

Dissertation Proposal

New Bounding Techniques for Discrete Optimization Problems

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Thursday, September 6th, 2012
1:30pm, 324 GSIA

Bounds on the optimal value are often indispensable for the practical solution of discrete optimization problems, as for example in branch-and-bound procedures. Such bounds are frequently obtained by solving a continuous relaxation of the problem, perhaps a linear programming relaxation of an integer programming model. In this thesis I explore two novel techniques for obtaining bounds for this class of problems.

The first such technique is the idea of obtaining valid inequalities for a 0-1 model from a constraint programming formulation of the problem. In particular, we formulate a graph coloring problem as a system of all-different constraints. By analyzing the polyhedral structure of all-different systems, we obtain facet-defining inequalities that can be mapped to valid cuts in the classical 0-1 model of the problem.

We employ a common strategy for generating problem-specific cuts: the identification of facet-defining cuts for special types of induced subgraphs, such as odd-holes, webs, and paths. We identify cuts that bound the objective function as well as cuts that exclude infeasible solutions.

One structure that we focus on is cyclic structures and show that the cuts we obtain are stronger than previously known cuts. For example, when an existing separation algorithm identifies odd-hole cuts, we can supply stronger cuts with no additional calculation. In addition, we generalize odd-hole cuts to odd-cycle cuts that are stronger than any collection of odd-hole cuts.

Secondly, we explore the idea of obtaining bounds on the optimal value of an optimization problem from a discrete relaxation based on binary decision diagrams (BDDs). We show how to construct a limited-width relaxed BDD, which represents a relaxation of the feasible set, and show how to obtain a bound for any additively separable objective function by solving a shortest (or longest) path problem in the BDD. We investigate methods that can tighten the bound provided by a BDD relaxation, much like cutting planes are used for continuous relaxations. We apply this technique to a class of set covering problems for which orders of magnitude computational speed-ups over state-of-the-art integer programming technology are observed.

In addition, as it is well known that the size of a BDD used to represent a set of solutions is dramatically impacted by the variable ordering chosen for the layers of the BDD, we explore variable ordering considerations in the context of relaxed BDDs. We consider variable orderings for the application of BDDs to the maximum independent set problem. Through this analysis we find a bound on the size of the exact BDD, in terms of the size of the graph. Also we show computationally that the bound provided by relaxed BDDs is dramatically altered by the variable ordering choice. We find that through these considerations, the relaxed BDD can deliver significantly tighter bounds, in far less computational time, than state-of-the-art integer programming software obtains for an integer programming formulation by solving a continuous relaxation augmented with cutting planes.

The use of BDDs for approximating the set of feasible solutions to binary optimization problems can be used for another purpose. Just as relaxed BDDs represent an over-approximation of the feasible set, restricted BDDs represent an under-approximation of the feasible set. These structures can be used to develop a new class of primal heuristics for binary optimization problems, since just as in the case of relaxed BDDs, among the set of solutions in a restricted BDD, the best (with respect to a given additively separable objective function) can be identified with a shortest (or longest) path calculation.

We have investigated the application of restricted BDDs to several classes of problems. I intend on continuing this line of work, by applying restricted BDDs to several classes of problems, looking at different aspects of the algorithm used to compile the restricted BDDs, and consider variable orderings.

Approximate BDDs can then be incorporated into a single framework. Relaxation bounds and primal heuristics are the main ingredients found in many algorithms that have been developed to tackle these computationally difficult problems. With the use of both relaxed and restricted BDDs, I propose to explore a novel branch-and-bound algorithm that utilizes both structures and branches on nodes within relaxed BDDs, enabling branching on not just value assignments to variables, but on groups of partial solutions. Many aspects of the algorithm need to be investigated, including branching decisions and maximum width for the approximate BDDs. This branch-and-bound algorithm requires little information to be passed to search tree nodes which may lead to a natural and efficient parallelization of the algorithm.