Decentralized Procurement and Inventory Management

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August 2012
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Abstract

Keeping input costs low and managing inventories are two critical components of firm strategy that comprise the basis for most cost accounting initiatives. The fact that many of the key decisions to this end are implemented by divisional managers with their own incentives in play is a commonly discussed roadblock in achieving success in cost management. This paper presents a contrast to conventional wisdom by showing that when it comes to procurement and inventory control, decentralized decision making can actually be a boon to firm efficiency. The reason decentralization proves beneficial is that it helps rein in a firm's natural tendency to hoard inventory in order to get an edge over its supplier in future negotiations. In effect, decentralized procurement conveys a lower willingness to use inventory to manipulate a supplier in future interactions which, in turn, convinces the supplier to soften its own stance in setting initial input prices. Besides providing support for the common practice of decentralized procurement, the results suggest that attempts to mimic centralized decision making in decentralized firms may, in fact, be counter productive. The paper also examines the design of divisional incentives in light of the fact that decentralized decision making can be used to boost intertemporal cooperation along the supply chain.
1. Introduction

Cost management is a primary goal of managerial accounting. As academics and practitioners have repeatedly stressed, managing costs is a delicate exercise due to the need to coordinate inside parties and influence outside parties. While it is tempting to presume that coordination of internal parties entails developing incentive structures that encourage the parties to closely mimic the behavior of an omniscient central planner, this view is incomplete at best. After all, the primary take away from the literature on strategic delegation, a field that spans several disciplines including accounting, economics, and marketing (see, e.g., Alles and Datar 1998; Fershtman and Judd 1987; McGuire and Staelin 1983; Sklivas 1987), is that a nuanced firm strategy must foresee and plan for the likely reaction of external competitors to changes in the firm's own behavior.

When it comes to inventory management and procurement, however, the view that effective incentive design is geared towards making a decentralized firm behave as if it were centralized remains virtually untarnished. Cases in point are the ever-present criticisms of using absorption costing and budget variances in evaluation, which are rooted in concern that doing so creates perverse incentives for inventory accumulation in divisions (Horngren et al. 2011; Zimmerman 2010). In this paper, we seek to examine the widely held perception that centralized procurement and inventory management represent the ideal. In particular, we note that procurement and inventory management inevitably bring a different outside constituency to the forefront, the supplier. Accounting for strategic multi-period interplay with the supplier, it turns out that a firm (buyer) may prefer delegating procurement and inventory management decisions to its individual divisions. The decentralized arrangement conveys a comforting posture wherein the firm is less exploitative of the supplier down-the-road via excessive inventory holdings, and the supplier reciprocates in the near term by offering more favorable pricing terms.
To elaborate, consider the canonical two-period strategic inventory model. In this setting, a firm initially purchases inputs from a supplier to offer them for sale as outputs. Any unsold units from among these initial purchases are carried forward in inventory to the subsequent period. In the second period, the firm can procure additional units from the supplier and once again offer units for sale. In such a setting, inventory plays an important strategic role: if the firm accumulates extra inventory, the stockpile communicates a lower willingness to pay for inputs in the future. Recognizing the supplier will lower its input price in response, the firm is eager to accumulate inventory. The supplier is well aware of this predilection on the firm's part and offers initial input purchases at a steep premium to stem inventory hoarding (Anand et al. 2008).

With this strategic inventory management problem as a backdrop, we ask what happens if procurement and inventory choices are made not by the firm itself but by its individual divisions, each of which is fixated on its own profits. The usual thinking is that decentralization, particularly at the procurement stage, only creates divergent incentives among the divisions, undercutting any potential economies of scale. What this usual thinking misses is that the firm's penchant for building inventory for strategic reasons comes at a cost and so some level of discord can be useful.

In particular, given the first period wholesale price, the benefit of raising inventory in any one division is much greater from the firm's perspective than from the division manager's perspective. After all, an increase in a division's inventory level puts downward pressure on the second period wholesale price, a benefit shared by all procuring divisions of the firm. While this externality is valuable to the firm, it is of no consequence to the self-interested division manager who is concerned only with his own divisional performance. As a consequence, decentralization leads to lower inventory levels. Stated a bit differently, since each division is only one of many procurers, its unilateral ability to influence the supplier's pricing is limited, and this curbs the division's incentives to build
inventory. The prospect of lower strategic inventory levels thus creates a détente of sorts between the supplier and the firm with both input prices and inventory levels kept in check.

Lest one think the result is rooted in decentralization simply minimizing an evil the firm would prefer to entirely eliminate, it is worth stressing that the role of strategic inventories is not all bad. The downward pressure inventories put on future supplier prices and the de facto two-tier pricing structure it creates for inputs used in second-period sales (some are purchased in period one and some are purchased in period two) means that strategic inventories can be beneficial provided they are used in moderation. Such moderation is precisely what decentralization provides; as evidence, the results demonstrate that when decentralization is preferred by the firm, it is also preferred to the just-in-time (no inventory) approach to procurement.

Given decentralization can actually enhance firm profits by effectively moderating inventory levels, we next consider optimal divisional incentives. After all, the usual view is that incentives should be designed so as to achieve an outcome that replicates (or at least approximates) outcomes under centralized decision-making. Since centralization is no longer the objective when strategic inventories are considered, incentive design too must change. We demonstrate that optimal incentive design balances consideration of firm-wide (global) incentives and divisional (local) incentives to best balance inventory levels. Not only that, we show that when the optimal incentive structure is in place, decentralization is guaranteed to be preferred to centralization, regardless of the number of divisions, extent of holding costs, or level of retail demand.

As a final consideration, we revisit the results when divisions face intra-firm competition at the retail level. That is, we consider what happens when one division's products directly compete (overlap) with those of another division. Of course, this introduces a stark downside of decentralization for the firm in that divisions will be more cutthroat in competition with one another than a central planner would prefer. That said, the upside of decentralization in moderating strategic inventories remains, thereby pointing
to a tradeoff in determining the preferred organizational structure. Once optimal divisional incentives are taken into account, however, decentralization again is always preferred (with greater competition pointing to a greater emphasis on firmwide profits).

While the focus here has been on strategic supply chain issues, extant research has examined other consequences of decentralized procurement and inventory management. For one, centralizing such decisions can be preferred because it allows a firm to negotiate quantity discounts, can help coordinate behavior, and diversify risks (e.g., Munson and Hu 2010). In line with such thinking, much of the literature takes decentralization as given and then asks how firms can improve coordination (e.g., Andersson and Marklund 2000; Anupindi et al. 2001; Bernstein and Federgruen 2005). Relatedly, there is the question of when delegation is costless in that contracts can be judiciously designed so as to replicate the ideal centralized outcome (e.g., Melumad and Reichelstein 1987).

In terms of when and why decentralized decisions can be preferred, extant explanations include the ability to exploit localized private information (e.g., Bushman et al. 2000; Vagstad 2000). In this vein, existing work has stressed that incomplete contracts and/or an inability to costlessly communicate information can point to decentralized arrangements being preferred (Demski Sappington 1987; Melumad et al. 1995). Decentralization and the rents it affords can also help alleviate attendant hold up problems (Baiman and Rajan 1995). Even in the absence of information and incentive problems, decentralization can be useful as a means of posturing to retail competitors (Alles and Datar 1998; Fershtman and Judd 1987; McGuire and Staelin 1983; Sklivas 1987).

The distinguishing feature of the present paper is that it looks at the value of decentralization of procurement and inventory management when there are strategic consequences along the supply chain. In particular, in the presence of a self-interested supplier, a firm's penchant for building inventories to influence input pricing creates a unique strategic interplay (Anand et al. 2008). As we show, this interplay is critically influenced by the organizational structure of the firm.
The remainder of this paper proceeds as follows. Section 2 presents the model. Section 3 details the results: 3.1 provides a one-period benchmark; 3.2 identifies equilibrium in the two-period model wherein inventory plays a nontrivial role; 3.3 details the preferred organizational structure; 3.4 examines optimal incentive provision; and 3.5 addresses the effects of inter-division competition. Section 4 concludes.

2. Model

We study a basic formulation of procurement and inventory management for a firm serving multiple markets. In each of two periods, the firm purchases inputs from a supplier and sells final products to end consumers. The firm consists of \( n, n \geq 2 \), segments (divisions), each of which serves a distinct market. The markets correspond to different geographical territories and/or distinct product offerings; denote the set of markets by \( N \). The (common knowledge) demand for the retail product in market \( i, i \in N \), in period \( t, t = 1,2 \), is given by \( p_t^i = a - q_t^i \), where \( p_t^i \) is the price paid by consumers and \( q_t^i \) is the quantity sold in the retail market. The supplier's production costs and the firm's conversion costs are each normalized to zero.

At the start of each period, the supplier sets its (per-unit) wholesale price, \( w_t \), for the firm. Subsequent to the supplier setting its terms, the firm procures \( Q_t^i \) units of the product and sells \( q_t^i \) units (equivalently, sets the retail price \( p_t^i \)) in retail market \( i, i \in N \). At the end of period 1, the firm carries forward any excess purchases as inventory; denote the inventory level for market \( i \) by \( I_t^i \), \( I_t^i = Q_t^i - q_t^i \). For each unit it carries in its inventory, the firm incurs a holding cost of \( h, h \geq 0 \).

The focus herein is on whether procurement and inventory decisions (i.e., \( Q_t^i \) and \( q_t^i \)) should be made centrally for all \( n \) markets or whether the firm should decentralize and leave procurement and inventory management decisions under the auspices of their respective divisions. Under decentralization, we initially employ the standard formulation
wherein divisions seek to maximize their own divisional profit; the issue of incentive
design is examined further in section 3.4.

The following timeline summarizes the sequence of events.

The supplier sets period one input price \( w_1 \).

The firm purchases \( Q^1_i \) units of input and sells \( q^1_i \) units of output in market \( i \); unsold units are carried in inventory.

The supplier sets period two input price \( w_2 \).

The firm purchases \( Q^2_i \) units of inputs and sells \( q^2_i \) units of output in market \( i \).

**FIGURE 1.** Timeline.

To formally compare the outcomes under centralization vs. decentralization, we first characterize the unique (subgame perfect) equilibrium of the game under each regime by employing backward induction. As is customary, throughout the paper we assume the regularity condition that the demand intercept \( a \) is sufficiently large that quantities, prices and inventory levels derived using the first-order approach are positive. In the model, this corresponds to \( a > 4nh \).

3. Results

As the key novelty of the present analysis is to consider how strategic inventory management influences the desirability of decentralization, we begin with a benchmark where inventory is a non-issue.

3.1. One Period Benchmark

Consider a one-period version of the model, so inventories are moot. Working backwards in the game, a centralized firm chooses sales quantities, \( q^i \), \( i \in N \), to solve

\[
\max_{q^1, i \in N} \sum_{i \in N} [(a - q^i)q^i - wq^i].
\]

This yields equilibrium quantities of \( q^i(w) = [a - w] / 2 \). The supplier, in turn, sets its input price to maximize its profit, taking into account all
output markets. In particular, it solves \( \max_w \sum_{i \in N} wq^i(w) \). Solving this problem yields the equilibrium input price of \( w = a / 2 \).

Under decentralization, output market quantities (retail sales) are chosen by individual divisions, each of which is focused only on its own profit. Again working backwards in the game, division \( i \) solves \( \max_{q^i} [(a - q^i)q^i - wq^i] \). The first-order condition of this problem yields \( q^i(w) = [a - w] / 2 \), precisely as in the centralized case. In other words, in a one-period (no inventory) setting, decentralization and centralization are equivalent. This claim is confirmed in the following proposition (all proofs are provided in the appendix).

**Proposition 1.** In a one-period model, there are no gains to decentralization.

With this benchmark in tow, we now consider how inventory changes preferences for organizational design.

### 3.2. Equilibrium with Inventory

In the two period setting, not only are two periods of retail sales considered, but more importantly, a time lag between input market purchases and output market sales can arise in that the firm can procure inputs and carry forward those purchases in inventory, thereby delaying their sale in the output market. When a firm enters a period holding inventory, the marginal benefit of, and thus the willingness to pay for, additional units is lower the more units are inventoried. Facing a firm with reduced willingness to pay, a supplier finds it optimal to reduce its input price. Thus, holding inventory enables the firm to parlay prior acquisitions into subsequent concessions. Realizing the firm's incentive to build inventory, the supplier strategically responds by boosting initial input prices as a means of reducing downstream inventory build-up. A high first-period wholesale price forces firms to trade off the costs of acquiring and holding inventories against the
concomitant supplier concessions. With these forces at work, the following subsections derive the equilibrium outcomes under centralization and decentralization, respectively.

### 3.2.1. Centralization

Working backwards in the game, the firm's period two choices potentially depend not only on the prevailing wholesale price, $w_2$, but also the inventory it has retained in each market, $I^i$. To be precise, the firm's period two retail sales quantity in market $i$, $q^i_2$, solves:

$$\max_{q^i_2, i \in N} \sum_{i \in N} [(a - q^i_2)q^i_2 - w_2(q^i_2 - I^i)].$$

In (1), the first term for each market reflects retail revenue, whereas the second reflects input market outlays. Solving (1) yields retail quantities in market $i$ of $q^i_2(w_2) = [a - w_2] / 2$ (and, associated input purchases of $q^i_2(w_2) - I^i$). Given this, the supplier's period two pricing problem is as in (2):

$$\max_{w_2} \sum_{i \in N} w_2[q^i_2(w_2) - I^i].$$

Solving (2) reveals the supplier's second period input price, $w_2(I) = a - \frac{1}{n} \sum I^i$, where $I$ denotes the vector of inventories held in each market. Note that the chosen input price is as in the one-period benchmark (i.e., $a/2$), less an adjustment for inventory levels. Intuitively, the greater the inventory carried forward by the firm, the less its willingness to pay for more units (since its inventory can service the high-demand consumers). This lower willingness to pay translates into leverage and thus a lower prevailing wholesale price. Since the input price reflects the average position of each market, the lower price due to inventory reflects the average inventory level held in the $n$ markets.

Given this period two outcome, the game unfolds in period one as follows. The firm's chosen retail sales and inventory levels in period one solve:
\[ \begin{align*}
\max_{q_i^1, I^i, i \in N} & \sum_{i \in N} \left[ (a - q_i^1)q_i^1 - w_1(q_i^1 + I^i) - hI^i \right] \\
+ & \sum_{i \in N} \left[ (a - q_i^2(w_2(I)))q_i^2(w_2(I)) - w_2(I)[q_i^2(w_2(I)) - I^i] \right] 
\end{align*} \] (3)

In (3), the first term reflects the aggregate profit in period one, while the second reflects ensuing aggregate profit in period two. First-order conditions of (3) reveal first-period retail sales and inventory levels, respectively, for market \( i \) equal \( q_i^1(w_1) = [a - w_1]/2 \) and \( \hat{I}^i(w_1) = [3a - 4(h + w_1)]/6 \); the "\( ^i \)" reflects the centralization outcome. Intuitively, period one retail sales are increasing in retail demand \( (a) \) and decreasing in the prevailing wholesale price \( (w_1) \). Similarly, the amount of purchases made in period one that are retained in inventory is increasing in future retail demand \( (a) \), and decreasing in the purchase cost \( (w_1) \) and holding cost \( (h) \). Denoting the vector of inventory levels \( (\hat{I}^i(w_1)) \) by \( \hat{I}(w_1) \), the supplier's first-period input price, taking into account the subsequent behavior of all parties, solves (4):

\[ \begin{align*}
\max_{w_1} & \sum_{i \in N} w_1[q_i^1(w_1) + \hat{I}^i(w_1)] + \sum_{i \in N} w_2(\hat{I}(w_1))[q_i^2(w_2(\hat{I}(w_1))) - \hat{I}^i(w_1)].
\end{align*} \] (4)

In (4), the first term reflects aggregate period one profit, while the second reflects the ensuing aggregate period two profit. Solving (4) yields period one wholesale price of \( \hat{w}_1 = [9a - 2h]/17 \). Intuitively, the price is increasing in demand \( (a) \). The price is also decreasing in inventory holding cost \( (h) \) – this reflects that higher holding costs reduce the supplier's concern that period one input sales will be retained as inventory and used as a strategic weapon against it in period two, thereby permitting it to reduce period one prices. Using this equilibrium price and back substituting yields the equilibrium outcome under centralization, as summarized in Proposition 2.
PROPOSITION 2. Under centralization, the equilibrium entails:

(i) Wholesale prices: \( \hat{w}_1 = \frac{[9a - 2h]}{17} \) and \( \hat{w}_2 = \frac{2[3a + 5h]}{17}; \)

(ii) Inventory: \( \hat{I} = \frac{5[a - 4h]}{34}, \ i \in N; \) and

(iii) Sales: \( \hat{q}_1^i = \frac{[4a + h]}{17} \) and \( \hat{q}_2^i = \frac{[11a - 10h]}{34}, \ i \in N. \)

Examining Proposition 2, the impact of inventory is clear. Absent inventory, the two-period setting is simply a repetition of the one-period benchmark. This is confirmed by presuming the limiting inventory holding cost that disables inventory, \( h = \frac{a}{4}. \) In that case, \( \hat{w}_1 = \hat{w}_2 = \frac{a}{2}, \) and \( \hat{q}_1^i = \hat{q}_2^i = \frac{a}{4}. \) With nontrivial inventory \( (h < \frac{a}{4}) \), the firm uses inventory to drive down period two wholesale price (lower \( h \) translates into lower \( \hat{w}_2 \)). The supplier, in turn, is forced to push up period one wholesale price to reduce the use of inventory for this purpose (lower \( h \) translates into higher \( \hat{w}_1 \)). As noted in Anand et al. (2008), this strategic role of inventory has offsetting effects on efficiency – it reduces double-marginalization in period two but increases it in period one. It also permits a degree of price discrimination for period two sales in that the first units sold in period two were bought at the higher \( \hat{w}_1 \)-rate while the later units sold are bought at the lower \( \hat{w}_2 \)-rate.

In the next section we study how this cross-period double marginalization tradeoff is impacted if the procuring firm is decentralized. After all, division managers may view this tradeoff in a different light than the firm which, in turn, can impact both the firm's aggregate inventory holdings and the strategic supplier's pricing terms.

3.2.2. Decentralization

With decentralization, each division's period two choice, \( q_2^i \), solves:

\[
\text{Max}_{q_2^i} \ [a - q_2^i]q_2^i - w_2[q_2^i - I^i].
\] (5)
Solving (5) yields retail quantities in market $i$ equivalent to the centralization case:

\[ q_2^i(w_2) = \left[ a - w_2 \right] / 2 \] (and input purchases of \( q_2^i(w_2) - I^i \)). Given this, the supplier's period two pricing problem is again as in (2), yielding a second period input price of \( w_2(I) = \frac{a}{2} - \frac{1}{n} \sum I^i \). In period one, division $i$'s chosen sales and inventory levels solve:

\[
\max_{q_1^i, I^i} (a - q_1^i)q_1^i - w_1 (q_1^i + I^i) - hI^i + [a - q_2^i(w_2(I))] q_2^i(w_2(I)) - w_2(I)[q_2^i(w_2(I)) - I^i].
\]  

In (6), the first term reflects the division's period one profit while the second term reflects the ensuing period two divisional profit. First-order conditions of (6) reveal first-period retail sales and inventory levels, respectively, for market $i$ equal to \( q_1^i(w_1) = \left[ a - w_1 \right] / 2 \) and \( \tilde{I}^i(w_1) = \left[ (2n + 1)a - 4n(h + w_1) \right] / [2(2n + 1)]; \) the "\( \sim \)" reflects the decentralization outcome. While period one retail sales are the same as under centralized procurement, inventory levels (and thus procurement levels) are not. In particular, \( \tilde{I}^i(w_1) \) and \( \hat{I}^i(w_1) \) are equivalent for the limiting case of $n = 1$ (as expected), but \( \tilde{I}^i(w_1) < \hat{I}^i(w_1) \) for $n > 1$. This reflects that the degree of inventory holding balances the costs (paying for inputs at the higher \( w_1 \)-rate and holding costs $h$) and the benefits (securing a lower \( w_2 \)). While division $i$ incurs all the costs of carrying its inventory, the benefits are shared with other divisions, since the \( w_2 \) reflects an averaging of market-level conditions. As a result, each division is more reluctant to carry inventory. In effect, decentralization engenders a free-rider problem, one that it is often criticized for. In this case, however, the free-rider problem also conveys a certain posture to the supplier that influences its initial pricing (\( w_1 \)).

Denoting the vector of inventory levels (\( \tilde{I}^i(w_1) \)) by \( \tilde{I}(w_1) \), the supplier's first-period input price solves (7):

\[
\max_{w_1} \sum_{i \in N} w_1[q_1^i(w_1) + \tilde{I}^i(w_1)] + \sum_{i \in N} w_2(\tilde{I}(w_1)) [q_2^i(w_2(\tilde{I}(w_1))) - \tilde{I}^i(w_1)].
\]
Solving (7) yields period one price of \( \tilde{w}_1 = \frac{(2n+1)^2a - 2nh}{8n(n+1) + 1} \). Using this equilibrium wholesale price and back substituting yields the equilibrium outcome under decentralization, as summarized in Proposition 3.

**PROPOSITION 3.** Under decentralization, the equilibrium entails:

(i) Wholesale prices: \( \tilde{w}_1 = \frac{(2n+1)^2a - 2nh}{8n(n+1) + 1} \) and \( \tilde{w}_2 = \frac{2n[a(2n+1) + h(4n+1)]}{8n(n+1) + 1} \);

(ii) Inventory: \( \tilde{I}^i = \frac{\lfloor a - 4nh \rfloor}{2\lfloor 8n(n+1)+1 \rfloor}, \; i \in N \); and

(iii) Sales: \( \tilde{q}^i_1 = \frac{n[2a(n+1) + h]}{8n(n+1)+1} \) and \( \tilde{q}^i_2 = \frac{a[2n(2n+3)+1] - 2nh[4n+1]}{2[8n(n+1)+1]} \), \; i \in N \).

We next consider the firm's preferred organization structure in light of its effect on inventory holding and supplier pricing.

### 3.3. Preferred Organizational Structure

As noted in the previous section, decentralization creates a free-rider problem in procurement – for a given wholesale price, each division procures less (and holds less inventory) than choices made by a centralized firm. While this clearly has coordination downsides, an interesting question is whether decentralization is ever preferred to centralization. The sagacious reader will probably have guessed the answer is affirmative. The reason decentralization can be preferred is that the free-rider problem also alters the supplier's period one pricing choice. Whereas under centralization the supplier is forced to charge an excessive price to dissuade excessive inventory holding, the uncoordinated behavior of separate divisions helps accomplish this task for the supplier, permitting a lower initial wholesale price. To see this effect most clearly, consider the case in which inventory concerns are most pronounced, \( h = 0 \). In this event, the more disjoint the
behavior under decentralization, the lower the period one wholesale price, i.e., $\tilde{w}_1$ is decreasing in $n$.

This effect suggests that some of the dangers of excessive inventory are naturally curbed when the firm employs decentralization. That said, it remains unclear whether this can go too far and, importantly, whether the firm can benefit from such curbs on its own behavior. A comparison of the firm's profit under centralization and decentralization confirms the following proposition. (Closed form expressions for $f(n)$, and similar functions employed in subsequent propositions, are provided in the appendix.)

**Proposition 4.** Decentralization is strictly preferred if and only if $h/a < f(n)$, where $f(n)$ is decreasing in $n$.

Following the above logic, low values of $h/a$ indicate a particularly disconcerting inventory problem – in that case, inventories held for strategic reasons excessively bump up period one input price. In such cases, decentralization is useful as it helps the firm precommit to not carrying excessive inventory which, in turn, ensures a more moderate period one wholesale price. The flip side of the equation is that the diffuse priorities of many divisions can create an excessively uncoordinated procurement policy, one in which the firm's inventory level is excessively low. Hence, provided discord is sufficiently small (i.e., $n$ is sufficiently low), the downsides of decentralization are not too severe. This notion is captured by the right-hand side of the condition in Proposition 4 being decreasing in $n$.

Lest one conclude that the upside of decentralization here is simply to reduce the evils of inventories, recall that inventories do have an upside. A better interpretation is thus that decentralization helps moderate inventory levels. This is best seen by comparing the outcome to the case where inventory is not permitted (say a precommitment to just-in-time purchases). If inventory were just an evil to be eradicated, and decentralization was imperfect at such eradication, a precommitment to no inventory would be preferable. As
the next proposition confirms, decentralization is also preferred to just-in-time purchases, pointing to its "goldilocks" feature of supporting just the right mix of inventories and sales.

PROPOSITION 5. Decentralization reduces, but does not eliminate, the use of strategic inventories by the firm. Moreover, decentralization is preferred by the firm to just-in-time purchases.

With the benefits of decentralization clearly delineated in our basic setting with the usual presumption of a simple linear wholesale price set unilaterally by the supplier, a digression on more general bargaining seems apropos. While simple pricing and negotiation arrangements form the crux of the literature on supply chain coordination, the literature has also stressed that the typical supply chain friction of double marginalization can often be eliminated under negotiated two-part tariff pricing arrangements (see, e.g., Cachon 2003). That begs the question of whether such negotiated pricing arrangements can disable the benefits of decentralization.

Consider the following (dynamic) pricing arrangement without rebates. In period $t$, the supplier offers a two-part tariff pricing schedule: the transfer in period $t$ is $F_t + w_t \sum_{i \in N} Q^i_t$, where $F_t$ is the fixed fee payment (allocated equally to all divisions) and $w_t$ is the incremental (per unit) price. Each period's pricing schedule is determined from a (generalized) Nash bargaining process Here, $\lambda \in (0,1)$ reflects the relative bargaining power of the firm, with the limiting case of $\lambda = 0$ corresponding to the supplier maintaining complete pricing authority.

Relegating the details to the appendix, it is worth noting that under centralization the inventory level in equilibrium is $\hat{I}^i = \frac{a}{2} - \frac{[2 - \lambda]t}{2[1 - \lambda]^2}$. Just as in Theorem 4 of Anand et al. (2008), strategic inventories ensure that the manufacturer cannot achieve first-best profit even with general contracts. The reason for general contracts failing to achieve supply chain coordination is that by carrying inventory forward from period one, the firm is able to gain bargaining traction since a failure to come to agreement in period two negotiations is
not overly costly to the firm (it can at least make use of its inventory reserves). The stronger posture inventory brings thus translates into a lower period two fixed fee being paid by the firm in equilibrium. This, in turn, implies that the supplier sets per unit price in period one greater than marginal cost in order to dissuade inventories. In short, with general contracts, the issue of strategic inventories remains prominent, but is manifest in a desire to influence fixed fees rather than unit fees.

Under decentralization, inventories are reduced to \( \bar{I}_i = \frac{a}{2} - \frac{[1 + n - \lambda]hn}{2[1 - \lambda]^2} \). This, in turn, can promote efficiency and higher firm profit. Again, the reason this occurs is because decentralization fosters a free-rider problem vis-a-vis inventories. In this case, each division is reluctant to incur substantial carrying costs and higher period one unit price in order to carry inventory, since the benefit of inventory (in lowering subsequent fixed fees) is shared among all divisions. As the firm would like to commit to a less aggressive posture with inventory, the decentralization fueled free-rider problem turns out to be a boon. With lower inventories, less wasteful carry forward occurs (the waste being due to holding cost \( h \)), and the gains from reduced waste are shared among the supply chain partners.

Roughly stated, in the previous section, decentralization helped moderate inventory levels which, in turn, induced the supplier to respond with more favorable unit wholesale prices. With two-part tariff arrangements, similar forces are again at work but the focus now is on inducing the supplier to set a lower fixed transfer. The fact that the main benefit of decentralization identified herein essentially persists under more general contracting and bargaining arrangements is confirmed in the following proposition.

**PROPOSITION 6.** With general bargaining over two-part tariffs, decentralization is strictly preferred for all \( h > 0 \).

With the newly identified and robust benefits of decentralization in mind, we now examine optimal divisional incentives.
3.4. Managerial Incentives under Decentralization

As alluded to previously, the traditional view of divisional incentives is one where the goal is to replicate centralized decision-making. That is, while practicalities may necessitate decentralization, ideal incentives are viewed as ensuing divisions do what an omniscient and omnipresent central planner would do. That said, in reality divisional evaluation varies in that some divisions are evaluated on divisional information, while others are evaluated based on both local as well as firm-wide information (see e.g. Keating 1997, Bushman, Indjejikian, and Smith 1996). The relative incentive weight to be placed on divisional versus firm-wide performance is a ubiquitous theme in the design of compensation systems for divisional managers.

An alleged shortcoming of divisional performance measurement is it provides division managers with insufficient consideration for the well-being of other divisions within the firm (see e.g. Holmstrom and Tirole 1991, Anctil and Dutta 1999). Firm-wide profit sharing is commonly thought to eliminate this concern but at the price of exposing managers to inordinate amounts of risk – at least if appropriate incentives for divisional effort are to be maintained. As demonstrated in the previous section, even in the absence of risk-sharing benefits, divisional performance measurement and the accompanying focus on local profits have an upside, that of moderating a firm's excessive inventories and reducing procurement costs. The focus in this subsection is to endogenize the degree of divisional myopia that is optimal.

To address the optimal mix of divisional incentives, say that the firm can design incentives so that each division maximizes a weighted average of firm profit and its own divisional profit. Placing a weight of $\beta \in [0,1]$ on divisional profit and weight of $(1 - \beta)$ on firm profit reflects this divisional incentive scheme most succinctly. Note that the previously examined decentralization regime reflects the limiting case of $\beta = 1$. The previous section noted that the upside of decentralization is that it can precommit the firm to
a softer stance in inventory hoarding. On the other hand, this softened stance can be too soft (i.e., the lack of coordination can be too costly). Consistent with this intuition, Proposition 7 confirms that the optimal incentive weight entails a nontrivial amount of concern for firmwide profit ($\beta < 1$) but also a degree of divisional myopia ($\beta > 0$).

**Proposition 7.** Under decentralization, the preferred divisional incentive weight is denoted $\beta^*(h/a,n)$. $\beta^*(h/a,n) \in (0,1)$ is the unique $\beta$-value that solves $h/a = g(\beta;n)$.

The precise value of $g(\beta;n)$ is detailed in the appendix. Perhaps more germane to the big picture, however, is its properties and how they affect the preferred incentive weights. This is reflected in the following corollary.

**Corollary.** $\beta^*(h/a,n)$ is (i) decreasing in $h/a$ and (ii) decreasing in $n$.

Intuitively, part (i) reflects that the greater the excessive inventory problem (i.e., lower $h/a$), the more the need to magnify divisional myopia (i.e., higher $\beta$). Part (ii) reflects that the greater the number of divisions, the more the concern with coordinating behavior and, hence, less the deviation away from firm profit in evaluation. Roughly stated, with many divisions, even a slight distortion in incentives has a large impact on behavior. Taking this into account, we can now revisit the question of optimal structure. If the firm has complete control over incentive structures, the downsides of decentralization can be managed so that it is (always) strictly preferred to centralization. This is confirmed in Proposition 8.

**Proposition 8.** With the preferred divisional incentive weight, decentralization is strictly preferred to centralization.

The essence of the results thus far are captured in Figure 2, which covers the case of $h = 0$. The left panel plots the firm's profit as a function of $\beta$, denoted $\Pi(\beta)$, for $n = 4$, capturing the pure centralization case of section 3.1 ($\beta = 0$), the pure decentralization
case of section 3.2 ($\beta = 1$); note that in this case decentralization is preferred. It also reflects the optimal incentive weight, $\beta^*(0,4)$, which surely beats centralization as in Proposition 8. The right panel then plots this preferred incentive weight as a function of $n$. As Proposition 7 notes, this weight is always positive; as the corollary notes, this weight is decreasing in $n$.

![Graph](image.png)

**Figure 2.** Firm profit as a function of $\beta$, and the optimal $\beta$-weight as a function of $n$.

### 3.5. Interdivisional Competition

In order to highlight the central forces affecting the role of strategic inventories and the implications for preferred organizational structure and divisional performance measures, we have maintained the implicit assumption that divisions are operating in geographically separated markets and/or sell independent goods. If these assumptions do not hold, a division manager with remuneration based on divisional profits will see other divisions as rivals and will act accordingly. Many consider this, the costly competition it engenders
within the firm, to be a key cost of decentralization. To include this feature, we now append the model to reflect interdivisional competition. To do so in a parsimonious fashion, say \( n = 2 \), and consumer demand for division \( i \)'s product in period \( t \) is 
\[
q_i^t = a - q_i - kq_i^t,
\]
where \( k \in [0,1] \) reflects the degree of interdivisional competition (again, we presume \( a \) is sufficiently large that positive quantities and prices are assured).

In this case, the one-period benchmark yields a strict benefit to centralization, since it permits quantity choices that reduce intra-firm competitive pressures. With inventories, however, a benefit of decentralization remains. It now just needs to be weighed against the (now steeper) costs of divisional myopia. Provided concerns of excessive inventories (relative to concerns of intra-firm competition) are sufficiently pronounced, however, decentralization can again be preferred.

**Proposition 9.** In the presence of interdivisional competition,

(i) In a one-period model, centralization is strictly preferred to decentralization; and

(ii) In the two-period model, decentralization is strictly preferred to centralization if and only if \( h/a < f^c(k) \).

Combining all of the paper's results, a firm with endogenous divisional incentives still prefers to engender a degree of myopia in order to reduce inventory incentives despite the fact that such a choice will boost intra-firm competition.

**Proposition 10.** Taking into account managerial incentives under decentralization,

(i) the preferred divisional incentive weight is \( \beta^c(h/a,k) \) where \( \beta^c(h/a,k) \in (0,1) \) is the unique \( \beta \)-value that solves \( h/a = g^c(\beta;k) \); and

(ii) decentralization with \( \beta^c(h/a,k) \) is strictly preferred to centralization for all \( (a,h,k) \).

Figure 3 provides a crisp summary of the results in this section, again for the case of \( h = 0 \). The left panel plots the firm's profit, \( \Pi(\beta,k) \), as a function of \( \beta \) for \( k = 1/4 \). As
in Proposition 9, profit under centralization ($\beta = 0$) is lower than under decentralization ($\beta = 1$), this despite the additional downside of intra-firm rivalries undermining profit. It also reflects the optimal incentive weight, $\beta^c(0,1/4)$, which surely beats centralization as in Proposition 10. The right panel then plots this preferred incentive weight as a function of $k$. Intuitively, the greater the concern over intra-firm rivalry (i.e., greater $k$), the lower the divisional incentive weight in optimal incentives (i.e., lower $\beta^c(0,k)$).

**FIGURE 3.** Firm profit as a function of $\beta$ (for $k = 1/4$), and the optimal $\beta$-weight as a function of $k$.

4. **Conclusion**

Textbook discussions of organizational structure typically view centralized decision-making as an ideal to strive for. Practicalities such as limited resources and time coupled with diffuse information typically form the basis for decentralization as an imperfect substitute. In this paper, we demonstrate that the seemingly imperfect nature of
decentralization may actually prove to be desirable when a firm relies on outsourcing and actively manages inventory.

In particular, when a firm procures inputs to be sold in a variety of markets, it has incentives to use inventory management as a strategic tool to extract surplus from its supplier. The supplier, in turn, must rely on its own strategic weapons (here, wholesale prices) to dissuade such behavior. A decentralized procurement policy serves as a natural salve on the otherwise strained supply chain relationship. Since individual divisions find large inventory build-up a substantial price to pay for influencing supplier pricing – the benefit of a price cut is shared by all divisions – the intense strategic posture is softened. We demonstrate that such softened strategic posture under decentralization can be preferred to both centralized procurement and a commitment to just-in-time purchasing. Besides providing some justification for the prevalence of decentralized procurement in practice, the results also provide intuitive comparative statics for the efficacy of decentralized decision making and for the optimal incentive structures of decentralized firms.
APPENDIX

Proof of Proposition 1. Consider the one-period centralized setting. Given supplier price $w$, the firm (buyer) chooses $q^i, i \in N$, to maximize its profit as follows:

$$\max_{q^i, i \in N} \sum_{i \in N} [(a - q^i)q^i - wq^i]. \quad (A1)$$

Solving the first-order conditions of (A1) yields equilibrium quantities as a function of the wholesale price: $q^i(w) = [a - w] / 2$. Thus, the supplier's problem is as in (A2):

$$\max_w w \sum_{i \in N} q^i(w) = \max_w nw[a - w] / 2. \quad (A2)$$

Solving (A2), yields $w = a / 2$ and, using (A1), the centralized firm profit is $\Pi = na^2 / 16$. Under decentralization, each division manager chooses his quantity focused solely on division profit. Thus, manager $i$ chooses $q^i$ for division $i$ as follows:

$$\max_{q^i} [(a - q^i)q^i - wq^i], \quad i \in N. \quad (A3)$$

From (A3), $q^i(w) = [a - w] / 2$ and, thus, the supplier problem is again as in (A2). As a result, under decentralization the outcome is identical to that under centralization, i.e., $w = a / 2, q^i = a / 4$, and $\Pi = na^2 / 16$. $\blacksquare$

Proof of Proposition 2. Consider the two-period outcome under centralization. Working backwards, given inventory level $I^i, i \in N$, and second-period wholesale price $w_2$, the firm chooses second-period sale quantities, $q^i_2, i \in N$, to solve:

$$\max_{q^i_2, i \in N} \sum_{i \in N} [(a - q^i_2)q^i_2 - w_2(q^i_2 - I^i)]. \quad (A4)$$

Solving the first-order conditions of (A4) yields equilibrium sales as a function of the wholesale price: $q^i_2(w_2) = [a - w_2] / 2$. Given the inventory it carries, the firm's second-period purchases are $\sum_{i \in N} [q^i_2(w_2) - I^i]$. Thus, the supplier's problem is as in (A5):

$$\max_{w_2} \sum_{i \in N} w_2[q^i_2(w_2) - I^i]. \quad (A5)$$

The first-order condition of (A5) yields the second-period wholesale price as a function of inventory $w_2(I^i, i \in N) = a / 2 - \frac{1}{n} \sum I^i$. Using this wholesale price and $q^i_2(w_2)$, in (A4) and (A5) yields second-period firm profit, $\Pi_2(I^i, i \in N)$, and second-period supplier profit, $\Pi^S_2(I^i, i \in N)$, respectively, as noted below:
\[\Pi_2(I^i, i \in N) = \frac{1}{16} \left[ na^2 + 12a \sum_{i \in N} I^i - \frac{12}{n} \left( \sum_{i \in N} I^i \right)^2 \right]\]

and

\[\Pi_2^S(I^i, i \in N) = \frac{1}{8n} \left[ na - 2 \sum_{i \in N} I^i \right]^2. \quad (A6)\]

Using (A6), the firm’s problem in period one is to choose \(q_1^i\) and \(I^i\), \(i \in N\), to maximize its two-period profit as follows:

\[\text{Max} \sum_{i \in N} \left[(a - q_1^i)q_1^i - w_1(q_1^i + I^i) - hI^i\right] + \Pi_2(I^i, i \in N). \quad (A7)\]

The first-order conditions of (A7) yield first-period sale quantities and inventory levels as a function of period-one wholesale price. Since the divisions are ex ante identical, without loss of generality the symmetric outcome is characterized by \(q_1^i(w_1) = \frac{[a - w_1]}{2}\) and \(\hat{I}_i(w_1) = \frac{[3a - 4(h + w_1)]}{6}\). Given this, the supplier sets the first-period wholesale price to solve:

\[\text{Max} \sum_{i \in N} w_1[q_1^i(w_1) + \hat{I}_i(w_1)] + \Pi_2^S(\hat{I}_i(w_1), i \in N). \quad (A8)\]

Solving (A8) yields \(\hat{w}_1 = \frac{[9a - 2h]}{17}\). Back substituting, \(q_1^i(\hat{w}_1) = \hat{q}_1^i\), \(\hat{I}_i(\hat{w}_1) = \hat{I}_i\), \(w_2(\hat{I}_i, i \in N) = \hat{w}_2\), and \(q_2^i(\hat{w}_2) = \hat{q}_2^i\), as presented in Proposition 2. Given \(a > 4nh\), the equilibrium inventory level, purchases, and sales are each non-negative. \(\blacksquare\)

**Proof of Proposition 3.** Consider the two-period outcome under decentralization. Given inventory level \(I^i\), and second-period wholesale price \(w_2\), manager \(i\) chooses second-period sale quantity, \(q_2^i\), to maximize his division's profit as follows:

\[\text{Max} [a - q_2^i]q_2^i - w_2[q_2^i - I^i], \quad i \in N. \quad (A9)\]

Solving the first-order condition of (A9) yields the same period-two equilibrium sales as a function of the wholesale price: \(q_2^i(w_2) = \frac{[a - w_2]}{2}\). Thus, the supplier's problem is again as in (A5), and this yields \(w_2(I^i, i \in N) = \frac{a}{2} - \frac{1}{n} \sum_{i \in N} I^i\). Thus, second-period firm and supplier profits are as in (A6). From (A9), the second-period profit for division \(i\), \(\pi_2^i(I^i, i \in N)\), equals:
\[
\pi_2^i(I^i, i \in N) = \left[ \frac{n^2a^2 + 4an\left(2nI^i + \sum_{j \in N} I^j\right) + 4 \sum_{j \in N}\left(\sum_{j \in N} I^j - 4nI^i\right)}{16n^2} \right].
\] (A10)

Using (A10), manager \(i\)'s problem in period one is to choose \(q_1^i\) and \(I^i\) to maximize division \(i\)'s two-period profit as follows:

\[
\text{Max}_{q_1^i, I^i} \left[ a - q_1^i - w_1[q_1^i + I^i] - hl^i + \pi_2^i(I^i, i \in N) \right].
\] (A11)

The first-order conditions of (A11) yield first-period sales and (symmetric) inventory levels as a function of period-one wholesale price. In particular, \(q_1^i(w_1) = [a - w_1] / 2\) and \(\tilde{I}^i(w_1) = [(2n + 1)a - 4n(h + w_1)] / [2(2n + 1)]\). Given this, the supplier sets the first-period wholesale price to solve:

\[
\text{Max}_{w_1} \sum_{i \in N} w_1[q_1^i(w_1) + \tilde{I}^i(w_1)] + \Pi_2^S(\tilde{I}^i(w_1), i \in N).
\] (A12)

Solving (A12) yields \(\tilde{w}_1 = [(2n + 1)^2a - 2nh] / [8n(n + 1) + 1]\). Back substituting, \(q_1^i(\tilde{w}_1) = \tilde{q}_1^i\), \(\tilde{I}^i(\tilde{w}_1) = \tilde{I}^i\), \(w_2(\tilde{I}^i(\tilde{w}_1), i \in N) = \tilde{w}_2\), and \(q_2^i(\tilde{w}_2) = \tilde{q}_2^i\), as presented in Proposition 3. Since \(a > 4nh\), the equilibrium inventory level, purchases, and sales are each non-negative.

**Proof of Proposition 4.** Using the solution in Proposition 2 and Proposition 3, the firm's profit under centralization and decentralization are as follows:

\[
\hat{\Pi} = \frac{n[155a^2 - 118ah + 304h^2]}{1156} \text{and}
\]

\[
\tilde{\Pi} = \frac{n}{4[8n(n + 1) + 1]^2} \left[ a^2(32n^4 + 80n^3 + 44n^2 - 1) - 2ah(32n^4 + 16n^3 + 4n^2 + 6n + 1) + 8nh^2(8n^3 + 20n^2 + 9n + 1) \right].
\] (A13)

From (A13), \(\hat{\Pi} - \tilde{\Pi} > 0\) if and only if the term \(T(h / a) > 0\) where:

\[
T(h / a) = [h / a]^2[76 + 714n + 1592n^2 - 240n^3] + [h / a][115 + 510n - 1272n^2 - 2736n^3] + [111 + 731n + 652n^2 - 168n^3].
\] (A14)

From (A14), \(T'(h / a) < 0\) for \(0 \leq h / a < 1 / [4n]\). This proves that decentralization is preferred if and only if \(h / a\) is sufficiently small, and the cutoff \(h / a\)-value is obtained by setting \(T(h / a) = 0\). Thus, from (A14), decentralization is preferred if and only \(h/a < f(n)\), where:
\[ f(n) = \frac{115 + 510n - 1272n^2 - 2736n^3 + 17[8n(n + 1) + 1]\sqrt{396n^2 - 324n - 71}}{4[120n^2 - 796n^2 - 357n - 38]} \] (A15)

From (A15), \( f'(n) < 0 \) for \( n \geq 2 \).

**Proof of Proposition 5.** From Proposition 2 and Proposition 3,

\[ \hat{I} - \tilde{I} = \frac{2[n - 1][a(10n + 3) + h(28n + 5)]}{17[8n(n + 1) + 1]} > 0. \] (A16)

From (A16), and \( a > 4nh \), \( \hat{I} > \tilde{I} > 0 \). Next, we contrast the solution under decentralization with the just-in-time purchase policy. In the latter, zero inventory case, the solution under either centralization or decentralization is merely a two-fold replication of the one-period solution as characterized in the proof of Proposition 1. Formally, this can be verified by repeating the backward induction process employed in the proofs of Propositions 2 and 3 with the added constraint that \( I \) is not a choice variable for the firm but is fixed at \( I = 0 \). With \( I = 0 \), in each period, the wholesale price is \( a / 2 \) and the firm's total sales equal \( na / 4 \). Using this solution, the two-period firm profit is \( na^2 / 8 \). Thus, decentralization is preferred to just-in-time if and only if \( \tilde{\Pi} < na^2 / 8 > 0 \) where \( \tilde{\Pi} \) is specified in (A13). It is easy to verify that:

\[ \frac{d[\tilde{\Pi} - na^2 / 8]}{dh} < 0 \quad \text{and} \quad [\tilde{\Pi} - na^2 / 8]_{h=a/4n} = 0. \] (A17)

From (A17), it follows that \( \tilde{\Pi} - na^2 / 8 > 0 \) for all \( 0 < h / a < 1 / [4n] \). \[\blacksquare\]

**Proof of Proposition 6.** Consider the two-part tariff arrangement under centralization. If negotiations break down in period two, the default (disagreement point) profit for the firm and the supplier, denoted \( \Pi^D \) and \( \Pi^{SD} \), equal \( \sum_{i \in \mathbb{N}} [a - I^i]l^i \) and 0, respectively. As in the proof of Proposition 2, \( q^i_2(w_2) = [a - w_2] / 2 \) and, hence, the second-period wholesale price and fixed tariff solve the generalization Nash product:

\[
\max_{w_2, F_2} \left[ \sum_{i \in \mathbb{N}} \left[ (a - q^i_2(w_2))q^i_2(w_2) - w_2[q^i_2(w_2) - I^i] \right] - F_2 - \Pi^D_2 \right]^\lambda \times \left[ \sum_{i \in \mathbb{N}} \left[ w_2[q^i_2(w_2) - I^i] \right] + F_2 - \Pi^{SD}_2 \right]^{1-\lambda}.
\]

(A18)

The solution to (A18) yields:
Using (A19), the second-period firm and supplier profits equal:

\[ \Pi_2(I^i, i \in \mathbb{N}) = \frac{na^2}{4} - F_2(I^i, i \in \mathbb{N}) \quad \text{and} \quad \Pi_2^S(I^i, i \in \mathbb{N}) = F_2(I^i, i \in \mathbb{N}). \]  

(A20)

Using (A20), the firm’s problem in period one is to choose \( q_1^i \) and \( I^i, i \in \mathbb{N} \), to maximize its two-period divisional profit as follows:

\[
\max_{q_1^i, I^i, i \in \mathbb{N}} \sum_{i \in \mathbb{N}} \left[ (a - q_1^i)q_1^i - w_1(q_1^i + I^i) - hI^i \right] - F_1 + \Pi_2(I^i, i \in \mathbb{N}).
\]  

(A21)

Since the divisions are ex ante identical, without loss of generality the symmetric outcome is characterized by \( q_1^i(w_1) = \frac{a - w_1}{2} \) and \( \hat{I}^i(w_1) = \frac{a}{2} - \frac{h + w_1}{2[1 - \lambda]} \). From (A19) and (A20), a breakdown of negotiations in the first period (so \( I^i = 0 \)) yields default profits for the firm and supplier of \( \Pi_1^D = \lambda na^2 / 4 \) and \( \Pi_1^{SD} = [1 - \lambda]na^2 / 4 \). Hence, the first-period wholesale price and fixed tariff solve the problem:

\[
\max_{w_1, F_1} \left[ \sum_{i \in \mathbb{N}} \left( (a - q_1^i(w_1))q_1^i(w_1) - w_1(q_1^i(w_1) + \hat{I}^i(w_1)) - h\hat{I}^i(w_1) \right) + \Pi_2(\hat{I}^i(w_1), i \in \mathbb{N}) - F_1 - \Pi_1^D \right]^\lambda \times \\
\left[ \sum_{i \in \mathbb{N}} w_1(q_1^i(w_1) + \hat{I}^i(w_1)) + \Pi_2^S(\hat{I}^i(w_1), i \in \mathbb{N}) + F_1 - \Pi_1^{SD} \right]^{1-\lambda}.
\]  

(A22)

Solving (A22) yields:

\[
\hat{w}_1 = \frac{h}{1 - \lambda} \quad \text{and} \quad \hat{F}_1 = \left[ \frac{n}{4[1 - \lambda]^3} \right] \left[ 2a^2(1 - \lambda)^4 - 2ah(1 - \lambda)^2(3 - 2\lambda + \lambda^2) + h^2(5 - 8\lambda + 6\lambda^2 - 2\lambda^3) \right].
\]

(A23)

Back substituting then yields firm profit under centralization with two-part tariff:

\[
\hat{\Pi} = \left[ \frac{n\lambda}{4[1 - \lambda]^2} \right] \left[ 2a^2(1 - \lambda)^2 - 2ah(1 - \lambda)^2 + h^2(3 - 2\lambda) \right].
\]

(A24)

Next consider the outcome under decentralization. The second-period solution is unaffected, so (A19) and (A20) hold. Division \( i \)'s problem in period one is to choose \( q_1^i \) and \( I^i \) to maximize its two-period divisional profit as follows:
\[ \text{Max}_{q_i, I'} \left[ (a - q_i^1)q_i^1 - w_1(q_i^1 + I^i) - h I^i - \frac{F_1}{n} + \frac{a^2}{4} - \frac{F_2(I^i, i \in N)}{n} \right]. \] (A25)

The first-order conditions of (A25) yield first-period sales and inventory levels as a function of period-one wholesale price. In particular, \( q_i^1(w_i) = \frac{(a - w_i)}{2} \) and \( \tilde{I}^i(w_i) = \frac{a - n[h + w_i]}{2[1 - \lambda]} \). Hence, the first-period wholesale price and fixed tariff solve the problem in (A22) with \( \tilde{I}^i(w_i) \) replaced by \( \tilde{I}^i(w_i) \). Solving this problem yields:

\[ \tilde{w}_1 = \frac{nh}{1 - \lambda} \]

and

\[ \tilde{F}_1 = \left[ \frac{n}{4[1 - \lambda]^2} \right] \left[ 2a^2(1 - \lambda)^4 - 2ah(1 - \lambda)^2(1 - 2\lambda + \lambda^2 + 2n) - nh^2(-2 - 6\lambda^2 + 2\lambda^3 - 4n + n^3 + \lambda(6 + 4n - 2n^2)) \right]. \] (A26)

Back substituting then yields firm profit under decentralization with two-part tariff:

\[ \tilde{\Pi} = \left[ \frac{n\lambda}{4[1 - \lambda]^2} \right] \left[ 2a^2(1 - \lambda)^2 - 2ah(1 - \lambda)^2 + nh^2(2 - 2\lambda + n) \right]. \] (A27)

Using (A24) and (A27), \( \tilde{\Pi} - \tilde{\Pi} = \frac{n\lambda[n - 1]h^2[3 + n - 2\lambda]}{4[1 - \lambda]^2} \), which is positive for all \( h > 0 \). This completes the proof of Proposition 6. \[ \square \]

**Proof of Proposition 7.** Given incentive weight \( \beta \), inventory level \( I^i \), and second-period wholesale price \( w_2 \), manager \( i \) chooses second-period sales, \( q_2^i \), to maximize his compensation as follows:

\[ \text{Max}_{q_2^i} \beta \left[ (a - q_2^i)q_2^i - w_2(q_2^i - I^i) \right] + [1 - \beta] \sum_{j \in N} \left[ (a - q_j^i)q_j^i - w_2(q_j^i - I^j) \right]. \] (A28)

The first-order conditions of (A28) yields \( q_2^i(w_2) = \frac{(a - w_2)}{2} \). Thus, the supplier's problem is again as in (A5), and this yields \( w_2(I^i, i \in N) = \frac{a}{2} - \frac{1}{n} \sum_{i \in N} I^i \) and second-period firm and supplier profits as in (A6); and second-period profit for division \( i \) is given in (A10). Thus, using (A6) and (A10), manager \( i \)'s problem in period one is to choose \( q_i^1 \) and \( I^i \) to maximize his two-period compensation as follows:

\[ \text{Max}_{q_i^1, I^i} \beta \left[ (a - q_i^1)q_i^1 - w_1(q_i^1 + I^i) - h I^i + \bar{\pi}_2^i(I^i, i \in N) \right] + \\
[1 - \beta] \left[ \left( \sum_{j \in N} \left[ (a - q_j^i)q_j^i - w_1(q_j^i + I^j) - h I^j \right] \right) + \Pi_2(I^i, i \in N) \right]. \] (A29)
The first-order conditions of (A29) yield first-period sales and (symmetric) inventory level for division $i$ as a function of period-one wholesale price. In particular, $q_i^1(w_1) = (a - w_1)/2$ and $I_i^1(w_1; \beta) = [(n(3 - \beta) + \beta)a - 4n(h + w_1)]/[2n(3 - \beta) + 2\beta]$. Given this, the supplier sets the first-period wholesale price to solve:

$$\max_{w_1} \sum_{i \in N} w_1[q_i^1(w_1) + I_i^1(w_1; \beta)] + \Pi_N^I(I_i^1(w_1; \beta), i \in N).$$

(A30)

Solving (A30) yields $w_1(\beta) = \frac{a[n(3 - \beta) + \beta]^2 - 2nh[n(1 - \beta) + \beta]}{2n[5 - \beta] + \beta^2 + n^2[17 - 10\beta + \beta^2]}$. Using this, $q_i^1(w_1(\beta)) = q_i^1(\beta)$; $I_i^1(w_1(\beta); \beta) = I_i(\beta)$; $w_2(I_i(\beta), i \in N) = w_2(\beta)$; $q_2^i(w_2(\beta)) = q_2^i(\beta)$. Thus, the firm's two-period profit, as a function of the incentive weight, $\Pi(\beta)$, equals:

$$\Pi(\beta) = n\left\{\left[a - q_1^i(\beta)\right]q_1^i(\beta) - w_1(\beta)\left[q_1^i(\beta) + I_i(\beta)\right] - hI_i(\beta) + [a - q_2^i(\beta)q_2^i(\beta) - w_2(\beta)\left[q_2^i(\beta) - I_i(\beta)\right]\right].$$

(A31)

Solving the first-order condition of (A31), i.e., $d\Pi(\beta)/d\beta = 0$, yields $h/a = g(\beta; n)$ where:

$$g(\beta; n) = \left\{-5\beta^4 - 4n\beta^3[14 - 5\beta] - 6n^2\beta^2[31 - 28\beta + 5\beta^2] + 4n^3\beta[-32 + 93\beta - 42\beta^2 + 5\beta^3]\right. +$$

$$n^4[199 + 128\beta - 186\beta^2 + 56\beta^3 - 5\beta^4] - [2n(5 - \beta)\beta + \beta^2 + n^2(17 - 10\beta + \beta^2)]\times$$

$$[-7\beta^4 + 4n\beta^3(-29 + 7\beta) - 6n^2\beta^2(99 - 58\beta + 7\beta^2) + 4n^3\beta(-233 + 297\beta - 87\beta^2 + 7\beta^3) +$$

$$n^4(1 + 932\beta - 594\beta^2 + 116\beta^3 - 7\beta^4)]^{1/2} + \left\{4[2n(7 - 2\beta)\beta^3 + \beta^4 + 6n^2\beta^2(12 - 7\beta + \beta^2) -$$

$$2n^3\beta(-77 + 72\beta - 21\beta^2 + 2\beta^3) + n^4(63 - 154\beta + 72\beta^2 - 14\beta^3 + \beta^4)]\right\}\bigg\}.$$  

(A32)

The fact that $\beta^*(h/a, n)$ is the unique $\beta$-value that solves $h/a = g(\beta; n)$, and that $\beta^*(h/a, n) \in (0, 1)$, follows from (i) $\partial g(\beta; n)/\partial \beta < 0$ for $0 \leq \beta \leq 1$ and $n \geq 2$, and (ii) $g(1; n) < 0 \leq h/a < 1/[4n] < g(0; n) = 13/18$. ■

**Proof of the Corollary.** $\beta^*(h/a, n)$ is decreasing in $h/a$ follows from the facts that $h/a = g(\beta^*(h/a, n); n)$ and $\partial g(\beta; n)/\partial \beta < 0$. Also, for $0 < \beta < 1$, $\partial g(\beta; n)/\partial n < 0$. The comparative statistics with respect to $n$ then follows:

$$\frac{d\beta^*(h/a, n)}{dn} = -\left.\frac{\partial g(\beta; n)/\partial n}{\partial g(\beta; n)/\partial \beta}\right|_{\beta = \beta^*} < 0.$$  

(A33)

This completes the proof of the Corollary. ■
Proof of Proposition 8. The proof follows from (i) $\beta^*(h/a,n)$ is the $\beta$-value that maximizes $\Pi(\beta)$ in (A31), (ii) $\beta^*(h/a,n) > 0$ as proved in the proof of Proposition 7, and (iii) $\Pi(0) = \hat{\Pi}$.

Proof of Proposition 9. The backward induction process as used in the Proofs of Proposition 1-3 applies except that now the (inverse) demand function in period $t$, $t = 1,2$, is $p_t^i = a - q_t^i - kq_j^i$, $i, j = 1, 2$; $i \neq j$. Rather than repeat the procedure, we merely summarize the equilibrium outcomes in each case.

One-period centralization: $w = a / 2$, $q^i = a / [2(2 + k)]$, and $\Pi = a^2 / [2(2 + k)^2]$.

Contrasting the two cases, the firm's profit under centralization exceeds that under decentralization by $a^2 k^2 / [8(1 + k)(2 + k)^2] > 0$ proving part (i). Turning to the two-period setting, the analogs to Propositions 2 and 3, respectively, are as follows (here, the nonnegativity condition is $a > 4h[2 + k]$):

Two-period centralization:

(i) $\hat{w}_1 = [9a - 2h] / 17$ and $\hat{w}_2 = 2[3a + 5h] / 17$;

(ii) $\hat{I}_i = [9a - 4h] / [34(1 + k)]$, $i \in N$; and

(iii) $\hat{I}_1^i = [4a + h] / [17(1 + k)]$ and $\hat{I}_2^i = [11a - 10h] / [34(1 + k)]$, $i \in N$.

Two-period decentralization:

(i) $\hat{w}_1 = [a(25 + 17k) - 4h(1 + k)] / [249 + 58k + 17k^2]$ and $\hat{w}_2 = [a(40 + 53k + 17k^2) + 4h[18 + 19k + 5k^2]] / [249 + 58k + 17k^2]$;

(ii) $\hat{I}_i = [9 + 5k][a - 4h(2 + k)] / [2 + k][49 + 58k + 17k^2]$, $i \in N$; and

(iii) $\hat{q}_1^i = [a(48 + 57k + 17k^2) + 4h[2 + 3k + k^2]] / [2 + k][49 + 58k + 17k^2]$ and $\hat{q}_2^i = [a(29 + 17k) - 4h(9 + 5k)] / [249 + 58k + 17k^2]$, $i \in N$.

Calculating the firm's profit under centralization and decentralization under the above solutions, and using the arguments utilized in the proof of Proposition 4, the firm prefers decentralization to centralization if and only if $h/a < f^c(k)$, where: 
Proof of Proposition 10.

This completes the proof of Proposition 9.

**Proof of Proposition 10.** The proof in this case follows the same rationale as the proofs of Propositions 7 and 8 except for the fact that \( n = 2 \) and the demand function incorporates \( k \). Given this, we simply present the critical (albeit tedious) \( g^C(\beta; k) \) function.

\[
g^C(\beta; k) = \frac{[A + BC^{1/2}]}{D}, \text{ where}
\]

\[
A = -50944 - 319616 k - 810880 k^2 - 1006208 k^3 - 488320 k^4 + 224896 k^5 + 420224 k^6 + 208000 k^7 + 36992 k^8 - 64\beta[1 + k]^3[256 - 1460 k + 595k^2] - 96\beta[1 + k]^2[-12 - 1141 k + 1682 k^2 + 833k^3] + 16\beta^3[1 + k]^4[-112 - 2968 k - 14019k^2 - 5450 k^3 + 11305 k^4] + 12\beta^5[1 + k]^2[-20 - 531 k - 3828 k^2 - 9058 k^3 - 3504 k^4 + 5117 k^5] - 8\beta^4[1 + k]^3[-10 - 735 k - 8524 k^2 - 2696 k^3 - 11570 k^4 + 18445 k^5]
\]

\[
B = 136[1 + k]^9 - 4\beta[1 + k]^2[10 + 51 k] - \beta^3[1 + 10 k + 17 k^2] + 2\beta^2[1 + 21 k + 71 k^2 + 51 k^3];
\]

\[
C = -8\beta^2[1 + k]^3[5 + 3 k][1 + 10 k + 17 k^2]^2 + 256[1 + k]^9[4 - 4348 k + 1089 k^2] + 1024\beta[1 + k]^7[466 + 2575 k + 8171 k^2 + 8414 k^3] - 256\beta^2[1 + k]^5[594 + 14049 k + 6933 k^2 + 159357 k^3 + 153095 k^4] + 512\beta^3[1 + k]^4[29 + 1892 k + 2481 k^2 + 104100 k^3 + 191041 k^4 + 143536 k^5] + 64\beta^5[1 + 30 k + 327 k^2 + 1652 k^3 + 4111 k^4 + 4814 k^5 + 2057 k^6] + 64\beta^4[1 + k]^4[57 + 4007 k + 7119 k^2 + 46341 k^3 + 1294065 k^4 + 1627443 k^5 + 804110 k^6] - 32\beta^4[1 + k]^4[14 + 2938 k + 88617 k^2 + 789628 k^3 + 2697720 k^4 + 4048994 k^5 + 2426953 k^6] - 16\beta^6[1 + k]^2[3 + 627 k + 22237 k^2 + 264133 k^3 + 1325725 k^4 + 3087565 k^5 + 3328547 k^6 + 1394843 k^7] + 64\beta^7 k^2[2 + 193 k + 4355 k^2 + 40283 k^3 + 182161 k^4 + 436303 k^5 + 564165 k^6 + 373749 k^7 + 99845 k^8] - \beta^9 k^2[-1 + 144 k + 2196 k^2 + 108928 k^3 + 735890 k^4 + 2485136 k^5 + 4311180 k^6 + 3634656 k^7 + 1182791 k^8];
\]

\[
D = -8[2 + (2 - \beta) k]^3[16(1 + k)^4(63 + 425 k) + \beta^4(1 + 3 k)(1 + 10 k + 17 k^2)^2 - 16\beta(1 + k)^2(77 + 596 k + 1071 k^2) + 48\beta^2(1 + k)^2(6 + 79 k + 292 k^2 + 323 k^3) - 4\beta^3(7 + 133 k + 894 k^2 + 2610 k^3 + 3555 k^4 + 1513 k^5)] - 4\beta^3(7 + 133 k + 894 k^2 + 2610 k^3 + 3555 k^4 + 1513 k^5).
\]

This completes the proof of Proposition 10.
REFERENCES


