v(g^N|_{S'' \cup \{i\}}) - v(g^N|_{S''}) \), where the inequality is due to the fact that \( S' \cup \{i\} \) and \( S' \) include the supply node \( j \) and \( \sum_{k \in S'' \cup \{i\}} E_k \geq \sum_{k \in S'} E_k \), and the equality is because \( S'' \cup \{i\} \) and \( S'' \) have no supply nodes.

(c) If \( j \in S'' \) (hence \( j \in S' \)), \( v(g^N|_{S'' \cup \{i\}}) - v(g^N|_{S'}) = v(g^N|_{S'' \cup \{i\}}) - v(g^N|_{S''}) = (r_i - u_j - t^\min_{ji})E_i \), where \( t^\min_{ji} \) is the minimum of the transportation costs from \( j \) to \( i \) among all possible paths. The equations hold because \( S' \cup \{i\}, S', S'' \cup \{i\}, \) and \( S'' \) include the supply node \( j \) with \( H_j \geq \sum_{k \in N \setminus \{i\}} E_k \). Q.E.D.
Remark: Suppose direct shipment incurs a minimum transportation cost among all paths between every pair of two firms and transshipment is always profitable (i.e., \( r_{\alpha} - u_{\beta} - t_{\beta \alpha} > 0 \) for all \( \alpha, \beta \in N \)). Then, the necessary and sufficient condition for the convexity to hold is that there is only one supply (resp., demand) node \( j \) with \( H_j \geq \sum_{k \in N \setminus \{j\}} E_k \) (resp., \( E_j \geq \sum_{k \in N \setminus \{j\}} H_k \)). Sufficiency is shown in the proof of Proposition 10. In the following, we show the necessity in two steps.

First, we show that the convexity does not hold when multiple demand nodes and multiple supply nodes exist in the complete network. We prove this by contradiction. Consider the complete network \( g^N \) with multiple demand nodes and multiple supply nodes. Without loss of generality, suppose \( E_1, E_2, H_3, \) and \( H_4 \) are all positive. If the convexity holds, then
\[
 v(g^N|\{1,2,3\} \cup \{4\}) - v(g^N|\{1,2,3\}) \geq v(g^N|\{1,2\} \cup \{4\}) - v(g^N|\{1,2\}).
\]
This indicates that \( E_1 + E_2 \geq H_3 + H_4 \) because otherwise there are more residual demands available within \( \{1,2\} \) than within \( \{1,2,3\} \) (i.e., \( E_1 + E_2 > E_1 + E_2 - H_3 \)) and
\[
 v(g^N|\{1,2,3\} \cup \{4\}) - v(g^N|\{1,2,3\}) < v(g^N|\{1,2\} \cup \{4\}) - v(g^N|\{1,2\}).
\]
Similarly, since
\[
 v(g^N|\{2,3,4\} \cup \{1\}) - v(g^N|\{2,3,4\}) \geq v(g^N|\{3,4\} \cup \{1\}) - v(g^N|\{3,4\}),
\]
\( E_1 + E_2 \leq H_3 + H_4 \). To satisfy these two inequalities simultaneously, \( E_1 + E_2 = H_3 + H_4 \). Further, since
\[
 v(g^N|\{2,3\} \cup \{4\}) - v(g^N|\{2,3\}) \geq v(g^N|\{2\} \cup \{4\}) - v(g^N|\{2\}),
\]
\( E_2 \geq H_3 + H_4 \). Because \( E_2 = H_3 + H_4 - E_1 \geq H_3 + H_4 \) and \( E_1 \geq 0 \), \( E_1 = 0 \), which contradicts our premise that \( E_1 > 0 \).
Second, we show that, given there exists only one supply node \( j \), the convexity does not hold when \( H_j < \sum_{k \in \mathcal{N} \setminus \{j\}} E_k \). (The case when only one demand node exists follows the same procedure, and hence omitted.) We prove this by finding \( S' \) and \( S'' \) (\( \subset S' \)) that do not satisfy \( v(g|_{S' \cup \{i\}}) - v(g|_{S''}) \leq v(g|_{S' \cup \{i\}}) - v(g|_{S'}) \).

Since \( H_j < \sum_{k \in \mathcal{N} \setminus \{j\}} E_k \), there exist \( S' \) and \( S'' \) such that \( H_j - \sum_{k \in S'} E_k < E_i \) and \( H_j - \sum_{k \in S''} E_k > 0 \) (\( S'' \) can be the empty set) for \( i \notin S' \). Then, \( v(g^N|_{S' \cup \{i\}}) - v(g^N|_{S'}) = (r_i - u_j - t_{ji}) \max(H_j - \sum_{k \in S'} E_k, 0) \) and \( v(g^N|_{S' \cup \{i\}}) - v(g^N|_{S''}) = (r_i - u_j - t_{ji}) \min(H_j - \sum_{k \in S''} E_k, E_i) \) because direct shipment incurs a minimum transportation cost and transshipment is always profitable. Therefore, \( v(g^N|_{S' \cup \{i\}}) - v(g^N|_{S'}) < v(g^N|_{S' \cup \{i\}}) - v(g^N|_{S''}) \) because \( H_j - \sum_{k \in S'} E_k < H_j - \sum_{k \in S''} E_k, H_j - \sum_{k \in S'} E_k < E_i, H_j - \sum_{k \in S''} E_k > 0 \) and \( E_i > 0 \).

**Proof of Proposition 11:** Consider the case when the hub is a supply node. (The proof for the case when the hub is a demand node follows the same procedure.) Similar to the proof of Proposition 10, we prove that the pair \( (g^H, \varphi^{MJW}(g^H)) \) is in the core by showing that \( v(g^H|_{S' \cup \{i\}}) - v(g^H|_{S''}) \leq v(g^H|_{S' \cup \{i\}}) - v(g^H|_{S'}) \) for any subsets \( S' \) and \( S'' \) of \( \mathcal{N} \) that satisfy \( S'' \subset S' \), and for any node \( i \notin S' \). We consider the following three cases: (Case 1) node \( i \) is the hub, (Case 2) node \( i \) is a supply node but not the hub, and (Case 3) node \( i \) is a demand node.
(Case 1): The proof is similar to that in (Case 1) of Proposition 10 because, given
i ≠ S', we have \( B_k(gH|S') = \emptyset \) ∀k ∈ S' and \( B_k(gH|S'') = \emptyset \) ∀k ∈ S'' so that
\( v(gN|S') = v(gN|S'') = 0. \)

(Case 2): Suppose \( j \neq i \) is the hub. There are three possibilities:

(a) If \( j \notin S' \) (hence \( j \notin S'' \)), then \( v(gH|S' \cup \{i\}) = v(gH|S'') = v(gH|S' \cup \{i\}) = v(gH|S') = 0 \) because \( B_k(gH|S' \cup \{i\}) = \emptyset \) ∀k ∈ S' ∪ \{i\} and \( B_k(gH|S'' \cup \{i\}) = \emptyset \) ∀k ∈ S'' ∪ \{i\} without the hub.

(b) If \( j \in S' \) and \( j \notin S'' \), then \( v(gH|S' \cup \{i\}) - v(gH|S') \geq 0 = v(gH|S'' \cup \{i\}) = v(gH|S'') \) where the inequality is due to the fact that \( S' \cup \{i\} \) and \( S'' \) do not include the hub, and the equality is because \( S'' \cup \{i\} \) and \( S'' \) do not include the hub.

(c) If \( j \in S'' \) (hence \( j \in S' \)), \( v(g^N|S' \cup \{i\}) - v(g^N|S') = v(g^N|S'' \cup \{i\}) - v(g^N|S'') = 0 \) because \( H_i \) generates zero value given \( H_j \geq \sum_{k \in N \setminus \{j\}} E_k. \)

(Case 3): The proof is similar to the proof of (Case 2) of Proposition 10, in which the hub acts as the supply node \( j \) which ships inventory to node \( i \). Q.E.D.

**Proof of Proposition 12:** For sufficiency, we show that the pair \((g^L, \varphi^{MJW}(g^L))\) is always in the core, so any subnetwork of \( g^L \) has no incentive to secede from \( g^L \) under the MJW value. Clearly, any subnetwork with no supply nodes or no demand nodes has no incentive to secede because they generate no profit. Thus, in the rest of the proof, we consider subnetworks that contain at least one supply node and one
Figure 5.1. Line $g^L$ and its subnetwork $g^L|_{\{i_{k-e+1},\ldots,i_{k+h}\}}$.

demand node. Without loss of generality, consider the subnetwork $g^L|_{\{i_{k-e+1},\ldots,i_{k+h}\}}$ of the line $g^L$ shown in Figure 5.1. Note that $g^L$ contains $k$ firms with $E_{ij} = 0$ for $j = 1, 2, \ldots, k$, and $(n - k)$ firms with $H_{ij} = 0$ for $j = k + 1, k + 2, \ldots, n$, while $g^L|_{\{i_{k-e+1},\ldots,i_{k+h}\}}$ contains $e$ ($\leq k$) firms with $E_{ij} = 0$ for $j = k - e + 1, \ldots, k$, and $h$ ($\leq n - k$) firms with $H_{ij} = 0$ for $j = k + 1, \ldots, k + h$, where $e = 1, 2, \ldots, k$, and $h = 1, 2, \ldots, n - k$.

To prove that $g^L|_{\{i_{k-e+1},\ldots,i_{k+h}\}}$ has no incentive to secede from $g^L$, we show that the allocations to the firms in $g^L|_{\{i_{k-e+1},\ldots,i_{k+h}\}}$ are non-decreasing when an outside firm is added to this subnetwork. Consider adding firm $i_{k-e}$ with no residual demand to this subnetwork. (The proof for adding a firm with no residual supply is similar, and hence omitted.) If $H_{i_{k-e}} = 0$, the allocations are unchanged, so we focus on the case when $H_{i_{k-e}} > 0$. First, consider the allocations to firm $i_j$ for $j = k - e + 1, \ldots, k$. We shall show that, when $i_{k-e}$ is added, the marginal contributions of firm $i_j$ are non-decreasing in all the orderings of the firms in $g^L|_{\{i_{k-e},i_{k-e+1},\ldots,i_{k+h}\}}$. Without loss of generality, consider two orderings before and after firm $i_{k-e}$ is
added: (1) ordering \( \{i'_1, \ldots, i'_{e+h}\} \), and (2) ordering \( \{i''_1, \ldots, i''_{e+h+1}\} \) with \( i''_\alpha = i'_{k-e}, i''_0 = i'_j \) for \( j < \alpha \), and \( i''_j = i'_{j-1} \) for \( j > \alpha \); i.e., \( \alpha \leq e+h+1 \) is the position of firm \( i_{k-e} \) in the second ordering and the two orderings have the same sequence of firms except \( i_{k-e} \). Suppose the positions of firm \( i_j \), whose allocations are under consideration, in the two orderings are \( \beta_1 \) and \( \beta_2 \), respectively. We can define the sets of firms before firm \( i_j \) in the two orderings, respectively, \( S' = \{i'_1, \ldots, i'_{\beta_1-1}\} \) and \( S'' = \{i''_1, \ldots, i''_{\beta_2-1}\} \). If \( i_{k-e} \notin S' \), then we have \( S' = S'' \), so \( v(g^L|_{S'\cup\{i_j\}}) - v(g^L|_{S'}) = v(g^L|_{S''\cup\{i_j\}}) - v(g^L|_{S''}) \). If \( i_{k-e} \in S'' \), it can be shown that \( v(g^L|_{S'}) = v(g^L|_{S''}) \) because firm \( i_{k-e} \) is not connected to any demand node without \( i_j \). Furthermore, we have \( v(g^L|_{S''\cup\{i_j\}}) \geq v(g^L|_{S'\cup\{i_j\}}) \) because more residual supplies are available within the subnetwork after firm \( i_{k-e} \) is added. Therefore, we obtain \( v(g^L|_{S''\cup\{i_j\}}) - v(g^L|_{S'}) \geq v(g^L|_{S'\cup\{i_j\}}) - v(g^L|_{S''}) \). Second, it can be shown in a similar way that, when \( i_{k-e} \) is added, the allocations to \( i_j \) for \( j = k + 1, \ldots, k + h \) are non-decreasing as well. As a result, no subnetwork in \( g^L \) has an incentive to secede.

For necessity, we show that if the pair \( (g, \varphi^{MJW}(g)) \) is in the core for any residual amounts (and thus for any numbers of supply and demand nodes), network \( g \) must be the line \( g^L \). We conduct the proof by showing the following two properties must hold for the network \( g \) such that the pair \( (g, \varphi^{MJW}(g)) \) is in the core for any residual amounts: (Property 1) every demand (resp., supply) node has at most
one link to supply (resp., demand) nodes, and (Property 2) every firm has at most two links. Then, the only network structure that satisfies these two properties for any residual amounts and any numbers of supply and demand nodes is the line.

First, we show that if $g$ does not satisfy Property 1, then $(g, \varphi^{M,JW}(g))$ is not always in the core for any residual amounts. For this proof, it suffices to find a network $g$ having a demand node with more than one link to supply nodes, and show that a subnetwork of $g$ has an incentive to secede from $g$ for certain quantities of residual supplies and demands. Consider $g$ that contains firm 1 with $E_1 = 1$ and $B_1(g) > 1$, and $(n - 1)$ firms with residual supplies. Let firm 2 denote a firm with the minimum value of $u_i + t_{i1}$ among all $i \in B_1(g)$, and assume $H_2 = 1$. The profit generated by transshipment is $v(g) = r_1 - u_2 - t_{21}$. If there exists any other firm $i \in B_1(g)$ such that $H_i > 0$ and $r_1 - u_i - t_{i1} > 0$, it receives a strictly positive allocation based on the M JW value according to (3.10) because firm $i$ makes a positive marginal contribution to $g$. Then the sum of allocations to firms 1 and 2 is strictly less than $r_1 - u_2 - t_{21}$ because the presence of another supply node $i$ reduces the marginal contribution of firm 2. On the other hand, if firms 1 and 2 secede, they can generate a profit of $r_1 - u_2 - t_{21}$ which is larger than the allocations they receive in the initial network $g$.

Second, similarly to Property 1, we show that if $g$ does not satisfy Property 2, then $(g, \varphi^{M,JW}(g))$ is not always in the core for any residual amounts. As in the
proof for Property 1, consider firm 1 with $E_1 = 1$ and $(n - 1)$ firms with residual supplies. Given firm 1 is only linked to firm 2 to prevent any subnetwork from seceding from $g$, we show that there exists at most one firm $i$ with $H_i > 0$ linked to firm 2; otherwise, there exists a subnetwork which has an incentive to secede. As shown in Figure 5.2(a), denote the firm $i \in B_2(g)$ in the network $g$ with the minimum value of $u_i + t_{i2}$ as firm 3. Set $H_2 = 0.5$ and $H_3 = 0.5$. Then the profit generated by transshipment is $v(g) = r_1 - 0.5u_2 - t_{21} - 0.5u_3 - 0.5t_{32}$. If there exists any other firm $i \in B_2(g)$ such that $H_i > 0$ and $r_1 - u_i - t_{i2} - t_{21} > 0$, it receives strictly positive allocation based on the MJW value, e.g. firm 4. On the other hand, if firms 1, 2 and 3 secede, they can still generate a profit of $r_1 - 0.5u_2 - t_{21} - 0.5u_3 - 0.5t_{32}$ which is greater than the allocations they receive in the initial network $g$. Next, we consider the subnetwork in Figure 5.2(b) to show that, other than firm 2, there exists at most one firm $i$ with $H_i > 0$ which is linked to firm 3. We denote firm 4 as the firm with the minimum value of $u_i + t_{i3}$ for $i \in B_3(g)$ in the network $g$. Set $H_2 = 0.5$, $H_3 = 0.25$ and $H_4 = 0.25$. By the same argument as above, we know firms 1, 2, 3 and 4 have an incentive to secede if firm 5 is also linked to firm 3. Repeating the same procedure for firms 4, ..., $n$, we can show that network $g$ must be a line if the pair $(g, \varphi^{MJW}(g))$ is always in the core. Q.E.D.
Figure 5.2. Subnetworks that have incentives to secede from the network $g$.

**Remark:** The chain does not satisfy Property 1 while it satisfies Property 2, because, in the case with only one demand (resp., supply) node, the demand (resp., supply) node is linked to two supply (resp., demand) nodes in the chain.

**Proof of Corollary 13:** First, since $\varphi_{i}^{M JW}(g^C) \leq v(g^C) - v(g^C|_{N \setminus \{i\}}) \forall i \in N$, we know that any subnetworks with $n - 1$ firms have no incentive to secede from $g^C$. Next, similar to the proof of the sufficiency of Proposition 12, we consider a subnetwork of $g^C$ with less than $n - 1$ firms and show that the allocations to the firms in this subnetwork are non-decreasing when an outside firm is added. For the subnetworks in which all the demand nodes are on one side and all the supply nodes are on the other side, the proof is the same as that for the line. Without loss of generality, we consider the subnetwork $g^C|_{\{i_1, i_{k+1}, i_{n-h_2+1}, \ldots, i_n\}}$ of the chain $g^C$ shown in Figure 5.3. Note that $g^C$ contains $k$ firms with $E_{ij} = 0$ for $j = 1, 2, \ldots, k$, and $(n - k)$ firms with $H_{ij} = 0$ for $j = k + 1, k + 2, \ldots, n$, while $g^C|_{\{i_1, i_{k+1}, i_{n-h_2+1}, \ldots, i_n\}}$ contains $k$ firms with $E_{ij} = 0$ for $j = 1, 2, \ldots, k$, and $h_1 + h_2$
Figure 5.3. Chain $g^C$ and its subnetwork $g^C|\{i_1, \ldots, i_{k_h + h_1}, i_{n - h_2 + 1}, \ldots, i_n\}$.

$(\leq n - k - 2)$ firms with $H_{ij} = 0$ for $j = k + 1, \ldots, k + h_1$ and $j = n - h_2 + 1, \ldots, n$ where $h_1 + h_2 = 2, 3, \ldots, n - k - 2$.

Consider adding firm $i_{n - h_2}$ to this subnetwork. If $E_{i_{n - h_2}} = 0$, the allocations are unchanged, so we focus on the case when $E_{i_{n - h_2}} > 0$. Adding residual demands does not reduce the marginal values of residual supplies and $i_{n - h_2}$ is not connected to any supply node without $i_j$ for $j = n - h_2 + 1, \ldots, n$. Thus, it can be shown in the same way as the proof of Proposition 12 that the allocations to firm $i_j$ for $j = 1, \ldots, k$ and $j = n - h_2 + 1, \ldots, n$ are non-decreasing when $i_{n - h_2}$ is added. In the rest of this proof, we show that the marginal contributions of firm $i_j$ for $j = k + 1, \ldots, k + h_1$ are also non-decreasing in all the orderings of the firms in $g^C|\{i_1, \ldots, i_{k_h + h_1}, i_{n - h_2 + 1}, \ldots, i_n\}$. Without loss of generality, consider two orderings before and after firm $i_{n - h_2}$ is added with the sequence of other firms being the same: (1) ordering $\{i_1', \ldots, i_{k + h_1 + h_2}'\}$, and (2) ordering $\{i_1'', \ldots, i_{\alpha}'', \ldots, i_{k + h_1 + h_2 + 1}''\}$ with $i''_{\alpha} =$
\[ i_{n-h_2}, i''_j = i'_j \text{ for } j < \alpha, \text{ and } i''_j = i'_{j-1} \text{ for } j > \alpha. \] Suppose the positions of firm \( i_j \), whose allocations are under consideration, in the two orderings are \( \beta_1 \) and \( \beta_2 \), respectively. We can define the sets of firms before firm \( i_j \) in the two orderings, respectively, \( S' = \{ i'_1, \ldots, i'_{\beta_1-1} \} \) and \( S'' = \{ i''_1, \ldots, i''_{\beta_2-1} \} \). If \( i_{n-h_2} \notin S'' \), then we have \( S' = S'' \), so \( v(g_C|S_{\cup(i_j)}) - v(g_C|S') = v(g_C|S_{\cup(i_j)}) - v(g_C|S') \). If \( i_{n-h_2} \in S'' \) and there exists \( i_m \notin S'' \) for \( m = 1, \ldots, k \), then \( v(g_C|S_{\cup(i_j)}) - v(g_C|S') = v(g_C|A_{ij}(i_j)) - v(g_C|A_{ij}(i_j)) = v(g_C|S_{\cup(i_j)}) - v(g_C|S') \) where \( A_{ij} \) is the set of firms that are directly or indirectly connected to \( i_j \) in \( g_C|S_{\cup(i_j)} \). The first equality holds because the marginal value of \( i_j \) depends only on the firms it is connected to. The second equality holds because \( i_j \) and \( i_{n-h_2} \) are not connected in \( g_C|S_{\cup(i_j)} \). Finally, if \( i_{n-h_2} \in S'' \) and \( i_m \in S'' \forall m = 1, \ldots, k \), then \( v(g_C|S_{\cup(i_j)}) - v(g_C|S') = v(g_C|S_{\cup(i_j)}) - v(g_C|S') \) because \( \sum_{i \in N} H_i = \sum_{i \in N} E_i \). Therefore, no subnetwork of \( g_C \) has an incentive to secede from \( g_C \). Q.E.D.

**Proof of Proposition 14:** Similar to the proof of Proposition 10, we can show that \( v(g + ij|S) - v(g|S) \geq 0 \) for all \( S \subseteq N \). When \( v(g + ij|S) - v(g|S) = 0 \) for all \( S \subseteq N \), \( \varphi_k^{M JW}(g + ij) - \varphi_k^{M JW}(g) = 0 \) for all \( k \in N \) due to (3.10). Therefore, \[ \sum_{k \in S} \varphi_k^{M JW}(g + ij) = \sum_{k \in S} \varphi_k^{M JW}(g) \geq v(g|S) = v(g + ij|S), \] where the inequality holds because the pair \( (g, \varphi^{M JW}(g)) \) is in the core. When \( v(g + ij|S) - v(g|S) > 0 \), \[ \sum_{k \in S} \varphi_k^{M JW}(g + ij) - v(g + ij|S) \geq \sum_{k \in S} \varphi_k^{M JW}(g) - v(g|S) \geq 0 \] because \( v(g + ij|S) - v(g|S) > 0 \).
Proof of Proposition 15: We prove that the complete network is pairwise Nash stable under the MJW value by showing that the complete network is pairwise stable (i.e., any firms $i$ and $j$ have no incentives to sever link $ij$), and that it is Nash stable based on backward induction. Consider the ordering $\{i_1, i_2, ..., i_\alpha, ..., i_n\}$ used to calculate the MJW value with $i_\alpha = i$. Firm $j$ can be anywhere in this ordering. Define set $S' = \{i_1, i_2, ..., i_{\alpha-1}\}$, i.e. the set of firms before firm $i$ in this ordering. Then we have $v(g^N|_{S'}) - v(g^N - ij|_{S'}) = 0$ because $S'$ does not contain firm $i$. Also, $v(g^N|_{S'\cup\{i\}}) - v(g^N - ij|_{S'\cup\{i\}}) \geq 0$ because $g^N|_{S'\cup\{i\}}$ has one more link than $g^N - ij|_{S'\cup\{i\}}$. With different orderings, $S'$ can be any subset of $N\setminus\{i\}$. Therefore, from the definition of the MJW value given in (3.10), we obtain $\varphi_i^{MJW}(g^N) - \varphi_i^{MJW}(g^N - ij) = \sum_{S' \subseteq N \setminus \{i\}} \left\{ v(g^N|_{S'\cup\{i\}}) - v(g^N - ij|_{S'\cup\{i\}}) \right\} \left\{ \frac{\#S'!(n-\#S'-1)!}{n!} \right\} \geq 0$. The same argument holds for firm $j$, so the complete network is pairwise stable under the MJW value. To prove Nash stability, consider the last link $i_mj_m$ in the ordering $(i_1j_1, i_2j_2, ..., i_mj_m)$. Using an argument similar to the above, we can show that $\varphi_{i_m}^{MJW}(g + i_mj_m) - \varphi_{i_m}^{MJW}(g) \geq 0$ and $\varphi_{j_m}^{MJW}(g + i_mj_m) - \varphi_{j_m}^{MJW}(g) \geq 0$, where $g$ is the network formed before link $i_mj_m$ appears in the ordering. In other words, firms $i_m$ and $j_m$ weakly prefer to form link $i_mj_m$ given $g$. Similarly,
anticipating that link $i_m j_m$ will be formed, firms $i_{m-1}$ and $j_{m-1}$ weakly prefer to form link $i_{m-1} j_{m-1}$. Repeating this process to links $i_{m-2} j_{m-2}, i_{m-3} j_{m-3}, \ldots, i_1 j_1$, by backward induction, the complete network is subgame-perfect Nash equilibrium.

Next, we show that no firm would find it weakly profitable to sever link $ij$ in the complete network if there exists $S \subseteq N$ such that the transshipment problem within $S$ given in (3.6)-(3.9) has a unique optimal transshipment pattern $Y^gN|s$ with $Y^gN|s > 0$. Consider the ordering \{i_1, i_2, \ldots, i_\alpha, \ldots, i_n\} with $i_\alpha = i$ and the set $S' = \{i_1, \ldots, i_{\alpha-1}\}$. When $S' = S$, we have $v(g^N|_{S' \cup \{i\}}) - v(g^N - ij|_{S' \cup \{i\}}) > 0$ because $Y^gN|s > 0$ implies that link $ij$ is used for transshipment in $g^N$ but cannot be used in $g^N - ij$. Thus, we obtain the following inequalities $\varphi_i^{MJW}(g^N) - \varphi_i^{MJW}(g^N - ij) \geq \left\{ v(g^N|_{S' \cup \{i\}}) - v(g^N - ij|_{S' \cup \{i\}}) \right\} \left\{ \frac{\#S'\setminus\{i\} - \#S'\setminus\{i\}!}{n!} \right\} > 0$, in which the first inequality follows from the first part of the proof that shows $v(g^N|_{S' \cup \{i\}}) - v(g^N - ij|_{S' \cup \{i\}}) \geq 0$ for any $S' \subseteq N\setminus\{i\}$. The same argument holds for firm $j$.

Finally, we show that no firm would find it weakly profitable to sever such link $ij$ in any network derived from the complete network. Consider the network $g^N - i'j'$ derived from the complete network $g^N$ by removing link $i'j'$ such that $\varphi_{i'}^{MJW}(g^N) \leq \varphi_{i'}^{MJW}(g^N - i'j')$ and $\varphi_{j'}^{MJW}(g^N) \leq \varphi_{j'}^{MJW}(g^N - i'j')$ according to the definition of the networks derived from the complete network. From the second part of the proof, we know $Y^gN-i'j'|s = Y^gN|s$ for any $S \subseteq N$ because otherwise
Proof of Proposition 16: First, we prove that, with small link cost, any pairwise Nash stable network derived from the complete network contains link $ij$ such that the transshipment problem within $S$ given in (3.6)-(3.9) has a unique optimal transshipment pattern $Y_{g^N|S}$ with $Y_{ij}^{g^N|S} > 0$ for some $S \subseteq N$. We can repeat this procedure (for example, consider the network derived from the complete network $g^N - i'j' - i''j''$ with $\varphi_{i'}^{M,JW}(g^N - i'j') \leq \varphi_{i''}^{M,JW}(g^N - i''j'')$ and $\varphi_{j'}^{M,JW}(g^N - i'j') \leq \varphi_{j''}^{M,JW}(g^N - i'j' - i''j'')$), and show that any network $g$ derived from the complete network contains such $ij$. Q.E.D.
\( \varphi_i^{MJW}(g^N) - l_{ij} \leq \varphi_i^{MJW}(g^N - i'j') \) and \( \varphi_j^{MJW}(g^N) - l_{ji} \leq \varphi_j^{MJW}(g^N - i'j') \). By the same argument as in the proof of Proposition 15: \( Y^{g^N}_{g^N - i'j'}|S = Y^{g^N}_{g^N}|S \) for any \( S \subseteq N \), and we can obtain \( \varphi_i^{MJW}(g^N - i'j') - \varphi_i^{MJW}(g^N - i'j' - ij) - l_{ij} > 0 \) and \( \varphi_j^{MJW}(g^N - i'j') - \varphi_j^{MJW}(g^N - i'j' - ij) - l_{ji} > 0 \). We can repeat this procedure (for example, consider the network derived from the complete network \( g^N - i'j' - i''j'' \) with \( \varphi_i^{MJW_{ij}}(g^N - i'j') - l_{ij} \leq \varphi_i^{MJW_{ij}}(g^N - i'j' - i''j'' \) and \( \varphi_j^{MJW_{ij}}(g^N - i'j') - l_{ji} \leq \varphi_j^{MJW_{ij}}(g^N - i'j' - i''j'') \), and show that \( \varphi_i^{MJW}(g) - \varphi_i^{MJW}(g - ij) - l_{ij} > 0 \) for any network \( g \) derived from the complete network.

Second, we show the existence of the subnetwork \( g^* \) stated in the proposition. From the premise of the proposition, the centralized transshipment problem has a unique optimal transshipment pattern \( Y^{g^N} \) with \( Y_{ij}^{g^N} = 0 \) and \( Y_{ji}^{g^N} = 0 \). Suppose that there is no link cost. Since link \( ij \) is not used in this problem, there exists an efficient network \( g^* \) such that \( ij \notin g^* \) and \( v(g^*) = v(g^N) \). This network \( g^* \) is a subnetwork of the pairwise Nash stable networks derived from the complete network because any pairwise Nash stable network derived from the complete network contains the links that are used in the centralized transshipment problem. Because \( l_{ij} > 0 \) or \( l_{ji} > 0 \), any pairwise Nash stable networks under the MJW value that contain \( ij \) are dominated by \( g^* \). Q.E.D.
Proof of Corollary 17: Under $\varphi^{FB}(g)$, the expected profit of firm $i$ from both stages is $\lambda_i \cdot E_D[\Pi(g, X, D)]$. Since $\lambda_i$ is a constant, the best-response functions of all the firms coincide with the first-order conditions in the centralized system. Q.E.D.

Proof of Corollary 18: Under $\varphi^{FB}(g)$, the ex-post profit of firm $i$ generated from both stages is $\lambda_i \cdot \Pi(g, X, D) = \lambda_i \left\{ \sum_{i \in N} (r_i \min\{X_i, D_i\} + u_i H_i - c_i X_i - \sum_{j \in B_i(g)} l_{ij}) + CTP(g, X, D) \right\}$. Since $\lambda_i$ is a constant, for given inventory profile $X$ and demand profile $D$, the objective of firm $i$ is to maximize $CTP(g, X, D) + \sum_{i \in N} (- \sum_{j \in B_i(g)} l_{ij}) = v(g) - \sum_{ij \in g} (l_{ij} + l_{ji})$. Therefore, the efficient network $g^*$, which maximizes $v(g) - \sum_{ij \in g} (l_{ij} + l_{ji})$, is pairwise Nash stable because there is no profitable deviation for any firm. The rest of the proof is similar to the proof of Corollary 17. Q.E.D.

Computing the MJW value in Table 1: In the following, we show how we have obtained the allocation to firm 1 in Table 1 based on the MJW value in $g' = \{12, 23\}$ under Scenario 1. The rest of the allocations presented in Table 1 can be calculated by following the same procedure. Consider the following 6 orderings of the three firms: (1) $\{1, 2, 3\}$, (2) $\{1, 3, 2\}$, (3) $\{2, 1, 3\}$, (4) $\{3, 1, 2\}$, (5) $\{2, 3, 1\}$, and (6) $\{3, 2, 1\}$. The marginal values of firm 1 are all zero in the orderings (1)-(3), i.e., $v(g'|_{S \cup \{1\}}) - v(g'|_S) = 0$, where $S$ is the set of firms before firm 1 in the orderings. This is because there exists no firm with residual supplies
in $S$, so $E_1$ generates no value. In the ordering (4), the marginal value of firm 1 is also zero because firms 1 and 3 are not connected with each other without firm 2. In the ordering (5), $v(g'|_{\{2,3\} \cup \{1\}}) = E_1 + E_2$ and $v(g'|_{\{2,3\}}) = E_2$ because $H_3 \geq E_1 + E_2$. So the marginal value of firm 1 is $E_1$. Similarly, in the ordering (6), the marginal value of firm 1 is $E_1$. Therefore, the allocation to firm 1 under the MJW value is the average marginal value, $(E_1 + E_1)/6 = E_1/3$.

Appendix E. Extension: Network Formation before Inventory Decisions

Consider a case in which a network $g$ is formed endogenously by following the same procedure as described in §3.5 before inventory decisions are made. This case may represent a situation in which firms set up physical networks and then use those networks for later transshipment. In this case, each firm $i$ decides whether to form a link based on its expected profit $E_D[\pi_i^{NE}(g)]$, where $\pi_i^{NE}(g)$ can be obtained by substituting Nash equilibrium inventory profile $X^{NE}$ into (3.13) as follows:

\[
\pi_i^{NE}(g) = r_i \min \{X_i^{NE}(g), D_i\} + u_i \max \{X_i^{NE}(g) - D_i, 0\} - c_i X_i^{NE}(g) + \phi_i(g, X^{NE}(g), D).
\]
Accordingly, a pairwise Nash stable network $g$ satisfies the following conditions:

$$E_D[\pi_i^{NE}(g)] \geq E_D[\pi_i^{NE}(g - ij)] + l_{ij} \text{ and } E_D[\pi_j^{NE}(g)] \geq E_D[\pi_j^{NE}(g - ij)] + l_{ji} \text{ for all } ij \in g;$$

if $E_D[\pi_i^{NE}(g + ij)] > E_D[\pi_i^{NE}(g)] + l_{ij}$, then $E_D[\pi_j^{NE}(g + ij)] < E_D[\pi_j^{NE}(g)] + l_{ji}$ for all $ij \notin g$.

Note that conditions above involve $E_D[\pi_i^{NE}]$, while the corresponding conditions (3.11) and (3.12) in §3.5 involve allocation $\varphi_i$.

The following proposition describes a set of pairwise Nash stable networks for the case of zero link costs (c.f. Proposition 15 in §3.5.1). We consider the case of positive link costs in a subsequent corollary.

**Proposition 25.** Suppose that a transshipment network is formed with no link costs before inventory decisions are made. There exists $\pi \geq 0$ such that, if $[X^{NE}(g) - X^{NE}(g - ij)][X^{NE}(g) - X^{NE}(g - ij)]^T \leq \pi$ for any $g$ and $ij \in g$, then the complete network $g^N$ is always pairwise Nash stable under the allocation based on the MJW value, and any pairwise Nash stable network $g$ derived from the complete network $g^N$ contains any link $ij$ with $Y_{ij}^{g^N|s} > 0$ for some realized demand $D$ and $S \subseteq N$, where $Y_{ij}^{g^N|s}$ is the unique optimal transshipment pattern of the transshipment problem within $S$ given in (3.6)-(3.9).

**Proof:** We first prove that the complete network is pairwise stable under the MJW value by showing that no firms have incentives to sever a link in the complete
network. The proof of Nash stability is similar to that in Proposition 15. From the
proof of Proposition 15, we know that, for any inventory decisions $X$ and realized
demand $D$, $\varphi_i^{MJW}(g^N, X, D) - \varphi_i^{MJW}(g^N - ij, X, D) \geq 0$ for any $i$ and $j$. Let
$\bar{\pi} = 0$ so that $X^{NE}(g^N) = X^{NE}(g^N - ij)$ for any $ij$. Then, for any realized demand
$D$, $\pi_i^{NE}(g^N) = r_i \min\{X_i^{NE}(g^N), D_i\} + u_i \max\{X_i^{NE}(g^N) - D_i, 0\} - c_i X_i^{NE}(g^N) +$
$\varphi_i^{MJW}(g^N, X^{NE}(g^N), D) \geq r_i \min\{X_i^{NE}(g^N - ij), D_i\} + u_i \max\{X_i^{NE}(g^N - ij) -$
$D_i, 0\} - c_i X_i^{NE}(g^N - ij) + \varphi_i^{MJW}(g^N - ij, X^{NE}(g^N - ij), D) = \pi_i^{NE}(g^N - ij)$.
Therefore, $E_D[\pi_i^{NE}(g^N)] \geq E_D[\pi_i^{NE}(g^N - ij)]$ for any $i$ and $j$.

Next, consider network $g$ that is derived from the complete network $g^N$. We
prove that no firms have incentives to sever a link with $Y_{ij}^{g^N|s} > 0$ for some realized
demand $D$. To prove this, we show that $\pi_i^{NE}(g) - \pi_i^{NE}(g - ij) > 0$ when $Y_{ij}^{g^N|s} > 0$
for some realized demand $D$, and that $\pi_i^{NE}(g) - \pi_i^{NE}(g - ij) = 0$ when $Y_{ij}^{g^N|s} = 0$
for some realized demand $D$.

Let us first consider the case when $Y_{ij}^{g^N|s} > 0$ for some realized demand $D$.
Let $x = [X^{NE}(g) - X^{NE}(g - ij)][X^{NE}(g) - X^{NE}(g - ij)]^T$. From (5.6), the first
three terms of $\pi_i^{NE}(g)$ are continuous in $X_i^{NE}$. Thus, $\lim_{x \to 0} \left[ r_i \min\{X_i^{NE}(g), D_i\} +$
$u_i \max\{X_i^{NE}(g) - D_i, 0\} - c_i X_i^{NE}(g) - r_i \min\{X_i^{NE}(g - ij), D_i\} - u_i \max\{X_i^{NE}(g -$
$ij) - D_i, 0\} + c_i X_i^{NE}(g - ij)\] = 0. From the proof of Proposition 15, we know that, for
any realized demand $D$ with $Y_{ij}^{g^N|s} > 0$, $\varphi_i^{MJW}(g, X, D) - \varphi_i^{MJW}(g - ij, X, D) > 0$. 


Since $\varphi_i^{MJW}$ is continuous in $X$, 
\[
\lim_{x \to 0} [\varphi_i^{MJW}(g, X^{NE}(g), D) - \varphi_i^{MJW}(g - ij, X^{NE}(g - ij), D)] = \varphi_i^{MJW}(g, \lim_{x \to 0} X^{NE}(g), D) - \varphi_i^{MJW}(g - ij, \lim_{x \to 0} X^{NE}(g - ij), D) > 0.
\]
Therefore, by putting together all four terms of $\pi_i^{NE}(g)$ in (5.6), 
\[
\lim_{x \to 0} \left[ \pi_i^{NE}(g) - \pi_i^{NE}(g - ij) \right] > 0.
\]
Next, for the case when $Y_{ij}^{gN|s} = 0$ for some realized demand $D$, we know from the proof of Proposition 15 that $\varphi_i^{MJW}(g, X, D) - \varphi_i^{MJW}(g - ij, X, D) = 0$. Thus, we can show that 
\[
\lim_{x \to 0} \left[ \pi_i^{NE}(g) - \pi_i^{NE}(g - ij) \right] = 0.
\]
Finally, using the results of both cases when $Y_{ij}^{gN|s} > 0$ and $Y_{ij}^{gN|s} = 0$ for some realized demand $D$ and taking the expectation of $\pi_i^{NE}(g)$ with respect to $D$, we obtain 
\[
\lim_{x \to 0} \{ E_D[\pi_i^{NE}(g)] - E_D[\pi_i^{NE}(g - ij)] \} > 0.
\]
Therefore, there exists $\bar{x} > 0$ such that if $x \leq \bar{x}$, $E_D[\pi_i^{NE}(g)] - E_D[\pi_i^{NE}(g - ij)] > 0$. Q.E.D.

Proposition 25 shows that over-connection in the pairwise Nash stable networks under the MJW value is still observed when firms build networks under uncertain demand. Compared to Proposition 15, Proposition 25 requires one additional condition that $[X^{NE}(g) - X^{NE}(g - ij)][X^{NE}(g) - X^{NE}(g - ij)]^T \leq \bar{x}$ (where $T$ denotes the transpose of a vector). This condition means that inventory decisions are not sensitive to the existence of a certain link in the transshipment network. This condition holds when the ex-post profit from satisfying local demand, $r_i \min\{X_i^{NE}(g), D_i\} + u_i \max\{X_i^{NE}(g) - D_i, 0\} - c_i X_i^{NE}(g)$, is sufficiently larger than the allocation from later transshipment, $\varphi_i(g, X^{NE}(g), D)$. Since each
firm satisfies its own demand first using its local inventory and then shares any excess inventories to satisfy unmet demands of other firms, this condition does not seem very restrictive.

Proposition 25 can be easily modified to the case where positive link costs exist (c.f. Proposition 16 in §3.5.2). The following corollary summarizes the result.

**Corollary 26.** Suppose that a transshipment network is formed before inventory decisions are made and there exist $S \subset N$ and a link $ij$ with $l_{ij} > 0$ or $l_{ji} > 0$ such that: (i) $v(g^N|S)$ has a unique optimal transshipment pattern $Y^{g^N|S}$ with $Y^{g^N|S}_{ij} > 0$ for some realized demand $D$, and (ii) $v(g^N)$ has a unique optimal transshipment pattern $Y^{g^N}$ with $Y^{g^N}_{ij} = 0$ and $Y^{g^N}_{ji} = 0$. Then there exist $\bar{l} > 0$ and $\bar{x} \geq 0$ such that, if $l_{ij} < \bar{l}, l_{ji} < \bar{l}$ and $[X^{NE}(g) - X^{NE}(g-ij)] [X^{NE}(g) - X^{NE}(g-ij)]^T \leq \bar{x}$ for all $ij \in g$, every pairwise Nash stable network $g$ derived from the complete network $g^N$ is not efficient under the allocation based on the MJW value because there is a subnetwork $g^*$ of $g$ which satisfies $E_D[\Pi(g, X^{NE}(g), D)] - \sum_{ij \in g} (l_{ij} + l_{ji}) < E_D[\Pi(g^*, X^{NE}(g^*), D)] - \sum_{ij \in g^*} (l_{ij} + l_{ji})$.

**Appendix F. Proofs of Analytical Results in Chapter 4**

**Proof of Proposition 19:** When coalition structure is symmetric, from (4.3) and (4.6), one can see that $\theta^{(1)} < \theta^{(0)}$ is equivalent to $\frac{c_{mn}}{m \cdot (m_n + (m_n - s_j) \gamma)} \left(1 - \frac{g}{m}\right)^{(x+1)} <$
\[
\frac{c x n}{\beta + (n-1)\gamma} (1 - \frac{g}{n})^{-(x+1)}
\]. We simplify the inequality and obtain the condition in the proposition.

When coalition structure is asymmetric, \(\theta^{(1)}\) satisfies the following equation

\[
(5.7) \quad l \sum_{j=1}^{m} n_j \left[ 1 - \left[ \frac{\theta n_j \{ \beta n_j + (n-n_j)\gamma \}}{c x n} \right]^{-\frac{1}{\beta + (n-1)\gamma}} \right] = g.
\]

If we substitute \(\theta = \frac{c x n}{\beta + (n-1)\gamma} (1 - \frac{g}{n})^{-(x+1)}\) in (5.7), the left-hand side becomes

\[
l \sum_{j=1}^{m} n_j \left[ 1 - \left[ \frac{n_j \{ \beta n_j + (n-n_j)\gamma \}}{\beta + (n-1)\gamma} \right]^{-\frac{1}{\beta + (n-1)\gamma}} \right] (1 - \frac{g}{n}) \geq 1 \text{ for } j = 1, \ldots, m \text{ and } \frac{n_j \{ \beta n_j + (n-n_j)\gamma \}}{\beta + (n-1)\gamma} > 1 \text{ for at least one } j,
\]

we obtain that \(l \sum_{j=1}^{m} n_j \left[ 1 - \left[ \frac{n_j \{ \beta n_j + (n-n_j)\gamma \}}{\beta + (n-1)\gamma} \right]^{-\frac{1}{\beta + (n-1)\gamma}} \right] (1 - \frac{g}{n}) = g\). Since the left-hand side of (5.7) is increasing in \(\theta\), \(\theta^{(1)} < \theta^{(0)}\). One can see that, if \(\frac{\beta}{\gamma} \geq 1\), \(\frac{n_j \{ \beta n_j + (n-n_j)\gamma \}}{\beta + (n-1)\gamma} > 1\) holds for \(n_j > 1\). Therefore, there exists a threshold \(t_{\text{joint}}\) such that, if \(\frac{\beta}{\gamma} > t_{\text{joint}}\), \(\theta^{(1)} < \theta^{(0)}\).

Q.E.D.

**Proof of Proposition 20:** When coalition structure is symmetric, from (4.3) and (4.8), one can see that \(\theta^{(2)} < \theta^{(0)}\) is equivalent to \(\frac{c x n}{\beta \gamma + (n-n_j)\gamma} (1 - \frac{g}{n})^{-(x/n_j+1)} < \frac{c x n}{\beta + (n-1)\gamma} (1 - \frac{g}{n})^{-(x+1)}\). We simplify the inequality and obtain the condition in the proposition.

When coalition structure is asymmetric, suppose the audit effort of each manufacturer is the same in one coalition. From (4.7), the optimal audit effort \(e^{(2)}_{i,B_j}\)
Thus, we obtain that \( e_{i,B_j}^{(2)} = 1 - \left[ \frac{\theta((1 - e_{i,B_j})n_j \{ \beta n_j + (n - n_j)\gamma \}}{cxn} \right]^{-\frac{1}{x+n_j}} \). Similar to (5.7), \( \theta^{(2)} \) satisfies the following equation

\[
(5.8) \quad l \sum_{j=1}^{m} n_j \left[ 1 - \left( \frac{\theta(\beta n_j + (n - n_j)\gamma)}{x+n_j} \right)^{\frac{n_j}{x+n_j}} \left( 1 - \frac{g}{nl} \right)^{\frac{n_j(x+1)}{x+n_j}} \right] = g.
\]

If we substitute \( \theta = \frac{cxn}{\beta+(n-1)\gamma} \left( 1 - \frac{g}{nl} \right)^{-\frac{1}{x+1}} \) in (5.8), the left-hand side becomes

\[
l \sum_{j=1}^{m} n_j \left[ 1 - \left( \frac{\beta n_j + (n - n_j)\gamma}{\beta+(n-1)\gamma} \right)^{\frac{n_j}{x+n_j}} \left( 1 - \frac{g}{nl} \right)^{\frac{n_j(x+1)}{x+n_j}} \right] < 1 - \frac{g}{nl},
\]

we obtain that, when \( \frac{\beta n_j + (n - n_j)\gamma}{\beta+(n-1)\gamma} > 1 \) for \( j = 1, ..., m \) and \( \frac{\beta n_j + (n - n_j)\gamma}{\beta+(n-1)\gamma} > 1 \) for at least one \( j \),

\[
l \sum_{j=1}^{m} n_j \left[ 1 - \left( \frac{\beta n_j + (n - n_j)\gamma}{\beta+(n-1)\gamma} \right)^{\frac{n_j}{x+n_j}} \left( 1 - \frac{g}{nl} \right)^{\frac{n_j(x+1)}{x+n_j}} \right] > l \sum_{j=1}^{m} n_j = g.
\]

Since the left-hand side of (5.8) is increasing in \( \theta \), \( \theta^{(2)} < \theta^{(0)} \). One can see that, if \( \frac{g}{\gamma} \geq 1 \), \( \frac{\beta n_j + (n - n_j)\gamma}{\beta+(n-1)\gamma} > 1 \) holds for \( n_j > 1 \). Therefore, there exists a threshold \( t_{\text{shared}} \) such that, if \( \frac{g}{\gamma} > t_{\text{shared}} \), \( \theta^{(2)} < \theta^{(0)} \). Q.E.D.

**Proof of Proposition 21:** When coalition structure is symmetric, we obtain the condition in the proposition by comparing (4.6) and (4.8). When coalition structure is asymmetric and is the same for joint audit and shared supplier information,
from (5.7) and (5.8), $\theta^{(1)}$ and $\theta^{(2)}$ satisfy the following equation

$$\left[\frac{\theta^{(2)}\{\beta n_j + (n - n_j)\gamma\}}{c_{xn}}\right]^{-\frac{n_j}{x+n_j}} = \left[\frac{\theta^{(1)}\{\beta n_j + (n - n_j)\gamma\}}{c_{xn}}\right]^{-\frac{1}{x+1}}.$$ 

We obtain that $\frac{\theta^{(1)}}{\theta^{(2)}} = \frac{1}{n_j} \left[\frac{\theta^{(2)}\{\beta n_j + (n - n_j)\gamma\}}{c_{xn}}\right]^{\frac{x(n_j - 1)}{x+n_j}}$. Since $\theta^{(2)}$ is increasing in $\varrho$ from (5.8), there exists a threshold $t_{\text{compare}}$ such that, if $\varrho > t_{\text{compare}}$, $\theta^{(1)} > \theta^{(2)}$. Q.E.D.

**Proof of Proposition 22:** We analyze whether the manufacturers in $S \subset N$ have incentives to secede from the grand coalition $N$ by comparing the allocation to $S$ under $B_N$ with that under another coalition structure $B$. In the following, we first prove condition (i) and then condition (ii).

From the definition of domination, in order for an allocation $\varphi$ to dominate $\varphi^{E_g}$ via $S$ (so that $S$ have incentives to secede), it requires that $\sum_{k \in S} \varphi_k \leq v_{B'}(S)$ for all $B' \supset S$. Thus, we consider $B$ that satisfies $B = \arg\min_{B' \supset S} v_{B'}(S)$ and $\sum_{k \in S} \varphi_k = v_B(S)$; otherwise, $\varphi$ is not feasible. From (4.4), $\sum_{k \in S} \varphi_k^{E_g} = n_s\alpha - \theta n_s\beta\left(1 - e^{(1)}_N\right) - \frac{n_s}{n}c(1 - e^{(1)}_N)^{-x}$, where $n_s$ is the number of manufacturers in $S$ and $e^{(1)}_N$ is the optimal audit effort under $B_N$. Similarly, $\sum_{k \in S} \varphi_k = n_s\alpha - \theta n_s\left\{\beta\left(1 - \frac{\sum_{j=1}^m n_j e^{(1)}_{B_j}}{n}\right) + \gamma\left(\frac{\sum_{j=1}^m n_j e^{(1)}_{B_j}}{n} - e^{(1)}_S\right)\right\} - c(1 - e^{(1)}_S)^{-x}$, where $e^{(1)}_S$ is the optimal audit effort of $S$ under $B$. Due to the optimality of $e^{(1)}_N$ given $B$, we
obtain that \( \sum_{k \in S} \varphi_k^{E_g} \geq n_s \alpha - \theta n_s \beta \left( 1 - e_S^{(1)} \right) - \frac{n_s}{n} c \left( 1-e_S^{(1)} \right)^x \). Therefore, by comparing the two allocations, if \( c \geq \theta n_s \left( 1- \frac{n_s}{n} \right)^{-1} \left( 1-e_S^{(1)} \right)^x (\beta - \gamma) \left( \frac{\sum_{j=1}^{m} n_j e_{B_j}^{(1)}}{n} - e_S^{(1)} \right) \),

\( \sum_{k \in S} \varphi_k^{E_g} \geq \sum_{k \in S} \varphi_k \) so that \( S \) have no incentives to secede.

For condition (ii), note from (4.5) that, when \( \frac{\beta}{\gamma} \geq 1 \), \( e_{B_j}^{(1)} \) is increasing in \( n_j \). So the coalition structure \( B \) which minimizes \( v_B(S) \) satisfies that \( n_j = 1 \) for \( B_j \neq S \); i.e., each coalition in \( B \) except \( S \) includes only one manufacturer. For such a coalition structure \( B \), \( \sum_{k \in S} \varphi_k^{E_g} - \sum_{k \in S} \varphi_k \geq \theta n_s (\beta - \gamma) \left( e_S^{(1)} - \frac{\sum_{j=1}^{m} n_j e_{B_j}^{(1)}}{n} \right) \geq 0 \). Q.E.D.

**Proof of Proposition 23:** Similar to the proof of Proposition 22, we analyze whether the manufacturers in \( S \subset N \) have incentives to secede from the grand coalition \( N \) by comparing the allocation to \( S \) under \( B^N \) with that under another coalition structure \( B \). In the following, we first prove the case in which \( \frac{\beta}{\gamma} \geq 1 \) and then the case in which \( \frac{\beta}{\gamma} < 1 \).

From (4.9), we know that \( \sum_{k \in S} \varphi_k^{E_g} = n_s \alpha \). We first show that any coalition \( S \), whose members fail to detect adulteration (i.e., \( s_k = 0 \) for all \( k \in S \)), has no incentive to secede from the grand coalition \( N \). For such a coalition \( S \), \( \sum_{k \in S} \varphi_k = n_s \alpha - \left\{ \beta \left( 1 - \frac{\sum_{k=1}^{m} n_k I(B_k)}{n} \right) + \gamma \left( \frac{\sum_{k=1}^{m} n_k I(B_k)}{n} \right) \right\} \leq n_s \alpha = \sum_{k \in S} \varphi_k^{E_g} \). Next, for the coalition \( S \) with at least one manufacturer \( i \) such that \( s_i = 1 \), when \( \frac{\beta}{\gamma} \geq 1 \),

\( \sum_{k \in S} \varphi_k = n_s \alpha - (\beta - \gamma) \left( 1 - \frac{\sum_{k=1}^{m} n_k I(B_k)}{n} \right) \leq n_s \alpha = \sum_{k \in S} \varphi_k^{E_g} \).
Suppose \( \frac{n-2}{2n-2} \leq \frac{\beta}{\gamma} < 1 \) and there exists only one manufacturer \( i \in N \) such that \( s_i = 1 \). For the coalition \( S \) that does not contain \( i \), \( \sum_{k \in S} \varphi_k^mo = n_s\alpha - \frac{n_s}{n} \{ \beta(n - 1) + \gamma \} \). Since \( \frac{\beta}{\gamma} < 1 \), the coalition structure \( B \) which minimizes \( v_B(S) \) satisfies that \( m = 2 \); i.e., all manufacturers that are not in \( S \) belong to one coalition.

Thus, \( \sum_{k \in S} \varphi_k = n_s\alpha - \frac{n_s}{n} \{ \beta n_s + \gamma(n - n_s) \} \leq \sum_{k \in S} \varphi_k^mo \). For the coalition \( S \) that contains \( i \), \( \sum_{k \in S} \varphi_k^mo = n_s\alpha + \{ 1 - \frac{n_s}{n} \} \{ n\beta + \gamma - \beta \} \). On the other hand, \( \sum_{k \in S} \varphi_k = n_s\alpha + (\gamma - \beta) \{ 1 - \frac{n_s}{n} \} \leq \sum_{k \in S} \varphi_k^mo \). Q.E.D.