Discussion of "Two views of a robust Ramsey planner"

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Theme

We have not succeeded in answering all our problems.

The answers we have found only serve to raise a whole set of new questions.

In some ways we feel we are as confused as ever, but we believe we are confused on a higher level and about more important things.

B. Øksendal, *Stochastic Differential Equations*

Policy design as Mechanism Design

- Government as Mechanism Designer.
- Continuum of identical private agents.
- Small agents in a smooth, concave world.
- Use first order approach.
- Agent FOC a constraint on government.
- In an Linear-Quadratic world, a linear constraint.

Robustness

- Classical mechanism design: Everyone is Bayesian.
- Hansen and Sargent introduce a concern for robustness.
- But there are many robustness configurations.

Robustness: Who's afraid of model mispecification?

- Mechanism Designer (MD):

  Hansen-Sargent (2011),
  Karantounias et al (2007),
  Kocherlakota-Phelan (2009), Chung-Ely (2006).

- Agents:

  Karantounias et al (2011) and Orlik (2011).

- Both:

  Van Nieuwerbergh (2001).

Robustness: What does the designer fear?

- Hansen-Sargent I: No confidence in beliefs about shocks or beliefs about the beliefs of agents (which are taken to be correct).
- Hansen-Sargent II: No confidence in beliefs about shocks, but confidence in beliefs about beliefs of agents (which are taken to be wrong).
- Chung-Ely: No confidence in beliefs about the beliefs of agents, but confidence in beliefs about shocks.
- Kocherlakota-Phelan: No confidence in beliefs about shocks or equilibrium that will be played.

Robust Policy Design

Question: How do different misspecification assumptions and concern for robustness shape policy?

- Details of the model are hard to follow for a non-expert. (Me!)
- Goal: Understand the basic mechanism and pose some questions.
- Approach: Write down simple problem and work through it.

Overview

- Static: Intratemporal output/inflation tradeoff.
- Approximating model: Intertemporal tradeoff.
  - Firms' beliefs about future shocks and policy affect current inflation.
- Robustness (Model II): Concerned high cost shock likely. Not concerned firms think them likely.
  - Relatively more period 2 stabilization in high cost shock states.
- Robustness (Model I): Concerned high cost shock likely + firms think them likely.
  - Relatively more use of period 2 policy to stabilize period 1 inflation.

Static problem

$$\max_{y_1} -\frac{1}{2} E[\hat{z}_1((\kappa y_1 + c_1 + c^*)^2 + (y_1 - y^*)^2)]$$

From FOC for y_1:

$$y_1 = \frac{y^*}{1+\kappa^2} - \frac{\kappa}{1+\kappa^2}(c_1 + c^*).$$

Stabilization: Output used to damp shocks independent of beliefs.

2-period Approximating Model, No robustness

$$\max_y -\frac{1}{2}((\kappa y_1 + c_1 + c^* + \beta E[(\kappa y_2 + c_2 + c^*)])^2 + (y_1 - y^*)^2)$$
$$-\frac{\beta}{2}E[((\kappa y_2 + c_2 + c^*)^2 + (y_2 - y^*)^2)]$$

From FOC for y_1 and y_2:

$$y_2 = \underbrace{\frac{1}{(1+\kappa^2)^2}y^* - \frac{\kappa}{1+\kappa^2}(c_2 + c^*)}_{\text{period 2 stabilization}}$$
$$\underbrace{-\frac{\kappa}{(1+\kappa^2)^2}((c_1 + c^*) + \beta E[(\kappa y_2 + c_2 + c^*)])}_{\text{extra deflation motive}}$$

Hansen-Sargent Model II, 2 -period version

Policymaker and agents share an approximating model, agents trust it, policymaker does not, and he is concerned about robustness.

$$\mathscr{S}^{II}(y,m) = -\frac{1}{2}((\kappa y_1 + c_1 + c^* + \beta E[(\kappa y_2 + c_2 + c^*)])^2 + (y_1 - y^*)^2) \\ -\frac{\beta}{2}E[\frac{\hat{z}_2}{\hat{z}_1}((\kappa y_2 + c_2 + c^*)^2 + (y_2 - y^*)^2)]$$

In this case:

$$\max_y \min_m \quad \mathscr{S}^{II}(y,m) + \beta\theta E[m \log m] \quad \text{s.t. } E[m] = 1.$$

where $m = \frac{\hat{z}_2}{\hat{z}_1}$

Hansen-Sargent Model II, 2 -period version *continued*

Solving inner min problem:

$$m = \frac{\exp\left(\sigma H(y_2, c_2)\right)}{E\left[\exp\left(\sigma H(y_2, c_2)\right)\right]}.$$

where,

$$\sigma = -\frac{1}{\theta} \qquad \text{and} \qquad H = \frac{1}{\beta}\mathscr{S}_m$$

H is (rescaled) derivative of government's objective modulo entropy component. In this case, it is government's continuation payoff.

$$H(y_2, c_2) = -\frac{1}{2}((\kappa y_2 + c_2 + c^*)^2 + (y_2 - y^*)^2).$$

Since $\sigma < 0$, m is decreasing in H.

Hansen-Sargent Model II, 2 -period version *continued*

Outer max problem: sub expression for m into government objective:

$$\max_{y} \quad -\frac{1}{2}((\kappa y_1 + c_1 + c^* + \beta E[(\kappa y_2 + c_2 + c^*)])^2 + (y_1 - y^*)^2)$$
$$+\frac{\beta}{\sigma} \ln E[\exp(\sigma H(y_2, c_2))].$$

The FOC for y_2 yields:

$$y_2 = \underbrace{\frac{1}{(1+\kappa^2)^2} y^* - \frac{\kappa}{1+\kappa^2}(c_2 + c^*)}_{\text{stabilization in period 2}}$$
$$-\frac{1}{m(y_2, c_2)} \underbrace{\frac{\kappa}{(1+\kappa^2)^2}((c_2 + c^*) + \beta E[(\kappa y_2 + c_2 + c^*)])}_{\text{extra deflation motive}}$$

Hansen-Sargent Model II, 2 -period version *continued*

$$
\begin{aligned}
y_2 \quad = \quad & \underbrace{\frac{1}{(1+\kappa^2)^2} y^* - \frac{\kappa}{1+\kappa^2}(c_2 + c^*)}_{\text{stabilization in period 2}} \\
& - \frac{1}{m(y_2, c_2)} \underbrace{\frac{\kappa}{(1+\kappa^2)^2}((c_2 + c^*) + \beta E[(\kappa y_2 + c_2 + c^*)])}_{\text{extra deflation motive}}
\end{aligned}
$$

- If $H(y_2, c_2) \downarrow \Longrightarrow \; m(y_2, c_2) \uparrow$
- In this state govt is relatively more "concerned" about period 2 stabilization, less willing to use deviations in period 2 output to stabilize period 1 inflation.

Contrast with the no misspecification/no robustness case:

$$y_2 = \underbrace{\frac{1}{(1+\kappa^2)^2}y^* - \frac{\kappa}{1+\kappa^2}(c_2+c^*)}_{\text{stabilization in period 2}} \underbrace{-\frac{\kappa}{(1+\kappa^2)^2}((c_2+c^*)+\beta E[(\kappa y_2+c_2+c^*)])}_{\text{extra deflation motive}}$$

Model II, concern for robustness

$$y_2 = \underbrace{\frac{1}{(1+\kappa^2)^2}y^* - \frac{\kappa}{1+\kappa^2}(c_2+c^*)}_{\text{stabilization in period 2}} \underbrace{-\frac{1}{m(y_2,c_2)}\frac{\kappa}{(1+\kappa^2)^2}((c_2+c^*)+\beta E[(\kappa y_2+c_2+c^*)])}_{\text{extra deflation motive}}$$

Hansen-Sargent Model I, 2 -period version

Govt believes agents know true model, distrusts approximating model.

$$\mathscr{S}^I(y,m) = \quad -\frac{1}{2}((\kappa y_1 + c_1 + c^* + \beta E[m(\kappa y_2 + c_2 + c^*)])^2 + (y_1 - y^*)^2)$$
$$-\frac{\beta}{2}E[m((\kappa y_2 + c_2 + c^*)^2 + (y_2 - y^*)^2)]$$

In this case:

$$\max_y \min_m \quad \mathscr{S}^I(y,m) + \beta\theta E[m \log m] \quad \text{s.t. } E[m] = 1.$$

Now m gives govt's beliefs about agents' beliefs.

Hansen-Sargent Model I, 2 -period version continued

Sub soln to min problem into objective. From FOC for y_2:

$$y_2 = \underbrace{\frac{1}{(1+\kappa^2)^2}y^* - \frac{\kappa}{1+\kappa^2}(c_2 + c^*)}_{\text{stabilization in period 2}} \underbrace{-\frac{\kappa}{(1+\kappa^2)^2}((c_1 + c^*) + \beta E[m(\kappa y_2 + c_2 + c^*)])}_{\text{extra deflation motive}}$$

- Govt concerned high cost-push shock likely and that firms think it's likely and hence inclined to set higher prices in period 1.
- Govt relatively "more" concerned about using period 2 output to damp inflation in period 1.

Active or Passive Policy?

Policymaker's concerns:

- Mismatch of policy to state.
  - Mandate for passive policy?
    - Ex: Optimality of Laissez-faire (Kocherlakota-Phelan)
- Bad state likely, agents believe bad state likely
  - Mandate for active policy?
    - Hansen-Sargent?

What kind of active policy?

Relative to approximating model,

- HS Model I :
  - More active in using period 2 output to damp period 1 inflation.
- HS Model II:
  - More active in using period 2 output to stabilize period 2 economy (if cost push shocks are bad.)

Big Picture

- HS clarified robustness interpretations.
- Pushed forward on the technical front.
- Closer contact with monetary policy literature.
- But, a menu of robustness configurations.
- Policy results seem sensitive to configuration chosen.
- A preferred configuration? Results that transcend configurations?

Details

- 2-period examples solves a particular problem
  1. Government picks (y_1, y_2) and then allows (λ_1, λ_2) to be determined by firms via their beliefs.
  2. Alternatively, both can be chosen by outer max problem. But, then m can be chosen to force a constraint violation unless inflation in period 1 is fully stabilized.
  3. It's not clear how issue in pt. 2 is avoided.
- What does continuous time deliver?
  1. More concavity? (Analogous to dynamic contracting)
  2. Simpler equations to solve?
  3. HS allude to some arguments made in CH. 16 of book, but details are incomplete.